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Full Length Research Paper

A new algorithm for selecting equip system based on fuzzy operations

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In this endeavor, the authors discuss the problem of defuzzification based on parametric interval approximation of fuzzy numbers and then suggest measure about these points of fuzzy numbers. The measure point of each fuzzy number plays an important role in fuzzy sets and systems, specifically in physics, mathematics and statistics. We provide the definition of the measure of fuzzy numbers as well as the definition of measures in probability theory. Some of their applications are mentioned.

Key words: Fuzzy number, measure, ranking, parametric interval approximation, evaluating.

INTRODUCTION

In statistics, measures of central tendency and measures dispersion of distribution are considered important. For fuzzy numbers, one of the most common and useful measures of central tendency is the mean of fuzzy numbers (Carlsson and Fuller, 2001; Fullér and Majlender, 2003), defined the weighted lower possibilistic and upper possibilistic mean values, crisp possibilistic mean value, the variance and covariance of fuzzy numbers. In this paper we introduce the parametric interval approximation of fuzzy numbers and their applications, for example, the measure and ranking of the fuzzy numbers. The main objective of this paper is to obtain fuzzy numbers, in such a way that they provide many applications in fuzzy sets and systems. In this work, a defuzzification as an interval approximation, the nearest interval approximation and measure of a fuzzy number is used. The main results of parametric interval approximation and the preference ordering of fuzzy numbers are new and interesting alternative justifications to the definitions of the parametric interval and measure value of a fuzzy number that is introduced by Fullér and Majlender (Carlsson and Fuller 2001; fuller et al., 2003; Saneifard et al., 2007).

BASIC DEFINITION AND NOTATION

The basic definitions of a fuzzy number are given in Saneifard (2009), (Saneifard and Ezatti, (2010), Wang and Kerre (2001), Baldwin and Guild (1979) as follows:

Definition 1: Let X be a universe set. A fuzzy set A of X is defined by a membership function $\mu_A(x) \to [0,1]$, where $\mu_A(x)$, $\forall x \in X$, indicates the degree of x in A.

Definition 2: A fuzzy subset A of universe set X is normal iff $\sup_{x\in X} \mu_A(x) = 1$.

Definition 3: A fuzzy set A is a fuzzy number iff A is normal and convex on X.

Definition 4: For fuzzy set A support function is defined as follows:

$$Supp\ (A) = \overline{\{x \mid \mu_A(x) > 0\}},$$

where $\overline{\{x|\mu_A(x) > 0\}}$ is closure of set $\{x|\mu_A(x) > 0\}$.
A space of all fuzzy numbers will be denoted by F, and this article recalls that $core\ A = \{x \in \Re | \mu_A(x) = 1\}$.

Definition 5: Assume that the fuzzy number $A \in F$ is represented by means of the following representation:

$$A = \bigcup_{\alpha \in [0,1]} (\alpha, A_\alpha). \tag{1}$$

Here, $A_\alpha = \{x : \mu_A(x) \geq \alpha\}$, is the α-level set of the fuzzy number A. This article considers normal and convex fuzzy numbers. Therefore the α-level sets may be represented in the form of a segment,

$$\forall \alpha \in [0,1] \quad : \quad A_\alpha = [L_A(\alpha), R_\alpha(\alpha)] \subset (-\infty, +\infty) \quad (2)$$

Here, $L:[0,1] \rightarrow (-\infty,+\infty)$ is a monotonically non-decreasing left-continuous and $R:[0,1] \rightarrow (-\infty,+\infty)$ is a monotonically non-increasing right-continuous functions. The functions $L(.)$ and $R(.)$ express the left and right sides of a fuzzy number, respectively. In otherwords,

$$L(\alpha) = \mu_{\uparrow}^{-1}(\alpha),\ R(\alpha) = \mu_{\downarrow}^{-1}(\alpha), \quad (3)$$

Where $L(\alpha) = \mu_{\uparrow}^{-1}(\alpha)$ and $R(\alpha) = \mu_{\downarrow}^{-1}(\alpha)$, denote quasi-inverse functions of the increasing g and decreasing parts of the membership functions $\mu(x)$, respectively. As a result, the decomposition representation of the fuzzy number A, called the $L-R$ representation, has the following form:

$$A = \bigcup_{\alpha \in [0,1]} (\alpha, [L_A(\alpha), R_A(\alpha)]) .$$

Definition 6: (Saneifard and Saneifard, 2011). The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number A:

$$I(A) = \int_0^1 (c\,L_A(\alpha) + (1-c)\,R_A(\alpha))\,p(\alpha)d\alpha, \quad (4)$$

