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Full Length Research Paper

Dynamic analysis for acceleration of crab shell after explosion

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In this research, the study of dynamical aspect of the crab shell, using assumption such as energy conservation in the nebula system, exiting of hydrogen gas all around the nebula and time dependant acceleration of expansion is assessed. We could found a relation for the velocity of expansion by drawing the graph of velocity, the initial velocity of expansion is calculated to be *v (0) = 196059 m/s*. Finally we have shown that the time relation of acceleration is much better with respect to constant acceleration for analyzing the expansion of the crab nebula.

Key words: Neutron star, pulsar, expansion of crab nebula, energy conservation.

INTRODUCTION

Messier list publication caused a new form of studying in the course of nature of crab nebula. Lord Ross was the first person who called it crab, due to what he observed in 1884. From what Chinese recorded in their history and their astronomers explained, we know it under the name of supernova remnant and represent it as SN1054 (Harry, 2001). Spherical expansion of the nebula caused the death of a heavy star, it is about $10\ M_{\Theta}$ and it is considered as type O or B. Crab pulsar is the first discovered pulsar which dispatches both optical and radio pulses; Period of this pulsar is *0.033 s*.

In optical frequencies, this source is not strong. Source magnitude is about 8.4 and this source is an extended and strong source at radio frequency. In X area, its radiation power is at least 100 times more than that in visible frequency which is about 1030 watt. In methods used for computing distance of nebula, this distance is 1930 ± 100 Parsec which is 6292 ± 326 light years (Harry, 2001; Lyne and Graham, 1998).

Expansion of crab nebula

As we know, an object with moment of inertia (I) which rotates around an axis with Angular velocity (Ω) has rotational energy. But if the angular velocity is not constant in time, then its time derivation will not be zero and its energy consumption rate in object will be

$$\dot{E}_{rot} = I\Omega\ \dot{\Omega}$$ (Phillips, 1999)

The value of Ω and $\dot{\Omega}$ are obtained due to measuring rotation time of pulsar and I is rotational moment of inertia. By means of photo techniques and different photos of nebula taken at different times, it can be supposed that nebula shell expands with constant acceleration [Figure 1]. Considering this assumption, the acceleration is related to expansion velocity (Bejger and Haensel, 2003) as follows:

$$\dot{V}_c = \frac{2\ \Delta\ V_p}{T^2} \quad (1)$$

On the other hand, if mass of the nebula shell be Mneb, then time derivation of kinetic energy for this shell will be:

$$\dot{E}_{\exp} = M_{neb}\, V\, \dot{V} + \frac{1}{2}\dot{M}_{neb}\ V^2 \quad (2)$$

Where, V is the radius velocity.

Peterson has calculated radiation energy rate in X, UV

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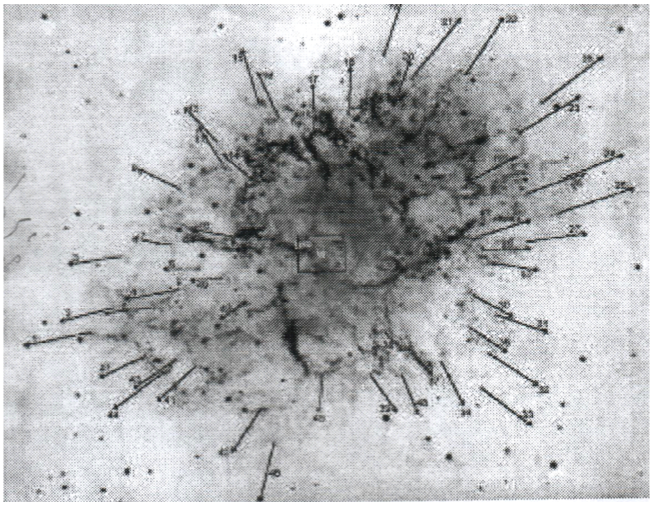

Figure 1. Moving the filaments after explosion during the time.

and visible areas as well as other radiation areas for nebula (Petersen, 1998) as follows:

$$\dot{E}_{rad} \cong 1.25 \times 10^{38} \left(\frac{d}{1.83\, kpc} \right) erg/s \quad (3)$$

In which d is the distance of nebula in terms of Kpc. Produced rotational energy by crab pulsar is used for all nebula radiation and energy required for accelerating and expanding nebula shell. This case occurs in inter star environment. So energy conservation of system involves having $\dot{E}_{rot} \geq \dot{E}_{\exp} + \dot{E}_{rad}$. Suppose the acceleration of the shell is constant and using current velocity of nebula obtained by spectrogram that is,

