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Full Length Research Paper

Hot symmetric nuclear and neutron matter properties in the Thomas-Fermi model

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Equation of state of hot nuclear and neutron matter are calculated in the frame of the Thomas-Fermi approximation using of the effective nucleon - nucleon interaction of Myers and Swiatecki in the new approach. The effect of temperature on effective mass, pressure, entropy and binding energy is discussed. A critical temperature of 17.4 MeV for symmetric nuclear matter is found and there is no phase transition in the neutron matter systems. The results of calculations are in good agreement with experimental prediction and other theoretical results.

Key words: Nuclear matter, nuclear structure models and methods, nuclear matter aspects of neutron stars, nucleon–nucleon interactions.

INTRODUCTION

The properties of hot and dense nuclear matter play an essential role in the understanding of high-energy heavy ion collision (Bao-An-Li et al., 2001), supernova explosions, and proto neutron stars (Lattimer and Prakash, 2001; Steiner et al., 2005; Horowitz and Piekarewicz, 2001). For that reason the problem of hot nuclear matter has been studied over the past decades in several investigations. The many-body theory at finite temperature has been developed along different lines and methods, such as Lattimer and Ravenhall (1985) and Su et al. (1987), have used phenomenological models of nuclear matter, the Chiral Sigma model has been applied by Jena and Singh (2004); Sahu and Jha et al. (2004), relativistic mean-field models have been used by Jiang et al. (2007), green function approach has been applied by Gad and Hassaneen (2007), lowest-order constrained variational method (LOCV) has been used by Modarres and Moshfegh (2005, 2002), relativistic Bruckner-Hartree-Fock theory at finite temperatures has been applied by Weber and Weigel (1988). So the results of our approach can be compared with those calculations. These models can be divided into two categories, namely: non-relativistic and relativistic potential models. The majority of the relativistic treatments are performed in the framework of the relativistic Hartree approximation (Weber and Weigel, 1988; Huber et al., 1998) and the majority of the theoretical treatments utilize the non-relativistic scheme, using either effective density dependent interactions or the Bruckner approach (Pethick et al., 1995; Das et al., 1992).

The disadvantage of such a microscopic treatment is the numerical complexity of the method. For that reason it is very tempting to use simpler models which are easier to deal with, and make comparisons with respect to the properties of finite nuclei, the parameters of the mass formula, neutron stars. For this purpose we selected the new Thomas-Fermi approach of Myers and Swiatecki (Myers and Swiatecki, 1994). The paper is organized as follows: we present a brief formalism of the Thomas-Fermi model for density and momentum dependent interactions. In this study we drive the EOS with zero and finite temperature for symmetric nuclear and neutron matter. The results of this model for binding energy, free energy and other thermodynamical properties of symmetric nuclear and neutron matter are given afterwards. Finally, the conclusions and summary are presented in the last section.

THOMAS-FERMI MODEL

Zero temperature

We shall not go into details concerning this model, since they are

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described in greater detail in numerous investigations prior to this one (Myers and Swiatecki, 1994; Strobel et al., 1999). We use the density and momentum dependent interaction of the following structure:

$$V_{12}=-\frac{2T_F}{\rho_0}f(\frac{r_{12}}{a})\{\frac{1}{2}(1\mp\xi)\alpha-\frac{1}{2}(1\mp\zeta)[\beta(\frac{p_{12}}{p_F})^2-\gamma(\frac{p_{12}}{p_F})^{-1}+\sigma(\frac{2\bar{\rho}}{\rho_0})^{\frac{2}{3}}]\} \quad (1)$$

The quantities ρ_0, p_F and T_F denote the baryon number density, Fermi momentum and the kinetic single-particle energy of symmetric nuclear matter at saturation, respectively and given by:

$$\rho_0=(\frac{4\pi}{3}r_0^3)^{-1}, \quad (2)$$

$$p_F=\hbar(\frac{3}{2}\pi^2\rho_0)^{\frac{1}{3}} \quad , \quad (3)$$

$$T_F=\frac{P_F^2}{2\bar{m}} \quad , \quad \bar{m}=\frac{1}{2}(m_n+m_p) \quad (4)$$

The potential's radial dependence, f , is chosen to be Yukawa type, that is:

$$f(\frac{r_{12}}{a})=\frac{1}{4\pi a^3}\frac{\exp(-\frac{r_{12}}{a})}{\frac{r_{12}}{a}} \quad (5)$$

and $\bar{\rho}$ is a mean density defined by:

$$\bar{\rho}^{\frac{2}{3}}=\frac{1}{2}(\rho_1^{\frac{2}{3}}+\rho_2^{\frac{2}{3}}) \quad (6)$$

Where ρ_1, ρ_2 are the relevant neutron or proton densities at point 1 and 2, respectively and $a=0.59542\ fm$.

