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Full Length Research Paper

Elastic-plastic transition stresses in an isotropic disc having variable thickness subjected to internal pressure

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Elastic-plastic transitional stresses in an isotropic disc having variable thickness under internal pressure have been derived by using Seth's transition theory. The effect of stresses in fully-plastic state for different values of k presented graphically and discussed. Disc made of compressible material and having variable thickness yields at some radius R_1 at a higher pressure as compare to disc made of incompressible material which yields at the outer surface. Flat disc made of incompressible material yields at internal surface at higher pressure as compare to disc made of compressible material. Circumferential stress is maximum at the outer surface of the disc having variable thickness.

Key words: Elastic-plastic, transition, isotropic, thickness, pressure.

INTRODUCTION

Circular disks under the action of external pressures have been investigated by several workers (Güven, 1993; Gamer, 1983, 1984)). It is well known that disc with variable thickness are frequently found in mechanical engineering. A literature survey indicates that several workers have analyzed circular discs with constant material properties under various conditions (Gamer, 1983). (Durban, 1987) found an exact solution for the internally pressurized elastic-plastic, strain-hardening, annular plate. Chaudhuri (1979) obtained stresses in a non-homogeneous rotating annulus by varying Poisson's ratio of the material. In analyzing the problem, these authors used some simplifying assumptions. First, the deformation is small enough to make infinitesimal strain theory applicable. Second, simplifications were made regarding the constitutive equations of the material like incompressibility of the material and a yield criterion. Incompressibility of the material is one of the most important assumptions which simplifies the problem.

In fact, in most of the cases, it is not possible to find a solution in closed from without this assumption. Seth's transition does not require these assumptions and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. Seth's transition theory utilizes the concept of generalized strain measure and asymptotic solution at turning points of the differential equations defining the deformed field and has been successfully applied to number of the problems (Seth, 1966; 1972; Seth, 1963; Hulsurkar, 1966; Gupta and Dharmani, 1979, 1980; Dharmani et al., 1979; Gupta and Kumari, 2005; Pankaj, 2008; Pankaj and Sonia, 2008; Pankaj and Gupta, 2007). Seth (1966) has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ij} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right], \text{ (i =1,2,3)} \quad (1)$$

where n is the measure and $\overset{A}{e}_{ii}$ is the principal Almansi finite strain components. In Cartesian framework we can readily write down the generalized measure in terms of any other measures. For uniaxial case, it is given by Seth [6] as $e = \frac{1}{n}\left[1 - \left(\frac{l}{l_0}\right)^n\right]$ where l_0 and l is the initial and strained length respectively. For n = 0, 1, 2, -1, -2, it gives the Hencky, Swainger, Almansi, Cauchy and green

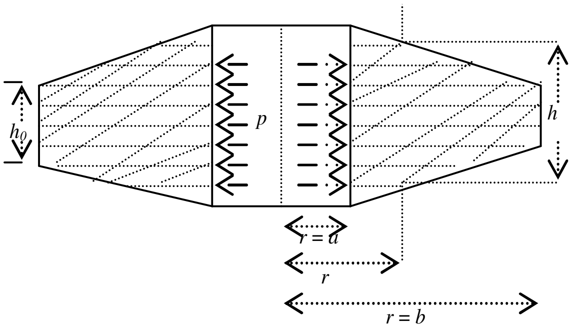

Figure 1. Isotropic disc having variable thickness subjected to internal pressure.

measures respectively. (Seth, 1966), has shown that the well known creep strain laws used in current literature such as Norton's law, Kachonov law, Odqvist law, Andrade's law etc. can be derived from the generalized measure. The generalized strain measure not only gives the well known strain measures as special cases, but it can also be used to find the creep stresses when it is combined with the transition point analysis of the governing differential equations. Seth has shown that the transition point analysis does not require the assumption of incompressibility. Seth showing that the asymptotic solution of the governing differential equations at the transition point gives the results which are obtained by assuming yield criteria when they exist. The most important contribution to be made by generalized measure is that it makes the use of semi-empirical laws and jump conditions unnecessary. If such law exists, they come out from the analytic treatment as a particular case. Thus, an important function of non-linear measure is to explain transition without assuming conditions to match the 2 solutions at transition. In this research paper we analyse elastic-plastic transition in an isotropic disc having variable thickness subjected to internal pressure. The thickness of the disc is assumed to vary along the radius in the form

$$h = h_0 (r/b)^{-k}, \qquad (2)$$

where h_0 is the constant thickness at r = b and k is the thickness parameter. Results obtained have been discussed numerically and depicted graphically.

