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Full Length Research Paper

Entropy production minimization in steady state heat conduction

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The rate of entropy generation during the steady conduction through a solid slab is investigated analytically. The effects of thermal conductivity and internal heat generation/sink on the total rate of entropy generation are discussed. Cases of uniform, spatial variable and temperature dependent thermal conductivity are considered. In addition, the case of introducing internal heat source/sink is discussed. It is concluded that the total rate of entropy generation per heat transfer rate in steady heat conduction through solid material is a function of only the rate of heat transfer, regardless of the distribution and or dependence of the thermal conductivity. The total rate of entropy generation during conduction in solid material can be eliminated by introducing internal heat generation/sink. The total rate of entropy generation during the transient heat conduction decreases with time and approaches its minimum value as the steady conduction is reached.

Key words: Entropy generation, entropy generation minimization, heat conduction.

INTRODUCTION

Method of the entropy generation minimization has been an important tool for optimization thermal systems to improve the performance of such systems (Bejan, 1979, 1996). Accordingly, the possibility of minimizing entropy generation in thermal conduction systems has been the subject of some recent investigations. Kolenda et al. (2004) studied entropy generation in steady-state heat conduction process. They stated that minimization of entropy generation in heat conduction process is always possible by introducing additional heat sources. However, when additional heat sources are added the entropy generation calculation should also take into account the additional term for the entropy generation due to the heat sources added. Ibanez et al. (2003) analyzed the minimization of the entropy generation of a solid slab with steady-state internal heating where the solid slab external surfaces are exposed to convective ambient with different Biot numbers (*Bi*). They found that by controlling the *Bi* in one of the surfaces, an optimum *Bi* for the second surface that minimizes the global entropy generation rate can be found. On the other hand, Bertola and Cafaro (2008) discussed the principle of minimum entropy production theorem and its application to heat and fluid flow. They showed that the minimum entropy production of system in a stationary state cannot be different from zero.

Entropy generation in transient heat conduction has also been studied. A second law analysis for hyperbolic heat conduction in a slab is carried out by Barletta and Zanchini (1997). Bautista et al. (2005a) studied entropy generation during transient heat conduction in a solid slab. They found that the spatial average entropy generation rate per unit volume is always a decreasing function with time. In another paper, Bautista et al. (2005b) studied the unsteady entropy generation rate due to an instantaneous internal heat generation in a solid slab. They showed that the entropy generation rate presents a very sensible dependence on heat conduction, heat convection and internal heat generation. Strub et al. (2005) investigated the periodic heat conduction through a wall using second law analysis. The two approaches that they developed lead to different, sometime opposite results; therefore, their study raises the question: which fundamental relations can exist between irreversibility and the

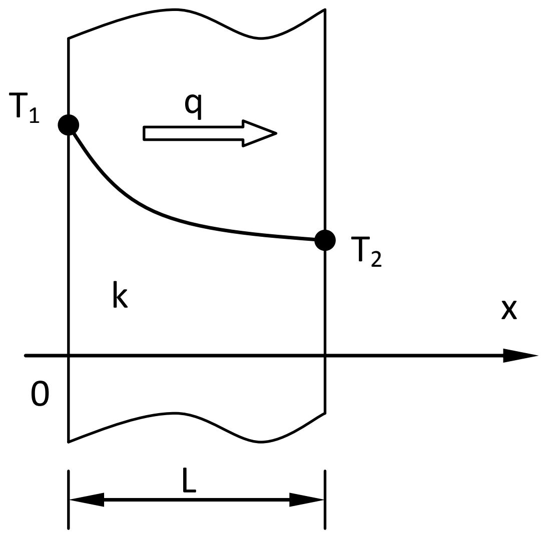

Figure 1. Heat conduction in a solid wall with fixed surface temperatures.

between irreversibility and the consumed energy?

Bisio (1990) studied the rate of entropy production in one-dimensional heat conduction considering temperature dependent thermal conductivity. Nuwayhid et al. (2000) studied the entropy generation in the thermoelectric generator. They found that the entropy generation minimization method is less straight forward than the power maximization technique requiring careful accounting of the different sources of irreversibility. Nuwayhid and Moukalled (2003) studied the entropy generation rate and power production in a power producing system taken as a temperature gap between two thermal reservoirs such as thermoelectric generator. They showed that there exists a possibility to enhance thermoelectric performance by tailoring the temperature profile. However they made no attempt to suggest practical means of achieving their results.