The parameters ξ and ζ in Equation 1 allow the interaction to be different for like (upper sign) and unlike (lower sign) pairs of particles. The choice $\xi \neq \zeta$ leads to a better description of asymmetric nuclear systems. With the parameter α one adjusts mainly the binding properties of nuclear matter. The repulsion is described by the momentum dependent term $\propto \beta p_{12}^2$. Since this repulsion turns out to be too strong for higher relative momenta, one corrects this deficit by the term $\propto \gamma|p_{12}|^{-1}$. The term proportional to $\sigma(\frac{2\bar{\rho}}{\rho_0})^{\frac{2}{3}}$ takes care of a better agreement with the nuclear optical potential (Strobel et al., 1999). The energy per nucleon (with the respect isospin) in the Thomas-Fermi model is given by:

$$u=\frac{E}{A}=\frac{1}{\rho}\frac{\nu}{h^3}\int_0^{p_F}d^3p_1(\frac{p_1^2}{2\bar{m}}+\frac{1}{2}V(p_1))\ \Theta(p_F-|p|) \quad (7)$$

In the aforementioned equation $\nu=4(2)$ is used for nuclear (neutron) matter. The single-particle potential is defined by:

$$V(p_1)=-\frac{\upsilon}{h^3}\frac{2T_F}{\rho_0}\times[\int_{-\infty}^{+\infty}d^3p_2(\alpha_l-\beta_l(\frac{p_{12}}{p_F})^2+\gamma_l\frac{p_F}{|p_{12}|}-\sigma_l(\frac{2\bar{\rho}}{\rho_0})^{\frac{2}{3}}) \quad (8)$$
$$\int_{-\infty}^{+\infty}d^3p_2(\alpha_u-\beta_u(\frac{p_{12}}{p_F})^2+\gamma_u\frac{p_F}{|p_{12}|}-\sigma_u(\frac{2\bar{\rho}}{\rho_0})^{\frac{2}{3}})]$$

Here, $\alpha_l,\beta_l,\gamma_l,\sigma_l\ and\ \alpha_u,\beta_u,\gamma_u,\sigma_u$ are defined as:

$\alpha_l=0.5(1-\xi),\alpha_u=0.5(1+\xi)$;

$\beta_l=0.5(1-\zeta),\beta_u=0.5(1+\zeta)$;

$\gamma_l=0.5(1-\zeta),\lambda_u=0.5(1+\zeta)$;

$\sigma_l=0.5(1-\zeta),\sigma_u=0.5(1+\zeta)$.

Where $\xi=0.44003$ and $\zeta=0.59778$ (Strobel et al., 1999).

Results of this model for cold matter are described in more detail in Myers and Swiatecki (1994) and agree rather well with the values of the semi empirical droplet mass formula.

Finite temperature

The calculation of the Thomas-Fermi model at finite temperature for symmetric nuclear matter follows exactly the calculation at zero temperature except that we use the Fermi-Dirac distribution function:

$$f(p_1,T,\rho)=(1+\exp(\frac{1}{T}(\varepsilon(p_1,T,\rho)-\mu(T,\rho))))^{-1} \quad (9)$$

With $\beta=\frac{1}{KT}$, instead of the step function $\Theta(p_F-|p|)$ in the case of zero temperature (T stands for temperature and $k\equiv1$ is the Boltzmann constant). In this equation, the single-particle energies are:

$$\varepsilon(p_1,T,\rho)=\frac{p_1^2}{2m(\rho,T)}+V(p_1) \quad (10)$$

And $\mu(T,\rho)$ is the chemical potential at a given temperature and density for a non-interacting system. We introduce an effective mass in single particle energy and simply use (Moshfegh and Modarres 2005; Moshfegh and Modarres, 1998):

$$\varepsilon(p_1,T,\rho)=\frac{p_1^2}{2m^*(\rho,T)} \quad (11)$$

Where $m^*(\rho, T)$ is the effective mass and it is taken as a variational parameter, chemical potential will be obtained by choosing density as follows:

$$\rho = \frac{\upsilon}{h^3}\int_{-\infty}^{+\infty} d^3 p_1 f(p_1) \tag{12}$$

One then obtains for the energy per nucleon for the symmetric nuclear matter:

$$u = \frac{E}{A} = \frac{\upsilon}{\rho h^3}\int_{-\infty}^{+\infty} d^3 p_1 \left(\frac{p_1^2}{2\bar{m}} + \frac{1}{2}V(p_1)\right) f(p_1) \tag{13}$$

