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Suggested formulas for half-life time of unstable nuclei: Effect of the isospin asymmetry

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Using the available data of half-life time of unstable nuclei given that the number of protons is greater than the number of neutrons, an elaboration was made on the dependence of half-life time of these nuclei on the value of the difference between the two numbers, which is referred to as the isospin asymmetry effect. The available data of half-life time was analyzed and two formulas of half-life time were suggested. The analysis of the data and the suggested formulas indicated that the half-life time of these nuclei and their decay mode are strongly dependent on the isospin asymmetry.

Key words: Half-life time formula, isospin asymmetry, nuclei with $Z > N$.

INTRODUCTION

The asymmetry effect is a quantum effect arising from the Pauli Exclusion Principle which only allows two protons or two neutrons (with opposite spin direction) in each energy state. Due to this effect, the binding energy of asymmetric nuclei decreases and is dependent on the asymmetry parameter $I = (N - Z)/A$, where Z is the atomic number, *N* is the number of neutrons and *A* is the mass number of nucleus. The *A* in the denominator reflects the fact that a given difference $(N - Z)$ is less significant for larger values of *A*. The physical properties of nuclei, such as their masses, neutron and proton density distributions and their mean radii depend on the asymmetry parameter I (Berdichevsky, 1984; Steiner et al., 2005). The energetics associated with the neutron-proton asymmetry can be characterized by the so-called symmetry energy.

The study of the nuclear matter symmetry energy that essentially characterizes the isospin-dependent part of the equation of state of asymmetric nuclear matter is currently an exciting topic of research in nuclear physics, for example (Centelles et al., 2009; Shetty and Yennello, 2010). Solution of some problems of nuclear collisions such as the isoscaling effect, the nuclear multi-fragmentation accompanied by the emission of asymmetric clusters, the isospin instability of nuclei at high temperatures and others, depend on our knowledge of the nuclear equation of state for the isospin symmetry energy, namely, its dependence on the particle density and the asymmetry parameter *I*, for example (Shetty et al., 2007; Tsang et al., 2009). Thus, we can say that the asymmetry parameter plays an important role in the nuclear structure and nuclear collision. Therefore, efforts have been made in this work, to discuss the dependence of half-life time $T_{1/2}$ of the radioactive nuclei on the asymmetry factor. Only the radioactive nuclei with $Z > N$ was consider and we will use the quantity $Z_{ex} = Z - N$ to represent the isospin asymmetry of the nucleus.

The half-life time is a fundamental property of

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radioactive nuclei, carrying important information on their intrinsic structure. It answers the question of the possibility of the observation of the nucleus. In the experimental and theoretical studies of nuclear decay and half-life time of nuclei, different models and semi-empirical formulas were used and suggested to explain the experimental data of half-life time of different types of decay (Barabash, 2010; Rath et al., 2010; Greiner, 2007; Dong et al., 2010; Santhosh et al., 2012).

Nearly, in all the previous works, the authors were concerned with nuclei where $N \geq Z$. The light and medium radioactive nuclei with $Z > N$ did not take a similar interest. Also, the asymmetry effect on the half-life time of radioactive nuclei has not been clearly represented in previous literatures. Therefore, in this work, the isospin asymmetry effect on the half-life time and this class of nuclei where $Z > N$, was studied. Only, cases where the asymmetry number $Z_{ex} = Z - N = 1, 2, 3, 4 \text{ and } 5$, were also considered. For $Z_{ex} \geq 6$ the radioactive nuclei, almost, does not exist in the data. Only with $Z_{ex} = 6$ we have two cases, $^{30}Ar_{18}$ and $^{46}Fe_{26}$ with the decay mode and half life time (p ; 20 ns) and (ε, εp ; 20 ms), respectively (Firestone and Ekström, 2004).

The asymmetry effect on the half-life time of considered nuclei was studied in two steps. The first is the classification of these nuclei into different sets with respect to the value of Z_{ex} (Hassan, 2009), so, we will discuss the dependence of the value of $T_{1/2}$ on the asymmetry number Z_{ex}. The second step is the suggestion of some formulas to describe the data of $T_{1/2}$ in each set and discuss the dependence of the parameterization of the suggested formulas on Z_{ex}. The data is taken from Firestone and Ekström (2004). In fact, we have found two formulas that described the data of Firestone and Ekström (2004). The first for $Z_{ex} = 1$ and ($Z_{ex} = 2 \text{ with even - } Z$) and the second for $Z_{ex} = (2 \text{ with odd - } Z), 3, 4 \text{ and } 5$. The suggested formulas depend on three parameters. At a certain value of Z_{ex}, these formulas give, in general, a good fit with the data of Firestone and Ekström (2004) with the same values of the parameters at each Z. For different values of Z_{ex} we have different values of the parameters or different parameterization. This reflects the dependence of the half-life time on the isospin asymmetry of nuclear matter. Due to the large fluctuations of the half-life time at some values of Z for some values of Z_{ex}, the χ^2-method is used to gives some indications about the initial values of the parameters of formulas.

Half-life time dependence on the isospin asymmetry: Data analysis

All nuclei with $Z > N$ are in the region of light or medium nuclei. Also, all these nuclei except the two nuclei $^1H_1$ and $^3He_2$ are unstable (Firestone and Ekström, 2004). The asymmetry number Z_{ex} classifies these nuclei into different sets. Each set of them is characterized by a certain value of Z_{ex}. The set with the asymmetry number $Z_{ex} = 2$ is divided into two different sets with respect to even- or odd-Z. This is because the behavior of the data in these two sets is clearly different. Thus, the total number of the sets for all considered nuclei is 6 sets. These sets of nuclei are presented with their half-life time values in the Tables 1 to 6 (Firestone and Ekström, 2004). From these tables we can see the strong dependence of half-life time on the asymmetry number Z_{ex}. In general, for two different sets, for a given value of Z, we can see that $T_{1/2}(Z_{ex} = i) \geq T_{1/2}(Z_{ex} = j)$, where $i \leq j$. Also, we must note that the order of values of $T_{1/2}$ for odd-Z nuclei, in general, is less than the order of $T_{1/2}$ for even-Z nuclei in the case of $Z_{ex} = 2$. For any table with certain value of the asymmetry number Z_{ex}, the half-life time $T_{1/2}$, can be considered as a function of the atomic number Z. Thus, the half-life time $T_{1/2}$ can be suggested as an explicit function of Z and its parameterization is dependent on Z_{ex}. More so, it must be noted that the type of decay in each set is related, in some way, to the asymmetry number Z_{ex} of the set. In fact, the majority of the decay mode in the first two sets where Z_{ex}=1 and (2 with even-Z) is $\varepsilon + \beta^+$ with 75% and 87.5%, respectively, (Tables 1 and 2). In the third set with $Z_{ex} = 2$ and odd-Z the situation is absolutely different, Table 3. This may seem related to the parity of Z. However, the clear difference between the possible decay modes in this set and the other sets with different asymmetry number Z_{ex}, gives some indication on the relation between the general feature of the decay modes in certain set and its asymmetry number. For $Z_{ex} = 3 \text{ and } 4$ the major decay mode is {$\varepsilon + \beta^+$, εp} with 70.4 and 52.94%, respectively, (Tables 4 and 5). In the last set with $Z_{ex} = 5$, we have 6 nuclei with 6 different decay modes (Table 6). Thus, we can conclude that, not only the half-life time is sensitive to the isospin asymmetry of nuclei, but, also the type of decay it is related to.

Table 1. The half-life time data for nuclei with $Z_{ex} = 1$ (Firestone and Ekström, 2004) 18 odd-even and 18 even-odd radioactive nuclei. The data with (*) are predicted using the formula (1).