Ghodoossi (2004) performed a second law analysis for the optimal uniform heat generating areas with different complexity levels of the tree network of heat conducting paths. He showed that the heat flow performance does not improve if the internal complexity of the heat generating area increases. Khaled (2008) studied analytically the effect of both the periodic heat flux and convective boundary condition on heat conduction and entropy transfer through semi-infinite and finite media. He found that the amplitude of the steady periodic noise in heat and entropy transfer decreases as *Bi* increases. He also found that the rate of entropy transfer to both media reaches maximum values at critical times lower than the time needed for both the applied heat flux. Recently Esfahani and Koohi-Fayegh (2009) studied entropy generation in one-dimensional semi-infinite conduction with constant surface temperature boundary condition. From the above cited literature, it is clear that there is no conclusive explanation for the existence of the minimum entropy generation in heat conduction systems. In the present work, the question for the possibility of minimizing the entropy generation rate for steady-state, heat conduction process is considered. The effect of the thermal conductivity variation as well as the internal heat generation on the entropy generation rate is studied.

ANALYSES

Consider the slab of thickness L with fixed surface temperatures, T_1 and T_2 respectively, as shown in Figure 1. The local entropy generation rate in the slab as a result of heat conduction depends on the temperature distribution in the slab. On the other hand, the temperature distribution in the slab depends on the thermal conductivity variation and the volumetric heat generation, if exists. In the absence of volumetric heat generation and for uniform thermal conductivity the temperature distribution in the slab is linear and therefore the entropy generation is positive non zero and depends on the constant temperature gradient. This entropy generation that corresponds to the linear temperature variation may not be necessarily the minimum. Search of the possible existence of another temperature variation which minimizes the entropy generation rate is the subject of the current subject of the current investigation. In the following, the analysis is carried out case by case and discussion is given.

Case 1: Uniform thermal conductivity and no internal heat generation

The differential equation governing the one-dimensional heat conduction with no internal heat generation is given by;

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0 \tag{1}$$

As a first step let us assume the thermal conductivity is constant and uniform. For constant thermal conductivity Equation (1) is reduced to;

$$\frac{d^2T}{dx^2}=0 \tag{2}$$

The local entropy generation in the slab is given by;

$$s'''=\frac{k}{T^2}\left(\frac{dT}{dx}\right)^2 \tag{3}$$

The total entropy generation in the slab is obtained by integrating Equation (3) along the thickness of the slab.

$$\sigma=\int_0^L s'''dx=\int_0^L \frac{k}{T^2}\left(\frac{dT}{dx}\right)^2 dx \tag{4}$$

The optimum temperature variation that minimizes the total entropy generation satisfies the Euler equation.

$$\frac{\partial F}{\partial T}-\frac{d}{dx}\left(\frac{\partial F}{\partial T_x}\right)=0 \tag{5}$$

where $F=\frac{k}{T^2}\left(\frac{dT}{dx}\right)^2$ and T_x denotes $\frac{dT}{dx}$. Carrying out the algebra in Equation (5) the required temperature distribution is found to satisfy the differential equation.

$$\frac{d^2T}{dx^2}-\frac{1}{T}\left(\frac{dT}{dx}\right)^2=0 \tag{6}$$

However, it is clear that the solution for the optimum temperature variation obtained from Equation (6) does not satisfy the heat conduction equation, Equation (2). In order to satisfy the heat conduction equation the first term in Equation (6) should vanish and thus the solution for the optimum temperature variation is obtained from the second term in Equation (6). Thus, the optimum temperature variation that satisfies both the heat conduction and the Euler equation becomes;

$$T=\text{const.} \tag{7}$$

which is a trivial solution which can occur only when $T_1=T_2$. Clearly both the rate of heat transfer and the entropy generation rate for this temperature distribution are zero.

Case 2: Uniform thermal conductivity and internal heat generation/sink

For a more meaningful solution for non-equal surface temperatures, Equation (6) may be compared with thermal conduction equation with internal heat generation.

$$\frac{d^2T}{dx^2}+\frac{q(x)}{k}=0 \tag{8}$$

where the internal heat generation (sink) is in the form of;

$$q(x)=-\frac{k}{T}\left(\frac{dT}{dx}\right)^2 \tag{9}$$

In this case the entropy generation rate becomes;

$$s'''=\frac{k}{T^2}\left(\frac{dT}{dx}\right)^2+\frac{q(x)}{T} \tag{10}$$

or

$$s'''=\frac{k}{T^2}\left(\frac{dT}{dx}\right)^2-\frac{k}{T^2}\left(\frac{dT}{dx}\right)^2=0 \tag{11}$$

that means, it is possible to eliminate the entropy generation rate by introducing a heat sink given by equation (9). The temperature variation in this case is the solution of equation (6) that satisfies the boundary conditions and can be shown as;

$$T(x)=T_1\left(\frac{T_2}{T_1}\right)^{x/L} \tag{12}$$

The temperature variation given in Equation (12) satisfies the heat conduction equation and yields zero entropy generation rate. The heat sink required to maintain this temperature variation is obtained by substituting Equation (12) into Equation (9) and is

$$q(x) = -\frac{kT_1}{L^2}\left[\ln\left(\frac{T_2}{T_1}\right)\right]^2 T_1\left(\frac{T_2}{T_1}\right)^{x/L} \tag{13}$$

