International Journal of Physical Sciences Vol. 7(10), pp. 1530 - 1539, 2 March, 2012
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS12.158
ISSN 1992 - 1950 ©2012 Academic Journals

Full Length Research Paper

Unsteady flow through porous medium induced by periodically rotating half-filled horizontal concentric cylindrical annulus with heat transfer

Bhupendra Kumar Sharma[1]*, Pawan Kumar Sharma[2] and R. C. Chaudhary[3]

[1]Department of Mathematics, Birla Institute of Technology and Science, Pilani-33303, India.
[2]Department of Applied Mathematics, Amity School of Engineering and Techonology, 580 Delhi-Palam Vihar Road, U&I Building, Bijwasan, New Delhi, India.
[3]Department of Mathematics, University of Rajasthan, Jaipur- 302004, India.

Accepted 2 November, 2011

This communication investigates the unsteady flow of viscous incompressible fluid through porous medium induced by periodically heated half filled concentric cylindrical annulus placed horizontally. The boundaries of the annulus are rotating periodically with different angular velocities in the same or opposite directions about their common axis. The governing equations are expressed in terms of stream function and vorticity functions. The analytical solutions have been obtained by expanding the variables in a power series of the annulus aspect ratio. The expressions for streamlines, temperature distribution and rate of heat transfer are obtained and the effects of various parameters upon them have been examined.

Key words: Incompressible fluid, porous medium.

INTRODUCTION

In recent years, the requirements of modern technology have stimulated interest in fluid flow studies, which involve the interaction of several phenomena. One such study is related to the flows of fluid through porous medium due to their applications in many branches in science and technology, viz. in the fields of agricultural engineering to study the underground water resources, seepage of water in river-beds, in petroleum technology to study the movement of natural gas, oil, water through the oil reservoirs and in chemical engineering for filtration and purification processes. Such problems have also important applications in geo-thermals reservoirs and geo-thermal energy extractions. It is obvious that in order to utilize the geo-thermal energy to maximum, one should have a complete and precise knowledge of the amount of perturbations needed to generate flow in geo-thermal fluids. Also, the knowledge of quantity of perturbations essential to initiate flow in the mineral fluid found in earth's crust helps, one to utilize minimal energy to extract the minerals. In view of this, Havstad et al. (1982) studied the convective heat transfer in vertical cylindrical annuli filled with a porous medium. Thermal convection in a horizontal eccentric annulus containing a saturated porous medium using perturbation technique was studied by Bau (1984). Stewart et al. (1992) had studied convection in a concentric annulus through porous media. Also, Barbosa et al. (1994 a) analyzed the natural convection in a porous horizontal cylindrical annulus. Natural convection flow through porous medium in horizontal eccentric annulus had studied by Barbosa et al. (1994 b). Barbosa et al. (1995) further studied the problem of natural convection in porous cylindrical annuli. Lee et al. (1995) presented oscillating flow through circular pipe by taking sinusoidal wall temperature. Transient free convection about a horizontal circular cylinder through porous medium with constant surface flux heating has also been studied by Pop et al. (1996). Kuznetsov (1996) analyzed non-thermal equilibrium fluid flow in a concentric tube annulus through a porous medium. Pulsating flow and heat transfer in an annulus

*Corresponding author. E-mail: bhupen_1402@yahoo.co.in.

partially filled with porous medium is studied by Guo et al. (1997). Johnson et al. (1997) had studied hydrodynamic stability of flow between rotating porous cylinder with radial and axial flow. Unsteady natural convective flow past a moving vertical cylinder with heat and mass transfer is studied by Ganesan et al. (2001). Recently, Abu-hijleh (2001, 2002) analyzed convection heat transfer from a cylinder with porous medium. The effect of porous inserts on the natural convection heat transfer in a vertical open-ended annulus has been numerically investigated by Kiwan and Al-Zahrani (2008). Recently, the problem of two-phase unsteady magnetohydro-dynamic (MHD) flow between two concentric cylinders of infinite length has been analysed by Jha et al. (2011) when the outer cylinder is impulsively started.

A few investigations have been reported in literature on the flow field in partially filled annulus between porous concentric cylinders with the inner one rotating. In this paper, we investigate theoretically, the flow in a half filled porous annulus with periodic boundary and oscillating temperature, when both the inner and outer cylinders are rotating. Analytical solutions have been obtained for the flow characteristics. The expressions for streamlines temperature distributions and rate of heat transfer are calculated. Numerical results are presented graphically and discussed.

MATHEMATICAL ANALYSIS

We consider the motion of a half- filled viscous incompressible fluid (Figure 1) between two infinite coaxial periodically rotating horizontal cylinders of radii $R_1^*, R_0^*\ (R_0^* > R_1^*)$, about their common axis with angular velocities $\Omega_1^*, \Omega_0^*,$ we take the z^* - axis along the cylinders. Since cylinder is infinite in z^* - direction, so all physical quantities are independent of z^*. Using cylindrical polar coordinate, the governing equations for unsteady, two-dimensional, laminar flow are

$$\frac{\partial u^*}{\partial r^*} + \frac{u^*}{r^*} + \frac{1}{r^*}\frac{\partial v^*}{\partial \theta} = 0 \quad , \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial r^*} + \frac{v^*}{r^*}\frac{\partial v^*}{\partial \theta} - \frac{v^{*2}}{r^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial r^*}$$

$$+\nu\left[\nabla^2 u^* - \frac{u^*}{r^{*2}} - \frac{2}{r^{*2}}\frac{\partial v^*}{\partial \theta}\right] - \frac{\nu u^*}{k^*} \quad , \tag{2}$$

$$\frac{\partial v^*}{\partial t^*} + u^*\frac{\partial v^*}{\partial r^*} + \frac{v^*}{r^*}\frac{\partial v^*}{\partial \theta} + \frac{u^* v^*}{r^*} = -\frac{1}{\rho}\frac{\partial p^*}{r^*\partial \theta}$$

$$+\nu\left[\nabla^2 v^* - \frac{v^*}{r^{*2}} + \frac{2}{r^{*2}}\frac{\partial u^*}{\partial \theta}\right] - \frac{\nu v^*}{k^*} \quad , \tag{3}$$

$$\frac{\partial q^*}{\partial t^*} + u^*\frac{\partial q^*}{\partial r^*} + \frac{v^*}{r^*}\frac{\partial q^*}{\partial \theta} = \alpha\,[\nabla^2 q^*] \quad , \tag{4}$$

where u and v are the velocity components in the radial and tangential directions, respectively and q is the temperature, t is the time, α is the thermal diffusivity, p is the pressure, ν is kinetic viscosity, ρ is density of fluid and k is the permeability parameter. The (*) stands for dimensional quantities. The Laplacian operator ∇^2 is given by

$$\nabla^2 = \frac{\partial^2}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial}{\partial r^*} + \frac{1}{r^{*2}}\frac{\partial^2}{\partial \theta^2} \tag{5}$$