Nucleus	$T_{1/2}$ and decay mode	Nucleus	$T_{1/2}$ and decay mode	Nucleus	$T_{1/2}$ and decay mode	Nucleus	$T_{1/2}$ and decay mode
$^{1}H_{1}$	Stable	$^{21}Na_{11}$	22.49 s±4 $\varepsilon + \beta^{+}$	$^{41}Sc_{21}$	596.3 ms±17 $\varepsilon + \beta^{+}$	$^{59}Zn_{30}$	182 ms±18 $\varepsilon + \beta^{+}$, εp
$^{3}He_{2}$	Stable	$^{23}Mg_{12}$	11.317 s±11 $\varepsilon + \beta^{+}$	$^{43}Ti_{22}$	509 ms±5 $\varepsilon + \beta^{+}$	$^{61}Ga_{31}$	0.15 s±3 $\varepsilon + \beta^{+}$
$^{5}Li_{3}$	56.1953 y* p	$^{25}Al_{13}$	7.183 s±12 $\varepsilon + \beta^{+}$	$^{45}V_{23}$	547 ms±6 $\varepsilon + \beta^{+}$	$^{63}Ge_{32}$	95 ms±23 $\varepsilon + \beta^{+}$
$^{7}Be_{4}$	53.12 d±7 ε	$^{27}Si_{14}$	4.16 s±2 $\varepsilon + \beta^{+}$	$^{47}Cr_{24}$	500 ms±15 $\varepsilon + \beta^{+}$	$^{65}As_{33}$	$0.19\,s^{+11}_{-7}$ $\varepsilon + \beta^{+}$
$^{9}B_{5}$	19.5727 h* $2\alpha p$	$^{29}P_{15}$	4.140 s±14 $\varepsilon + \beta^{+}$	$^{49}Mn_{25}$	382 ms±7 $\varepsilon + \beta^{+}$	$^{67}Se_{34}$	$60\,ms^{+17}_{-11}$ $\varepsilon + \beta^{+}$, εp
$^{11}C_{6}$	20.39 m±2 $\varepsilon + \beta^{+}$	$^{31}S_{16}$	2.572 s±13 $\varepsilon + \beta^{+}$	$^{51}Fe_{26}$	305 ms±5 $\varepsilon + \beta^{+}$	$^{71}Kr_{36}$	$64\,ms^{+8}_{-5}$ $\varepsilon + \beta^{+}$, εp
$^{13}N_{7}$	9.965 m±4 $\varepsilon + \beta^{+}$	$^{33}Cl_{17}$	2.511 s±3 $\varepsilon + \beta^{+}$	$^{53}Co_{27}$	240 ms±20 $\varepsilon + \beta^{+}$	$^{75}Sr_{38}$	$71\,ms^{+71}_{-24}$ $\varepsilon + \beta^{+}$, εp

Table 1. Contd.

$^{15}O_8$	122.24 s±16 $\varepsilon+\beta^+$	$^{35}Ar_{18}$	1.775 s±4 $\varepsilon+\beta^+$	$^{53m}Co_{27}$	247 ms±12 $\varepsilon+\beta^+$, p	$^{77}Y_{39}$	69 ms* $\varepsilon+\beta^+$, εp
$^{17}F_9$	64.49 s±16 $\varepsilon+\beta^+$	$^{37}K_{19}$	1.226 s±7 $\varepsilon+\beta^+$	$^{55}Ni_{28}$	212.1 ms±38 $\varepsilon+\beta^+$		
$^{19}Ne_{10}$	17.22 s±2 $\varepsilon+\beta^+$	$^{39}Ca_{20}$	859.6 ms±14 $\varepsilon+\beta^+$	$^{57}Cu_{29}$	199.4 ms±32 $\varepsilon+\beta^+$		

Table 2. The half-life time data for nuclei with $Z_{ex}=2$ and even- Z (Firestone and Ekström, 2004) 16 even-even nuclei. The data with (*) are predicted using the formula (1).

Nucleus	$T_{1/2}$ and decay mode	Nucleus	$T_{1/2}$ and decay mode
$^{6}Be_4$	5.4816 d*, 2pα	$^{38}Ca_{20}$	0.440 s±0.008, ε+β$^+$
$^{10}C_6$	19.255 s±53, ε+β$^+$	$^{42}Ti_{22}$	0.199 s±0.006, ε+β$^+$
$^{14}O_8$	70.606 s±18, ε+β$^+$	$^{46}Cr_{24}$	0.26 s±6, ε+β$^+$
$^{18}Ne_{10}$	1.672 s±0.008, ε+β$^+$	$^{50}Fe_{26}$	0.150 s±0.030, ε+β$^+$, β$^+$p
$^{22}Mg_{12}$	3.857 s±9, ε+β$^+$	$^{54}Ni_{28}$	0.10126 s* , ε+β$^+$
$^{26}Si_{14}$	2.234 s±13, ε+β$^+$	$^{58}Zn_{30}$	0.065 s±0.009 , ε+β$^+$
$^{30}S_{16}$	1.178 s±5, ε+β$^+$	$^{62}Ge_{32}$	0.058247 s* , ε+β$^+$
$^{34}Ar_{18}$	0.8445 s±0.034, ε+β$^+$	$^{66}Se_{34}$	0.044066 s* , ε+β$^+$

Table 3. The half-life time data for nuclei with $Z_{ex}=2$ and odd-Z (Firestone and Ekström, 2004) 14 odd-odd nuclei. The data with (*) are predicted using the formula (3).

Nucleus	$T_{1/2}$ s and decay mode	Nucleus	$T_{1/2}$ s and decay mode
$^{8}B_{5}$	0.77 s±0.003, ε+β$^+$, ε2α	$^{32}Cl_{17}$	0.298 s±0.001, ε+β$^+$, εα, εp
$^{12}N_{7}$	0.011 s±0.016, ε+β$^+$, ε3α	$^{36}K_{19}$	0.342 s± 0.002, ε+β$^+$, εp, εαα
$^{16}F_{9}$	0.59005 s* , p	$^{40}Sc_{21}$	0.1823 s±0.007, ε+β$^+$, εα, εp
$^{20}Na_{11}$	0.4479 s±0.023, ε+β$^+$, εα	$^{44}V_{23}$	0.090 s±0.025, ε+β$^+$, εα
$^{24}Al_{13}$	2.053 s±4, ε+β$^+$, εα	$^{44m}V_{23}$	~0.150 s, ε+β$^+$
$^{24m}Al_{13}$	0.1313 s±0.025, ε+β$^+$, εα	$^{48}Mn_{25}$	0.1581 s±0.022, ε+β$^+$, εp, εα
$^{28}P_{15}$	0.2703 s±0.005, ε+β$^+$, εp, εα	$^{52}Co_{27}$	0.018 s±0.013, ε+β$^+$

The suggested formulas

In this section we try to suggest some functions of Z to describe the half-life time of considered nuclei. Also, we will discuss the dependence of this function and it's parameterizations on the asymmetry number Z_{ex}.

The first suggested formula

The data of nuclei where $Z_{ex}=1$, which are given in Table 1, and the data of Table 2, where $Z_{ex}=2$ and even-Z are presented in Figures 1a and 2a, respectively. These figures are of semi-log scale. The behavior of the data, in the two cases, in this semi-log scale, is similar to the behavior of the function $\frac{1}{Z}$. Therefore, we suggest the following formula for half-life time of nuclei in these two sets

$$T_{1/2}(Z)=A\exp\{\frac{B}{Z}\}+C, \quad (1)$$

Where *A, B* and *C* are fitting parameters. The good fit with the data are obtained with $A=0.018\text{s}$, $B=76$ and $C=-0.055\text{s}$ for $Z_{ex}=1$, (Figure 1a). Also, with $A=0.0135\text{s}$ $B=69.5$ and $C=-0.06\text{s}$ for $Z_{ex}=2$ and even-Z a good fit with the data is obtained (Figure 2a). The χ^2 values for the results in the two Figures 1a and 2a are given in Table 8. χ^2 is defined as $\chi^2=\sum_i[T_{1/2}^{\exp}(z_i)-T_{1/2}^{theo}(z_i)]^2$. Also, given in the same table, the sum of experimental errors square $\Delta^2=\sum_i[\Delta_i]^2$, where Δ_i is the experimental error at Z_i. Since the difference between the two cases is the value of Z_{ex}, the different values of the parameters in the two cases can be considered as a result of the asymmetry effect in nuclei, that is, we can say that the quantities *A, B* and *C* are dependent on Z_{ex} as a parameter.