With the temperature dependent single-particle potential:

$$V(p_1) = -\frac{2}{h^3}\frac{2T_F}{\rho_0} \times \left[\int_{-\infty}^{+\infty} d^3 p_2 \left(\alpha_l - \beta_l\left(\frac{p_{12}}{p_F}\right)^2 + \gamma_l \frac{p_F}{|p_{12}|} - \sigma_l\left(\frac{2\bar{\rho}}{\rho_0}\right)^{\frac{2}{3}}\right) f(p_2)\right.$$
$$\left. \int_{-\infty}^{+\infty} d^3 p_2 \left(\alpha_u - \beta_u\left(\frac{p_{12}}{p_F}\right)^2 + \gamma_u \frac{p_F}{|p_{12}|} - \sigma_u\left(\frac{2\bar{\rho}}{\rho_0}\right)^{\frac{2}{3}}\right) f(p_2)\right] \tag{14}$$

At finite temperature we must calculate the Helmholtz free energy per particle, that is:

$$F = u - Ts \tag{15}$$

And the entropy per S particle is written as (Fetter and Walecka, 1971):

$$s = -\frac{4}{\rho h^3}\int_{-\infty}^{+\infty} d^3 p_1 [f(p_1)\ln f(p_1) + (1 - f(p_1))\ln(1 - f(p_1))] \tag{16}$$

In other to calculate symmetric nuclear matter properties in this model at finite temperatures we performed in the following way. We calculated free energy per nucleon for a given temperature, density and different masses of m^* so that:

$$0 \le \frac{m^*}{m} \le 1, \left(m = \bar{m} = \frac{1}{2}(m_n + m_p)\right)$$

The mass for which F is minimized will be considered as the effective mass. For a given effective mass by calculation, it is very easy to calculate, the pressure, entropy, free energy, binding energy and other properties of the Thomas-Fermi equation of state. The calculated free energy at a given density, temperature and effective mass is used to obtain the isothermal pressure, $P(T, \rho, m^*)$, by differentiating $F(\rho, T, m^*)$, that is:

$$P(T, \rho, m^*) = \rho^2 \left.\frac{\partial f(T, \rho, m^*)}{\partial \rho}\right|_{\rho = \rho_0} \tag{17}$$

From the definition of specific heat per nucleon that is:

$$c_v = \left(\frac{\partial u}{\partial T}\right)_\rho \tag{18}$$

We can calculate c_v.

RESULTS AND DISCUSSION

We presented a macroscopic calculation of symmetric nuclear and neutron matter in the frame of the Thomas-Fermi approximation using a recent modern parameterization of the effective nucleon –nucleon interaction of Myers and Swiateski in the new approach. We compared our results with the EOS in this model, obtained by Strobel et al. (1999). They have been the critical temperature for symmetric nuclear matter is $T_C = 20.8\ MeV$, but we have found a lower critical temperature. It turned out that, despite its simplicity, the modern TF model shows the same features with respect to the EOS of symmetric nuclear and neutron matter as complicated non-relativistic variational calculations. The symmetric nuclear and neutron matter properties for TF are in excellent agreement with the body of presently existing data (Figure 1). As we would expect, u is a rapidly increasing function of density beyond the nuclear matter saturation density, but it is satisfactorily small up to this density. From this figure, we observed that the EOS gets stiffer with increasing temperature. For small density one obtains as expected, the behavior of a free Fermi gas with a linear temperature dependence, since the nucleon-nucleon force has a small range (Strobel et al., 1997). For increasing density the EOS exhibits a quadratic temperature dependence. At zero temperature it has a minimum at the nuclear saturation density ρ_0 which corresponds to a binding energy per nucleon as resulted in Jena and Singh (2004) and Sahu et al. (2004). With the increase of temperature the minimum shifts toward higher densities and for higher temperatures the minimum of the curve becomes positive (Figure 1). From these graphs, we find that at constant density, the free energy decreases with the increase of density, the free energy shows an increasing trend for a given temperature (Figure 2).

It is observed that entropy is non-zero even at vanishing baryon density at a temperature. At lower temperature s decreases slowly as compared to higher temperatures and the minimum value of s increases as the temperature increases which is similar to the results of Jena and Singh (2004), Sahu et al. (2004) and Randrup and Medeiros (1992) and agrees with the experimental situation of Jacak et al. (1983) and Li et al. (1994) (Figure 3).