$V_p \approx 1.5 \times 10^6 \frac{m}{s}$, we will have:

$$\dot{V}_C = 0.82 \times 10^{-3}\, cm/s^2$$

Also using the data and current values and replacing these values in energy conservation law, we obtain the inertia parameter. By substituting the parameters such as

$$\Omega \left| \dot{\Omega} \right| = 4.459 \times 10^{-7}$$

$V = 1.5 \times 10^6 \frac{m}{s}$ $R_{neb} = 1.25\, pc$ and $M_{neb} = 4.6 M_{\Theta}$ the limit for rotational inertia obtain as $I = 3.04 \times 10^{38}\, kg.m^2$.

Expansion due to the time dependant acceleration

We suppose that expansion acceleration of shell is not constant but that it depends on time. So, we must use approximation related to moment of inertia of pulsar. In the case of $P \phi P_i = 19\, ms$, correlation between P and I, in contrast to time, is significant. So we have:

$$\dot{V}(t = 938\ yr) = 0.82 \times 10^{-5} \frac{m}{s^2}$$

The relationship between angular velocity and its derivation for Neutron stars is linear (Manchester and Talor, 1977).

$$\dot{\Omega} = -K \Omega^n \quad (4)$$

In this equation, n and k values are constant. n is obtained by measuring time. Then by replacing values of observational data of crab pulsar,

$$\Omega_p = 188101 s^{-1}, \dot{\Omega}_p = -2.3707 \times 10^{-9} s^{-2}$$ (Taylor, 1993)

pulsar parameters will be obtained k (n = 2.509 ± 0.001 K = 4.66 x 10^{-15}). After integrating and replacing n value, angular velocity of pulsar will be obtained by following equation:

$$\Omega(t) = \left(1.509kt + 1.6159 \times 10^{-4}\right)^{-2.3254} \quad (5)$$

Mass increases during expansion which occurs from outer space to the nebula shell. It can be computed in a spherical system with Rneb:

$$\dot{M} = 4\pi R_{neb}\, n_H\, m_H\, V \quad (6)$$

nH is density of hydrogen atoms around nebula and mH is mass of hydrogen atom. Manchester and Tailor have computed $n_H = 0.2\, cm^3$.

Replacing equations 2, 3, 5 and 6 in conversation law, we will have differential equation:

$$Ik(1.509\, kt + 1.6159 \times 10^{-4})^{-2.325} = M_{neb} V(t) \frac{dV(t)}{dt} + \alpha V(t)^3 + \beta \quad (7)$$

In this equation, the coefficients $\alpha = 2\Pi R_{neb} n_H^2 m_H$ and $\beta = 1.25 \times 10^{31}$ causes incongruity of differential