The sensitivity of $T_{1/2}$ with respect to the values of the Parameters *A* and *B* is clear at all values of *Z*, Figures 1b, 2b, 1c and 2c, respectively. The parameter *C* plays a role at $20<Z\leq 39$ in the first set and at $20<Z\leq 34$ in the second set, Figures 1d and 2d, respectively. Thus, from the previous paragraph, this sensitivity reflects, in some way, sensitivity with respect to the asymmetry number Z_{ex}. From these figures we can introduce and accept the concept of the range of the parameters. We can consider that this concept is, in our case, the theoretical analog for the experimental error. Of course, we will consider the range of only one parameter. We choose this parameter with the help of the figures. In our two cases we can take the range of the parameter *A* (or *B*) to obtain a theoretical box that contains most of the data. For nuclei with $Z_{ex}=1$, we can take $A=0.01\rightarrow 0.028\text{s}$

Table 4. The half-life time data for nuclei with $Z_{ex} = 3$ (Firestone and Ekström, 2004) 16 even-odd nuclei and 11 odd-even nuclei. The data with (*) are predicted using the formula (3).

Nucleus	$T_{1/2}$ s and decay mode	Nucleus	$T_{1/2}$ s and decay mode
$^{7}B_{5}$	0.089233 s* Xp	$^{35}K_{19}$	0.19 s ±0.03, ε+β$^+$, εp
$^{9}C_{6}$	0.1265 s±0.009, ε+β$^+$, εp2α	$^{37}Ca_{20}$	0.1811 s±0.01, ε+β$^+$, εp
$^{11}N_{7}$	0.120578 s*, p	$^{41}Ti_{22}$	0.08 s±0.002, ε+β$^+$, εp
$^{13}O_{8}$	0.00858 s±0.005, ε+β$^+$, εp	$^{43}V_{23}$	>0.800 s , ε+β$^+$
$^{15}F_{9}$	0.153306 s*, p	$^{45}Cr_{24}$	0.050 s±0.006 , ε+β$^+$, εp
$^{17}Ne_{10}$	0.1092 s±0.006, ε+β$^+$, εp, εα	$^{47}Mn_{25}$	0.100 s±0.05 , ε+β$^+$, εp
$^{19}Na_{11}$	0.182461 s*, P	$^{49}Fe_{26}$	0.070 s±0.003, ε+β$^+$, εp
$^{21}Mg_{12}$	0.122 s±0.003, ε+β$^+$, εp	$^{53}Ni_{28}$	0.045 s±0.015, ε+β$^+$, εp
$^{23}Al_{13}$	0.47 s±3, ε+β$^+$, εp	$^{55}Cu_{29}$	0.042574s* , ε+β$^+$
$^{25}Si_{14}$	0.22 s±0.003, ε+β$^+$, εp	$^{57}Zn_{30}$	0.04 s±0.01, ε+β$^+$, εp
$^{27}P_{15}$	0.260 s±0.08, ε+β$^+$, εp	$^{61}Ge_{32}$	0.04 s±0.015, ε+β$^+$, εp
$^{29}S_{16}$	0.187 s±0.004, ε+β$^+$, εp	$^{65}Se_{34}$	0.017069 s*, ε+β$^+$, εp
$^{31}Cl_{17}$	0.15 s±0.025, ε+β$^+$, εp	$^{73}Sr_{38}$	0.011492 s*, ε+β$^+$, εp
$^{33}Ar_{18}$	0.173 s±0.02, ε+β$^+$, εp		

Table 5. The half-life time data for nuclei with $Z_{ex} = 4$ (Firestone and Ekström, 2004) 12 even-even nuclei and 5 odd-odd nuclei. The data with (*) are predicted using the formula (3).

Nucleus	$T_{1/2}$ s and decay mode	Nucleus	$T_{1/2}$ s and decay mode
$^{8}C_{6}$	0.047s*, Xp	$^{32}Ar_{18}$	0.098 s±0.002, ε+β$^+$, εp
$^{12}O_{8}$	0.063s*, 2p	$^{36}Ca_{20}$	0.102 s±0.002, ε+β$^+$, εp
$^{14}F_{9}$	0.071s*, p	$^{40}Ti_{22}$	0.05 s±0.015, ε+β$^+$

Table 5. Contd.

$^{16}Ne_{10}$	0.079s*, 2p	$^{44}Cr_{24}$	0.053 s±0.004, ε+β$^+$,εp
$^{20}Mg_{12}$	$0.095s^{+0.08}_{-0.05}$, ε+β$^+$, εp	$^{46}Mn_{25}$	0.041 s±0.007 ,ε, εp
$^{22}Al_{13}$	$0.07s^{+0.05}_{-0.035}$, ε+β$^+$, εp+ ε2p	$^{48}Fe_{26}$	0.044 s±0.007,ε+β$^+$, εp
$^{24}Si_{14}$	0.102 s±0.035, ε+β$^+$, εp	$^{50}Co_{27}$	0.044 s±0.004,ε+β$^+$, εp
$^{26}P_{15}$	$0.02s^{+0.035}_{-0.015}$, ε+β$^+$, εp+ε2p	$^{52}Ni_{28}$	0.038 s±0.005, ε+β$^+$, εp
$^{28}S_{16}$	0.125 s±0.010, ε+β$^+$, εp		

Table 6. The half-life time data for nuclei with $Z_{ex} = 5$ (Firestone and Ekström, 2004) 6 even-odd nuclei.

Nucleus	$T_{1/2}$ s and decay mode
$^{27}S_{16}$	0.021s±0.004, ε+β$^+$, ε2p, εp
$^{31}Ar_{18}$	0.0151 s±0.012, ε+β$^+$, εp, ε2p, ε3p
$^{35}Ca_{20}$	0.05 s±0.03, ε+β$^+$, ε2p
$^{39}Ti_{22}$	$0.026s^{+0.008}_{-0.007}$, ε , εp+ ε2p
$^{43}Cr_{24}$	0.021s±0.004, ε+β$^+$, εp, εα
$^{47}Fe_{26}$	$0.027s^{+0.032}_{-0.010}$, ε+β$^+$, εp

(or $B = 66 \rightarrow 86$), (Figure 1b and 1c). In the second case with $Z_{ex} = 2$ and even-Z, we can take $A = 0.01 \rightarrow 0.017$s (or $B = 65 \rightarrow 75$), (Figure 2b and 2c). In the latter case, we have two points at $Z = 6$ and $Z = 10$ outside the theoretical box. However, we can obtain the exact value of $T_{1/2}$ at these values of Z using the formula (1) and certain values of the parameters. In fact, for $^{18}Ne_{10}$ the exact value of $T_{1/2}$ =1.672s (Firestone and Ekström, 2004), can be obtained using $A = 0.012$s, $B = 50$ and $C = -0.109$s. Also, we can use $A = 0.013$s, $B = 44$ and $C = -0.64$ s to obtain the exact value of $T_{1/2}$ for $^{10}Ne_6$, where $T_{1/2} = 19.255$ s (Firestone and Ekström, 2004). This means that the suggested formula is a very good representation of the half-lives of nuclei in the two considered sets where $Z_{ex} = 1$ and ($Z_{ex} = 2$ with even-Z). Finally, some radioactive nuclei in the previous two cases are given in the reference, Firestone and Ekström (2004) without half-life time values. Therefore, the predicted values of half-life time for these nuclei, using formula (1) and the same values of the fitting parameters of the Figures 1a and 2a are given in