The figure shows that at zero temperature, the pressure first decreases, then increases and passes through $p = 0$ at $\rho = \rho_0$ (saturation density), where the binding energy per nucleon is a minimum. When the temperature increases, the region of mechanical instability decreases and disappears at the critical

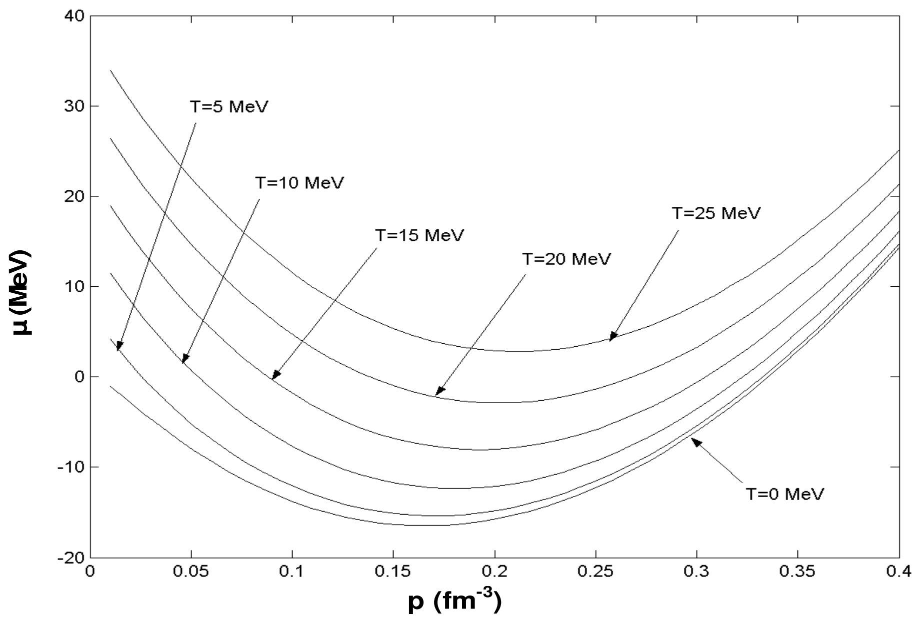

Figure 1. Energy per nucleon versus density for symmetric nuclear matter $(\delta = 0)$ at different temperatures.

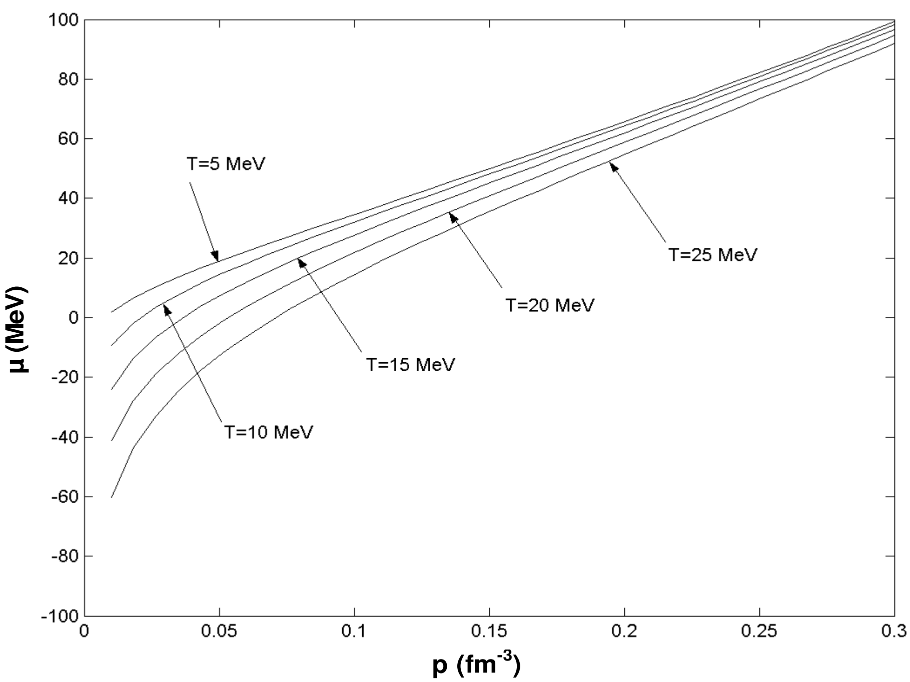

Figure 2. Chemical potential as function of density for symmetric nuclear matter at different temperatures.

temperature T_C, which is determined by $\left.\frac{\partial p}{\partial \rho}\right|_{T_c} = \left.\frac{\partial^2 p}{\partial^2 \rho}\right|_{T_c} = 0$. The critical point, where the liquid phase and the gas phase merge, correspond to the maximum of the coexistence line in the density-pressure plane. The thermodynamic conditions for the coexistence of the two phases are:

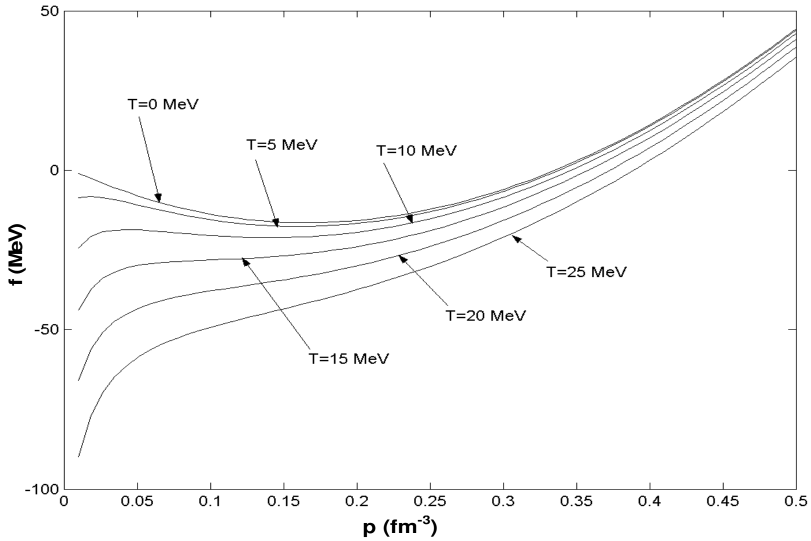

Figure 3. Free energy per baryon versus density for symmetric nuclear matter.

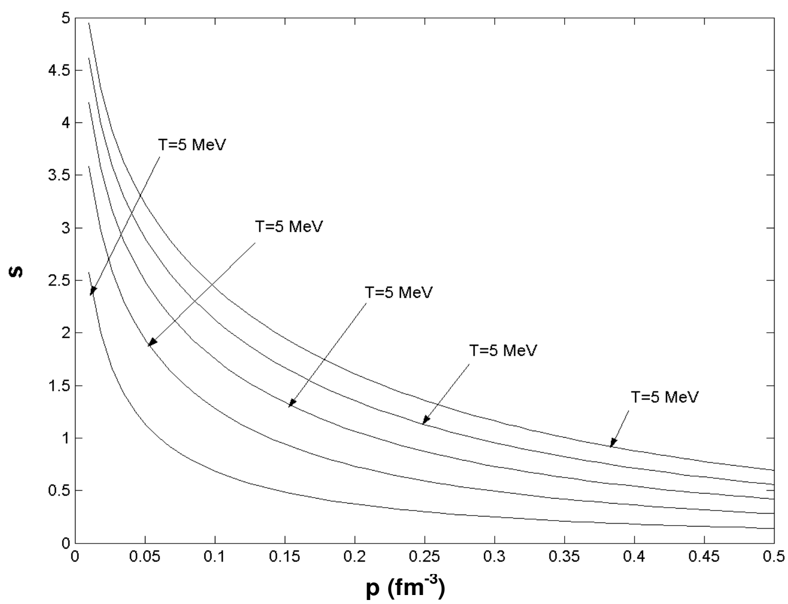

Figure 4. The entropy as a function of density at different temperatures for symmetric nuclear matter.

$$\mu_{liquid}(\rho,T)=\mu_{gas}(\rho,T), \quad p_{liquid}(\rho,T)=p_{gas}(\rho,T)$$

The value of the critical temperature obtained from the Thomas-Fermi model in the new approach is $T_C = 17.4\ MeV$, which is in fair agreement with the results obtained in other studies. For temperature $T > T_c$, only the gas phase can exist. Due to the importance of the liquid-gas phase transitions the transitions are predicted with various models. The value for the critical temperature depends strongly on the choice of the forces (and approximations). This result can be compared with the experimental result $17.5 \pm 1\ MeV$ of Jacak et al. (1983) and $13.1 \pm 0.6\ MeV$ of Li et al. (1994), $17.2\ MeV$ and $14\ MeV$ of the theoretical prediction of Sahu et al. (2004) and (Zeng-Hua et al., 2004) respectively (Figure 4). It is observed that in all