Governing equations

Consider an isotropic thin disc of variable thickness with internal radius and external radius b subjected to internal pressure p as shown in Figure 1. The disc is taken to be sufficiently small so that the disc is effectively in a state of plane stress, that is, the axial stress T_{zz} is zero.

The displacement components in cylindrical polar coordinate are given by Seth (1966):

$$u = r(1-\beta),\ v = 0,\ w = dz, \qquad (3)$$

where β is position function, depending on $r = \sqrt{x^2 + y^2}$ only and d is a constant.

The finite strain components are given by Seth (1966) as

$$\overset{A}{e}_{rr} = \frac{1}{2}\left[1 - \left(\beta + r\beta'\right)^2\right],$$

$$\overset{A}{e}_{\theta\theta} = \frac{1}{2}\left[1 - \beta^2\right],$$

$$\overset{A}{e}_{zz} = \frac{1}{2}\left[1 - (1-d)^2\right],$$

$$\overset{A}{e}_{r\theta} = \overset{A}{e}_{\theta z} = \overset{A}{e}_{zr} = 0, \qquad (4)$$

where $\beta' = d\beta/dr$ and meaning of superscripts "A" is Almansi.

By substituting equation (4) into equation (1), the generalized components of strain are

$$e_{rr} = \frac{1}{n}\left[1 - \left(\beta + r\beta'\right)^n\right],$$

$$e_{\theta\theta} = \frac{1}{n}\left[1 - \beta^n\right],$$

$$e_{zz} = \frac{1}{n}\left[1 - (1-d)^n\right],$$

$$e_{r\theta} = e_{\theta z} = e_{zr} = 0, \qquad (5)$$

The stress –strain relations for isotropic media is given by Sokolinikoff (1956).

$$T_{ij} = \lambda\delta_{ij} I_1 + 2\mu e_{ij},\ (i, j = 1, 2, 3) \qquad (6)$$

Where T_{ij} and e_{ij} are the stress and strain components, λ and μ are lame's constants and $I_1 = e_{kk}$ is the first strain invariant, δ_{ij} is the Kronecker's delta.

Equations (6) for this problem becomes

$$T_{rr} = \frac{2\lambda\mu}{\lambda + 2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2\mu e_{rr},$$

$$T_{\theta\theta} = \frac{2\lambda\mu}{\lambda + 2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2\mu e_{\theta\theta},$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0. \qquad (7)$$

By substituting equations (5) into equations (7), one gets

$$T_{rr} = \frac{2\mu}{n}\left[3-2c-\beta^n\left\{1-c+(2-c)\left(\frac{r\beta'}{\beta}+1\right)^n\right\}\right],$$

$$T_{\theta\theta} = \frac{2\mu}{n}\left[3-2c-\beta^n\left\{2-c+(1-c)\left(\frac{r\beta'}{\beta}+1\right)^n\right\}\right],$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0, \qquad (8)$$

where c is compressibility factor of the material in term of Lame's constant, there are given by

$$c = 2\mu/(\lambda+2\mu).$$

Equations of equilibrium are all satisfied except

$$\frac{d}{dr}(rT_{rr}h) - hT_{\theta\theta} = 0. \qquad (9)$$

By substituting equations (8) into equation (9), one gets a non-linear differential equation with respect to β

$$(2-c)n\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta} = \left[\begin{array}{l} \frac{rh'}{h}\left[3-2c-\beta^n\left\{1-c+(2-c)(P+1)^n\right\}\right] \\ +\beta^n\left[1-(P+1)^n\right]-np\beta^n\left[\begin{array}{l}1-c\\+(2-c)(P+1)^n\end{array}\right]\end{array}\right] \qquad (10)$$

where $h' = dh/dr$ and $r\beta' = \beta P$ (P is function of β and β is function of r).

