

Full Length Research Paper

Some electromagnetic transition properties and mixing ratios of tellurium isotopes

Harun Reşit Yazar and İhsan Uluer

Kırıkkale University, Faculty of Art and Science, Kırıkkale-Turkey.

Accepted 26 January, 2007

IBM-2 calculation is presented for the low-lying states in the even-even $^{122-128}\text{Te}$ core nucleus. We developed the projection by using the F-spin formalism from the operator of IBA-2 model over the operator of IBA-1 model space. With the help of this projection IBA-2 Hamiltonian parameters are obtained and we explore the energy levels and the electric quadrupole transition probabilities $B(E2; I_i \rightarrow I_f)$ and γ -ray E2/M1 mixing ratios for selected transitions. It was found that the calculated positive parity low spin state energy spectra of the even-even $^{122-128}\text{Te}$ isotopes agree quite well with the experimental data.

Key words: Interacting boson model, the electric quadrupole transition probability, mixing ratios, Low spin states, PACS: 21.60. Fw, 23.20.Lv, 27.70. +q.

INTRODUCTION

In recent years many works have been done on the structure of tellurium nucleus in recent years; Lopac (1970) studied on semi-microscopic description of even tellurium isotopes, Degrieck and Berghe (1974) determined structure and electromagnetic properties of the doubly even Te isotopes, Subber et al. (1987) apply the dynamic deformation model to the tellurium isotopes, Sambataro (1982) calculated the some of electromagnetic properties of Tellurium and Cadmium isotopes with the framework of the interacting boson approximation, Rikovska et al. (1987). Studied dynamical symmetries in even-even Te nuclides, Yazar et al. (2005) explore the energy levels and the electric quadrupole transition probabilities $B(E2; I_i \rightarrow I_f)$ and γ -ray E2/M1 mixing ratios for selected transitions of some even-even erbium isotopes with the help of this projection.

The interacting boson approximation represents a significant step forward in our understanding of nuclear structure. It offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, and is founded on rather general algebraic group theoretical techniques which have also found recent application to problems in atomic, molecular, and

high-energy physics. The application of this model to deformed nuclei is currently a subject of considerable interest and controversy.

In the first version, IBA-1, no distinction is made between neutron and proton degrees of freedom. An unsatisfactory aspect of this model is that there is no clear connection with a microscopic structure of nucleus. The microscopic theory strongly suggests that it is important to treat the neutron and proton degrees of freedom independently. This has led to the introduction of the second, generalized, version of the IBA-model, IBA-2. In the second version, the neutron and proton degrees of freedom are treated explicitly. In this model the nucleus is described explicitly in terms of neutron (s_v, d_v) and proton (s_π, d_π) bosons. From calculations in the IBA-2 model it appears that the lowest levels are symmetric under the interchange of neutrons and protons. This symmetry is most easily discussed in terms of a variable called F-spins (Scholten, 1980). In the case of boson systems F-spin plays a role similar to that of isospin in the case of fermion systems.

The relation between the IBA-1 and IBA-2 model is obtained by identifying states of the former to the fully symmetric that is maximal F-spin states of the latter model. Since the space of the IBA-1 model can be regarded as a subspace of the IBA-2 model there is a unique