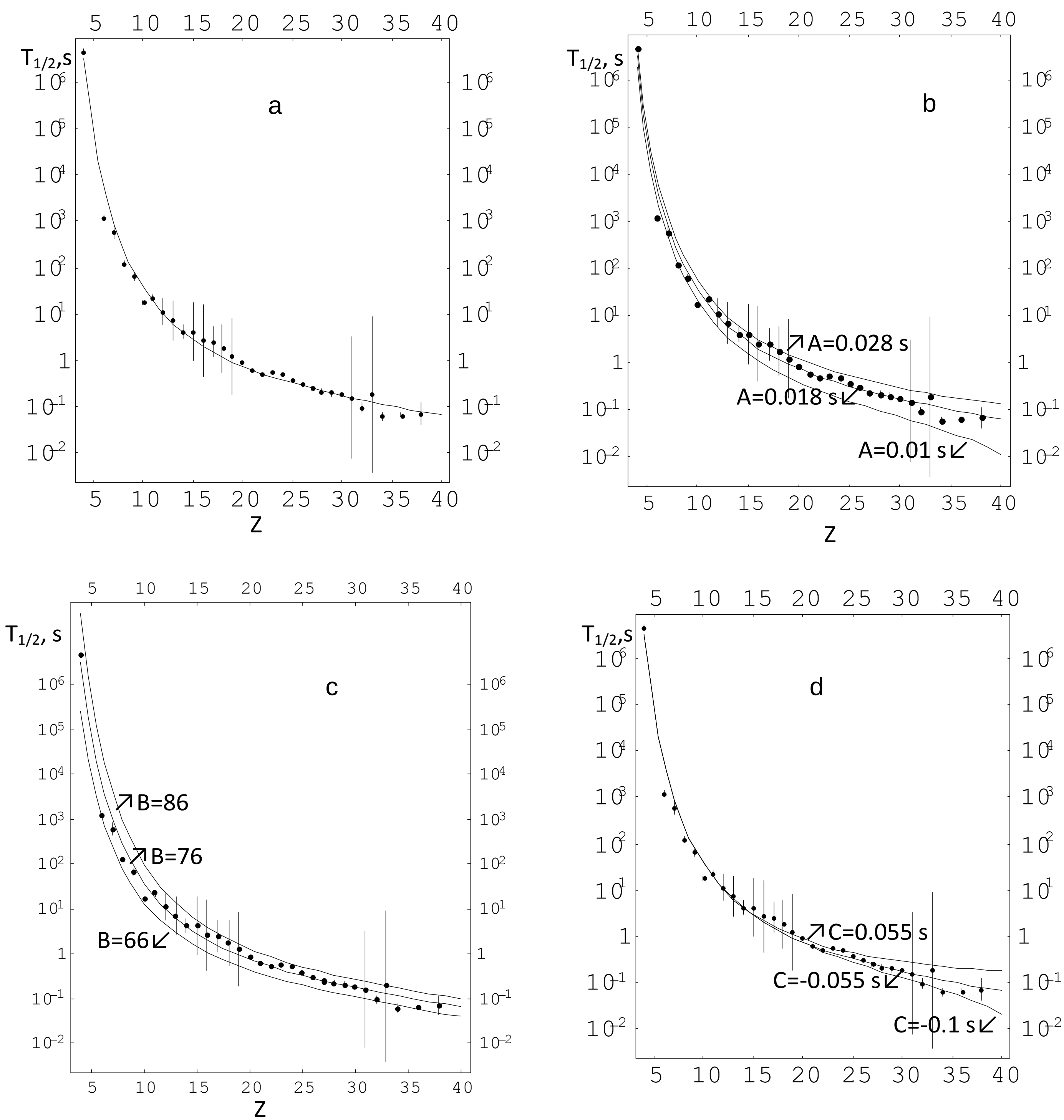

Figure 1. (a) $T_{1/2}$ for nuclei with Z_{ex}=1. The solid curves represent the results of the formula (1), where A= 0.018 s, B=76 and C=-0.055 s. The data are taken from (Firestone and Ekström, 2004). (b) Same as Figure (a) except A=0.01, 0.018 and 0.028 s. (c) Same as Figure (b) except A=0.018 s, B=66, 76 and 86. (d) Same as Figure (c) except B=76 and C=-0.1, -0.055 and 0.055 s.

the Tables 1 and 2.

From the formula (1), we get the rate of decay of nuclei in these two sets as:

$$\lambda(Z) = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{A\exp\{\frac{B}{Z}\} + C} \qquad (2)$$

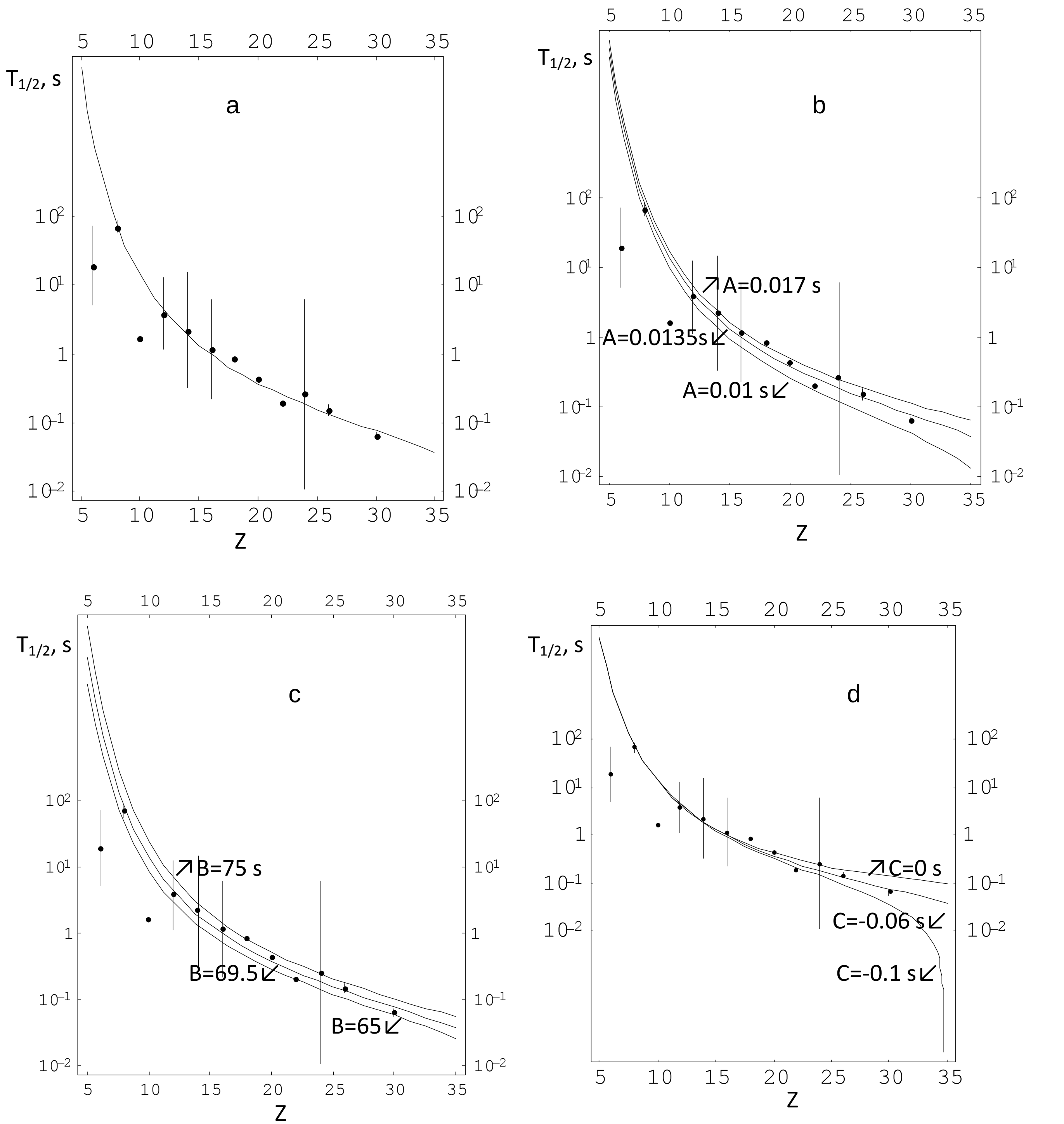

Figure 2. (a) $T_{1/2}$ for nuclei with Z_{ex}= 2 and even-Z. The solid curve represents the results of the formula (1) where A =0.0135 s, B=69.5, and C=-0.06 s. The data are taken from (Firestone and Ekström, 2004). (b) Same as Figure (a) expect A =0.01, 0.0135 and 0.017 s. (c) Same as Figure (b) expect A=0.0135 s, B=65, 69.5 and 75. (d) Same as Figure (c) expect B=69.5 and C=0, -0.06 and -0.1s.

