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*Full Length Research Paper*

# Anti-synchronization of uncertain Rikitake systems via active sliding mode control

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**In this paper, an active sliding mode controller is presented for a class of master-slave anti-synchronization of uncertain Rikitake systems. Uncertainties are considered in linear and nonlinear parts of the Rikitake systems. Analysis of the stability for the proposed method is derived based on the Lyapunov stability theory. Numerical simulations are performed to evaluate effectiveness of the analytical results.**

**Key words:** Rikitake systems, uncertainties, anti-synchronization, active sliding mode control.

## INTRODUCTION

Chaos synchronization has received increasing attention, and has been extensively investigated theoretically, numerically and experimentally in many chaotic systems (Pecora and Carroll, 1990). So far, different types of chaos synchronization have been studied, such as anti-synchronization (Li, 2005), generalized synchronization (Ghosh, 2009), phase synchronization (Ghosh, 2009), projective synchronization (Lee and Park, 2010a) and lag synchronization (Lee et al., 2010). Among them, anti-synchronization is another interesting synchronization phenomenon (Li, 2005). Various anti-synchronization methods have been proposed, such as backstepping control (Lin et al., 2009), feedback control (Lee and Park, 2010b; Li and Liao, 2006), adaptive control (Al-sawalha and Noorani, 2009; Elabbasy and El-Dessoky, 2009), active control (Wang et al., 2007; Njah and Vincent, 2009) and sliding mode control (Feki, 2009; Roopaei et al., 2009). Among the aforementioned methods, active control (Wang et al., 2007; Njah and Vincent, 2009) and sliding mode control (Feki, 2009; Roopaei et al., 2009) have been widely recognized as two powerful design methods to anti-synchronize chaotic systems. One of the new anti-synchronization methods is the active sliding mode control, which is a combination of active control and sliding mode control, and has been successfully applied to several chaotic systems (Zhao, 2009; Zhao and Wang, 2009; Liu and Song, 2008). The Rikitake system (Vincent, 2005) describes the chaotic behaviour exhibited by the reversal of the earth's magnetic field. Wu et al. (2008) studied the chaos synchronization problems of the Rikitake system via passive control. The aim of this article is to design an active sliding mode controller to anti-synchronize the uncertain Rikitake systems, which include linear and nonlinear uncertain parts.

The rest of this paper is organized as follows: First, the Rikitake system and the anti-synchronization of uncertain Rikitake systems are introduced respectively, after which an active sliding mode controller is designed to anti-synchronize the Rikitake systems with uncertainties. Next, numerical simulations are given for illustration and verification, before conclusions of the study are finally drawn.

## DESCRIPTION OF THE RIKITAKE SYSTEM

The Rikitake system (Vincent, 2005; Wu et al., 2008) consists of two connected identical frictionless disk dynamos, which describes the reversal of the earth's magnetic field. The equations describing the system are given by the nonlinear dynamical system:

$$\begin{cases} \dot{x}_1 = -bx_1 + x_2 x_3 \\ \dot{x}_2 = -bx_2 + (x_3 - a)x_1 \\ \dot{x}_3 = 1 - x_1 x_2 \end{cases}, \qquad (1)$$

where $x_1$, $x_2$ and $x_3$ are the state variables, and $b$ and *a* are the positive real constants. The Rikitake system (1)

