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Phasors from the linear algebra perspective applied to RLC circuits

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Phasors provide a simple way to analyze sinusoidally excited linear circuits. Solutions of those circuits would be undoable otherwise. Besides, measurement units in phasors are a technological resource that powers with precision the observation of the electric power system dynamic state. In the last technological research in electronics, through this units tension phasors and current phasors are obtained in a synchronized way. Since the analysis of complex circuits with resistors, inductors and capacitors for sinusoidal entry types is time consuming, the sinusoidal analysis by phasors is a simple way to analyze such circuits without solving the differential equations. This applies to the case of the sinusoidal entries to a given frequency and once the system is in a stable state. This analysis is shown in this article.

Key words: Sinusoidally excited linear circuit analysis, tension phasors, current phasors.

INTRODUCTION

In the first engineering classes, sometimes linear algebra (Grossman, 2005) is presented without applications in engineering of vector spaces. On the other hand, in these semesters, in the electricity and magnetism physics (Steinmetz, 1983), the concept of phasors is introduced, sometimes without a rigorous foundation.

In Araújo and Tonidandel (2013), it is said that the history of phasors starts in 1868 with the RLC circuits studies by Maxwell (1868), but that the phasor concept was introduced by Steinmetz in 1889 (Serway, 2009), In addition, phasor measurement units are a technological resource that enables very accurately the dynamic state of the electric power system (Lozano and Castro, 2012).

Phasors are widely used in the analysis of electrical circuits. For instance, in the electrical circuits book by Nilson and Riedel (2005), they said that a phasor is a complex number that provides information of amplitude and phase angle of a sinusoidal function. The concept of phasor is based on the identity of Euler that relates the exponential function with the trigonometrical function (Zhang et al., 2010).

In this paper, the space of the $A\,cos(x+\ \emptyset)$ will be tackled first, then the space of complex numbers, later an example of RLC circuits will be shown; the article ends with impedance.

THE Acos(χ+∅) FUNCTION TYPE SPACE

The set of functions in $x \in \Re$

$$V_x = \{Acos(x+\emptyset) | A \in R, 0 \leq \emptyset < 2\}, \quad (1)$$

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is formed by all cosine functions with different phase and amplitude.

With appropriate units, an element of the set V_x could represent a sound similar to the sound of a flute (Dagle, 2010). The amplitude A represents the volume of the sound or how close the instrument is and the phase $\emptyset$ represents the direction in which the instrument is placed.

In principle, the set V_x does not have the appearance of a vector space. However, using the following identity, another perspective of the same set is gained,

$$A cos(x + \emptyset) = A[cos(x)cos(\emptyset) - sin(x)sin(\emptyset)], \quad (2)$$

As A and $\emptyset$ are constants, then the new constants are defined: A_c and A_s,

$A_c = Acos(\emptyset)$ and $A_s = Asin(\emptyset)$.

Therefore, the set V_x becomes

$$V_x = \{A_c cos(x) - A_s \sin(x) \mid A_c, A_s \in R\}, \quad (3)$$

Which corresponds to the vector space generated by the functions {cos(x), −sin(x)}

$$V_x = Gen\{cos(x), -\sin(x)\}$$

We should remember that n functions derivable n times are lineally independent if the Wronskian is different to zero. For the case of the cos(x) and -sin(x) functions, the Wronskian would be

$$\begin{vmatrix} \cos(x) & -\sin(x) \\ -\sin(x) & -\cos(x) \end{vmatrix} = -1 \neq 0$$

The previous equation allows us to conclude that the functions $\{\cos(x), -\sin(x)\}$ for a base for the set V_x and that the dimension of V_x is 2. This implies that V_x and R^2 are isomorphs.