The boundary conditions of the problem are

$$\left.\begin{array}{ll}
\left.\begin{array}{l} r^* = R_1^* : \ u^* = 0, \ v^* = R_1^*\Omega_1^*(1 + \varepsilon e^{i\omega^* t^*}) \\ \qquad q^* = q_1^* + \varepsilon (q_0^* - q_1^*) e^{i\omega^* t^*} \end{array}\right] & (6a) \\
\left.\begin{array}{l} r^* = R_0^* : \ u^* = 0, \ v^* = R_0^*\Omega_0^*(1 + \varepsilon e^{i\omega^* t^*}) \\ \qquad q^* = q_0^* + \varepsilon (q_0^* - q_1^*) e^{i\omega^* t^*} \end{array}\right], & (6b) \\
\text{and } \theta = 0, \ \pi, \ v^* = 0 = \dfrac{\partial u^*}{\partial \theta} = \dfrac{\partial q^*}{\partial \theta} & (6c)
\end{array}\right\} \tag{6}$$

Introducing the stream function $\psi^*(r, \theta, 0)$ and z- component of the vorticity vector as

$$u^* = \frac{1}{r^*}\frac{\partial \psi^*}{\partial \theta}, \qquad v^* = -\frac{\partial \psi^*}{\partial r^*}, \qquad w_z^* = -\nabla^2 \psi^* \qquad (7)$$

The forms of the velocity component u* and v* in Equation 7 are so chosen that the equation of continuity (Equation 1) is satisfied. Substituting Equation 7 into Equations 2 to 4 and eliminating the pressure term from Equations 2 and 3 we get the following equations:

$$\frac{1}{r^*}\frac{\partial^3 \psi^*}{\partial \theta^2\,\partial t^*} + \frac{\partial^2 \psi^*}{\partial t^*\,\partial r^*} + r^*\frac{\partial^3 \psi^*}{\partial t^*\,\partial r^{*2}} - \frac{2}{r^{*3}}\frac{\partial \psi^*}{\partial \theta}\frac{\partial^2 \psi^*}{\partial \theta^2}$$

$$+ \frac{1}{r^{*2}}\frac{\partial \psi^*}{\partial \theta}\frac{\partial^3 \psi^*}{\partial r^*\,\partial \theta^2} - \frac{1}{r^{*2}}\frac{\partial \psi^*}{\partial r^*}\frac{\partial^3 \psi^*}{\partial \theta^3} - \frac{1}{r^*}\frac{\partial \psi^*}{\partial r^*}\frac{\partial^2 \psi^*}{\partial r^*\,\partial \theta}$$

$$+ \frac{\partial \psi^*}{\partial \theta}\frac{\partial^3 \psi^*}{\partial r^{*3}} - \frac{\partial \psi^*}{\partial r^*}\frac{\partial^3 \psi^*}{\partial r^{*2}\,\partial \theta} - \frac{1}{r^{*2}}\frac{\partial \psi^*}{\partial \theta}\frac{\partial \psi^*}{\partial r^*} + \frac{1}{r^*}\frac{\partial \psi^*}{\partial \theta}\frac{\partial^2 \psi^*}{\partial r^{*2}}$$

$$= \nu\left[\frac{4}{r^{*3}}\frac{\partial^2 \psi^*}{\partial \theta^2} + \frac{2}{r^*}\frac{\partial^4 \psi^*}{\partial r^{*2}\,\partial \theta^2} + \frac{1}{r^{*3}}\frac{\partial^4 \psi^*}{\partial \theta^4} + \frac{2\partial^3 \psi^*}{\partial r^{*3}}\right.$$

$$\left. + r^*\frac{\partial^4 \psi^*}{\partial r^{*4}} + \frac{1}{r^{*2}}\frac{\partial \psi^*}{\partial r^*} - \frac{1}{r^*}\frac{\partial^2 \psi^*}{\partial r^{*2}}\right] - \frac{\nu}{k^*}\left[\frac{1}{r^*}\frac{\partial^2 \psi^*}{\partial \theta^2} + \frac{\partial \psi^*}{\partial r^*} + r^*\frac{\partial^2 \psi^*}{\partial r^{*2}}\right], \qquad (8)$$

$$\frac{\partial q^*}{\partial t^*} + \frac{1}{r^*}\frac{\partial \psi^*}{\partial \theta}\frac{\partial q^*}{\partial r^*} - \frac{1}{r^*}\frac{\partial \psi^*}{\partial r^*}\frac{\partial q^*}{\partial \theta} = \alpha\left[\nabla^2 q^*\right]. \qquad (9)$$

The boundary conditions are such that the flow satisfies the no-slip condition on the solid walls and zero-shear stress condition on the free surface. In the present analysis, free surfaces are assumed. The boundary conditions (Equations 6a and 6b) are the usual no-slip condition at the boundaries. The conditions (Equations 6c) are valid when edge effects are not taken into account and we are considering the case of moderate rotating velocities and as such the edge effects are neglected. Now introducing the following dimensionless parameters

$$A(Aspect\ ratio) = \frac{R_0^* - R_1^*}{R_1^*}, \quad r = \frac{r^* - R_1^*}{R_0^* - R_1^*}, \quad u = \frac{u^*}{R_1^*\,\Omega_1^*},$$

$$v = \frac{v^*}{R_1^*\,\Omega_1^*}, \qquad t = \omega^*\,t^*, \qquad \omega = \frac{\omega^*\,R_1^{*2}}{\nu},$$

$$\psi = \frac{\psi^*}{R_1^*\,\Omega_1^*\,(R_0^* - R_1^*)}, \quad k = \frac{k^*}{R_1^{*2}}, \quad T = \frac{(q^* - q_1^*)}{(q_0^* - q_1^*)}, \quad Pr\ (Prandtl\ Number) = \frac{\nu}{\alpha}.$$