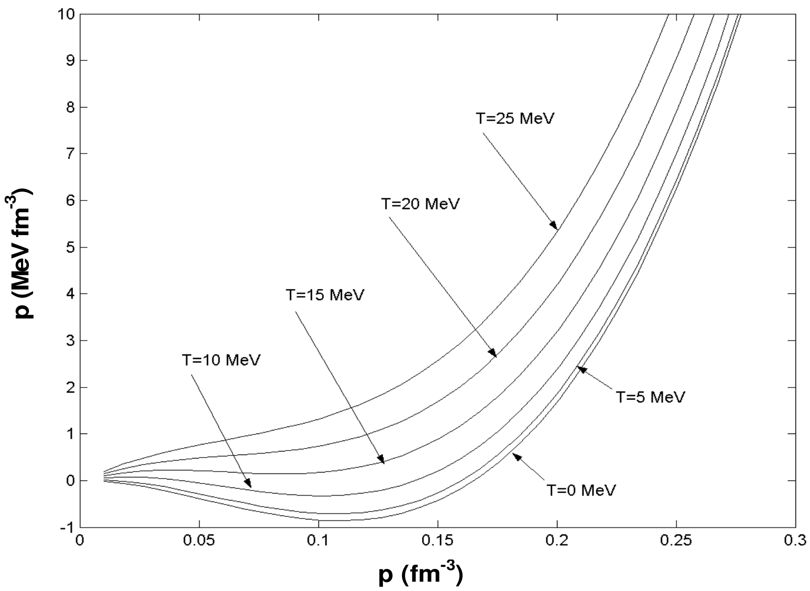

Figure 5. The pressure of symmetric nuclear matter as function of baryon density at different temperatures.

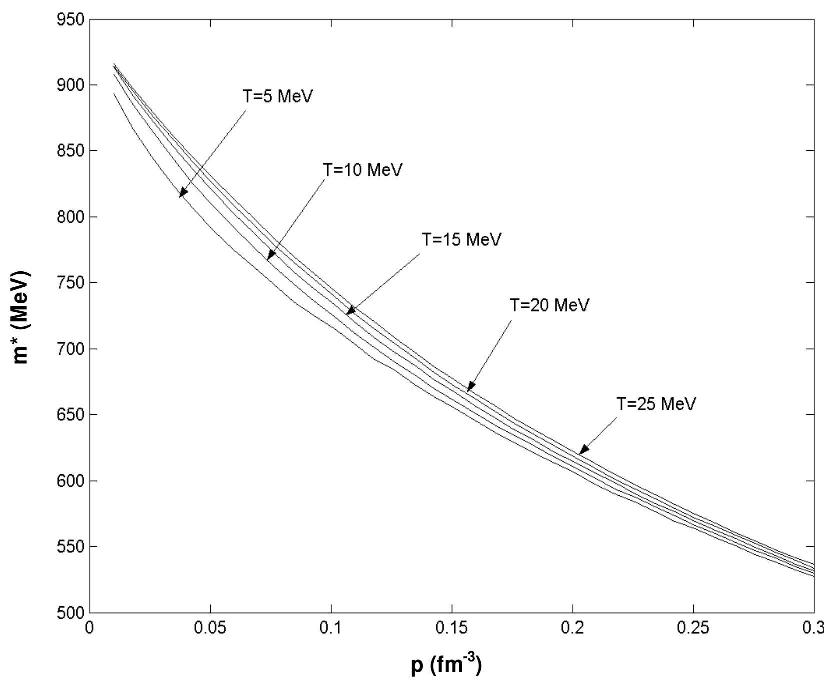

Figure 6. The effective mass versus number density at different temperatures for symmetric nuclear matter.

temperatures m^* decreases with the increase of ρ . Also it is clear from the figure that m^* increases gradually with T . There is an agreement between our results and those given in Jena and Singh (2004) and Sahu et al. (2004) (Figure 5).

From the simple Fermi-gas model we expect that the specific heat behaves linearly with respect to T at low temperatures and densities (Figure 6).

The results of calculation for neutron matter are

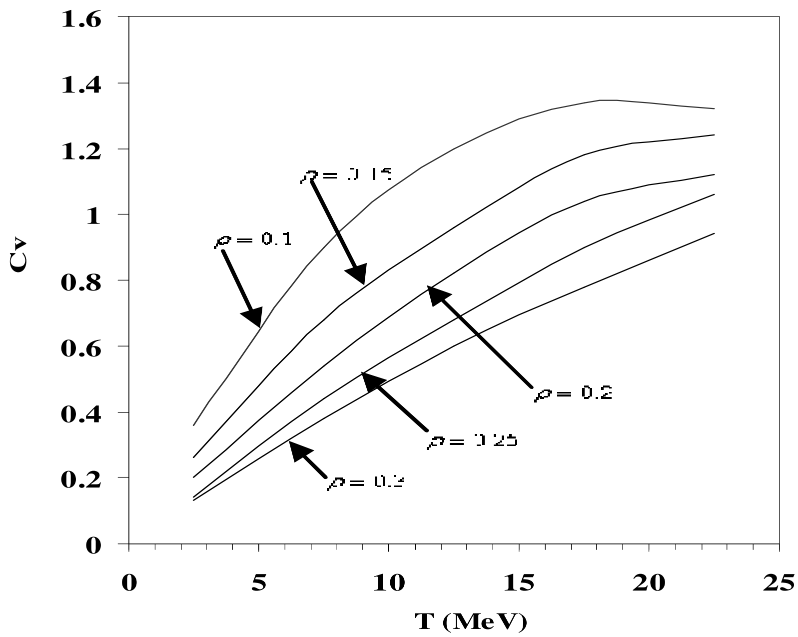

Figure 7. The total specific heat at various densities versus temperature.