The total heat sink is obtained by integration Equation (13) along the thickness of the slab as

$$Q_s = \int_0^L q(x)dx = -\frac{kT_1}{L}\ln\left(\frac{T_2}{T_1}\right)\left(\frac{T_2 - T_1}{T_1}\right) \tag{14}$$

On the other hand the net heat input to the slab through both the surfaces of the slab is,

$$Q_i = \left(-k\frac{dT}{dx}\right)\bigg|_{x=0} - \left(-k\frac{dT}{dx}\right)\bigg|_{x=L} = \frac{kT_1}{L}\ln\left(\frac{T_2}{T_1}\right)\left(\frac{T_2 - T_1}{T_1}\right) \tag{15}$$

So, from Equations (14) and (15), $Q_s = -Q_i$ and the conservation of energy is satisfied.

In conclusion, entropy generation minimization (elimination) is possible only when an internal heat sink variation in the form given by Equation (9) is introduced. However, the practical application of this internal heat sink is questionable.

Case 3: Temperature dependent thermal conductivity, no internal heat generation

Thermal conductivity is a temperature dependent property for most materials. Thus, we consider a possible temperature relation of the thermal conductivity k(T) for optimum entropy generation in this case. The thermal conduction equation is

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] = 0 \tag{16}$$

or

$$\frac{dk}{dT}\left(\frac{dT}{dx}\right)^2 + k\frac{d^2T}{dx^2} = 0 \tag{17}$$

The local volumetric entropy generation rate is

$$s''' = \frac{k(T)}{T^2}\left(\frac{dT}{dx}\right)^2 \tag{18}$$

The total entropy generation in the slab is obtained by integrating Equation (18) along the thickness of the slab

$$\sigma = \int_0^L \frac{k(T)}{T^2}\left(\frac{dT}{dx}\right)^2 dx \tag{19}$$

For optimum temperature distribution in the slab the Euler equation must be satisfied

$$\frac{\partial F}{\partial T} - \frac{d}{dx}\left(\frac{\partial F}{\partial T_x}\right) = 0 \tag{20}$$

where $F = \frac{k(T)}{T^2}\left(\frac{dT}{dx}\right)^2$. Performing the algebraic procedures this yields

$$\frac{dk}{dT}\left(\frac{dT}{dx}\right)^2 + k\frac{d^2T}{dx^2} - \frac{1}{2}\frac{dk}{dT}\left(\frac{dT}{dx}\right)^2 - \frac{k}{T}\left(\frac{dT}{dx}\right)^2 = 0 \tag{21}$$

The first two terms must vanish in order to satisfy the heat conduction equation, equa£tion (17), thus, the necessary condition for the optimum solution is obtained to be

$$-\frac{1}{2}\frac{dk}{dT}\left(\frac{dT}{dx}\right)^2 - \frac{k}{T}\left(\frac{dT}{dx}\right)^2 = 0 \tag{22}$$

or

$$\frac{1}{2}\frac{dk}{dT} + \frac{k}{T} = 0 \tag{23}$$

The solution of Equation (23) suggests that the thermal conductivity dependence to the temperature should be in the form;

$$k(T) = \frac{C}{T^2} \tag{24}$$

where C is a constant. The constant C can be evaluated if the thermal conductivity of the material at ambient temperature $T=T_o$ is known as $k=k_o$, that is, $C=k_oT_o^2$. When the thermal conductivity is in the form given by Equation (24) the temperature distribution can be determined by solving the heat conduction equation, Equation (16). The solution of Equation (16) that satisfies the boundary conditions becomes;

$$T(x) = \frac{T_1 T_2 L}{(T_1 - T_2)x + T_2 L} \tag{25}$$

The entropy generation rate can be evaluated by substituting Equations (24) and (25) into Equation (18). It can be shown that the entropy generation rate in this case is obtained to be uniform and is

$$s''' = k_0 T_0^2 \frac{1}{L^2} \left(\frac{T_1 - T_2}{T_1 T_2} \right)^2 = \text{const.} \tag{26}$$

The local heat transfer rate, on the other hand, is evaluated to be;

$$q = -k(T)\frac{dT}{dx} = \frac{k_0 T_0^2}{L} \frac{T_1 - T_2}{T_1 T_2} \tag{27}$$