Also it is assumed that $R_1^* >> (R_0^* - R_1^*)$ or $A << 1$, so that the streamlines are parallel, we seek a solution which is independent of ‘θ’, that is

$$\frac{\partial^n \psi}{\partial \theta^n} = 0 \quad \text{and} \quad \frac{\partial^n T}{\partial \theta^n} = 0, \qquad n = 1, 2, \ldots\ldots\ldots \quad (10)$$

Equations 8 and 9 are reduce to

$$\omega(Ar+1)^2 \frac{\partial^2 \psi}{\partial t\, \partial r} + \frac{\omega(Ar+1)^3}{A} \frac{\partial^3 \psi}{\partial t\, \partial r^2}$$

$$= 2\frac{(Ar+1)^2}{A^2} \frac{\partial^3 \psi}{\partial r^3} + \frac{(Ar+1)^3}{A^3} \frac{\partial^4 \psi}{\partial r^4} + \frac{\partial \psi}{\partial r}$$

$$- \frac{(Ar+1)}{A} \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{k}\left[(Ar+1)^2 \frac{\partial \psi}{\partial r} + \frac{(Ar+1)^3}{A} \frac{\partial^2 \psi}{\partial r^2}\right], \quad (11)$$

$$\omega \frac{\partial T}{\partial t} = \frac{1}{Pr}\left[\frac{1}{A^2} \frac{\partial^2 T}{\partial r^2} + \frac{1}{A(Ar+1)} \frac{\partial T}{\partial r}\right]. \quad (12)$$

The corresponding boundary conditions are reduce to

$$r = 0, \quad \psi = 0, \quad \frac{\partial \psi}{\partial r} = -(1 + \varepsilon e^{it}), \quad T = \varepsilon e^{it},$$

$$r = 1, \quad \psi = 0, \quad \frac{\partial \psi}{\partial r} = -(A+1)\frac{\Omega_0^*}{\Omega_1^*}(1 + \varepsilon e^{it}), \quad T = 1 + \varepsilon e^{it}, \quad (13)$$

$$\theta = 0, \pi, \quad \psi = 0, \quad \frac{\partial^2 \psi}{\partial \theta^2} = 0, \quad \frac{\partial T}{\partial \theta} = 0.$$

Now, we assume that the amplitude ε ($<< 1$), so we take

$$\left.\begin{aligned} \psi &= \psi_0 + \varepsilon e^{it}\psi_1 + \varepsilon^2 e^{it}\psi_2 + \cdots\cdots \\ T &= T_0 + \varepsilon e^{it} T_1 + \varepsilon^2 e^{it} T_2 + \cdots\cdots \end{aligned}\right\} \quad (14)$$

Substituting Equation 14 in Equations 11 and 12 and comparing the coefficient of identical powers of ε, neglecting those of ε^2, ε^3 etc. we get

$$2A(Ar+1)^2 \frac{\partial^3 \psi_0}{\partial r^3} + (Ar+1)^3 \frac{\partial^4 \psi_0}{\partial r^4} + A^3 \frac{\partial \psi_0}{\partial r} - A^2(Ar+1)\frac{\partial^2 \psi_0}{\partial r^2} - \frac{1}{k}\left[A^3(Ar+1)^2 \frac{\partial \psi_0}{\partial r} + A^2(Ar+1)^3 \frac{\partial^2 \psi_0}{\partial r^2}\right] = 0 \quad (15)$$

$$i\omega A^3(Ar+1)^2 \frac{\partial \psi_1}{\partial r} + i\omega A^2(Ar+1)^3 \frac{\partial^2 \psi_1}{\partial r^2} = 2A(Ar+1)^2 \frac{\partial^3 \psi_1}{\partial r^3} + (Ar+1)^3 \frac{\partial^4 \psi_1}{\partial r^4} + A^3 \frac{\partial \psi_1}{\partial r} - A^2(Ar+1)\frac{\partial^2 \psi_1}{\partial r^2}$$

$$- \frac{1}{k}\left[A^3(Ar+1)^2 \frac{\partial \psi_1}{\partial r} + A^2(Ar+1)^3 \frac{\partial^2 \psi_1}{\partial r^2}\right], \quad (16)$$

$$(Ar+1)\frac{\partial^2 T_0}{\partial r^2} + A\frac{\partial T_0}{\partial r} = 0 \quad (17)$$

$$\text{or} \quad (Ar+1)\frac{\partial^2 T_1}{\partial r^2} + A\frac{\partial T_1}{\partial r} - i\omega\, Pr A^2 (Ar+1) T_1 = 0. \quad (18)$$

The corresponding boundary conditions reduce to

$$\left.\begin{aligned} &r = 0, \quad \psi_0 = 0 = \psi_1, \quad \frac{\partial \psi_0}{\partial r} = -1 = \frac{\partial \psi_1}{\partial r}, \quad T_0 = 0, \quad T_1 = 1, \\ &r = 1, \quad \psi_0 = 0 = \psi_1, \quad \frac{\partial \psi_0}{\partial r} = -(A+1)\frac{\Omega_0^*}{\Omega_1^*} = -(A+1)\Omega, \\ &\qquad \frac{\partial \Psi_1}{\partial r} = -(A+1)\Omega, \quad T_0 = 1, \quad T_1 = 1. \end{aligned}\right\} \quad (19)$$

Now, for the solution of Equations 15 to 18, we seek the asymptotic solution for the dependent variables, by a regular expansion in terms of A, that is, for $A << 1$

$$\psi_0 = \psi_{01} + A\psi_{02} + A^2\psi_{03} + \cdots\cdots \quad (20)$$

$$\psi_1 = \psi_{11} + A\psi_{12} + A^2\psi_{13} + \cdots\cdots \quad (21)$$

$$T_0 = T_{01} + AT_{02} + A^2 T_{03} + \cdots\cdots \quad (22)$$

$$T_1 = T_{11} + AT_{12} + A^2 T_{13} + \cdots\cdots \quad (23)$$

Substituting Equations 20 to 23 in Equations 15 to 18 and equating terms of like power of A, we get

$$O(A^0), \quad \frac{\partial^4 \psi_{01}}{\partial r^4} = 0, \quad (24)$$

$$O(A^1),\ 2\frac{\partial^3 \psi_{01}}{\partial r^3} + 3r\frac{\partial^4 \psi_{01}}{\partial r^4} + \frac{\partial^4 \psi_{02}}{\partial r^4} = 0, \quad (25)$$