Transition points of β in equation (10) are $P=-1$ and $P=\pm\infty$. The boundary conditions are

T_{rr} = -p at r = a and T_{rr} = 0 at r = b. (11)

Solution through the principal stress

For finding the plastic stress, the transition function is taken through the principal stress (Seth, 1972, 1963; Hulsurkar, 1966; Gupta and Dharmani, 1979, 1980; Dharmani et al., 1979, Gupta and Kumari, 2005; Pankaj, 2008) at the transition point $P \to \pm\infty$. We take the transition function R as:

$$R^* = T_{\theta\theta} = \frac{2\mu\beta^n}{n}\left[3-2c-\beta^n\left\{\begin{array}{l}2-c\\+(1-c)(1+P)^n\end{array}\right\}\right] \qquad (12)$$

and by taking the logarithmic differentiation of equation (12) with respect to r, one gets:

$$\frac{d\left(\log R^*\right)}{dr} = \frac{nP}{r\left[1-(P+1)^n\right]}\left\{\left[\begin{array}{l}1-(P+1)^n\\-\beta(P+1)^{n-1}\frac{dP}{d\beta}\end{array}\right]\right\} \qquad (13)$$

By substituting the value of $dP/d\beta$ from equation (10) into equation (13), one gets

$$\frac{d\left(\log R^*\right)}{dr} = \frac{2\mu}{rnR}\left(\frac{-rh'}{h}\right)\left(\frac{1-c}{2-c}\right)\left\{3-2c-\beta^n\left[\begin{array}{l}1-c\\+(2-c)\\(1+P)^n\end{array}\right]\right\} - \left(\frac{1-c}{2-c}\right)\beta^n\left[1-(P+1)^n\right]+n\beta^nP\left(\frac{2c-3}{2-c}\right) \qquad (14)$$

Asymptotic value of equation (14) as $P\to\pm\infty$ is, and using equation (2), one gets after integration,

$$R^* = T_{\theta\theta} = \frac{Ar^{-1/(2-c)}}{h} \qquad (15)$$

where A is a constant of integration.

Substituting equation (15) in equation (9), one gets after integration

$$rhT_{rr} = A\left(\frac{2-c}{1-c}\right)r^{(1-c/2-c)} + B, \qquad (16)$$

where B is a constant of integration.

Substituting equation (11) in equation (16), we obtained

$$A = \frac{pah(a)(1-c)}{(2-c)\left[b^{(1-c/2-c)}-a^{(1-c/2-c)}\right]}$$

and $$B = \frac{-pah(a)b^{(1-c/2-c)}}{\left[b^{(1-c/2-c)}-a^{(1-c/2-c)}\right]}.$$

Substituting the value of A and B in equations (15) and (16), one gets the transitional stresses as:

Table 1. Pressure required for initial yielding (P_i) and fully plastic state (P_f) for an Isotropic disc having variable thickness for different values of *k* and *c*.

$0.5 \leq R \leq 1$	c	k	Yielding Occurs at	Isotropic Disc having variable thickness $\left[h = h_0 \left(r/b\right)^{-k}\right]$		Pressure required for yielding p_i	Pressure required for fully-plastic p_f	Percentage increase in pressure from initial yielding to fully plastic state $P = \left(\dfrac{p_f}{p_i} - 1\right) \times 100$
				$r = a$	$r = b$			
	0	1.273459	R_1 = 0.5	$h = h_0$(2.417405)	$h = h_0$	0.4530818	0.484640742	6.965386786 %
	0.25	1.316213		$h = h_0$ (2.490116)	$h = h_0$	0.4466273	0.47048926	5.342693651 %
	0.5	1.374582		$h = h_0$ (2.592927)	$h = h_0$	0.4381275	0.451833986	3.128427988 %
	0	1.316058	R_1 = 0.6	$h = h_0$ (2.489848)	$h = h_0$	0.4512678	0.470539811	4.270626831 %
	0.25	1.363723		$h = h_0$ (2.573484)	$h = h_0$	0.4446324	0.45524772	2.387442092 %
	0.5	1.428203		$h = h_0$ (2.691113)	$h = h_0$	0.4359177	0.453081839	3.93748271 %
	0	1.381854	R_1 = 0.75	$h = h_0$ (2.606031)	$h = h_0$	0.4424812	0.453081839	2.395723618 %
	0.25	1.437485		$h = h_0$ (2.708483)	$h = h_0$	0.4349341	0.453081839	4.172531641 %
	0.5	1.512061		$h = h_0$ (2.852173)	$h = h_0$	0.4251179	0.453081839	6.57791428 %
	0	1.427605	R_1 =0.85	$h = h_0$ (2.689997)	$h = h_0$	0.4381106	0.453081839	3.417219549 %
	0.25	1.489106		$h = h_0$ (2.80715)	$h = h_0$	0.4300967	0.453081839	5.344189641 %
	0.5	1.571265		$h = h_0$ (2.971651)	$h = h_0$	0.4134543	0.453081839	9.584492404 %
	0	1.5	R_1 = 1	$h = h_0$ (2.828427)	$h = h_0$	0.4142136	0.453081839	9.383632136 %
	0.25	1.571429		$h = h_0$ (2.971989)	$h = h_0$	0.4035501	0.453081839	12.27399866 %
	0.5	1.666667		$h = h_0$ (3.174802)	$h = h_0$	0.3898815	0.453081839	16.21014514 %