*Corresponding author. E-mail: yazar@kku.edu.tr

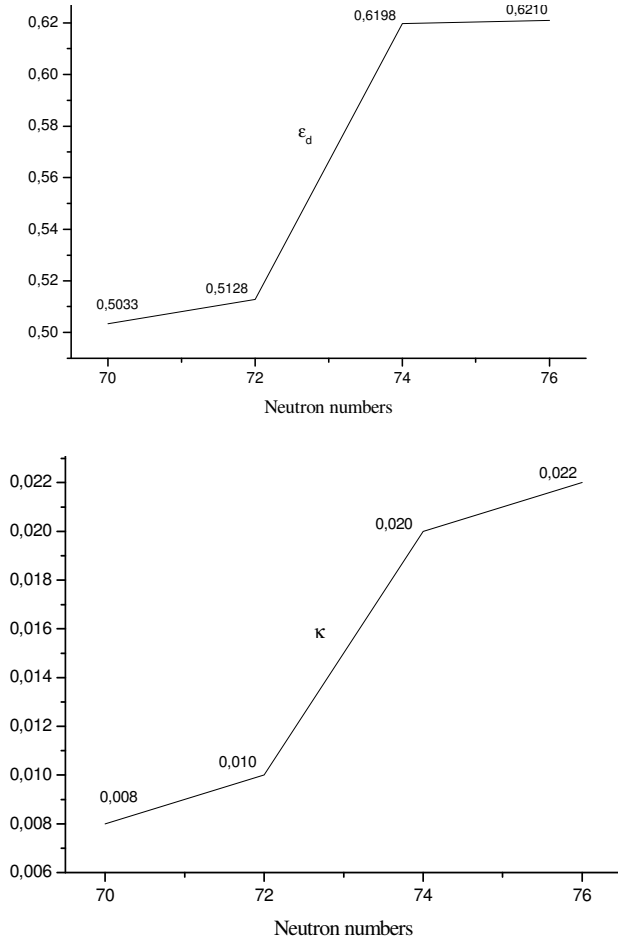


Figure 1. The parameters ϵ_d and κ employed for the IBA-2 calculations for tellurium isotopes with even neutron numbers 70 up to 76.

way to “Project” the operators of the IBA-2 model onto those of IBA-1. This projection can be carried out by using the F-spin formalism (Bardeen et al., 1957; Voight and Dudek, 1983; Iachello and Scholten, 1979).

From these considerations it follows that IBA-1 and IBA-2 model can be related to each others and the states of the IBA-1 model can be identified with the fully symmetric states in the IBA-2 model. It is the purpose of this work to study these relations and applied to $^{122-128}\text{Te}$ even mass isotopes.

The Project approximation used in this study has been extensively described by Scholten and Yazar for the neodymium, samarium, erbium and gadolinium isotopes (Yazar and Uluer, 2005; Scholten, 1985; Yazar, 2005). We shall present here only the results of calculation and refer the reader to that work of the Project approximation for details (Scholten, 1985; Yazar, 2005; Yazar and Uluer, 2005). In section 2 we study the positive parity spectra of the $^{122-128}\text{Te}$ isotopes. In the same section E2 and M1 transition probabilities and electric quadrupole transition

probabilities $B(\sigma L; i_i \rightarrow i_f)$ are analysed. Finally, the work is summarised in section 3.

THEORY AND METHOD OF CALCULATION

The IBM (Talmi, 1981; Bohr and Mottelson, 1969; Iachello and Isacker, 1991) provides a unified description of collective nuclear states in terms of a system of interacting bosons. The $^{122-128}\text{Te}$ isotopes have 52 protons 70 - 76 neutrons, which fill the orbits below major shell closure at $N=82$, characterized by six neutron holes. Within the IBM, these structure or shape changes correspond to the system moving between the vibrational SU(5) and γ -unstable O(6) limits. The $^{122-128}\text{Te}$ nucleus has been considered as a transitional nucleus from SU(5) to O(6) (Iachello and Isacker, 1991).

In the present work IBA-2 Hamiltonian parameters are normalized with the help of IBA-1 model hamiltonian. In the IBA-2 calculation the lowest states are indeed fully symmetric; the calculation with the help of this projection gave good results for the excitation energies. Because of the admixtures of non-fully-symmetric states in IBA-2 model space, the projection gave some difficulties and we had to renormalize the parameter (ϵ) and κ as shown in Figure 1.

In IBA-2 the neutron and proton degrees of freedom are treated explicitly. This has the advantage of being closer to a microscopic theory. The matrices that have to be diagonalized are, however, much larger. One can regard the IBA-1 model space, in which neutron and proton degrees of freedom are not distinguished, as a subspace of the IBA-2 hamiltonian one can thus Project out its IBA-1 pieces (Scholten, 1980; Yazar, 2005). In the present work the relevant terms in the IBA-2 hamiltonian

$$H = \epsilon(n_{d_p} + n_{d_n}) + \kappa(Q_{v,\pi}) + V_{vv} + V_{\pi\pi} \quad (1)$$