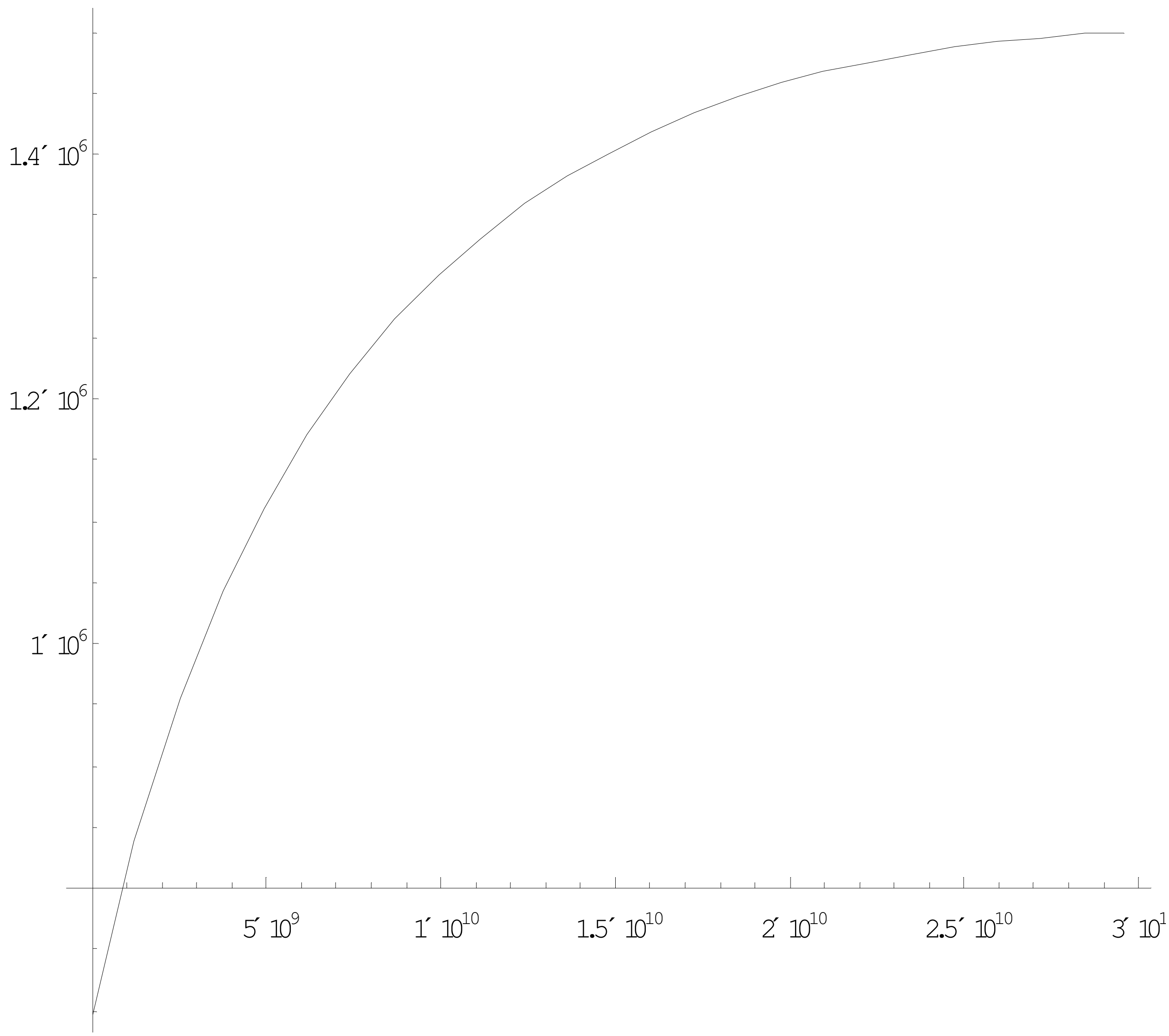

Figure 2. The graph of velocity changes against time

equation. We examined this differential equation in various software and the results showed that this differential equation has no analytical answer. In other words, it was not possible to find velocity function in terms of time, but using mathematic software and special boundary condition which is current velocity of expansion ($V(t=938\,yr)=1.5\times10^{6}$), then we could be able to draw the graph of velocity changes against time spent areas (Figure 2).

Horizontal and vertical axes represent time (s) and velocity expansion (m/s), respectively. Studying this graph during initial times showed that initial velocity of expansion was $V(0) = 196059 \quad m/s$. Another point in this graph is that expansion velocity will not change in future times. In other words it seems that velocity is stable and it is almost constant.

We could be able to derive the acceleration equation from (Equation 7)

$$\dot{V}(t) = \frac{1}{\sqrt{M_{neb}}}\frac{Ik\Omega^{3.509} - \beta}{\sqrt{I\Omega^{2} - 2\beta T - f}} \quad (8)$$

By substituting current parameters, we obtained the constant of integral as $f = -1.458\times10^{43}$, so the initial and current acceleration will be obtained:

$\dot{V}(0) = 0.443\times10^{-4}\,m/s^{2}$, $\dot{V}(t=38\,yr) = 0.82\times10^{-5}\,m/s^{2}$

It seems that the acceleration is decreasing during the time; so we calculate the average of acceleration:

$$\overline{a} = \frac{1}{T}\int^{T} \dot{V}(t)dt \qquad (9)$$

With numerical solution of this equation we will have:

$$\bar{a} = 0.195 \times 10^{-5} m/s^{2}$$

Conclusion

For expanding crab shell, we introduced a model with time dependent acceleration. We could find differential equation by using energy conservation law of system. So, initial velocity of expansion obtained was V (0) =196059 m/s. However Bejger and haensel calculated the initial velocity as the variance which is obtained from the difference between two models for studying acceleration. (Constant acceleration and related to time acceleration)

We conclude that velocity difference between Vi and Vp is significant and can not be neglected. Therefore, optical motion of nebula must have a negative acceleration based on time. It means that the magnitude of initial acceleration is more than the current acceleration. Current velocity of expansion is several times as much as initial velocity too.

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