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Full Length Research Paper

Inextensible curves in the Galilean space

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In this paper, we investigated inextensible flow of curves in Galilean space. Conditions for an inextensible curve flow were expressed as a partial differential equation involving the curvature and torsion.

Key words: Galilean space, inextensible curves, Frenet formulas.

INTRODUCTION

Recently, the study of the motion of inextensible curves has arisen in a number of diverse engineering applications. The flow of a curve is said to be inextensible if the arc length is preserved. Physically, inextensible curve flows give rise to motions in which no strain energy is induced.

Kwon et al. (1999; 2005) studied the inextensible flows of curves and developable surface in R^3. Moreover Latifi et al. (2008) studied inextensible flows of curves in Minkowski 3-space.

A Galilean space may be considered as the limit case of a pseudo-Euclidean space in which the isotropic cone degenerates to a plane. This limit transition corresponds to the limit transition from the special theory of relativity to classical mechanics in Yaglom (1979). Differential geometry of the Galilean space G_3 has been largely developed in Kamenarovic (1991), Öğrenmiş et al. (2007; 2009) and Röschel (1986).

In this paper, we derived inextensible flows of curves in Galilean space G_3. Conditions for an inextensible curve flow were expressed as a partial differential equation involving the curvature and torsion. We used some idea from Kwon et al. (1999; 2005) and Latifi et al. (2008) in this paper.

PRELIMINARIES

The Galilean space is a three dimensional complex projective space P_3 in which the absolute figure $\{w, f, I_1, I_2\}$ consists of a real plane w (the absolute plane), a real line $f \subset w$ (the absolute line) and two complex conjugate points $I_1, I_2 \in f$ (the absolute points).

We shall take, as a real model of the space G□, a real projective space P_3 with the absolute $\{w, f\}$ consisting of a real plane $w \subset G_3$ and a real line $f \subset w$ on which an elliptic involution has been defined. We introduce homogeneous coordinates in G_3 in such a way that the absolute plane w is given by $x_0 = 0$, the absolute line f by $x_0 = x_1 = 0$ and elliptic involution by $(x_0{:}x_1{:}x_2{:}x_3) = (1{:}x{:}y{:}z)$, the distance between the points $Pi = (x_i, y_i, z_i),\ i = 1,2,$ is defined by

$$d(P_1,P_2) = \begin{cases} |x_2 - x_1|, & if \quad x_1 \neq x_2 \\ \sqrt{((y_2-y_1)^2 + (z_2-z_1)^2)}, & if \quad x_1 = x_2 \end{cases}$$

Let C be a curve in G_3, defined by arc length $\alpha: I \to G_3$ and parametrized by the invariant parameter $s \in I$, given in the coordinate form

$$\alpha(s) = (s, y(s), z(s)).$$

Then the curvature $\kappa(s)$ and the torsion $\tau(s)$ are defined by

$$\kappa(s) = \sqrt{(y''^2(s) + z''^2(s))}$$

$$\tau(s) = \frac{det(\alpha'(s), \alpha''(s), \alpha'''(s))}{\kappa^2(s)}$$

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and associated moving trihedron is given by

$$T(s) = \alpha'(s) = (1, y'(s), z'(s))$$

$$N(s) = \frac{1}{\kappa(s)}\alpha''(s) = \frac{1}{\kappa(s)}(0, y''(s), z''(s))$$

$$B(s) = \frac{1}{\kappa(s)}(0, -z''(s), y''(s)).$$

The vectors *T, N, B* are called the vectors of the tangent, principal normal and binormal line of α, respectively. For their derivatives the following Frenet formulas hold:

$$\begin{aligned} T'(s) &= \kappa(s)N(s) \\ N'(s) &= \tau(s)B(s) \\ B'(s) &= -\tau(s)N(s). \end{aligned} \qquad (1)$$

More about the Galilean geometry can be found in Kamenarovic (1991) and Röschel (1986).

MAIN RESULTS

In this section, first we adopt the definitions of Euclidean and Minkowskian expressed in Kwon et al. (1999; 2005) and Latifi et al. (2008).

Inextensible curve flows in Galilean space

Throughout this paper, we assume that $F:[0,l]\times[0,t^{\infty}] \to G_3$ is a one parameter family of smooth curves in Galilean space G_3, where l is the arc length of the initial curve. Let u be the curve parametrization variable, $0 \leq u \leq l$.

The arc length of *F* is given by

$$s(u) = \int_0^u \left|\frac{\partial F}{\partial u}\right| du$$

Where $\left|\frac{\partial F}{\partial u}\right| = |\langle\frac{\partial F}{\partial u}, \frac{\partial F}{\partial u}\rangle|^{\frac{1}{2}}$.