$$O(A^2),\ 4r\frac{\partial^3 \psi_{01}}{\partial r^3} + 2\frac{\partial^3 \psi_{02}}{\partial r^3} + \frac{\partial^4 \psi_{03}}{\partial r^4} + 3r\frac{\partial^4 \psi_{02}}{\partial r^4}$$

$$+ 3r^2\frac{\partial^4 \psi_{01}}{\partial r^4} - \frac{\partial^2 \psi_{01}}{\partial r^2} - \frac{1}{k}\frac{\partial^2 \psi_{01}}{\partial r^2} = 0. \quad (26)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} r = 0:\ & \psi_{01} = 0 = \psi_{02} = \psi_{03},\ \frac{\partial \psi_{01}}{\partial r} = -1,\ \frac{\partial \psi_{02}}{\partial r} = \frac{\partial \psi_{03}}{\partial r} = 0, \\ r = 1:\ & \psi_{01} = 0 = \psi_{02} = \psi_{03},\ \frac{\partial \psi_{01}}{\partial r} = -\Omega = \frac{\partial \psi_{02}}{\partial r},\ \frac{\partial \psi_{03}}{\partial r} = 0, \end{aligned}\right\} \quad (27)$$

and $O(A^0),\ \dfrac{\partial^4 \psi_{11}}{\partial r^4} = 0$, (28)

$$O(A^1),\ \frac{2\partial^3 \psi_{11}}{\partial r^3} + 3r\frac{\partial^4 \psi_{11}}{\partial r^4} + \frac{\partial^4 \psi_{12}}{\partial r^4} = 0, \quad (29)$$

$$O(A^2),\ 4r\frac{\partial^3 \psi_{11}}{\partial r^3} + 2\frac{\partial^3 \psi_{12}}{\partial r^3} + \frac{\partial^4 \psi_{13}}{\partial r^4} + 3r\frac{\partial^4 \psi_{12}}{\partial r^4}$$

$$+ 3r^2\frac{\partial^4 \psi_{11}}{\partial r^4} - \frac{\partial^2 \psi_{11}}{\partial r^2} - \frac{1}{k}\frac{\partial^2 \psi_{11}}{\partial r^2} - i\omega\frac{\partial^2 \psi_{11}}{\partial r^2} = 0. \quad (30)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} r = 0:\ & \psi_{11} = \psi_{12} = \psi_{13} = 0,\ \frac{\partial \psi_{11}}{\partial r} = -1, \frac{\partial \psi_{12}}{\partial r} = \frac{\partial \psi_{13}}{\partial r} = 0, \\ r = 1:\ & \psi_{11} = \psi_{12} = \psi_{13} = 0,\ \frac{\partial \psi_{11}}{\partial r} = -\Omega = \frac{\partial \psi_{12}}{\partial r},\ \frac{\partial \psi_{13}}{\partial r} = 0, \end{aligned}\right\} \quad (31)$$

and

$$O(A^0),\ \frac{\partial^2 T_{01}}{\partial r^2} = 0, \quad (32)$$

$$O(A^1),\ r\frac{\partial^2 T_{01}}{\partial r^2} + \frac{\partial^2 T_{02}}{\partial r^2} + \frac{\partial T_{01}}{\partial r} = 0, \quad (33)$$

$$O(A^2),\ r\frac{\partial^2 T_{02}}{\partial r^2} + \frac{\partial^2 T_{03}}{\partial r^2} + \frac{\partial T_{02}}{\partial r} = 0. \quad (34)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} r = 0:\ & T_{01} = T_{02} = T_{03} = 0, \\ r = 1:\ & T_{01} = 1,\quad T_{02} = T_{03} = 0, \end{aligned}\right\} \quad (35)$$

and

$$O(A^0),\ \frac{\partial^2 T_{11}}{\partial r^2} = 0, \quad (36)$$

$$O(A^1),\ r\frac{\partial^2 T_{11}}{\partial r^2} + \frac{\partial^2 T_{12}}{\partial r^2} + \frac{\partial T_{11}}{\partial r} = 0, \quad (37)$$

$$O(A^2),\ r\frac{\partial^2 T_{12}}{\partial r^2} + \frac{\partial^2 T_{13}}{\partial r^2} + \frac{\partial T_{12}}{\partial r} - i\omega \Pr T_{11} = 0. \quad (38)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} r = 0:\ & T_{11} = 1,\quad T_{12} = T_{13} = 0, \\ r = 1:\ & T_{11} = 1,\quad T_{12} = T_{13} = 0. \end{aligned}\right\} \quad (39)$$

Equations 24 to 26, 28 to 30, 32 to 34, 36 to 38 are ordinary differential equations and therefore can be solved by direct integration with the help of their boundary conditions. The solutions are

$$\psi_{01} = -(\Omega+1)r^3 + (\Omega+2)r^2 - r, \quad (40)$$

$$\psi_{02} = \frac{1}{2}(\Omega+1)r^4 - (2\Omega+1)r^3 + \frac{1}{2}(3\Omega+1)r^2, \quad (41)$$

$$\psi_{03} = -\left[7(\Omega+1) + \frac{(\Omega+1)}{k}\right]\frac{r^5}{20} + \left[13\,\Omega + 8 + \frac{(\Omega+2)}{k}\right]\frac{r^4}{12} - \left[67\,\Omega + 17 + \frac{(\Omega+11)}{k}\right]\frac{r^3}{60} + \left[23\,\Omega - 2 - \frac{(\Omega-4)}{k}\right]\frac{r^2}{60}, \quad (42)$$

$$\psi_{11} = -(\Omega+1)r^3 + (\Omega+2)r^2 - r, \quad (43)$$

$$\psi_{12} = \frac{1}{2}(\Omega+1)r^4 - (2\Omega+1)r^3 + \frac{1}{2}(3\Omega+1)r^2, \quad (44)$$

$$\psi_{13} = -\left[7(\Omega+1) + \frac{(\Omega+1)}{k}\right]\frac{r^5}{20} + \left[13\,\Omega + 8 + \frac{(\Omega+2)}{k}\right]\frac{r^4}{12} - \left[67\,\Omega + 17 + \frac{(\Omega+11)}{k}\right]\frac{r^3}{60} + \left[23\,\Omega - 2 - \frac{(\Omega-4)}{k}\right]\frac{r^2}{60}$$
$$+ i\omega\left[-(\Omega+1)\frac{r^5}{20} + (\Omega+2)\frac{r^4}{12} - (\Omega+11)\frac{r^3}{60} - (\Omega-4)\frac{r^2}{60}\right] \quad (45)$$