$$T_{rr} = pR_0^{1-k} R^{k-1} \left[\frac{R^{(1-c/2-c)} - 1}{1 - R_0^{(1-c/2-c)}} \right], \quad (17)$$

$$T_{\theta\theta} = pR_0^{1-k} \left(\frac{1-c}{2-c} \right) \frac{R^{\left[(1-c/2-c)+k-1\right]}}{\left[1 - R_0^{(1-c/2-c)}\right]}, \quad (18)$$

where R = r/b and $R_0 = a/b$.

From equations (17) and (18), one gets

$$T_{\theta\theta} - T_{rr} = \left[\frac{pR_0^{1-k}}{\left[1 - R_0^{(1-c/2-c)}\right]} R^{k-1} \left\{ \begin{array}{l} -\dfrac{1}{(2-c)} R^{1-c/2-c} \\ +1 \end{array} \right\} \right]. \quad (19)$$

Initial yielding

The maximum value of $\left|T_{\theta\theta} - T_{rr}\right|$ occurs at radius $R = \left[\dfrac{(k-1)(2-c)^2}{k(2-c)-1} \right]^{(2-c/1-c)} = R_1$ (say), which depends upon the values of k and c. For example if we take c = 0, 0.25, 0.5 yielding starts at the internal surface for values of k = 1.273459, 1.316213, 1.374582 respectively and for values of k = 1.5, 1.571429, 1.666667 yielding starts at the external surface (Table 1). For yielding at $R = R_1$, equation (19) becomes

$$\left|T_{\theta\theta} - T_{rr}\right|_{R=R_1} = \left| \frac{(1-c)\, pR_0^{1-k} \left[\dfrac{(k-1)}{(2-c)^2} \right]^{(2-c)(k-1)/(1-c)}}{\left[k(2-c)-1\right]^{k(2-c)-1/(1-c)} \left[\begin{array}{l} 1 \\ -R_0^{(1-c/2-c)} \end{array} \right]} \right| \equiv Y(say). \quad (20)$$

and the required pressure for yielding is

$$p_i = \frac{p}{Y} = \frac{\left[1 - R_0^{(1-c/2-c)}\right] \left[k(2-c)-1\right]^{(2-c)k-1/(1-c)}}{(1-c) R_0^{1-k} \left[(k-1)(2-c)^2\right]^{(2-c)(k-1)/(1-c)}} \quad (21)$$

Using equation (21) in equations (17) and (18), we get the transitional stresses in non- dimensional form as

$$\sigma_r = \frac{T_{rr}}{Y} = p_1 R_0^{1-k} R^{k-1} \left[\frac{R^{(1-c/2-c)} - 1}{1 - R_0^{(1-c/2-c)}} \right] \qquad (22)$$

$$\sigma_\theta = \frac{T_{\theta\theta}}{Y} = p_1 R_0^{1-k} \left(\frac{1-c}{2-c} \right) \frac{R^{[(1-c/2-c)+k-1]}}{\left[1 - R_0^{(1-c/2-c)} \right]} \qquad (23)$$