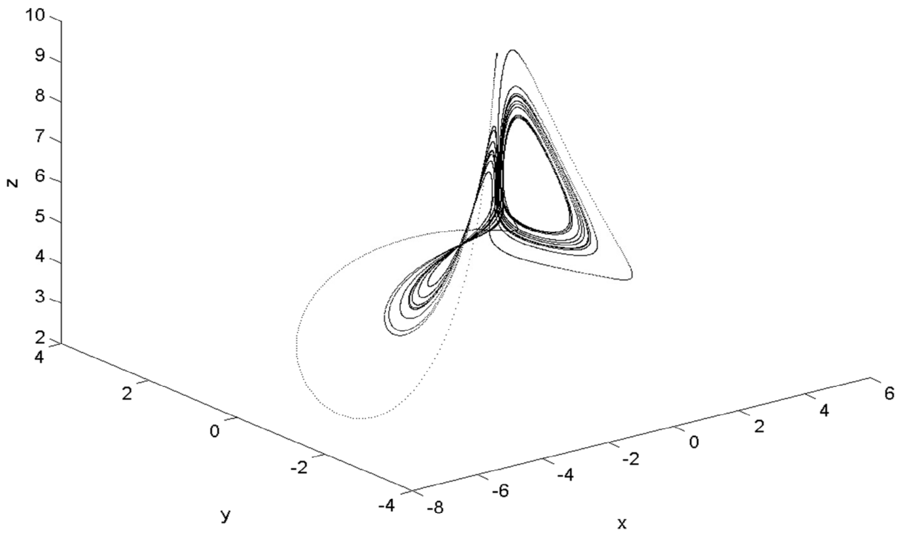


**Figure 1.** Chaotic attractor ( $b = 2$ and *a* = 5).

exhibits a chaotic attractor for $b = 2$ and *a* = 5 as shown in Figure 1.

## ANTI-SYNCHRONIZATION OF RIKITAKE SYSTEMS WITH UNCERTAINTIES

Consider an uncertain Rikitake system described by the following nonlinear differential equation as the master system:

$$\dot{x} = (Q + \Delta Q)x + f(x) + \Delta f(x) . \quad (2)$$

while the response system should be considered as follows:

$$\dot{y} = (Q + \Delta Q)y + f(y) + \Delta f(y) + u(t) , \quad (3)$$

where $x(t), y(t) \in R^3$ denotes state vectors of two systems. $Q \in R^{3\times3}$ and $f : R^3 \to R^3$ represent the linear and nonlinear parts of the systems dynamics, respectively. $\Delta Q \in R^{3\times3}$ are unknown linear parts of systems (2) and (3) which satisfy $\|\Delta Q\| \le \eta$, where $\eta > 0$. $\Delta f : R^3 \to R^3$ are the unknown nonlinear parts of systems (2) and (3). The controller $u(t) \in R^3$ has been added to the response system in order to anti-synchronize its states $y(t)$ with the states of the master system $x(t)$.

In defining the anti-synchronization error as $e = y + x$, the study's goal is to design an appropriate active sliding mode controller $u(t)$ such that:

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|y + x\| = 0, \quad (4)$$

where $\|\cdot\|$ is the Euclidean norm.

## ACTIVE SLIDING MODE CONTROLLER DESIGN

From Equations (3) + (2), we get the error dynamical system as follows:

$$\dot{e} = (Q+\Delta Q)x + f(x) + \Delta f(x) + (Q+\Delta Q)y + f(y) + \Delta f(y) + u(t) . \quad (5)$$

In accordance with the active control design strategy (Ho and Hung, 2002; Agiza and Yassen, 2001), the control input vector-function is chosen as follows:

$$u(t) = H(t) - f(x) - f(y) - (Q + \lambda I)x - (Q + \lambda I)y , \quad (6)$$

where $\lambda$ is a positive constant and $H(t)$ is designed based on a sliding control law (Zhao, 2009). From Equations (5) and (6), the error system is derived as:

$$\dot{e} = -\lambda e + H(t) + N(x, y) \quad (7)$$

where $N(x, y)$ represents the uncertain part of the dynamics and is given by:

$$N(x, y) = \Delta Q x + \Delta Q y + \Delta f(x) + \Delta f(y) . \quad (8)$$

We assume that the unknown nonlinear parts of $\Delta f(x)$, $\Delta f(y)$ is Lipschitz with coefficients $\theta$ respectively, that is, $|\Delta f(x)| \le \theta |x|$, $|\Delta f(y)| \le \theta |y|$. Therefore, one can assume $\Delta f(x) = Lx$, $\Delta f(y) = Ly$ where $\|L\| = \theta$.