And

$$D(A) = \int_0^1 (R_A(\alpha) - L_A(\alpha))\,p(\alpha)\,d\alpha \ . \quad (5)$$

Here $0 \le c \le 1$ denotes an "optimism/pessimism" coefficient in conducting operations on fuzzy numbers. The function $p:[0,1] \rightarrow [0,+\infty)$ denotes the distribution density of the importance of the degrees of fuzziness, where $\int_0^1 p(\alpha)\,d\alpha = 1$. In particular cases, it may be assumed that $p(\alpha) = (k+1)\alpha^k, k = 0,1,\cdots$.

Definition 7: (Saneifard and Saneifard, 2011). For arbitrary fuzzy numbers A and B the quantity

$$d_b(A,B) = \sqrt{[I(A) - I(B)]^2 + [D(A) - D(B)]^2} \quad (6)$$

is called the parametric distance between the fuzzy numbers A and B.

Definition 8: (Grzegorzewski, 2002). An operator $I : F \rightarrow (Set\ of\ Closed\ Intervals\ in\ \Re)$ is called an interval approximation operator if for any $A \in F$

(a') $I(A) \subseteq Supp\ (A)$,
(b') $core(A) \subseteq I(A)$,
(c') $\forall(\varepsilon > 0)\exists(\delta > 0)\ s.t\ d(u,v) < \delta \Rightarrow d(I(u),I(v)) < \varepsilon$,

where $d : F \rightarrow [0,+\infty[$, is a metric defined in the family of all fuzzy numbers.

Definition 9: (Grzegorzewski, 2002). An interval approximation operator satisfying in condition (c') for any $A, B \in F$ is called the continuous interval approximation operator.

The measure of interval number

The measure of interval is given first which is different from the measure of traditional interval number, such as the length of interval number.

Generally, interval number is denoted as $A(a_1,a_2) = [a_1,a_2]$, where a_1 and a_2 are respectively called left end point and right end point, $a_1 \le a_2$. Particularly, if $a_1 = a_2$, $A(a_1,a_2)$ denotes real number a_1. Let, $A(a_1,a_2)$ and $B(b_1,b_2)$ are arbitrary interval numbers, here in $A(a_1,a_2) = B(b_1,b_2)$ if and only if $a_1 = b_1$ and $a_2 = b_2$.

Definition 10: (Yang and Gao, 2002). Let, $A(a_1,a_2)$ is arbitrary interval number. The measure of interval number A defines as follows:

$$M_I(A) = sign(a_1)\left|a_1.a_2\right|. \quad (7)$$

Note that the geometric meaning of the measure that we defined here is monotone function of a triangle area which is constituted by segment $l(a_1,a_2)$ and two axes. The implication of the symbol function is that we can compare the size between two interval numbers when the end point of interval numbers is a negative number.

PARAMETRIC INTERVAL APPROXIMATION

Various authors in Saneifard (2009), Chakrabarty et al. (1998), Saneifard et al. (2001) have studied the crisp approximation of fuzzy sets. They proposed a rough theoretic definition of that crisp approximation, called the nearest ordinary set and nearest interval approximation of a fuzzy set. Here, the authors propose another approximation called the parametric interval approximation.

Let A be an arbitrary fuzzy number and $[L_A(\alpha), R_A(\alpha)]$ be its α-cut set. This effort attempts to find a closed interval $C_{d_p}(A)$, which is the parametric interval to A with respect to metric d_p. Since each interval with constant α-cuts for all $\alpha \in (0,1]$ is a fuzzy number, hence, suppose $C_{d_p}(A) = [L_C, R_C]$, , that is $C_{d_p}(A) = [L_C, R_C]$, $\forall \alpha \in (0,1]$. So, this article has to minimize

$$d_p(A, C_{d_p}(A)) = \left(\left[I(A) - I(C_{d_p}(A))\right]^2 + \left[D(A) - D(C_{d_p}(A))\right]^2\right)^{\frac{1}{2}}, \quad (8)$$

with respect to L_C and R_C, where

$$I(C_{d_p}(A)) = \int_0^1 \left(cL_C + (1-c)R_C\right)p(\alpha)d\alpha, \; D((C_{d_p}(A))) = \int_0^1 \left(R_C - L_C\right)p(\alpha)d\alpha.$$