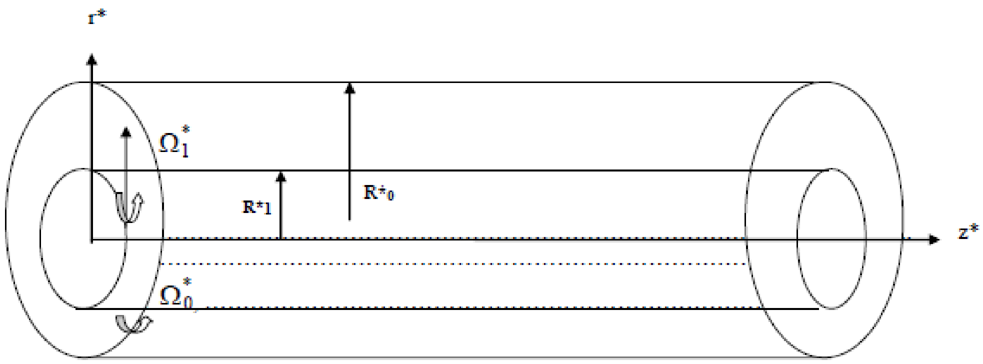

Figure 1. Physical configuration and coordinate system.

$$T_{01} = r : T_{02} = \frac{1}{2}\left[r - r^2\right], \quad T_{03} = \frac{1}{12}\left[4r^3 - 3r^2 - r\right] \tag{46}$$

$$T_{11} = 1 : \quad T_{12} = 0, \quad T_{13} = \frac{i\omega p_r}{2}\left[r^2 - r\right]. \tag{47}$$

RESULTS AND DISCUSSION

Substituting equations 40 to 45 in Equations 2) and 2), we get

$$\psi_0 = \left[-(\Omega+1)r^3 + (\Omega+2)r^2 - r\right] + A\left[\frac{1}{2}(\Omega+1)r^4 - (2\Omega+1)r^3 + \frac{1}{2}(3\Omega+1)r^2\right] + A^2\left[-\left\{7(\Omega+1) + \frac{(\Omega+1)}{k}\right\}\frac{r^5}{20} + \frac{r^4}{12}\left\{13\Omega + 8 + \frac{(\Omega+2)}{k}\right\} - \left\{67\Omega + 17 + \frac{(\Omega+11)}{k}\right\}\frac{r^3}{60} + \left\{23\Omega - 2 - \frac{(\Omega-4)}{k}\right\}\frac{r^2}{60}\right],$$

$$\psi_1 = \left[-(\Omega+1)r^3 + (\Omega+2)r^2 - r\right] + A\left[\frac{1}{2}(\Omega+1)r^4 - (2\Omega+1)r^3 + \frac{1}{2}(3\Omega+1)r^2\right]$$
$$+ A^2\left[-\left\{7(\Omega+1) + \frac{(\Omega+1)}{k}\right\}\frac{r^5}{20} + \left\{13\Omega + 8 + \frac{(\Omega+2)}{k}\right\}\frac{r^4}{12} - \left\{67\Omega + 17 + \frac{(\Omega+11)}{k}\right\}\frac{r^3}{60} + \left\{23\Omega - 2 - \frac{(\Omega-4)}{k}\right\}\frac{r^2}{60}\right]$$
$$+ A^2 i\omega\left[-(\Omega+1)\frac{r^5}{20} + (\Omega+2)\frac{r^4}{12} - (\Omega+11)\frac{r^3}{60} - (\Omega-4)\frac{r^2}{60}\right]$$

The flow in a half filled porous annulus with periodic boundary and oscillating temperature, when both the inner and outer cylinders are rotating has been carried out in preceding sections. In order to get physical insight into the problem, the numerical calculations for the distribution of the stream function, temperature profile and rate of heat transfer for various values of the parameter have been done. The behavior of the non-dimensional stream function Ψ of the fluid with changes in the annulus aspect ratio A, the frequency (ω), the time t, the rotation parameter Ω and the permeability parameter k are depicted in Figure 2a to c.

It is observed that stream function increases with an increase in ω and t both, while reverse phenomena is observed near the boundary of outer cylinder. The stream function changes its characteristic in the central region by increasing r and A both. It is further noticed that Ψ increases with increasing the permeability parameter k.
Now, substituting expressions 46 and 47 in Equations 1), the temperature field is given by

$$T = \left[r + \frac{A}{2}\left(r - r^2\right) + \frac{A^2}{12}\left(4r^3 - 3r^2 - r\right)\right] + \varepsilon\left[\cos t - \frac{A^2\omega Pr}{2}\left(r^2 - r\right)\sin t\right].$$

The temperature profiles for air (Pr = 0.71) are presented in Figure 3a. It is observed that temperature was not affected by increasing ω, while it increases with increasing the annular aspect ratio A. It is also observed that temperature decreases with increasing t.

The temperature profiles for water (Pr = 7) are presented in Figure 3b. It is observed that temperature increases with increasing A and ω both, while it decreases with increasing t. It is also observed that temperature increases with increasing r in both the situations [Pr = 0.71 (air) and Pr = 7 (water)]. Now, after knowing the temperature field, we can calculate the rate of heat transfer at the surface of inner and outer rotating cylinders as:

$$-q_1 = \left(\frac{\partial T}{\partial r}\right)_{r=o} = 1 + \frac{A}{2} - \frac{A^2}{12} + \frac{\varepsilon A^2 \omega Pr}{2}\sin t,$$

$$-q_2 = \left(\frac{\partial T}{\partial r}\right)_{r=1} = 1 - \frac{A}{2} + \frac{5}{12}A^2 - \frac{\varepsilon A^2 \omega Pr}{2}\sin t.$$