With these assumptions, one can show that $N(x, y)$ is linearly bounded by the error signal e:

$$N(x, y) = (\Delta Q + L)e \quad (9)$$

The control input $H(t)$ is defined as:

$$H(t) = Kw(t), \quad (10)$$

where $K = [k_1, k_2, k_3]^T$ is a constant gain vector and $w(t) \in R$ is the control input that satisfies:

$$w(t) = \begin{cases} w^+(t) & s(e) \ge 0 \\ w^-(t) & s(e) < 0 \end{cases}, \quad (11)$$

in which $s = s(e)$ is a switching surface that prescribes the desired sliding dynamics. The error dynamics is then realized as:

$$\dot{e} = -\lambda e + Kw(t) + N(x, y). \quad (12)$$

The sliding surface can be chosen as follows:

$$s(e) = Ce, \quad (13)$$

where $C = [c_1, c_2, c_3]$ is a constant vector. In sliding mode, the controlled system satisfies the following conditions:

$$s(e) = 0, \quad (14a)$$

and

$$\dot{s}(e) = 0. \quad (14b)$$

To design the sliding mode controller, the constant together with the proportional rate reaching law are considered (Zhao, 2009), that is:

$$\dot{s} = -q\,\mathrm{sgn}(s) - rs, \quad (15)$$

where $\mathrm{sgn}(\cdot)$ denotes the sign function. The gains $q > 0$ and $r > 0$ are determined such that the sliding conditions are satisfied and the sliding mode motion occurs.

From Equations (12) and (13), it can be found that:

$$\dot{s} = C[-\lambda e + Kw(t) + N(x, y)]. \quad (16)$$

Therefore, according to Equations (15) and (16), we have

$$w(t) = -(CK)^{-1}[(r - \lambda)Ce + q\,\mathrm{sgn}(s) + CN(x, y)]. \quad (17)$$

For simplicity, let $\lambda = r$. In the practical engineering applications, $N(x, y)$ is the uncertain part of the dynamics and the implemented control input is described by

$$w(t) = -(CK)^{-1} q\,\mathrm{sgn}(s). \quad (18)$$

From Equations (12) and (18), we can get the following error dynamics:

$$\dot{e} = -(rI - \Delta Q - L)e - K(CK)^{-1} q\,\mathrm{sgn}(s). \quad (19)$$

**Theorem 1**

The drive system (2) and the response system (3) with bounded uncertainties can approach anti-synchronization asymptotically with the controllers (6), if $r$ satisfies $r \ge \eta + \theta$.

**Proof**

Consider the following Lyapunov function:

$$V = \frac{1}{2}s^2. \quad (20)$$

The time derivative of $V$ is

$$\begin{aligned}\dot{V} &= s\dot{s} = sC\dot{e} = sC[-(rI - \Delta Q - L)e - K(CK)^{-1} q\,\mathrm{sgn}(s)] \\ &\le -sq\,\mathrm{sgn}(s) - (r - \|\Delta Q + L\|)s^2 \\ &\le -sq\,\mathrm{sgn}(s) - (r - \eta - \theta)s^2 \end{aligned} \quad (21)$$