In order to minimize d_p it suffices to minimize

$$\overline{D}_p(L_C, R_C) = d_p^2(L_C, R_C). \quad (9)$$

It is clear that, the parameters L_C and R_C which minimize Equation (9) must satisfy

$$\nabla \overline{D}_p(L_C, R_C) = \left(\frac{\partial \overline{D}_p}{\partial L_C}, \frac{\partial \overline{D}_p}{\partial R_C}\right) = 0 \cdot$$

Therefore, the following equations are utilized in this endeavor:

$$\begin{cases} \dfrac{\partial \overline{D}_p(L_C, R_C)}{\partial L_C} = -2c\displaystyle\int_0^1 (c(L_A(\alpha) - L_C) + (1-c)(R_A(\alpha) - R_C))p(\alpha)d\alpha \\ +2\displaystyle\int_0^1 ((R_A(\alpha) - R_C) - (L_A(\alpha) - L_C))p(\alpha)d\alpha = 0, \\ \dfrac{\partial \overline{D}_p(L_C, R_C)}{\partial R_C} = -2(1-c)\displaystyle\int_0^1 (c(L_A(\alpha) - L_C) + (1-c)(R_A(\alpha) - R_C))p(\alpha)d\alpha \\ -2\displaystyle\int_0^1 ((R_A(\alpha) - R_C) - (L_A(\alpha) - L_C))p(\alpha)d\alpha = 0, \end{cases} \quad (10)$$

The parameters L_C associated with the left bound and R_C associated with the right bound of the parametric interval can be found by using Equation (10) as follows:

$$\begin{cases} L_C = \displaystyle\int_0^1 L_A(\alpha)p(\alpha)d\alpha, \\ R_C = \displaystyle\int_0^1 R_A(\alpha)p(\alpha)d\alpha. \end{cases} \quad (11)$$

Remark 1: Since,

$$\det\begin{bmatrix} \dfrac{\partial^2 \overline{D}_p(L_C, R_C)}{\partial L_C^2} & \dfrac{\partial^2 \overline{D}_p(L_C, R_C)}{\partial R_C \partial L_C} \\ \dfrac{\partial^2 \overline{D}_p(L_C, R_C)}{\partial L_C \partial R_C} & \dfrac{\partial^2 \overline{D}_p(L_C, R_C)}{\partial R_C^2} \end{bmatrix} = \det\begin{bmatrix} 2c^2 & 2c(1-c)-2 \\ 2c(1-c)-2 & 2(1-c)^2+2 \end{bmatrix} = 4 > 0, \text{ and}$$

$\forall c \in [0,1], \dfrac{\partial \overline{D}_p^2(L_C, R_C)}{\partial L_C^2} = 1 > 0$, therefore L_C and R_C given by (11), minimize $d_p(A, C_{d_p}(A))$. Therefore, the interval

$$C_{d_p}(A) = \left[\int_0^1 L_A(\alpha)p(\alpha)d\alpha, \int_0^1 R_A(\alpha)p(\alpha)d\alpha\right], \quad (12)$$

is the nearest parametric interval approximation of fuzzy number A with respect to metric d_p.

Remark 2: Whenever, in the distribution density function $p(\alpha) = (k+1)\alpha^k$, where $k = 1$, $C_{d_p}(A)$ denotes the weighted interval-value possibilistic mean (Carlsson and Fuller, 2001).

Remark 3: If, in distribution density function $p(\alpha) = (k+1)\alpha^k$, assuming $k = 0$, therefore $C_{d_p}(A)$ is the expected interval (Grzegorzewski, 2002).

However, intention of this article is to approximate a fuzzy number using a crisp interval. Thus, the authors have used an operator $C_{d_p}(A) : F \to (Set\ of\ Closed\ Intervals\ in\ \Re)$ which transforms fuzzy numbers into a family of closed intervals on the real line.

Theorem 1: (Saneifard and Ezatti, 2010). The operator $C_{d_p}(A) : F \to (Set\ of\ Closed\ Intervals\ in\ \Re)$ is an interval approximation operator (that is, $C_{d_p}(A)$ is a continuous interval approximation operator).

THE PREFERENCE ORDERING OF FUZZY NUMBERS

Here, the authors propose a novel technique for ranking of fuzzy numbers associated with the parametric interval.