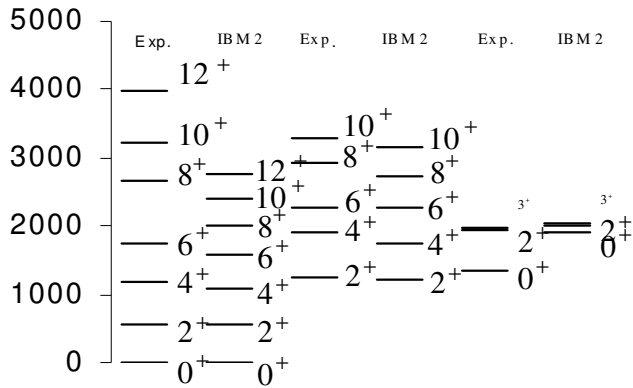
Where the dot denotes a scalar product. The first term represents the single-boson energies for proton and neutron bosons and n_{d_p} is the number of d-bosons where p corresponds to π (proton) or v (neutron) bosons. The second term denotes the main part of the boson-boson interaction, which is the quadrupole-quadrupole interaction between neutron and proton bosons with strength κ . The quadrupole operator is

$$Q_p = [d_p^+ s_p + s_p^+ \tilde{d}_p]^{(2)} + \chi_p [d_p^+ \tilde{d}_p]^{(2)} \quad (2)$$

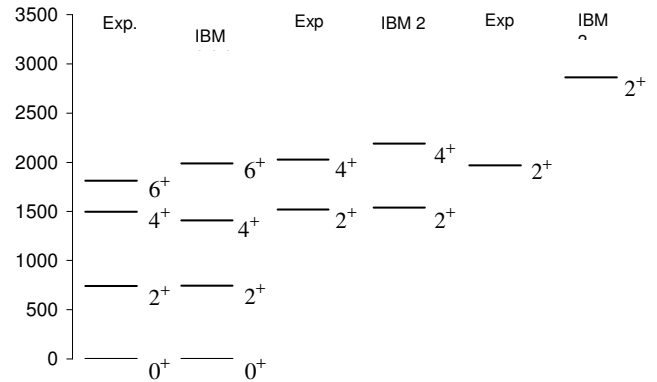
Where p corresponds to π (proton) or v (neutron) bosons and χ_p determines the structure of the quadrupole operator and is determined empirically. Where χ_p determines the structure of the quadrupole operator and is determined empirically. The square brackets in (2) denote angular momentum coupling.

Table 1. IBM-2 parameters; all parameters in MeV except χ_v, χ_π

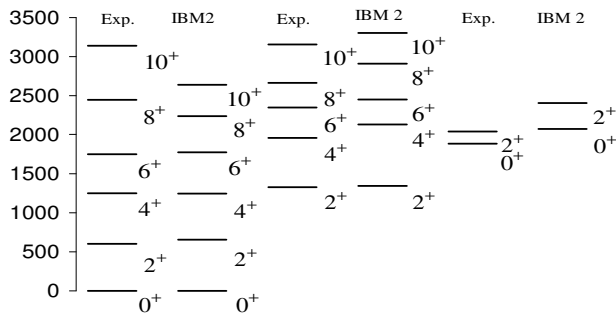
	ϵ	κ	χ_v	χ_π	$C_{\pi\nu L}(0,2,4)$
¹²² Te	0.503	0.008	0.01	0.002	0.002 0.003 0.002
¹²⁴ Te	0.508	0.010	0.01	0.002	0.002 0.003 0.002
¹²⁶ Te	0.602	0.020	0.02	0.003	0.002 0.003 0.002
¹²⁸ Te	0.621	0.022	0.02	0.003	0.002 0.003 0.002



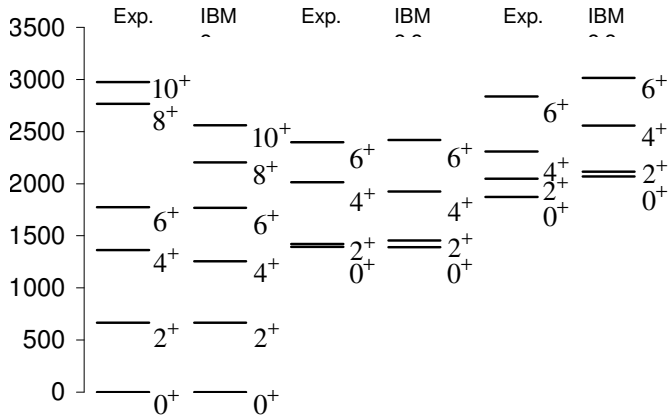
(a)



(d)



(b)



(c)