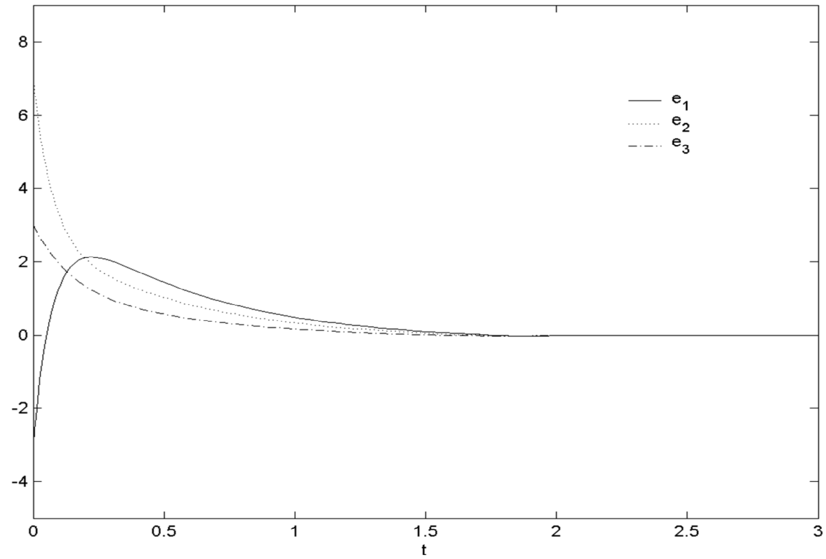


**Figure 2.** Anti-synchronization errors between systems (2) and (3).

Since the expression $-sq\,\mathrm{sgn}(s)$ is always negative when $e \neq 0$, the inequality of $\dot{V} = s\dot{s} < 0$ holds if the following condition is satisfied:

$$r \geq \eta + \theta. \tag{22}$$

Thus, for any value of $r$ that satisfies $r \geq \eta + \theta$, $\dot{V}$ will be negatively definite for all values of $s$ which implies asymptotic stability of the switching surface. In other words, the error system described by Equation (19) is asymptotically stable. Therefore, the response system (3) can anti-synchronize the drive system (2) asymptotically, which completes the proof.

## NUMERICAL SIMULATION

To verify and demonstrate the effectiveness of the proposed method, we discuss the anti-synchronization of Rikitake systems. In the numerical simulations, the fourth-order, which is the Runge–Kutta method, is used to solve the Rikitake systems with a time step size of 0.001.

In the following, an investigation was done on the active sliding mode control approach used for the uncertain Rikitake system through the numerical experiments:

$$\dot{x} = (Q + \Delta Q)x + f(x) + \Delta f(x), \tag{23}$$

where

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad Q = \begin{pmatrix} -b & 0 & 0 \\ -a & -b & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad f(x) = \begin{pmatrix} x_2 x_3 \\ x_1 x_3 \\ 1 - x_1 x_2 \end{pmatrix}. \tag{24}$$

Without loss of generality, the uncertainties of the Rikitake system (23) are assumed as follows:

$$\Delta Q = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 3 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}. \tag{25}$$

By simple calculation, $\eta + \theta = 8.71$ was obtained. In accordance with Theorem 1, $r = 9$ was selected and it satisfied the stability condition, that is, $r \geq \eta + \theta$.

According to the foregoing design method, the gain vector $K = [1, 2, 2]^T$, the control parameter $q = 0.35$ and the sliding surface $s(e) = Ce = [-1.5, 5, -3]e$ were chosen. As such, the sliding mode control input was:

$$w(t) = -0.14\,\mathrm{sgn}(-1.5e_1 + 5e_2 - 3e_3). \tag{26}$$

In this study, the initial conditions of $x_1(0) = 3$, $x_2(0) = 4$, $x_3(0) = -5$ and $y_1(0) = -6$, $y_2(0) = 3$, $y_3(0) = 8$ were utilized. Hence, the error system had the initial values of $e_1(0) = -3$, $e_2(0) = 7$ and $e_3(0) = 3$.

The simulation results are shown in Figure 2. From Figure 2, we can see that the error vector $e$ converges to zero as $t \to \infty$. This shows that the response system (3) can anti-synchronize the drive system (2) asymptotically.

## Conclusions

This paper presents a method that is used to design an active sliding mode controller for the anti-synchronization of Rikitake systems with system uncertainties. Based on the Lyapunov stability theory and Lipschitz condition, the active sliding mode controller and the selection scope of the controller parameters for anti-synchronization are designed. According to the simulations, the proposed method can be successfully applied to anti-synchronization problems of uncertain Rikitake systems. Moreover, the proposed control method can be easily extended to control and synchronize other chaos systems and is suited for systems with uncertainties and disturbances.

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