Definition 11: Let A be an arbitrary fuzzy number and $[L_A(\alpha), R_A(\alpha)]$ be its α-cut and $C_{d_p}(A) = [I_1, I_2]$ is its the parametric interval, where $I_1 = \int_0^1 L_A(\alpha)\, p(\alpha)\, d\alpha$ and $I_2 = \int_0^1 R_A(\alpha)\, p(\alpha)\, d\alpha$. According to definition 10, the measure of $C_{d_p}(A)$ which is an interval number is as $M_I(C_{d_p}(A)) = sign\,(I_1(\alpha))|I_1.I_2|$. We define the *measure* of fuzzy number A as follows:

$$M(A) = \int_0^1 p(\alpha) M_I(C_{d_p}(A))\, d\alpha. \qquad (13)$$

Obviously, if $L-R$ fuzzy numbers become interval numbers, then $M(A)$ will be the measure of the interval number which can be denoted as $M_I(A)$. For a certain fuzzy numbers, we can obtain $M(A)$ by definite integral. But it is not easy to compute definite integral sometimes. For trapezoid fuzzy numbers and triangular fuzzy numbers, the calculation formulas for the indices are given in the paper.

Proposition 1: If $A = (a,b,c,d)$ is a trapezoidal fuzzy number, the measure $M(A)$ can be denoted as follows:

$$M(A) = \frac{B}{2} + \frac{2C}{3} + D \qquad (14)$$

where, $B = (b-a)(c-d)$, $C = db - 2ad + ac$ and $D = ad$.

Since every measure can be used as a crisp approximation of a fuzzy number, therefore, the resulting value is used to rank the fuzzy numbers. Thus, $M(A)$ is used to rank fuzzy numbers.

Let A and $B \in F$ be two arbitrary fuzzy numbers, and $M(A)$ and $M(B)$ be the measures of A and B, respectively. Define the ranking of A and B by $M(.)$ on F, that is

(1) $M(A) = M(B)$ if only if $A \sim B$,
(2) $M(A) < M(B)$ if only if $A \prec B$,
(3) $M(A) > M(B)$ if only if $A \succ B$.

Then, this article formulates the order $\succeq$ and $\preceq$ as $A \succeq B$ if and only if $A \succ B$ or $A \sim B$, $A \preceq B$ if and only if $A \prec B$ or $A \sim B$.

Proposition 2: Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ are two $L-R$ fuzzy numbers

(1) If $a_1 \le b_1$, $a_2 \le b_2$, $a_3 \le b_3$ and $a_4 \le b_4$, then $A \preceq B$,
(2) If $a_1 = b_1$, $a_2 = b_2$, $a_3 \le b_3$ and $a_4 \le b_4$, then $A \preceq B$,
(3) If $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$ and $a_4 \le b_4$, then $A \preceq B$.

Proofs: (1) Let $[L_A(\alpha), R_A(\alpha)]$ and $[L_B(\alpha), R_B(\alpha)]$ are α-cuts of them. If $a_1 \le b_1$ and $a_2 \le b_2$ then $L_A(\alpha) \le L_B(\alpha)$ and if $a_3 \le b_3$ and $a_4 \le b_4$, then $R_A(\alpha) \le R_B(\alpha)$. Since, A_α and B_α are two interval numbers, so $M_I(A_\alpha) \le M_I(B_\alpha)$, thus $M(A) \le M(B)$. That is $A \preceq B$.

Similarly, we can prove (2) and (3).

Remark 4: If $A \preceq B$, then $-A \succeq -B$. Hence, this article can infer ranking order of the images of the fuzzy numbers.

NUMERICAL EXAMPLES

Here, this study compares the proposed method with others methods. Also, the authors assumed $k = 1$.

Example 1: Consider the following sets, (Yao and Wu, 2000).

Set 1: $A = (0.4, 0.5, 1)$, $B(0.4, 0.7, 1)$, $C = (0.4, 0.9, 1)$.
Set 2: $A = (0.3, 0.4, 0.7, 0.9)$ (trapezoidal fuzzy number), $B = (0.3, 0.7, 0.9)$, $C = (0.5, 0.7, 0.9)$.
Set 3: $A = (0.3, 0.5, 0.7)$, $B = (0.3, 0.5, 0.8, 0.9)$ (trapezoidal fuzzy number), $C = (0.3, 0.5, 0.9)$.
Set 4: $A = (0.3, 0.5, 0.8, 0.9)$ (trapezoidal fuzzy number), $B = (0.2, 0.5, 0.9)$, $C = (0.1, 0.6, 0.8)$.

Table 1 shows a comparison with other methods.

Table 1. Comparative results of Example (1).