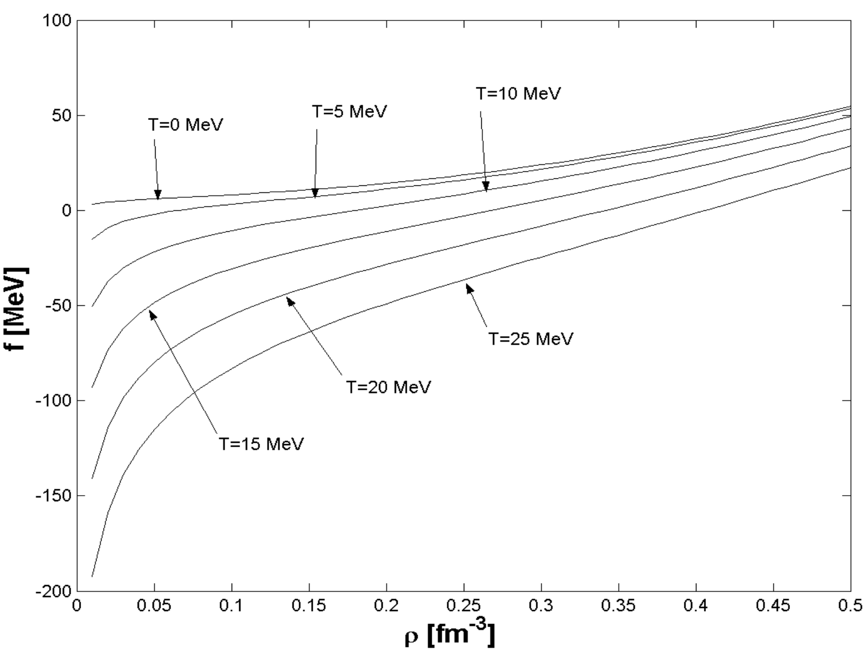

Figure 8. The energy per nucleon versus density for neutron matter $(\delta = 1)$ at different temperatures.

presented in Figures 7 to 11.

SUMMARY AND CONCLUSION

We have done calculations for cold and hot nuclear and neutron matter at different temperatures using the nucleon-nucleon interaction of Myers and Swiatecki in the new approach with effective mass. We have presented variations of pressure, effective nucleon mass, entropy, free energy, chemical potential and energy per nucleon with respect to density for various temperatures. We also

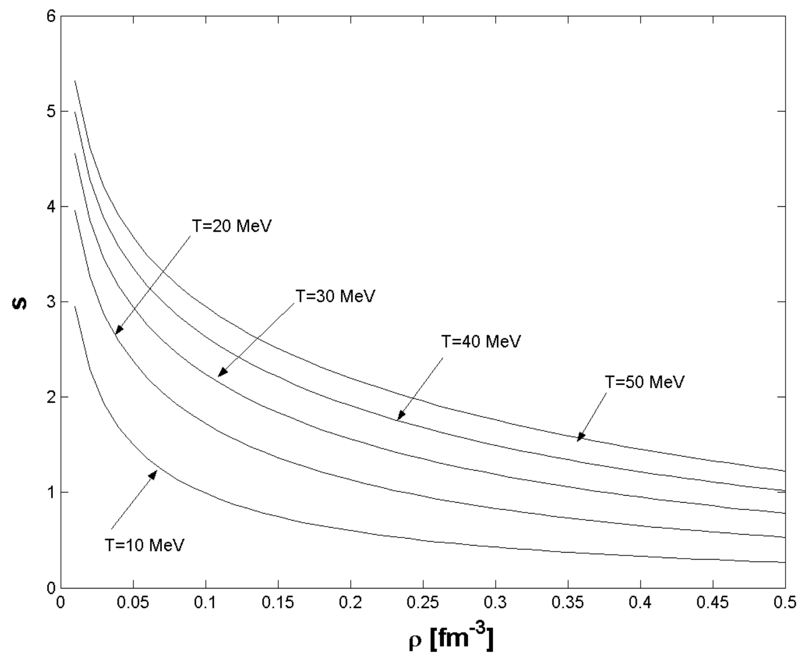

Figure 9. The chemical potential as function of density for neutron matter at different temperature.