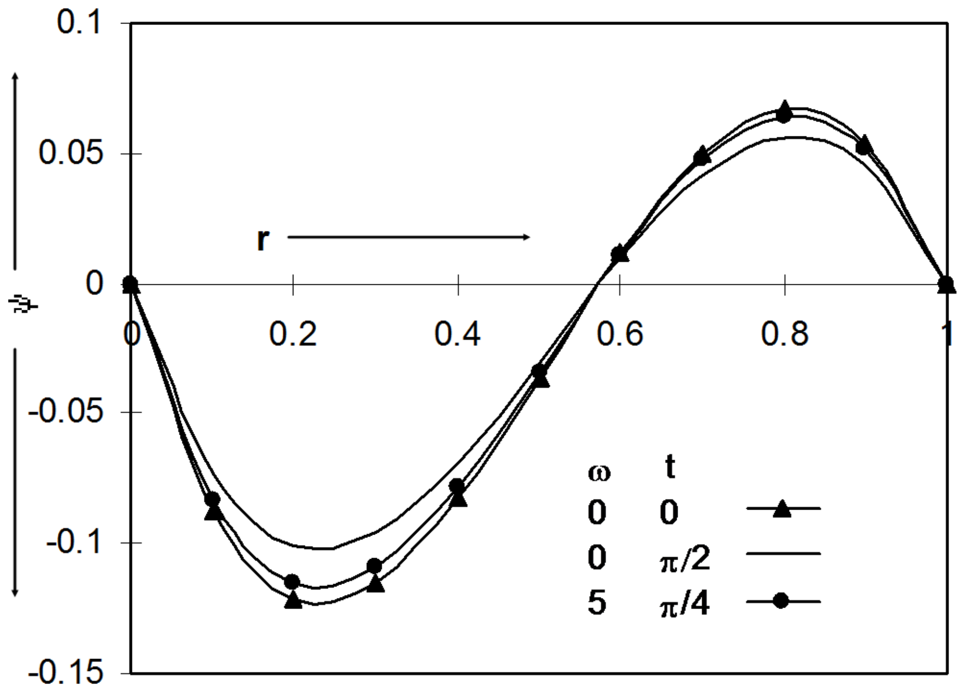

Figure 2a. The streem function (Ψ) for Ω = 0.5, k = 0.5, A = 0.3 and ε = 0.2.

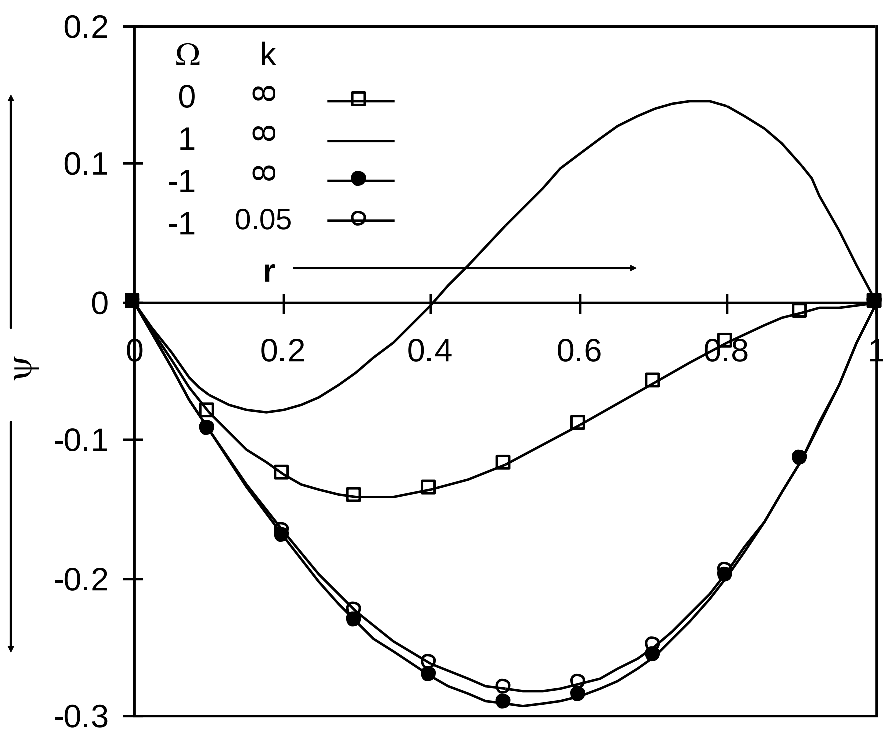

Figure 2b. The streem function (Ψ) for ω = 5, A = 0.3, ε = 0.2 and t = π/2.

The rates of heat transfer at the surface of inner and outer cylinder are presented in Figure 4a and b. The rate of heat transfer increases with increasing ω at the surface of inner cylinder, while decreases at the surface of outer

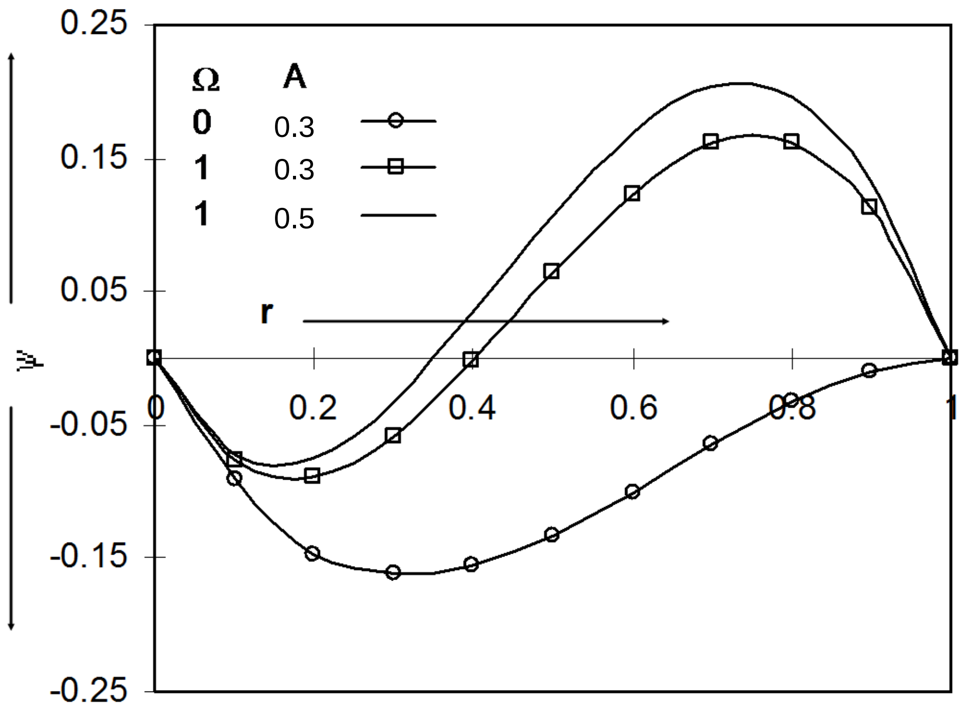

Figure 2c. The streem function (Ψ) for ω = 5, k = 0.5, ε = 0.2 and t = π/4.