Authors	Fuzzy number	Set 1	Set 2	Set 3	Set 4
	A	0.3	0.28	0.24	0.36
Proposed method	B	0.47	0.43	0.36	0.24
	C	0.68	0.48	0.27	0.28
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
	A	1.2000	1.1500	1.0000	0.0950
Sing distance method with p = 1	B	1.4000	1.3000	1.2500	1.0500
	C	1.6000	1.4000	1.1000	1.0500
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
	A	0.8869	0.8756	0.7257	0.7853
Sing distance method with p = 2	B	1.0194	0.9522	0.9416	0.7958
	C	1.1605	1.0033	0.8165	0.8386
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
	A	0.6	0.575	0.5	0.475
Distance minimization	B	0.7	0.65	0.625	0.525
	C	0.9	0.7	0.55	0.525
Result		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
	A	0.5334	0.5584	0.5000	0.5250
Abbasbandy and Hajjari	B	0.7000	0.6334	0.6416	0.5084
	C	0.8666	0.7000	0.5166	0.5750
Result		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$B \prec A \prec C$
	A	0.3333	0.5480	0.3330	0.5000
Choobineh and Li	B	0.5000	0.5830	0.4164	0.5833
	C	0.6670	0.6670	0.5417	0.6111
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
	A	0.2990	0.2847	0.2500	0.2440
Chu and Tsao	B	0.3500	0.3247	0.3152	0.2624
	C	0.3993	0.3500	0.2747	0.2619
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
	A	0.6000	0.5750	0.5000	0.4750
Yao and Wu	B	0.7000	0.6500	0.6250	0.5250
	C	0.8000	0.7000	0.5500	0.5250
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
	A	0.0272	0.0328	0.0133	0.0693
Cheng CV uniform distribution	B	0.0214	0.0246	0.0304	0.0385
	C	0.0225	0.0095	0.0275	0.0433
Results		$A \prec C \prec B$	$A \prec B \prec C$	$B \prec C \prec A$	$A \prec C \prec B$
	A	0.0183	0.0260	0.0080	0.0471
Cheng CV proportional distribution	B	0.0128	0.0146	0.0234	0.0236
	C	0.0137	0.0057	0.0173	0.0255
Results		$A \prec C \prec B$	$A \prec B \prec C$	$B \prec C \prec A$	$A \prec C \prec B$

Table 2. Comparative results of Example (2).

Fuzzy number	New approach	Sign distance with p = 1	Sign distance with p = 2	Distance minimization	Chu and Tsao
A	6.2	3	2.16	2.50	0.74
B	6.5	3	2.70	2.50	0.74
C	4.8	3	2.70	2.50	0.75
Results	$C \prec A \prec B$	$C \sim A \sim B$	$C \prec A \sim B$	$C \sim A \sim B$	$A \sim B \prec C$

Example 2: Consider the three fuzzy numbers $A = (1,2,5)$, $B = (0,3,4)$ and $C = (2,2.5,3)$. By using this new approach $M(A) = 6.2$, $M(B) = 6.5$ and $M(C) = 4.83$. Hence, the ranking order is $C \prec B \prec A$ too. Table 2 shows a comparison with some of the other methods in Saneifard and Ezatti, (2010), Saneifard and Asgari (2011) and Chu and Tsao (2002).

All the aforeseen examples show the results of this effort to be more efficient and consistent than the previous ranking methods, and overcome the short-comings of other methods.

Using proposed ranking method in selecting army equip system

From experimental results, the proposed method with some advantages: (a) without normalizing process, (b) fit all kind of ranking fuzzy number, (c) correct Kerre's concept. Therefore we can apply measure of fuzzy ranking method in practical examples. In the following, the algorithm of selecting equip systems is proposed, and then adopted to ranking an army example.

An algorithm for selecting equip system

We summarize the algorithm for evaluating equip system as follows:

Step 1: Construct a hierarchical structure model for equip system.

Step 2: Build a fuzzy performance matrix $\widetilde{A}$. We compute the performance score of the sub factor, which is represented by triangular fuzzy numbers based on expert's ratings, average all the scores corresponding to its criteria. Then, build a fuzzy performance matrix $\widetilde{A}$.

Step 3: Build a fuzzy weighting matrix $\widetilde{W}$. According to the attributes of the equip systems, experts give the weight for each criterion by fuzzy numbers, and then form a fuzzy weighting matrix $\widetilde{W}$.