Defining $v = \left|\frac{\partial F}{\partial u}\right|$, the operator $\frac{\partial}{\partial u}$ is given by

$$\frac{\partial}{\partial s} = \frac{1}{v}\frac{\partial}{\partial u},$$

While the arc length parameter is $ds = v.du$ (Latifi et al. 2008) for a review of curve theory. Any flow of *F* can be represented as

$$\frac{\partial F}{\partial t} = fT + gN + hB.$$

Letting the arc length variation be in the Euclidean space, the requirement that the curve not be subjected to any elongation or compression expressed by condition

$\frac{\partial}{\partial t}s(u,t) = \int_0^u \frac{\partial v}{\partial t}du = 0$ for all $u \in [0,l]$.

Definition

A curve evolution $F(u,t)$ and its flow $\frac{\partial F}{\partial t}$ in Galilean space *G*□ are said to be inextensible if $\frac{\partial}{\partial t}\left|\frac{\partial F}{\partial u}\right| = 0$.

Condition for inextensible flow in Galilean space *G*□ are given by the following theorem.

Theorem

Let $\frac{\partial F}{\partial t} = fT + gN + hB$ be a smooth flow of the curve *F* in Galilean space *G*□. Let the flow is inextensible then *f* is constant.

Proof

According to definition of *F*, we have $v^2 = \langle\frac{\partial F}{\partial u}, \frac{\partial F}{\partial u}\rangle$. $\frac{\partial}{\partial u}$ and $\frac{\partial}{\partial t}$ commute since u and t are independent coordinates. So we have

$$2v\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}\langle\frac{\partial F}{\partial u}, \frac{\partial F}{\partial u}\rangle$$

$$= 2\langle\frac{\partial F}{\partial u}, \frac{\partial}{\partial u}(\frac{\partial F}{\partial t})\rangle$$

$$= 2\langle\frac{\partial F}{\partial u}, \frac{\partial}{\partial u}(fT + gN + hB)\rangle$$

$$= 2\langle T, \frac{\partial f}{\partial u}T + fv\kappa N + \frac{\partial g}{\partial u}N + gv\tau B + \frac{\partial h}{\partial u}B - hv\tau N\rangle$$

$$= 2v\frac{\partial f}{\partial u}$$

Thus we get

$$\frac{\partial v}{\partial t} = \frac{\partial f}{\partial u}$$

$$\frac{\partial f}{\partial s} = 0$$

$$f = const.$$

We now restrict ourselves to arc length parametrized curves. That is, v=1, and the local coordinate u corresponds to the curve arc length s. We require the following lemma.

Lemma

$$\frac{\partial T}{\partial t} = \left(f\kappa + \frac{\partial g}{\partial s} - h\tau\right)N + \left(g\tau + \frac{\partial h}{\partial s}\right)B,$$

$$\frac{\partial N}{\partial t} = -\left(f\kappa + \frac{\partial g}{\partial s} - h\tau\right)T + \psi B,$$

$$\frac{\partial B}{\partial t} = -\left(\frac{\partial g}{\partial s} + h\tau\right)T - \psi N,$$

Where $\psi = \langle \frac{\partial N}{\partial t}, B\rangle$.

Proof

Using Equation (1) and last theorem, we calculate

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial s}\right) = \frac{\partial}{\partial s}(fT + gN + hB)$$
$$= \frac{\partial f}{\partial s}T + f\kappa N + \frac{\partial g}{\partial s}N + g\tau B + \frac{\partial h}{\partial s}B - h\tau N$$
$$= (f\kappa + \frac{\partial g}{\partial s} - h\tau)N + (g\tau + \frac{\partial h}{\partial s})B.$$

Now differentiate the Frenet frame by t:

$$0 = \frac{\partial}{\partial t}\langle T, N\rangle = \langle \frac{\partial T}{\partial t}, N\rangle + \langle T, \frac{\partial N}{\partial t}\rangle$$
$$= (f\kappa + \frac{\partial g}{\partial s} - h\tau) + \langle T, \frac{\partial N}{\partial t}\rangle,$$

$$0 = \frac{\partial}{\partial t}\langle T, B\rangle = \langle \frac{\partial T}{\partial t}, B\rangle + \langle T, \frac{\partial B}{\partial t}\rangle$$
$$= (g\tau + \frac{\partial h}{\partial s}) + \langle T, \frac{\partial B}{\partial t}\rangle,$$

$$0 = \frac{\partial}{\partial t}\langle N, B\rangle = \langle \frac{\partial N}{\partial t}, B\rangle + \langle N, \frac{\partial B}{\partial t}\rangle$$
$$= \psi + \langle N, \frac{\partial B}{\partial t}\rangle.$$

From the above we obtain

$$\frac{\partial N}{\partial t} = -(f\kappa + \frac{\partial g}{\partial s} - h\tau)T + \psi B$$

and

$$\frac{\partial B}{\partial t} = -(g\tau + \frac{\partial h}{\partial s})T - \psi N,$$

Since $\langle \frac{\partial N}{\partial t}, N\rangle = \langle \frac{\partial B}{\partial t}, B\rangle = 0$.