Fully plastic state

Stresses for fully-plastic state are obtained from equation (17) and (18) by taking $c \rightarrow 0$.
There are 2 plastic zones:

Inner-plastic zone: $R_0 \leq R \leq R_1$

Outer-plastic zone: $R_1 \leq R \leq 1$

For Inner-plastic zone, equation (19) becomes

$$\left| T_{\theta\theta} - T_{rr} \right|_{R=R_0} = \left| \frac{p\left(2-\sqrt{R_0}\right)}{2\left(1-\sqrt{R_0}\right)} \right| \equiv Y^*(say) \qquad (24)$$

and the required pressure for fully plastic state is given by

$$p_1^* \equiv \frac{p}{Y^*} = \frac{2\left(1 - \sqrt{R_0}\right)}{\left(2 - \sqrt{R_0}\right)} \qquad (25)$$

Using equation (25) in equations (17) and (18), we get the stresses for the inner plastic zone as,

$$\sigma_r^* = \frac{T_{rr}}{Y^*} = \frac{p_1^* R_0^{1-k} R^{k-1} \left(\sqrt{R} - 1 \right)}{\left(1 - \sqrt{R_0} \right)}, \qquad (26)$$

$$\sigma_\theta^* = \frac{T_{\theta\theta}}{Y^*} = \frac{p_1^* R_0^{1-k} R^{k-\frac{1}{2}}}{2\left(1 - \sqrt{R_0}\right)}. \qquad (27)$$

For Outer-plastic zone, equation (19) becomes

$$\left| T_{\theta\theta} - T_{rr} \right|_{R=1} = \left| \frac{pR_0^{1-k}}{2\left(1-\sqrt{R_0}\right)} \right| \equiv Y^{**}(say) \qquad (28)$$

and the required pressure is

$$p_1^{**} \equiv \frac{p}{Y^{**}} = \frac{2\left(1-\sqrt{R_0}\right)}{R_0^{1-k}} \qquad (29)$$

Substituting equation (29) in equations (17) and (18), we get the stresses for outer plastic zone as

$$\sigma_r^{**} = \frac{T_{rr}}{Y^{**}} = \frac{p_1^{**} R_0^{1-k} R^{k-1}\left(\sqrt{R}-1\right)}{\left(1-\sqrt{R_0}\right)}, \qquad (30)$$

$$\sigma_\theta^{**} = \frac{T_{\theta\theta}}{Y^{**}} = \frac{p_1^{**} R_0^{1-k} R^{k-\frac{1}{2}}}{2\left(1-\sqrt{R_0}\right)} \qquad (31)$$

Particular case

For a flat disc (k = 0) elastic-plastic transitional stresses (17) and (18) becomes

$$T_{rr} = \frac{pR_0}{R} \left[\frac{R^{(1-c/2-c)} - 1}{1 - R_0^{(1-c/2-c)}} \right], \qquad (32)$$

$$T_{\theta\theta} = pR_0 \left(\frac{1-c}{2-c} \right) \frac{R^{-1/(2-c)}}{\left[1 - R_0^{(1-c/2-c)} \right]}, \qquad (33)$$

It is seen that $\left| T_{\theta\theta} - T_{rr} \right|$ is maximum at the internal surface and yielding take place at the bore, we have

$$\left| T_{\theta\theta} - T_{rr} \right|_{R=R_0} = \left| \frac{p}{\left[1 - R_0^{(1-c/2-c)} \right]} \left[- \frac{1}{(2-c)} R_0^{(1-c)/(2-c)} + 1 \right] \right| \equiv Y_1 \ (say). \qquad (34)$$

The necessary pressure P_i required for initial yielding is given by

$$P_i = \frac{p}{Y_1} = \frac{\left[1 - R_0^{(1-c/2-c)} \right]}{\left[-(1/2-c) R_0^{(1-c)/(2-c)} + 1 \right]}. \qquad (35)$$

Using equation (35) in equations (32) and (33), one gets the transitional stresses as:

Table 2. Pressure required for initial yielding (P_i) and fully plastic state (P_f) for an Isotropic disc having variable thickness (k = 1.5) and flat disc (k =0) for different values of c.