With the base $\{\cos(x), -\sin(x)\}$ of V_x, we can represent any function of the form $Acos(x + \emptyset)$ with the coordinate $\begin{bmatrix} A_c \\ A_s \end{bmatrix}$ of the plane R^2. For instance, the function $3\cos(x + 45°)$ has coordinates in R^2

$$\begin{bmatrix} A_c \\ A_s \end{bmatrix} = \begin{bmatrix} 3\cos(45°) \\ 3\sin(45°) \end{bmatrix} \cong \begin{bmatrix} 2{,}12 \\ 2{,}12 \end{bmatrix}$$

As $A_c^2 + A_s^2 = A^2cos^2(\varphi) + A^2sin^2(\varphi) = A^2[cos^2(\varphi) + sin^2(\varphi)]$, (4)

Then $A = \sqrt{A_c^2 + A_s^2}$

Constant A coincides with the magnitude of vector $\begin{bmatrix} A_c \\ A_s \end{bmatrix}$ as

$$\frac{A_s}{A_c} = \frac{Asin(\varphi)}{Acos(\varphi)} = \tan(\varphi), \quad (5)$$

then

$$\varphi = \arctan\left(\frac{A_s}{A_c}\right)$$

The diphase $\emptyset$ coincides with the angle of the vector $\begin{bmatrix} A_c \\ A_s \end{bmatrix}$.

For instance, the vector (3, 4) are the coordinates of the function

$$3\cos(x) - 4\sin(x) = \sqrt{9} + 16\cos(x + \arctan\left(\frac{4}{3}\right)) \cong 5\cos(x + 53{,}13°)$$

(Figure 1).

The complex plane

The complex plane is the set $C = \{a + jb \mid a, b \in R, j^2 = -1\}$ that with the usual operations form a vector space, which has the canonical base {1, j}. This implies that the complex plane has dimension 2 and, therefore, is isomorphic with R^2. Besides, with this canonical base, a complex number $a + bj$ corresponds to the vector of coordinates a, b.

Due to the fact that the set V_x of the previous section is isomorphic with R^2, then it is also isomorphic with set C and, therefore, the complex number $a + bj$ can be represented by the function $a\cos(x) - bsin(x)$.

Application in RLC circuits

It is known that when a current *i* passes through a resistor (R), an inductor (L) and a capacitor (C), the following potential differences are generated, respectively,

$$vR = iR, \quad vL = L\frac{di}{dt}, \quad vc = \int \frac{i}{C} dt, \quad (6)$$

If we have a power source $i(t) = i_0\cos(wt)$, then the respective voltages are:

$$vR(t) = Ri_0\cos(\omega t), \quad vL(t) = -\omega Li_0\sin(\omega t), \quad vc(t) = \frac{1}{\omega C} i_0 \sin(\omega t), \quad (7)$$