Of course, by the definition, the rate of decay is also dependent on the isospin asymmetry of nucleus. In the considered two sets with $Z_{ex}=1$ and ($Z_{ex}=2$ with even-Z) we can neglect the parameter C for $Z \le 20$ and the

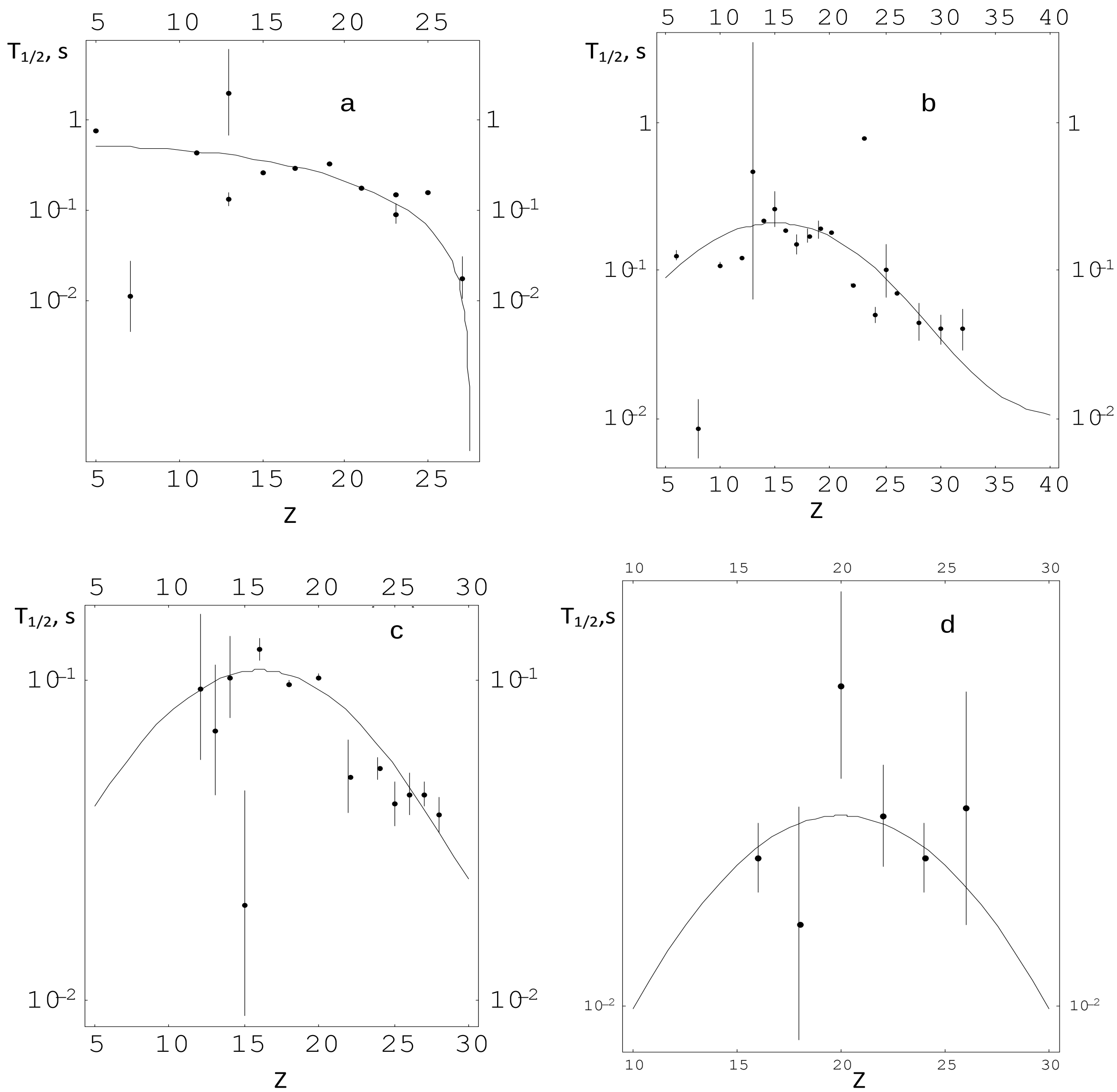

Figure 3. (a) $T_{1/2}$ for nuclei with Z_{ex} =2 and odd-Z. The solid curves represent the results of the formula (3) where A=0.76 s, B=0.0022, C=-0.25 s and Z_{max}=5. The data are taken from (Firestone and Ekström, 2004). (b) Same as Figure (a) expect Z_{ex}=3, A=0.2 s, B=0.00926, C=0.01 s and Z_{max}=15. (c) Same as Figure (a) expect Z_{ex}=4, A=0.1 s, B=0.00926, C=0.0075 s and Z_{max}=16. (d) Same as Figure (a) expect Z_{ex}=5, A=0.025 s, B=0.01, C=0.001 s and Z_{max}=20.

Equations (1) and (2) take the forms $T_{1/2}(Z) = A\exp\{\frac{B}{Z}\}$ and $\lambda(Z) = (\ln 2/A)\exp\{-\frac{B}{Z}\}$, respectively.

The second formula

The representations of the half-life time for nuclei where $Z_{ex} = 2$ with odd-Z, 3, 4 and 5; as seen in Tables 3, 4, 5, and 6, respectively; are given in the Figure 3. The general shape of these data in all cases, approximately, has two characters: concave down and has a maximum at some points. We can say that the shape of the data is similar to a parabola with an up vertex. Since the figures have a semi-log scale, we can suggest the following formula for the half-life time of nuclei in these four cases:

$$T_{1/2}(Z) = A\exp\{-B(Z - Z_{\max})^2\} + C, \quad (3)$$

where $Z_{\max}$ is the value of Z at a maximum of $T_{1/2}$ such

Table 7. The values of the parameters which give a good agreement with the data of $T_{1/2}$ for different last four cases of Z_{ex}.

Parameter	$Z_{ex} = 2, odd - Z$	$Z_{ex} = 3$	$Z_{ex} = 4$	$Z_{ex} = 5$
A, s	0.76	0.2	0.1	0.025
B	0.0022	0.00926	0.00926	0.01
C, s	-0.25	0.01	0.0075	0.001
Z_{max}	5	15	16	20
Range of *A*, s	0.60→1	0.1→0.3	0.08→0.12	0.02→0.03
Range of *C*, s	-0.3→ -0.15	-0.045→+o.o45	-0.02→+0.02	0→0.002

Table 8. The values of χ^2 and Δ^2 for different cases of Z_{ex}.