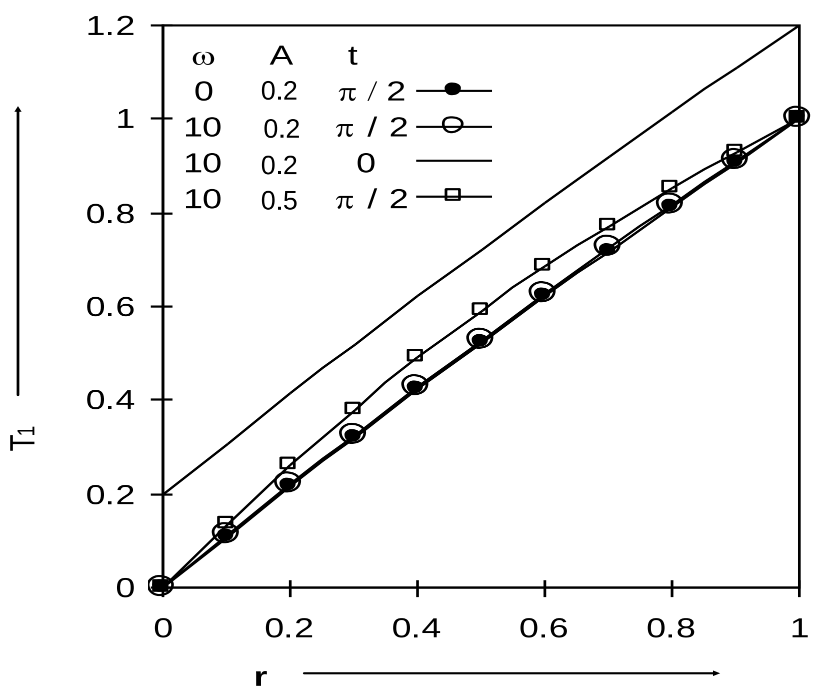

Figure 3a. The temperature profiles for Pr = 0.71 and ε = 0.2.

cylinder. The rate of heat transfer for the case of water (Pr = 7) rapidly increases with increasing A at the surface of inner cylinder, while it rapidly decreases with increasing A at the surface of outer cylinder.

It is also observed that the rate of heat transfer for the case of air (Pr = 0.71) increases with increasing A and t

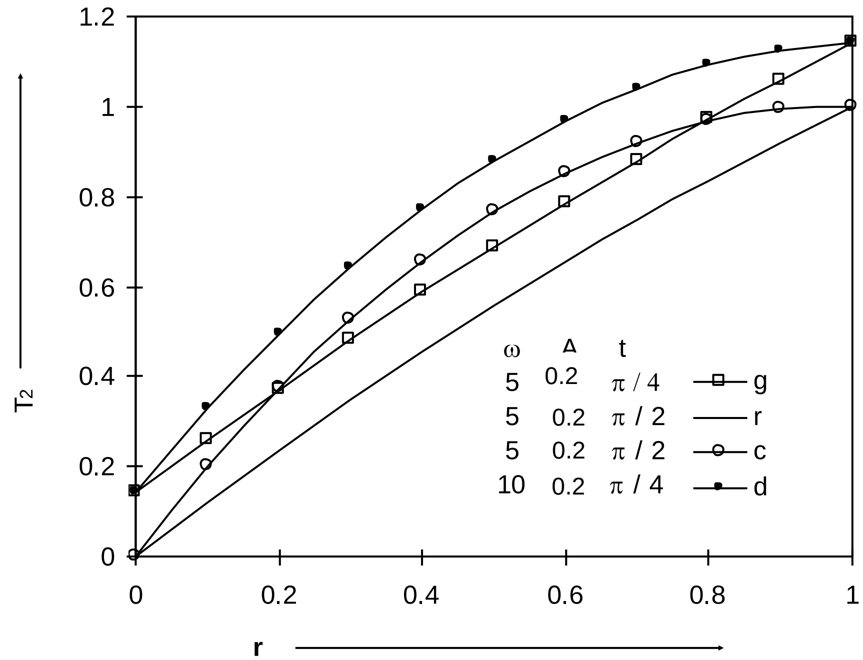

Figure 3b. The temperature profiles for Pr = 7 and ε = 0.2.

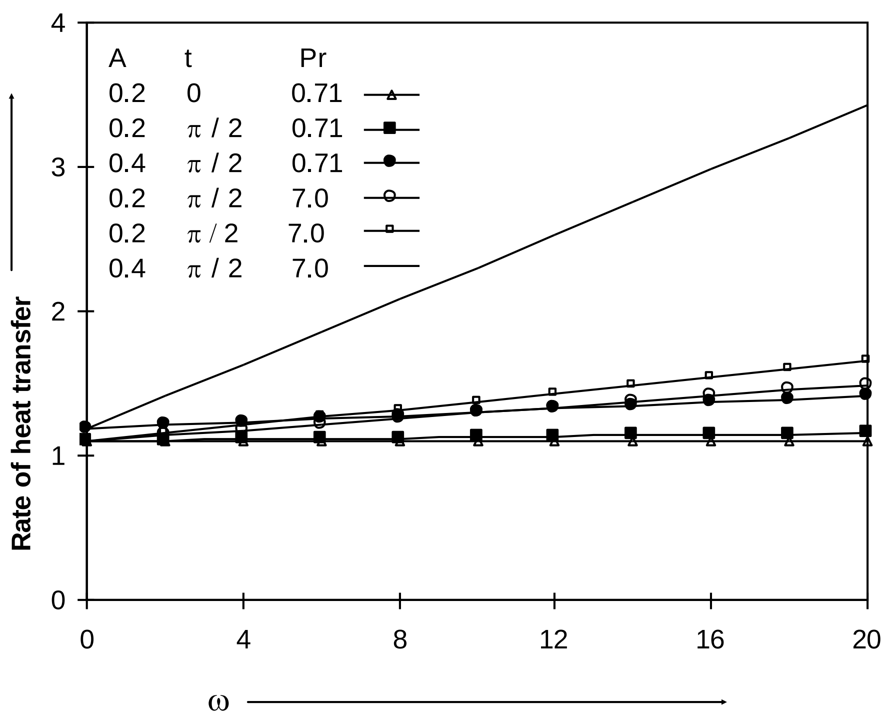

Figure 4a. The rate of heat transfer at the surface of inner cylinder for ε = 0.2.