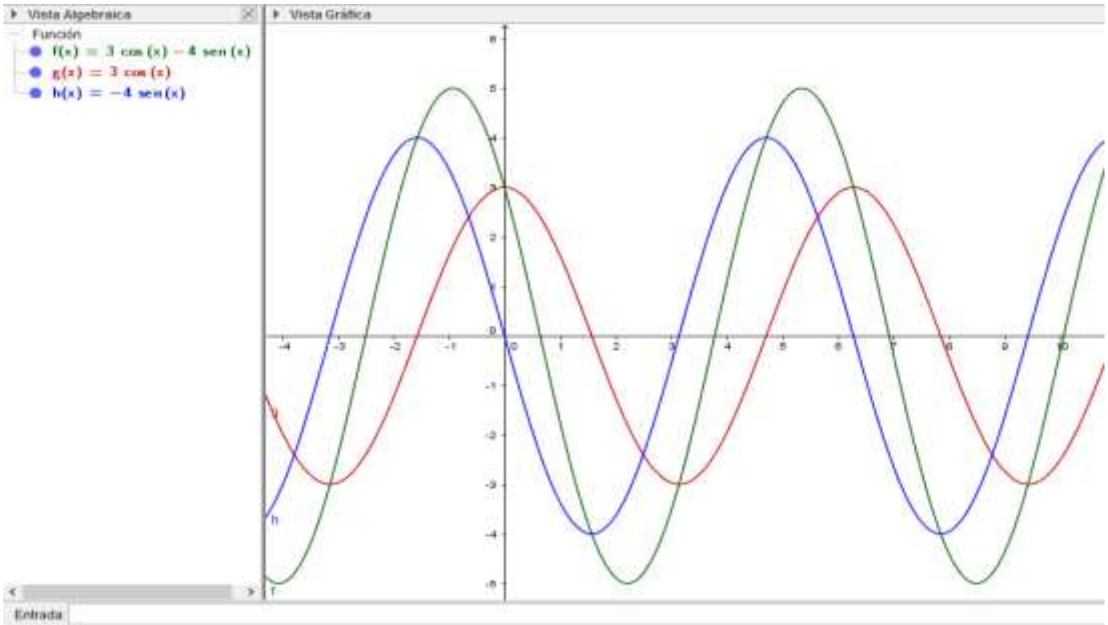

Figure 1. Relationship between phasors: $f(x) = 3\cos(x) - 4\sin(x)$ and $\cos(x + 53{,}13°)$.

According to $A\cos(\chi+\varnothing)$ function type space, the coordinates of each one of those functions in the $V_{\omega t}$ space are:

$$\overline{vR} = \begin{bmatrix} Ri_0 \\ 0 \end{bmatrix},\quad \overline{vL} = \begin{bmatrix} 0 \\ \omega L i_0 \end{bmatrix},\quad \overline{vc} = \begin{bmatrix} 0 \\ -i_0/(\omega C) \end{bmatrix}, \tag{8}$$

which correspond to the following complex numbers:

$$Ri_0, \qquad jwLi_0, \qquad -ji_0/(\omega C)$$

If the resistor, the inductor and the capacitor are in series, then the same current passes through every one of them and the total voltage is the sum of each one of the voltages (Figure 2).

Series Circuit R-L-C

$$v_{RLC}(t) = i_0\left[R\cos(\omega t) + \left(\frac{1}{\omega C} - \omega L\right)\mathrm{sen}(\omega t)\right], \tag{9}$$

As the isomorphisms between V_{wt} with R^2 and C preserve the sum, then the total voltage can also be represented in such spaces as the sum of partial voltages.

$$\begin{bmatrix} Ri_0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega L i_0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{i_0}{\omega C} \end{bmatrix} = i_0\begin{bmatrix} R \\ -\omega L + \frac{1}{\omega C} \end{bmatrix} \tag{10}$$

Impedance

Impedance is the relation between voltage and current (Maxwell, 1868). If we assume a current that varies in time like a function of space V_{wt}, which has a coordinates: $\bar{\imath} = \begin{bmatrix} i_c \\ i_s \end{bmatrix}$, where i_c and i_s are constant, then this current is

$$i(t) = i_c cos(\omega t) - i_s sin(\omega t), \tag{11}$$

and, therefore, the respective voltages are:

$$vR(t) = Ri_c cos(\omega t) - Ri_s sin(\omega t), \tag{12}$$

$$vL(t) = -\omega L i_c cos(\omega t) - \omega L i_s sin(\omega t), \tag{13}$$

$$L(t) = \frac{1}{\omega C} i_c sin(\omega t) + \frac{1}{\omega C} i_s cos(\omega t), \tag{14}$$

When writing the coordinates in R^2 of the voltages, we obtain:

$$\overline{vR} = \begin{bmatrix} Ri_c \\ Ri_s \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}\begin{bmatrix} i_c \\ i_s \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}\bar{\imath}, \tag{15}$$

$$\overline{vL} = \begin{bmatrix} -\omega L i_s \\ \omega L i_c \end{bmatrix} = \begin{bmatrix} 0 & -\omega L \\ \omega L & 0 \end{bmatrix}\begin{bmatrix} i_c \\ i_s \end{bmatrix} = \begin{bmatrix} 0 & -\omega L \\ \omega L & 0 \end{bmatrix}\bar{\imath}, \tag{16}$$