Z_{ex}	χ^2	Δ^2	The total number of data	The number of included data
1	0.21 m^2	0.2958 m^2	33	29
2, even-Z	0.01 m^2	0.0864 m^2	12	9
2, odd-Z	0.17 s^2	0.0022 s^2	13	11
3	0.022 s^2	0.0117 s^2	20	17
4	0.0099 s^2	0.0118 s^2	13	12
5	0.00075 s^2	0.0022 s^2	6	6

that this maximum in the flow of the data, *A, B* and *C* are fitting parameters. In general, a good fit with the data in different cases is obtained with different values of the parameters, (Figure 3). The values of the parameters are given in Table 7. The values of χ^2 and Δ^2 for the results in the Figures 3a, b, c and d are given in the Table 8. Also, as in the case of formula (1), since the only considered difference between the four cases is the value of Z_{ex}, the different values of the parameters in the these cases can be considered as a result of the isospin asymmetry in nuclei, that is, we can say that the quantities A, B, C and Z_{max} are dependent on Z_{ex} as a parameter. Then, the dependence of $T_{1/2}$ on these parameters reflects the isospin asymmetry effect on the half-life time of the considered nuclei. It is clear that $T_{1/2}$ is sensitive to the parameters A, B, C and the value of Z_{max} in all cases, Figures 4a, 5a, 6a, 7a; 4b, 5b, 6b, 7b; 4c, 5c, 6c, 7c; and 4d, 5d, 6d, 7d, respectively. Also, considering a range for the parameter A (or C), all the data can be approximately contained in the theoretical box; Figures 4a (or 4c), 5a (or 5c), 6a (or 6c) and 7a (or 7c). The ranges of the parameters are given in Table 7. Also, as in the case of formula (1), we can obtain the exact value of $T_{1/2}$ for any nucleus in these four sets of Tables 3, 4, 5 and 6 using formula (3) with certain values of the parameters. This means that the formula (3) is a good representation of $T_{1/2}$ in these four sets.

We have noted that the effect of the parameter C in the formula (1) appears only at the values of $Z \geq 20$, while its action is clear at all values of *Z* for the most calculations of the formula (3). This is due to the small values of the correction parameter *C*, at the same time, the values of $T_{1/2}$, which are calculated by the formula (1), are relatively large except at the values of $Z \geq 20$. On the other hand, the values of $T_{1/2}$ which are calculated by formula (3) are relatively small at the most values of Z.

Also, some radioactive nuclei in the considered four cases are given in Firestone and Ekström (2004) without half-life time values. Therefore, the predicted values of half-life time for these nuclei, using formula (3) and the values of the parameters in Table 7 are given in Tables 3, 4 and 5.

From the formula (3), for the total rate of decay we get:

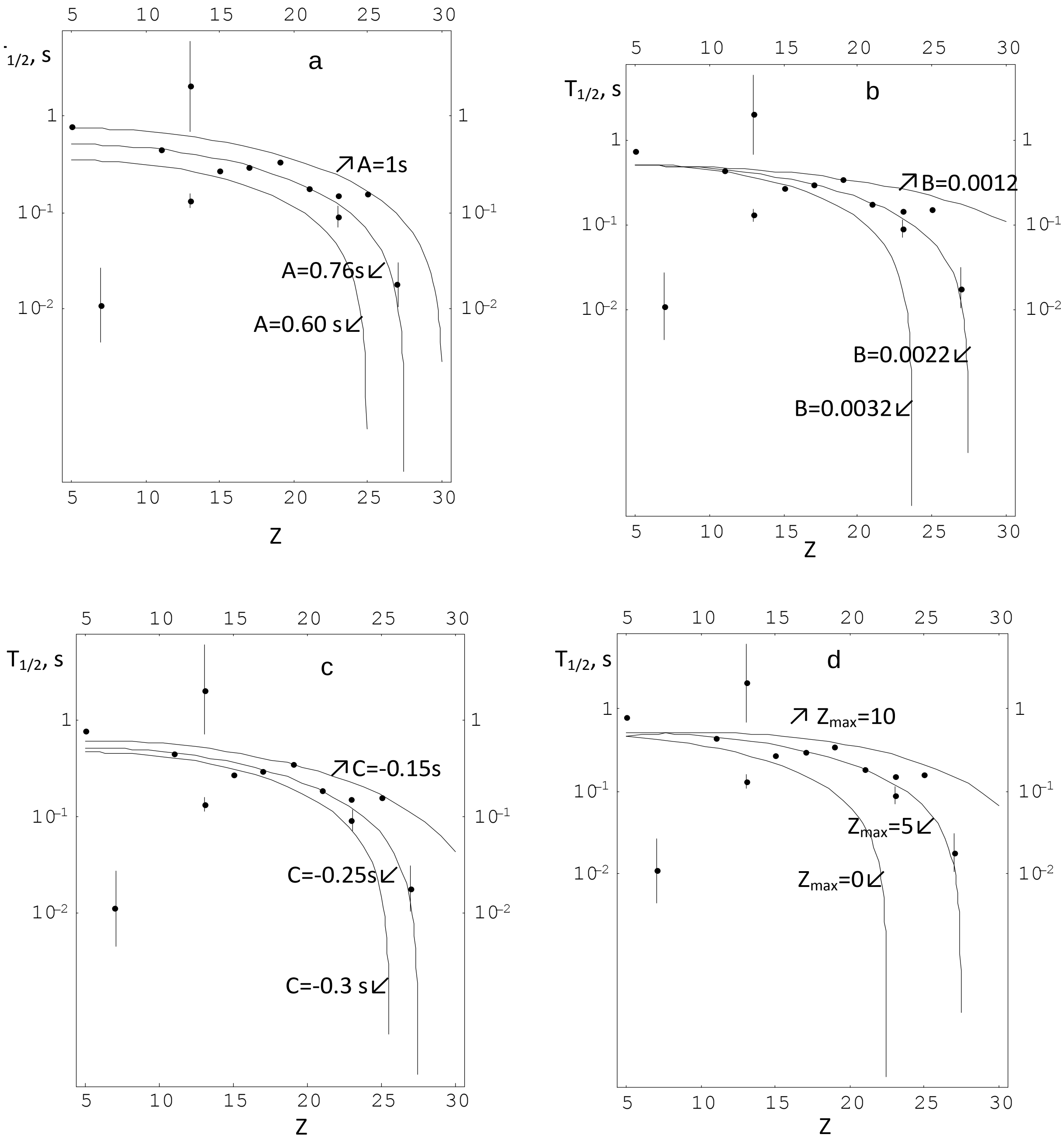

Figure 4. (a) $T_{1/2}$for nuclei with Z_{ex}=2 and odd-Z. The solid curves represent the results of the formula (3) where A=0.60, 0.76 and 1 s, B=0.0022, C=-0.25 s and Z_{max}=5. The data are taken from (Firestone and Ekström, 2004). (b) Same as Figure (a) except A=0.76 s, B=0.0012, 0.0022 and 0.0032 (c) Same as Figure (b) except B=0.0022 and C=-0.15, -0.25 and -0.3 s. (d) Same as Figure (c) except C=-0.25 s and Z_{max}=0, 5 and 10.

$$\lambda(Z) = \frac{\ln 2}{A\exp\{-B(Z-Z_{\max})^2\}+C} \qquad (4)$$

In the formulas (3) and (4) we cannot neglect any parameter where all parameters play significant roles at all values of *Z*.