Theorem

Suppose the curve flow $\frac{\partial F}{\partial t} = (fT + gN + hB)$ is inextensible. Then the following system of partial differential equations holds:

$$\frac{\partial \kappa}{\partial t} = \frac{\partial}{\partial s}(f\kappa) + \frac{\partial^2 g}{\partial s^2} - \frac{\partial}{\partial s}(h\tau) - g\tau^2 - \tau\frac{\partial h}{\partial s},$$
$$\frac{\partial \tau}{\partial t} = \kappa(g\tau + \frac{\partial h}{\partial s}) + \frac{\partial \psi}{\partial s},$$
$$\kappa\psi = \tau(f\kappa + \frac{\partial g}{\partial s} - h\tau) + \frac{\partial}{\partial s}(g\tau) + \frac{\partial^2 h}{\partial s^2}.$$

Proof

Noting that $\frac{\partial}{\partial s}\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\frac{\partial T}{\partial s}$,

$$\frac{\partial}{\partial s}\frac{\partial T}{\partial t} = \frac{\partial}{\partial s}[(f\kappa + \frac{\partial g}{\partial s} - h\tau)N + (g\tau + \frac{\partial h}{\partial s})B$$
$$= [\frac{\partial}{\partial s}(f\kappa) + \frac{\partial^2 g}{\partial s^2} - \frac{\partial}{\partial s}(h\tau)]N + (f\kappa + \frac{\partial g}{\partial s} - h\tau)\tau B$$
$$+ \left[\frac{\partial}{\partial s}(g\tau) + \frac{\partial^2 h}{\partial s^2}\right]B + \left(g\tau + \frac{\partial h}{\partial s}\right)(-\tau N),$$

While

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial s} = \frac{\partial}{\partial t}(\kappa N)$$

$$= N\frac{\partial \kappa}{\partial t} + \kappa[-(f\kappa + \frac{\partial g}{\partial s} - h\tau)T + \psi B].$$

Hence we see that

$$\frac{\partial \kappa}{\partial t} = \frac{\partial}{\partial s}(f\kappa) + \frac{\partial^2 g}{\partial s^2} - \frac{\partial}{\partial s}(h\tau) - g\tau^2 - \tau\frac{\partial h}{\partial s}$$

and

$$\kappa\psi = \tau(f\kappa + \frac{\partial g}{\partial s} - h\tau) + \frac{\partial}{\partial s}(g\tau) + \frac{\partial^2 h}{\partial s^2}.$$

Since $\frac{\partial}{\partial s}\frac{\partial B}{\partial t} = \frac{\partial}{\partial t}\frac{\partial B}{\partial s}$, we have

$$\frac{\partial}{\partial s}\frac{\partial B}{\partial t} = \frac{\partial}{\partial s}[-(g\tau + \frac{\partial h}{\partial s})T - \psi N]$$
$$= -[\frac{\partial^2 h}{\partial s^2} + \frac{\partial}{\partial s}(g\tau)]T - (g\tau + \frac{\partial h}{\partial s})\kappa N - \frac{\partial \psi}{\partial s}N - \psi\tau B$$

While

$$\frac{\partial}{\partial t}\frac{\partial B}{\partial s} = \frac{\partial}{\partial t}(-\tau N)$$
$$= -[\frac{\partial \tau}{\partial t}N + \tau[-\left(f\kappa + \frac{\partial g}{\partial s} - h\tau\right)T + \psi B]].$$

Thus we get

$$\frac{\partial \tau}{\partial t} = \kappa\left(g\tau + \frac{\partial h}{\partial s}\right) + \frac{\partial \psi}{\partial s}.$$

RESULTS

Kwon et al. (1999, 2005) studied inextensible flows of curves and developable surface in R^3. Moreover Latifi et al. (2008) studied inextensible flows of curves in Minkowski 3-space. We derived inextensible flows of curves in Galilean space G_3. Conditions for an inextensible curve flow were expressed as a partial differential equation involving the curvature and torsion.

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