Case 4: Variable thermal conductivity, no internal heat generation

The solution for the optimum thermal conductivity in the previous case can be combined with the temperature profile to obtain the variation of the thermal conductivity along the thickness of the slab. Thus substituting Equation (25) into Equation (24) the variation of the thermal conductivity k(x) is obtained as

$$k(x) = k_0 T_0^2 \left(\frac{T_1 - T_2}{T_1 T_2} \frac{x}{L} + \frac{1}{T_1} \right)^2 \tag{28}$$

Equation (28) indicates a quadratic variation of thermal conductivity along the thickness of the slab that yields a steady-state temperature distribution in the form given in Equation (25). The resultant entropy generation rate and heat transfer rate are given in Equations (26) and (27), respectively.

In order to find out the advantage of arranging the thermal conductivity in the form given in Equation (28) for the reduction on the rate of entropy generation in the slab, let us compare the rates of entropy generation for both the uniform and variable thermal conductivity models for the same amount of heat transfer rate.

When the thermal conductivity is uniform the temperature distribution in the slab is linear and the rate of heat transfer is given by;

$$q = k \frac{T_1 - T_2}{L} \tag{29}$$

Comparing Equation (29) with Equation (27), equal rate of heat transfer requirement is satisfied for:

$$k_0 T_0^2 = k T_1 T_2 \tag{30}$$

On the other hand, substituting Equation (30) into Equation (26) the rate of local entropy generation is found to be

$$s''' = k \frac{1}{L^2} \frac{(T_1 - T_2)^2}{T_1 T_2} \tag{31}$$

Integrating Equation (31) along the thickness of the slab, the total entropy generation in the slab is found to be

$$\sigma = \int_0^L s''' dx = \frac{k}{L} \frac{(T_1 - T_2)^2}{T_1 T_2} \tag{32}$$

which is exactly equal to the total entropy generation for the case of uniform thermal conductivity distribution. Thus, the total entropy generation per unit heat transfer in steady state conduction process is independent from the temperature distribution and/or the variation of thermal conductivity in the solid material. In fact, regardless of the volumetric variation within the solid material, the total entropy generation rate can be calculated by considering the entropy balance on the thermodynamic system that includes the solid slab shown in Figure 1.

$$\sigma = -\frac{q}{T_1} + \frac{q}{T_2} = \frac{k}{L} \frac{(T_1 - T_2)^2}{T_1 T_2} \tag{33}$$

The total rate of entropy generation during the transient heat conduction process could be much higher than that given in Equation (33). As the heat conduction approaches the steady-state condition the rate of entropy generation decreases and approaches its minimum value given by Equation (33) asymptotically. Therefore, the total rate of entropy generation in steady heat conduction process is always the minimum when compared with that in the transient part of the heat conduction process. This depends on the total rate of heat transfer through the solid material for fixed surface temperatures with no internal heat generation and is independent from the variation of the thermal conductivity of the solid material and the resultant temperature variation in it.

Conclusions

Rate of entropy generation during steady one-dimensional conduction heat transfer through solid slab material has been considered. Possible ways of minimizing the entropy generation rate have been explored case by case. The following conclusions can be derived from the present work:

1. In steady conduction heat transfer through solid material minimization of entropy generation rate is only possible when the temperature distribution is uniform. However, in this case the heat transfer is zero.
2. Minimization of the total rate of entropy generation in conduction is possible by introducing a distributed local rate of heat sink given by Equation (13). However, the practical application of this heat sink may be difficult in general.
3. The other possibility of minimizing the total rate of entropy generation is through the use of temperature dependent thermal conductivity given by Equation (24). However, the practical application of this case is also limited.
4. On the other hand, arranging the spatial variation of the thermal conductivity as given in Equation (28) the total rate of entropy generation can be minimized to that for the case of temperature dependent thermal conductivity.
5. However, considering the total rate of entropy generation per unit rate of heat transfer through the solid slab, it is concluded that neither the dependence of the thermal conductivity on the temperature nor the spatial variation of it do not have any influence on the total rate of entropy generation.
6. The total rate of entropy generation per unit rate of heat transfer in conducting heat is only related to the heat transfer rate through the solid material, regardless of the temperature and or thermal conductivity distribution in the solid material. Therefore, the total rate of entropy generation during transient conduction process always decreases and approaches its minimum as the process approaches the steady-state.

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NOMENCTLATURE

k: Thermal conductivity, W/m K
L: Thickness of slab, m
q: Internal heat generation rate, W/m^3
s’’’: Local entropy generation rate, W/m^{3}K
T: Temperature, K
x: Coordinate axis, m
σ: Total entropy generation rate, W/K
i: Input
o: Reference value
s: Sink

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