DISCUSSION

In this work we have tried to answer the question, does the asymmetry number Z_{ex} has a role in the evaluation of half-life time $T_{1/2}$?. Classifying the data with respect to the value of Z_{ex} we have 6 sets, each of which has a certain

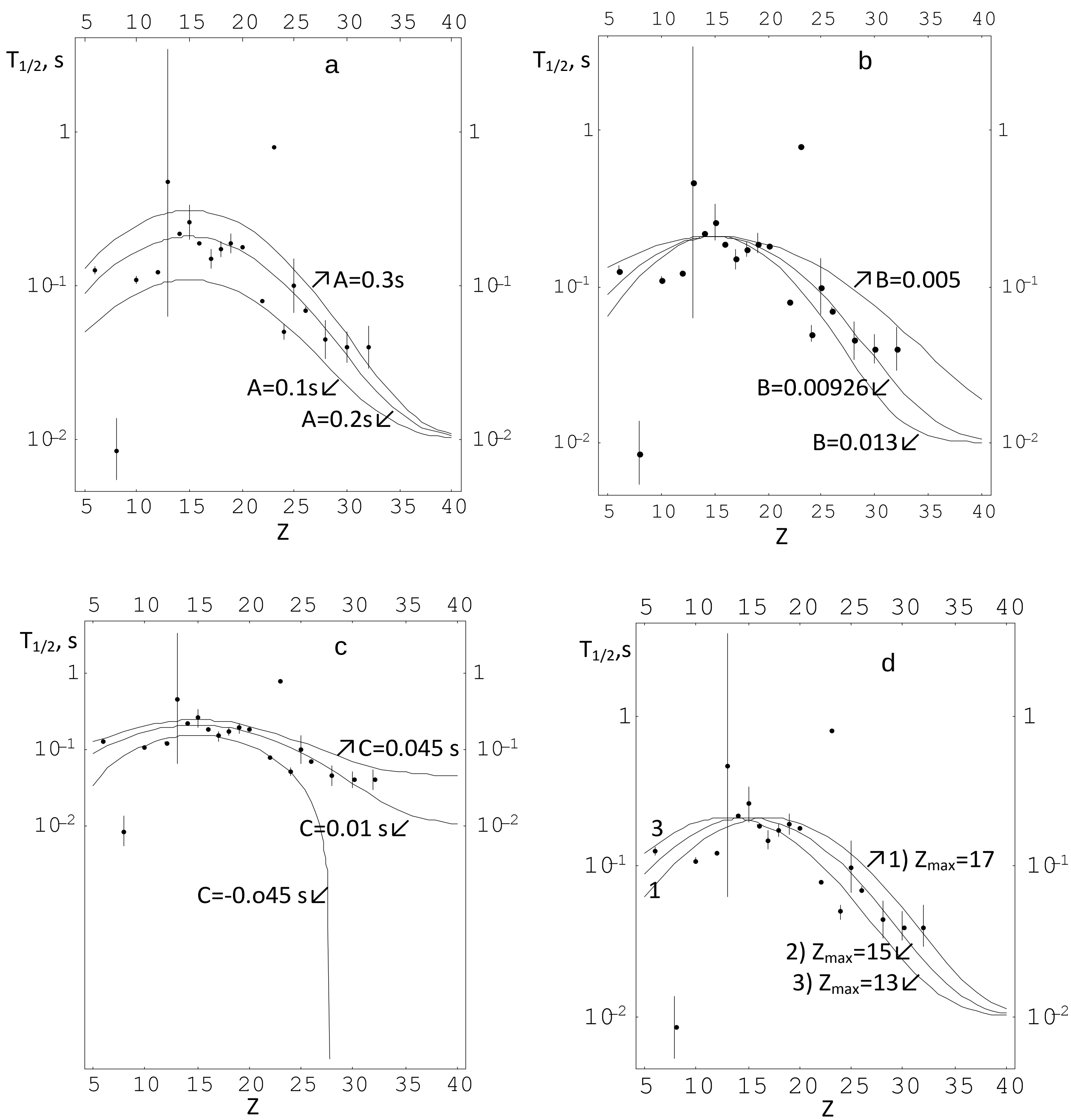

Figure 5. (a) $T_{1/2}$ for nuclei with Z_{ex}=3. The solid curve represents the results of the formula (3), where A=0.1, 0.2 and 0.3 s, B=0.00926, C=0.01 s and Z_{max}=15. The data are taken from (Firestone and Ekström, 2004). (b) Same as Figure (a) except A=0.2 s, B=0.005, 0.00926 and 0.013. (c) Same as Figure (b) except B=0.00926 and C=-0.045, 0.01 and 0.045 s. (d) Same as Figure (c) except C=0, 0.01 and Z_{max}=13, 15 and 17.

value of Z_{ex}. The elements in each set have a common isospin asymmetry property, the same value of Z_{ex}. The different values of the asymmetry number Z_{ex} lead to different physical properties between the sets. This is because many fundamental quantities, equations and reactions are dependent on the isospin asymmetry of nuclei (Berdichevsky, 1984; Steiner et al., 2005; Centelles et al., 2009; Shetty and Yennello, 2010; Shetty et al., 2007; Tsang et al., 2009).

In general, we can say that the value order and the behaviors of $T_{1/2}$ are dependent on the value of Z_{ex}. Also, the data indicate some relation between the type of decay and the asymmetry number Z_{ex}. At the same time,

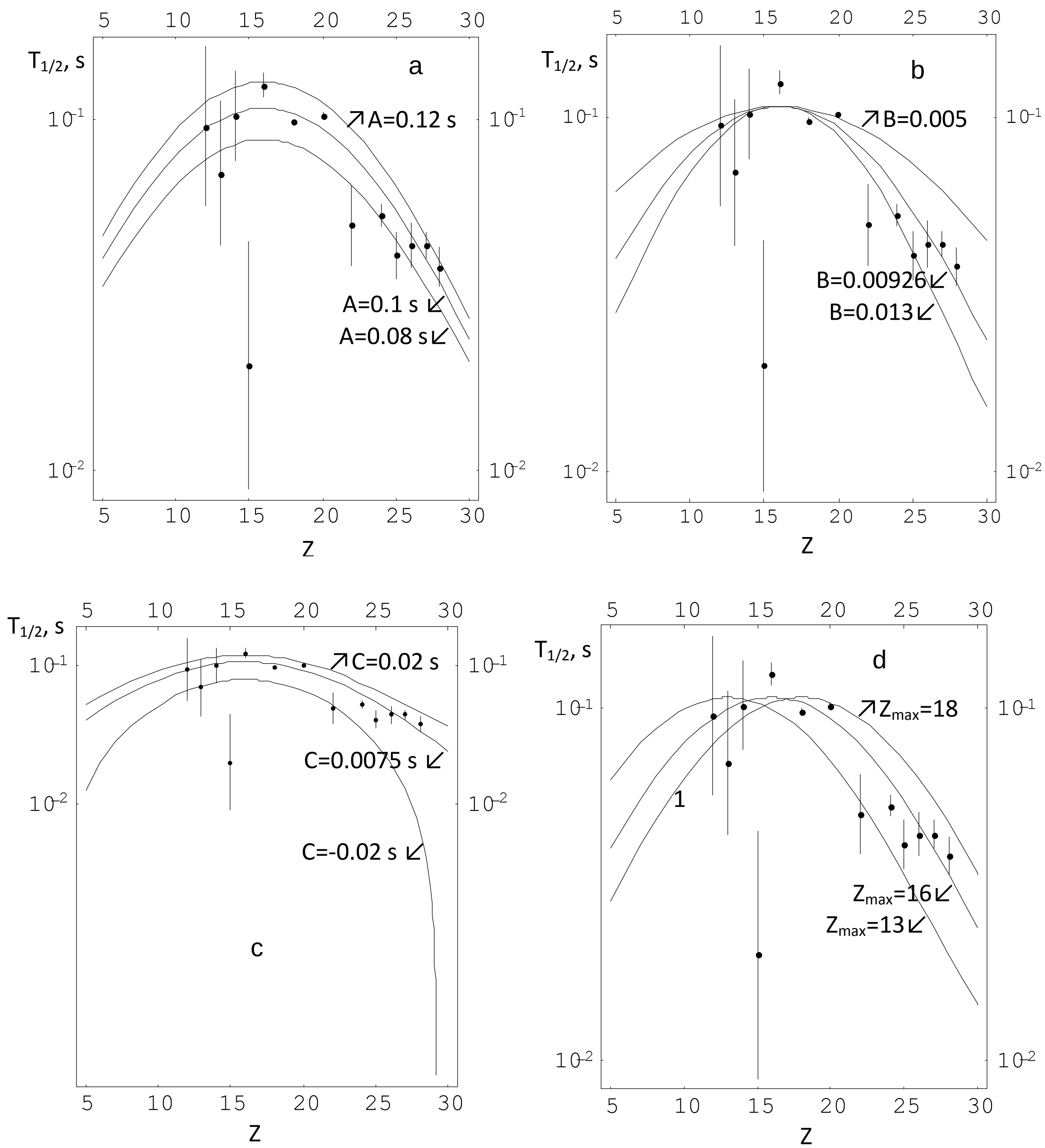

Figure 6. (a) $T_{1/2}$ for nuclei with Z_{ex}=4. The solid curve represents the results of the formula (3) where A=0.08, 0.1 and 0.12 s, B=0.00926, C=0.0075 s and Z_{max}=16. The data are taken from (Firestone and Ekström, 2004). (b) Same as Figure (a) except A=0.1 s, B=0.005, 0.00926 and 0.013. (c) Same as Figure (b) except B=0.00926, C=-0.02, 0.0075 and 0.02 s. (d) Same as Figure (c) except C=0.0075 s, Z_{max}=13, 16 and 18.