Step4: Aggregateevaluation.Tomultiplefuzzy performance matrix and fuzzy weighting matrix $\widetilde{W}$, then get fuzzy aggregative evaluation matrix $\widetilde{R}$ (that is, $\widetilde{R} = \widetilde{A} \otimes \widetilde{W}^t$).

Step 5: Determinate the best alternative. After step 4, we can get the fuzzy aggregative performance for each alternative, and then rank fuzzy numbers by measure of fuzzy numbers.

Selecting optimal self-propelled howitzers

From Armour (2001), we construct a practical example for selecting equip system to illustrate our proposed method. The three self propelled howitzers alternatives are: America M109-A6 (S1), England AS90 (S2) and South Africa G6 (S3).

Step 1: According to literature review and experts opinions, the hierarchical structure diagram about the equip system is constructed as shown in Figure 1.

Step 2: Compute the performance score of the sub factor, which is represented by triangular fuzzy numbers based on expert's opinions in Table 3, then average all the scores corresponding to its criteria. Table 4 depicts the performance sore of three self-propelled Howitzers. In addition, we transfer the real value to triangular fuzzy numbers by experts. Then, we form a fuzzy performance matrix $\widetilde{A}$ as shown in Table 5.

Step 3: From experts give criteria weight to build a fuzzy weighting matrix $\widetilde{W}$ in Table 6.

Step 4: Aggregate evaluation. We multiple fuzzy performance matrix $\widetilde{A}$ and fuzzy weighting matrix $\widetilde{W}$, then we can get fuzzy aggregate evaluation matrix $\widetilde{R}$.

According to the fuzzy aggregate evaluation matrix, we can get aggregative triangular fuzzy numbers of three alternatives, which are $S_1 : \widetilde{r}_1$; $S_2 : \widetilde{r}_2$ and $S_3 : \widetilde{r}_3$.

Step 5: Determined the best alternative.

From step 4 result, we apply fuzzy ranking method to obtain the best alternative. Therefore, we use the new measure method to rank fuzzy numbers of alternatives for decision maker.

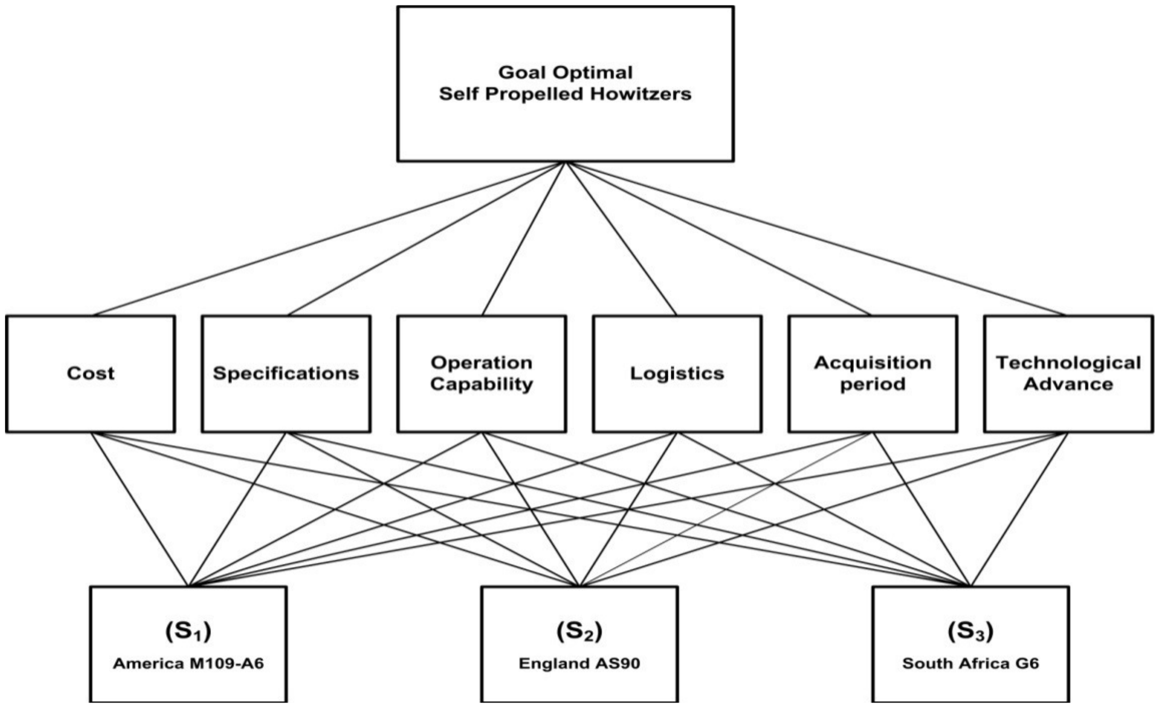

Figure 1. The structure model of evaluating three self propelled Howitzers.