$0.5 \le R \le 1$	c	k	Yielding Occurs at	Isotropic Disc having variable thickness $\left[h = h_0\left(r/b\right)^{-k}\right]$ r = a	r = b	Pressure required for yielding p_i	Pressure required for fully-plastic p_f	Percentage increase in pressure from initial yielding to fully plastic state $P = \left(\frac{p_f}{p_i} - 1\right) \times 100$
	0	1.5	$R_1 = 1$	$h = h_0(2.828427)$	$h = h_0$	0.4142136	0.45308189	9.383632136 %
	0.25	1.5	$R_1 = 0.974854$	$h = h_0(2.828427)$	$h = h_0$	0.4220108	0.45308189	7.362626636 %
	0.5	1.5	$R_1 = 0.965489$	$h = h_0(2.828427)$	$h = h_0$	0.4271293	0.45308189	6.076037342 %
	0	0.0000		$h = h_0$	$h = h_0$	0.453082	1.1716	158.5786 %
	0.25	0.0000	$R_0 = 0.5$	$h = h_0$	$h = h_0$	0.446627	1.1716	162.3155 %
	0.5	0.0000		$h = h_0$	$h = h_0$	0.435243	1.1716	169.1764 %

$$\sigma_r = \frac{T_{rr}}{Y_1} = \frac{P_i R_0}{R}\left[\frac{R^{(1-c/2-c)} - 1}{1 - R_0^{(1-c/2-c)}}\right], \quad (36)$$

$$\sigma_\theta = \frac{T_{\theta\theta}}{Y_1} = P_i R_0 \left(\frac{1-c}{2-c}\right)\frac{R^{-1/(2-c)}}{\left[1 - R_0^{(1-c/2-c)}\right]}. \quad (37)$$

For fully-plastic state ($c \to 0$) at the external surface (R = 1), we have

$$\left|T_{rr} - T_{\theta\theta}\right|_{R=1} = \left[\frac{pR_0}{2\left[1 - \sqrt{R_0}\right]}\right]. = Y_1^* \quad (38)$$

and pressure P_f required for fully plastic state is:

$$P_f \equiv \frac{p}{Y_1^*} = \frac{2\left(1 - \sqrt{R_0}\right)}{R_0}. \quad (39)$$

Using equation (39) in equations (32) and (33), one gets the stresses for fully plastic state as

$$\sigma_r = \frac{T_{rr}}{Y_1^*} = \frac{P_f R_0}{R}\left[\frac{\sqrt{R} - 1}{1 - \sqrt{R_0}}\right], \quad (40)$$

$$\sigma_\theta = \frac{T_{\theta\theta}}{Y_1^*} = \frac{P_f R_0 R^{-1/2}}{2\left[1 - \sqrt{R}\right]}. \quad (41)$$

Numerical illustration and discussion

In Table 1 pressure required for initial yielding and fully plastic state for an isotropic disc having variable thickness for different values of c has been given, it can be seen from the table that yielding occurs at any radius $R = R_1$ or at the internal surface (R_1 = 0.5) or at the external surface (R_1 = 1) of the disc depending upon values of k and c. For example yielding occurs at the internal surface of the disc made of compressible material (c = 0.25) at a pressure 0.4466247 for k = 1.3174582 whereas yielding occurs at the outer surface at a pressure 0.4035501 for k = 1.571429. It is also seen from Table 1 that disc having variable thickness and made of incompressible material yields at a higher pressure as compare to disc made of compressible material. In Table 2, pressure required for initial yielding P_i and fully plastic P_f for an isotropic disc having variable thickness (k = 1.5) and flat disc (k = 0) for different values of c has been given. It can be seen from Table 2, that for an isotropic disc made of compressible material and having variable thickness (k = 1.5) yields at some radius R_1 at a higher pressure as compare to disc made of incompressible material which yields at the outer surface whereas reverse is the case for flat disc, that is, flat disc made of incompressible material yields at internal surface at higher pressure as compare to disc made of compressible material. Disc made of incompressible material and having variable thickness requires higher % increase in pressure from initial yielding to fully-plastic state as compare to disc made of compressible material. In the case of flat disc it requires less % increase in pressure from initial yielding to fully plastic state as compare to disc made of compressible material and much higher percenttage increase in pressure is required to become fully plastic as compare o the disc having variable thickness. In Figure 2, curves have been drawn between stresses