the parameterization of the suggested formulas for $T_{1/2}$ and the values of the contained parameters are dependent on the value of the asymmetry number Z_{ex}. We can see this fact if we consider isotopes of an element. With the same Z and different Z_{ex}, each isotope belong to different one of the 6 sets. To obtain the value of $T_{1/2}$, we will use the formula (1) for the isotopes in the first two sets with two different sets of values of the parameters. The formula (3) is used for the isotopes in the other four sets with different values of the parameters for each set. For example, the isotopes of Ne_{10}, Ca_{20} and P_{15}. Thus, the data of Firestone and Ekström (2004) and the suggested formulas lead to the same conclusion, which

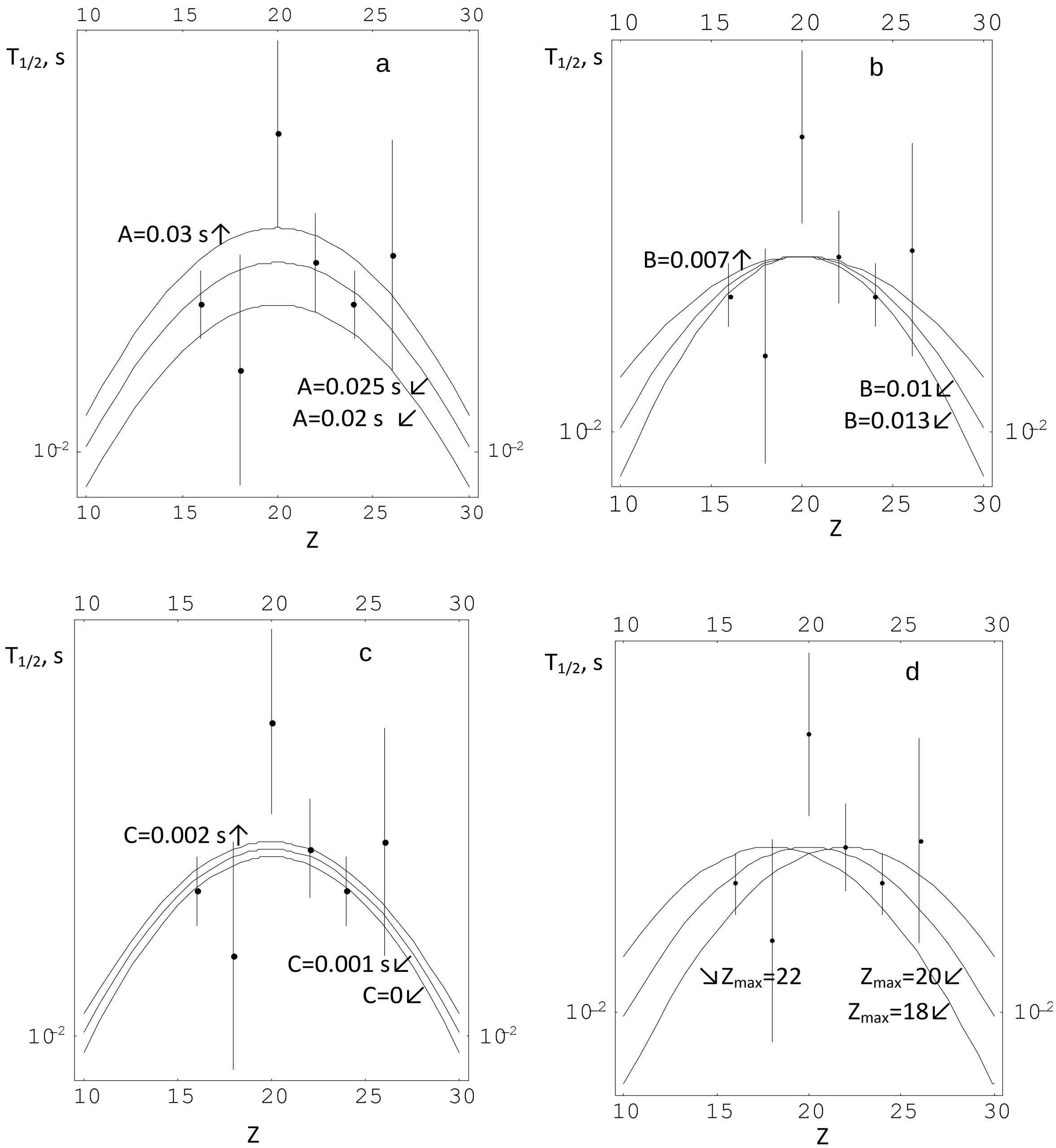

Figure 7. (a) $T_{1/2}$ for nuclei with Z_{ex}=5. The solid curve represents the results of the formula (3), where A=0.02, 0.025 and 0.03 s, B=0.01, C=0.001 s and Z_{max}=20. The data are taken from (Firestone and Ekström, 2004). (b) Same as Figure (a) except A=0.025 s, B=0.007, 0.01 and 0.013. (c) Same as Figure (b) except B=0.01, C=0, 0.001 and 0.002 s. (d) Same as Figure (c) except C=0.001, Z_{max}=18, 20 and 22.

is, that the isospin asymmetry of nuclei plays important role in nuclei half-life time evaluation and that a connection between the asymmetry number Z_{ex} and the type of decay does exist.

In general, we can consider that the formulas obtained herein describe well the half-life time of nuclei in each one of 6 considered sets, especially, if we consider the suggested theoretical box. However, we cannot determine the reasons of the existence of some nuclei outside the theoretical box. But, this may be due to the decay modes of these nuclei that are, in some cases, different from the major decay mode of the set. Also, our formulas are in a good agreement with the last measurements of half-life time of some nuclei. For example, in Triambak et al. (2012) the measurement of the half-life time of $^{19}Ne_{10}$, where $Z_{ex}=1$, equals to 17.262 s, which can be obtained using the formula (1) with *A*= 0.017452s, *B*=69 and *C*=-0.055s. In Iacob et al. (2010) the measurement of the half-life time of $^{26}Si_{14}$, where $Z_{ex}=2$ and Z is even, equals to 2.2453 s, which can be obtained using the formula (1) with *A*=0.016098s, *B*=69.5 and *C*=-0.06s.

Conclusions

In conclusion, the half-life time and decay mode of radioactive nuclei with $Z>N$ is sensitive to the isospin asymmetry of nuclei which is represented by the asymmetry number Z_{ex}. Any one of suggested formulas of $T_{1/2}$ is a function of Z and dependent on Z_{ex} as a parameter, that is, we can write $T_{1/2}=T_{1/2}^{Z_{ex}}(Z),\ Z_{ex}=1,2,3,4,5$.,

or $T_{1/2}=T_{1/2}^{Z_{ex}}(Z)=\begin{cases} A\exp\{\frac{B}{Z}\}+C,\ Z_{ex}=1,\ (2\ with\ even\ Z) \\ A\exp\{-B(Z-Z_{max})^2\}+C,\ Z_{ex}=(2\ with\ odd\ Z),3,4,5 \end{cases}$.

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