Table 3. The membership function of $\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}$ and $\tilde{9}$.

Fuzzy number	Triangular fuzzy number
$\tilde{1}$	(1,1,3)
$\tilde{x}$	$(x-2, x, x+2)$ for $x = 3,5,7.$
$\tilde{9}$	(7,9,9)

Table 4. The performance score of three self propelled Howitzers.

Criteria/Attribute	America (S_1)	England (S_2)	Africa (S_3)
Cost			
Purchase cost	$\tilde{3}$	$\tilde{5}$	$\tilde{7}$
Operational maintenance cost	$\tilde{7}$	$\tilde{3}$	$\tilde{5}$
Average fuzzy numbers	(3,5,7)	(2,4,6)	(4,6,8)
Specifications			
Fire range (km)	$\tilde{5}$	$\tilde{5}$	$\tilde{7}$
Fire rate (r/m)	$\tilde{5}$	$\tilde{7}$	$\tilde{5}$
Speed (km/h)	$\tilde{7}$	$\tilde{5}$	$\tilde{9}$
Weight (ton)	$\tilde{9}$	$\tilde{5}$	$\tilde{5}$
Ammunition	$\tilde{5}$	$\tilde{7}$	$\tilde{7}$
Average fuzzy numbers	(4.2,6.2,7.8)	(3.8,5.8,7.8)	(4.6,6.6,8.2)

Table 4. Contd.

Operation capability			
Accuracy	$\tilde{7}$	$\tilde{5}$	$\tilde{5}$
Gun life	$\tilde{5}$	$\tilde{7}$	$\tilde{7}$
Gun elevation/depression	$\tilde{7}$	$\tilde{5}$	$\tilde{7}$
Reaction time	$\tilde{7}$	$\tilde{5}$	$\tilde{5}$
Pilot night vision system	$\tilde{7}$	$\tilde{7}$	$\tilde{5}$
Operation	$\tilde{7}$	$\tilde{5}$	$\tilde{7}$
N.B.S protection	$\tilde{7}$	$\tilde{9}$	$\tilde{5}$
Average fuzzy numbers	(4.7,6.7,8.7)	(4.1,6.1,7.8)	(3.8,5.8,7.8)
Logistic			
Maintenance ability	$\tilde{7}$	$\tilde{7}$	$\tilde{5}$
Crew	$\tilde{7}$	$\tilde{5}$	$\tilde{3}$
Train	$\tilde{7}$	$\tilde{5}$	$\tilde{7}$
Reliability	$\tilde{7}$	$\tilde{5}$	$\tilde{3}$
Average fuzzy numbers	(5,7,9)	(3.5,5.5,7.5)	(2.5,4.5,6.5)
Acquisition			
Acquisition period	$\tilde{5}$	$\tilde{3}$	$\tilde{9}$
Service country	$\tilde{9}$	$\tilde{5}$	$\tilde{3}$
International relation	$\tilde{7}$	$\tilde{5}$	$\tilde{3}$
Average fuzzy numbers	(5,7,8.3)	(2.3,4.3,6.3)	(3,5,6.3)
Technological advance			
Automatic fire control system	$\tilde{9}$	$\tilde{7}$	$\tilde{5}$
Gun control system	$\tilde{5}$	$\tilde{9}$	$\tilde{5}$
Global positioning system	$\tilde{9}$	$\tilde{7}$	$\tilde{5}$
Average fuzzy numbers	(5.6,7.6,8.3)	(5.6,7.6,9)	(3,5,7)

Table 5. Fuzzy performance matrix $\tilde{A}$.

Alternative	Criteria					
	Cost	Specification	Operation capability	Logistics	Acquisition period	Technological advance
S_1	(3,5,7) (3,5,7)	(4.2,6.2,7.8) (4.2,6.2,7.8)	(4.7,6.7,8.7)	(4.7,6.7,8.7)	(5,7,8.3)	(5.6,7.6,8.3) (5.6,7.6,8.3)
S_2	(2,4,6) (2,4,6)	(3.8,5.8,7.8) (3.8,5.8,7.8)	(4.1,6.1,7.8)	(3.5,5.5,7.5)	(2.3,4.3,6.3) (2.3,4.3,6.3)	(5.6,7.6,9)
S_3	(4,6,8) (4,6,8)	(4.6,6.8,8.2)	(3.8,5.8,7.8)	(2.5,4.5,6.5) (2.5,4.5,6.5)	(3,5,6.3)	(3,5,7)

Table 6. Fuzzy weighting matrix $\tilde{W}$.