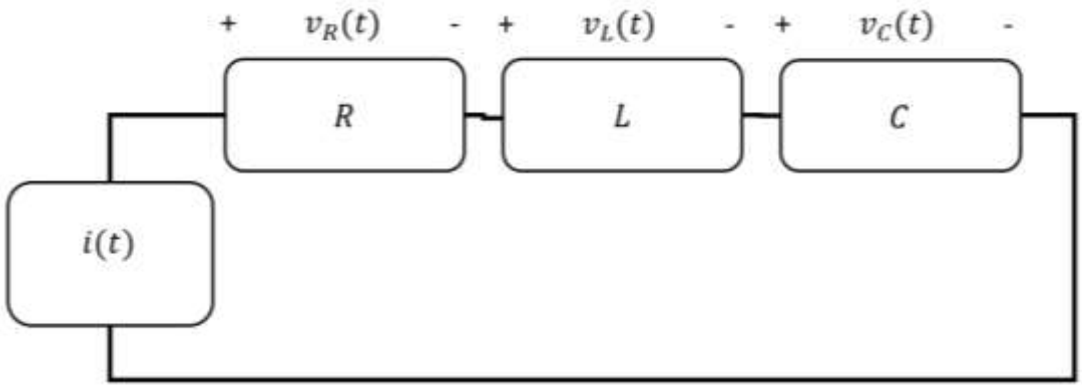

Figure 2. Series circuit, resistance, inductance and condenser.

$$\overline{vL} = \begin{bmatrix} \frac{i_s}{\omega C} \\ \frac{i_c}{\omega C} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\omega C} \\ -\frac{1}{\omega C} & 0 \end{bmatrix} \begin{bmatrix} i_c \\ i_s \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\omega C} \\ -\frac{1}{\omega C} & 0 \end{bmatrix} \bar{\imath}, \quad (17)$$

The previous expressions allow to represent the impedances as matrixes of 2×2

$$Z_R = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}, \quad Z_L = \begin{bmatrix} 0 & -\omega L \\ \omega L & 0 \end{bmatrix}, \quad Z_C = \begin{bmatrix} 0 & \frac{1}{\omega C} \\ -\frac{1}{\omega C} & 0 \end{bmatrix}, \quad (18)$$

Next, we present how the relation would be between current and voltage in complex numbers. Given the following complex numbers

$$V = V_c + jv_s, \quad Z = Z_c + jZ_s, \quad I = i_c + ji_s$$

that represent voltage, impedance and current, respectively. These quantities should relate in the following manner:

$$V = ZI, \quad (19)$$

$$V_c + jv_s = (Z_c + jZ_s)(i_c + ji_s), \quad (20)$$

$$V_c + jv_s = (Z_c i_c - Z_s i_s) + j(i_c Z_s + i_s Z_c), \quad (21)$$

These complex numbers can be represented in R^2 by the following coordinates:

$$\begin{bmatrix} v_c \\ v_s \end{bmatrix} = \begin{bmatrix} Z_c i_c & -Z_s i_s \\ i_c Z_s & i_s Z_c \end{bmatrix} = \begin{bmatrix} Z_c & -Z_s \\ Z_s & Z_c \end{bmatrix} \begin{bmatrix} i_c \\ i_s \end{bmatrix}, \quad (22)$$

which can represent impedances also as 2×2 matrices:

$$Z = \begin{bmatrix} Z_c & -Z_s \\ Z_s & Z_c \end{bmatrix}$$

Then, it allows to represent the impedances of the resistor, the inductor and the capacitor, which will represented in the following manner:

$$Z_R = R + j0, \quad Z_L = 0 + j\omega L, \quad Z_c = 0 - j\frac{1}{\omega C}.$$

With the complex number notation, linear equations systems can be posed and solved in an analogous way to resistive equations systems, which simplifies the solution of differential equations produced by the inductors and capacitors for the concrete case of sinusoidal signals.

CONCLUSIONS

In this article, we present another way, from linear algebra, that do not require the Euler identity, to show the relationship between the sinusoidal function, complex numbers and plane R^2. This approach allows the student to have another perspective of phasors, bettering her or his understanding of this theme.

The proposed path is an application of the isomorphism between vector spaces, which illustrates the importance of this concept.

Conflict of Interests

The authors have not declared any conflict of interest

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