both at the surface of inner cylinder, while reverse effect is observed at the surface of outer cylinder. It is interesting to note that it increases with increasing t at the surface of inner cylinder for water (Pr= 7), while reverse phenomena is observed at the surface of outer cylinder. Furthermore, when Pr = 0.71 (air), which means the

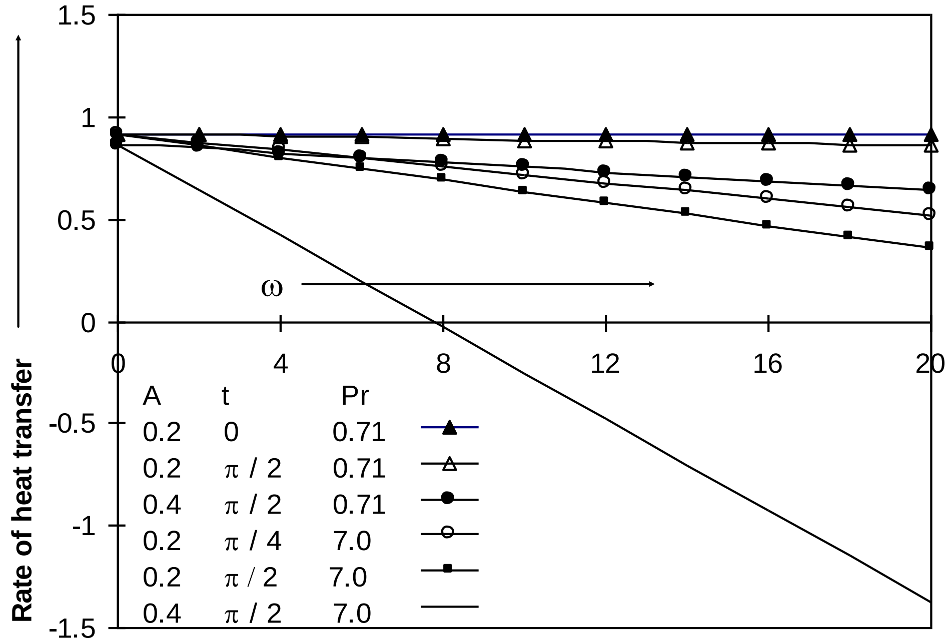

Figure 4b. The rate of heat transfer at the surface of outer cylinder for ε = 0.2.

viscosity is small but the thermal conductivity is finite, the heat transfer is greater at the surface of the outer cylinder, while, reverse effect may be observed at the surface of the inner cylinder. However, when the kinematic viscosity is large in comparison to the thermal diffusivity (Pr = 7.0, in the case of water), the heat transfer reduces at the surface of outer cylinder while, the reverse phenomena is observed at the surface of inner cylinder.

REFERENCES

Abu-hijleh B (2001). Natural convection heat transfer from a cylinder covered with an orthtotropic porous layer. Numer. Heat Transfer, 40: 767-782.

Abu-hijleh B (2002). Natural convection heat transfer from a cylinder with high conductivity permeable fins. ASME J. Heat Transfer, 125: 282-288.

Barbosa MJP, Saatdjian E (1994a). Natural convection in a porous horizontal cylindrical annulus (ASME). J. Heat Transfer, 116: 621-626.

Barbosa MJP, Le Provost JF, Puons E, Saatdjian E (1994b). Nztural convection in porous, horizontal eccentric annuli. Rugby, Heat Transfer, Inst. Chem. Eng., 5: 435-440.

Barbosa MJP, Saatdjian E (1995). Natural convection in porous cylindrical annuli. Int. J. Numer. Methods Heat Fluid Flow, 5: 3-17.

Bau HH (1984). Thermal convection in a horizontal eccentric annulus containing a saturated porous medium. An extended perturbation expansion, Int. J. Heat Mass Transfer, 27: 2277-2287.

Ganesan P, Loganathan P (2001). Unsteady natural convective flow past a moving vertical cylinder with heat and mass transfer. Heat Mass Transfer, 37: 59-65.

Guo Z., Sung HJ, Hyan JM (1997). Pulsating flow and heat transfer in a annulus partially filled with porous media. Numer. Heat Transfer, 31: 517-527.

Havstad MA, Burns PJ (1982). Convective heat transfer in vertical cylindrical annuli filled with a porous medium. Int. J. Heat Mass Transfer, 25: 1755-1766.

Johnson EC, Lueptow RM (1997). Hydrodynamic stability of flow between rotating porous cylinders with radial and axial flow. Phys. Fluids, 9: 3687-3699.

Kuznetsov AV (1996). Analysis of a non-thermal equilibrium fluid flow in a concentric tube annulus filled with a porous medium. Int. Commun. Heat Mass Transfer, 23: 929-938.

Lee DY, Parz SJ, Ro ST (1995). Heat transfer by oscillating flow in a circular pipe with a sinusoidal wall temperature distribution. Int. J. Heat. Mass Transfer, 38: 2529-2538.

Pop I, Ingham DB, Braden R (1996). Transient free convection about a horizontal circular cylinder in a porous medium with constant surface flux heating. Acta Mech., 119: 79-91.

Stewart WE, Burns AS (1992). Convection in a concentric annulus with heat generating porous media and a permeable inner boundary. Int. Commun. Heat Mass Transfer, 19: 859-868.

Kiwan S, Al-Zahrani MS (2008). Effect of porous inserts on natural convection heat transfer between two concentric vertical cylinders, Numer. Heat Transfer Part A: Appl., 53: 870–889.

Jha BK, Apere CA (2011). Unsteady MHD two-phase Couette flow of fluid-particle suspension in an annulus. AIP Advances 1**,** 042121, doi:10.1063/1.3657509.