Alternative	Criteria					
	Cost	Specification	Operation capability	Logistics	Acquisition period	Technological advance
$\tilde{W}$	$\tilde{7}$	$\tilde{5}$	$\tilde{9}$	$\tilde{7}$	$\tilde{3}$	$\tilde{7}$

First, we use Equations (4) and (5) to compute the weighted averaged that is, I and the weighted width that is, D of $\tilde{r}_1$, $\tilde{r}_2$ and $\tilde{r}_3$. We also use Equation (12) to compute the nearest parametric interval approximation of fuzzy numbers $\tilde{r}_1$, $\tilde{r}_2$ and $\tilde{r}_3$. Then we use Equation (13) to compute $M(.)$ of the three triangular fuzzy numbers $\tilde{r}_1$, $\tilde{r}_2$ and $\tilde{r}_3$, and the results are as follows:

$$M(\tilde{r}_1) = 601,$$
$$M(\tilde{r}_2) = 457,$$
$$M(\tilde{r}_3) = 425.$$

Therefore, the ranking order is $S_3 < S_2 < S_1$. Through the algorithm for selecting equip system and proposed measure method, we can make sure the best alternative is S_1 (that is, America M109-A6).

$$\tilde{R} = \tilde{A} \otimes \tilde{W}^t =$$

$$\begin{bmatrix} (3,5,7) & (4.2,6.2,7.8) & (4.7,6.7,8.7) & (5.0,7.0,9.0) & (5.0,7.0,8.3) & (5.6,7.6,8.3) \\ (2,4,6) & (3.8,5.8,7.8) & (4.1,6.1,7.8) & (3.5,5.5,7.5) & (2.3,4.3,6.3) & (5.6,7.6,9.0) \\ (4,6,8) & (4.6,6.6,8.2) & (3.8,5.8,7.8) & (2.5,4.5,6.5) & (3.0,5.0,6.3) & (3.0,5.0,7.0) \end{bmatrix} \otimes \begin{bmatrix} \tilde{7} \\ \tilde{5} \\ \tilde{9} \\ \tilde{7} \\ \tilde{3} \\ \tilde{7} \end{bmatrix}$$

$$\tilde{r}_1 = (3,5,7) \otimes \tilde{7} \oplus (4.2,6.2,7.8) \otimes \tilde{5} \oplus (4.7,6.7,8.7) \otimes \tilde{9} \oplus (5,7,9) \otimes \tilde{7} \oplus (5,7,8.3) \otimes \tilde{3} \oplus (5.6,7.6,8.3) \otimes \tilde{7},$$
$$\tilde{r}_2 = (2,4,6) \otimes \tilde{7} \oplus (3.8,5.8,7.8) \otimes \tilde{5} \oplus (4.1,6.1,7.8) \otimes \tilde{9} \oplus (3.5,5.5,7.5) \otimes \tilde{7} \oplus (2.3,4.3,6.3) \otimes \tilde{3} \oplus (5.6,7.6,9) \otimes \tilde{7},$$
$$\tilde{r}_3 = (4,6,8) \otimes \tilde{7} \oplus (4.6,6.6,8.2) \otimes \tilde{5} \oplus (3.8,5.8,7.8) \otimes \tilde{9} \oplus (2.5,4.5,6.5) \otimes \tilde{7} \oplus (3,5,6.3) \otimes \tilde{3} \oplus (3,5,7) \otimes \tilde{7}.$$

Then

$$\tilde{r}_1 = (118.5,249.5,393.1),$$
$$\tilde{r}_2 = (97.9,216.5,358.8),$$
$$\tilde{r}_3 = (90.9,208.7,352.6).$$

CONCLUSION

In this study, the authors discuss the problems of parametric interval approximation of fuzzy numbers and instead, propose a novel ranking approach. Furthermore, it is shown that this interval can be used as a crisp approximation (measure) with respect to a fuzzy quantity, can effectively rank various fuzzy numbers and their images, there by overcoming the weaknesses of previous techniques.

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