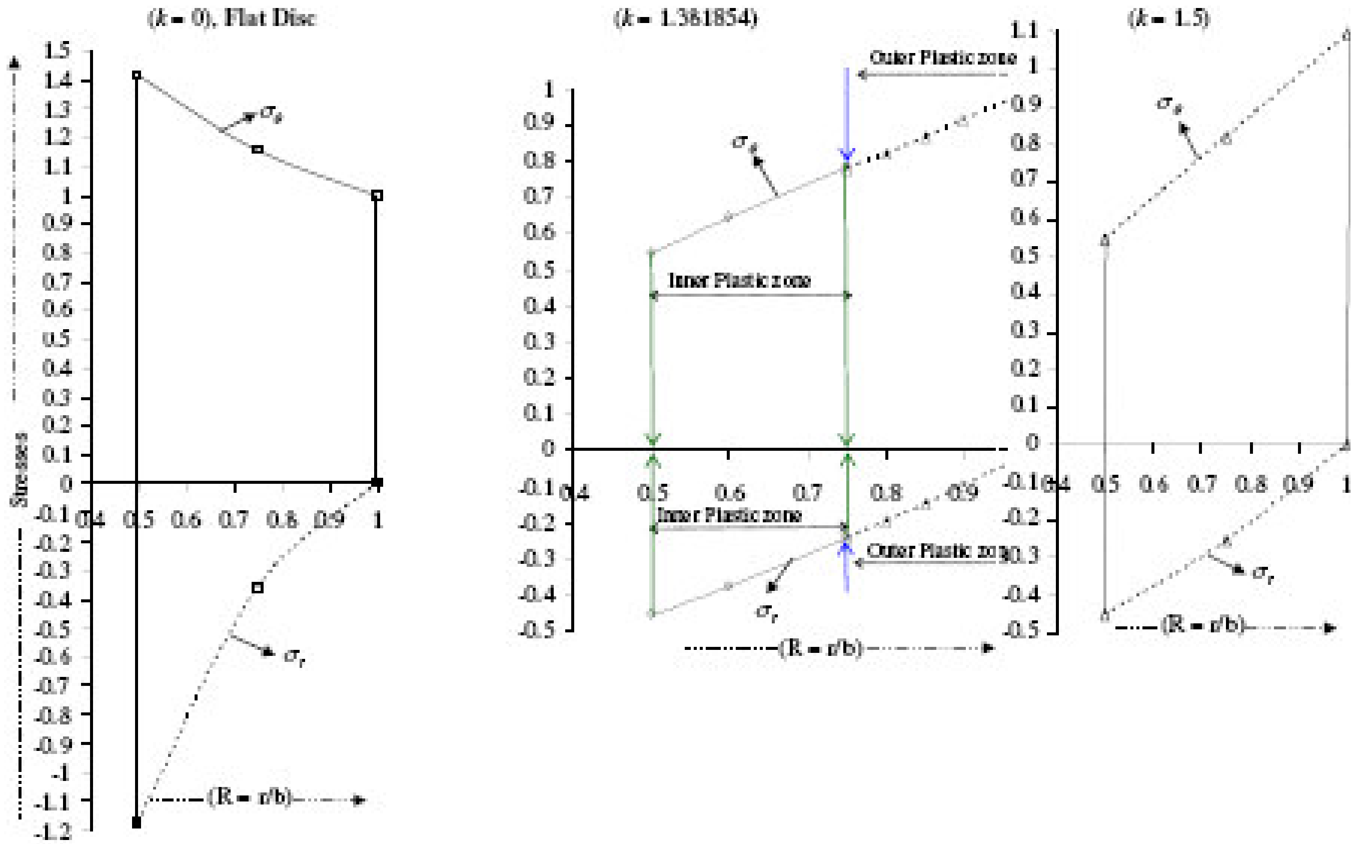

Figure 2. Stresses at fully - plastic state for different values of *k* with respect to radii ratio R = r/b.

and radii ratio r/b for fully-plastic state. It is seen that circumferential stress is maximum at the internal surface of the flat disc whereas it is maximum at the outer surface of the disc having variable thickness.

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