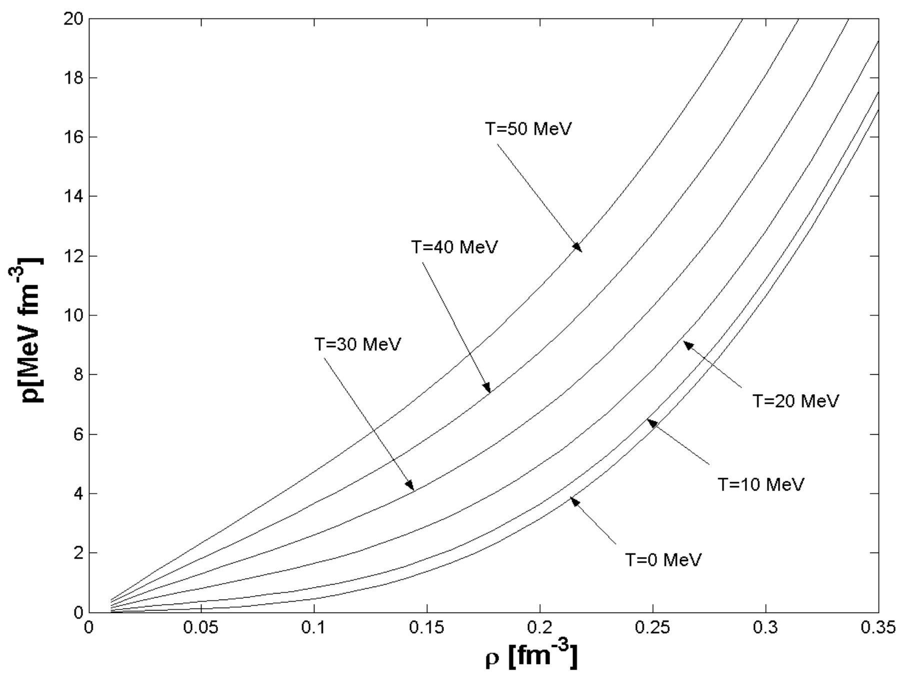

Figure 10. Free energy per baryon versus density for neutron matter at different temperatures.

found that the model under investigation indicates a liquid-gas phase transition and the critical temperature is found to be $T_c = 17.4\ MeV$. Almost all non-relativistic calculations give critical temperatures in the range of $14 - 22\ MeV$. The results are in excellent agreement with the outcome of calculations performed for a broad collection of sophisticated non-relativistic as well as relativistic models for the equation of state.

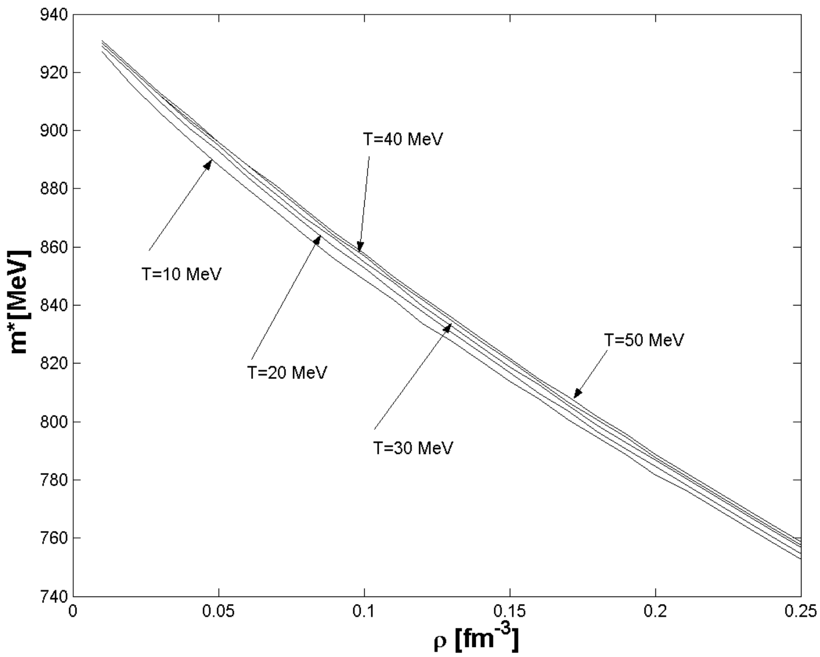

Figure 11. Entropy per baryon versus density for neutron matter at different temperatures.

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