Figure 2. The three lowest rotational bands in spectra of (a) ¹²²Te and (b) ¹²⁴Te and (c) ¹²⁶Te and (d) ¹²⁸Te. In each band the experimental data are plotted on the left and calculated values on the right. (Energy in keV)

The terms $V_{\pi\pi}$ and $V_{\nu\nu}$, correspond to interactions between like-bosons. They are of the form

$$V_{\rho\rho} = \frac{1}{2} \sum_{L=0,2,4} C_L^\rho \left([d_\rho^+ d_\rho^+]^{(L)} \cdot [\tilde{d}_\rho \tilde{d}_\rho]^{(L)} \right) \quad (3)$$

The isotopes ¹²²⁻¹²⁸Te have $N_\pi = 1$, and N_ν varies from 4 and 5, while the parameters κ, χ_v, χ_π and ϵ were treated as free parameters and their values were estimated by fitting to the measured level energies. This procedure was made by selecting the “traditional” values of the parameters and then allowing one parameter to vary while keeping the others constant until a best fit was obtained. This was carried out iteratively until an overall fit was achieved. The best fit values for the hamiltonian parameters are given in Table 1.

The numerical diagonalization has been carried out by using the code (Otsuka and Yoshida, 1985). The values of the main parameters of the Hamiltonian are given in Table 1. The calculated excitation energies for ¹²²⁻¹²⁸Te isotopes as well as the experimental ones are compared in Figure 2(a)-(b)-(c)-(d). The general agreement between experiment and model is quite good.

A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but

Table 2. $B(E2; I \rightarrow I-2)$ values for ground state bands of $^{122-128}\text{Te}$ isotopes.

N	B(E2) (in e^2b^2)						B(E2) ratios		
	$4_1 \rightarrow 2_1$			$2_1 \rightarrow 0_1$			$(4_1 \rightarrow 2_1) / (2_1 \rightarrow 0_1)$		
Exp. Rikovska et al., 1987; (ENSDF).	Theory	Ref. Küçük bursa and Yörük, 1999	Exp. Rikovska et al., 1987; (ENSDF)	Theory	Ref. Küçük bursa and Yörük, 1999	Exp. Rikovska et al., 1987; (ENSDF)		Theory	Ref. Küçük bursa and Yörük, 1999
70	0.18	0.20	0.19	0.12	0.13	0.132±0012	1.50	1.53	1.46
72	0.16	0.18	0.1630	0.11	0.11	0.1138±0.0015	1.45	1.63	1.43
74	0.16	0.12	0.159	0.09	0.07	0.094±0.004	1.77	1.71	1.68
76	0.12	0.12	-	0.06	0.07	0.076±0.006	2.0	1.71	-

also of its electromagnetic properties. The most important electromagnetic features are the E2 transitions. The B (E2) values were calculated by using the E2 operator. The E2 transition operator must be a hermitian tensor of rank two and therefore the number of bosons must be conserved. Since, with these constraints the general E2 operator can be written as (Iachello and Isacker, 1991).

$$T_m(E2) = e_\pi Q_\pi + e_\nu Q_\nu \quad (4)$$

Where Q_p are the Q_π and Q_ν boson quadrupole operators and e_π and e_ν are the "effective charges" for the proton bosons and the neutron bosons. For simplicity the "effective charges" e_π and e_ν were taken as equal ($e=0.23 eb$). The B(E2) strength for the E2 transitions is given by:

$$B(E2; L_i \rightarrow L_f) = 1/(2L_i + 1)^{1/2} \left| \langle L_f || T(E2) || L_i \rangle \right|^2 \quad (5)$$

Some calculated B (E2) values from the ground state band and B(E2) ratios are given in Table 2.

Since tellurium nucleus has a rather vibrational-like character, taking into account of the dynamic symmetry location of the even-even tellurium nuclei at the IBM phase triangle where their parameter sets are at the O(6)-SU(5) transition region and closer to O(6) γ -unstable character and we used the multiple expansion form of the Hamiltonian for our approximation. In order to find the value of the effective charge we have fitted the calculated absolute strengths B (E2) of the transitions within the ground state band to the experimental ones. The best agreement is obtained with the value $e_\pi = e_\nu = e = 0.23 eb$, as shown in Table 2. The B (E2) values depend quite sensitively on the wave functions, which suggest that the wave functions obtained in this work are reliable.

E2:MI multipole mixing ratios; In the nucleus, an electromagnetic exchange connecting a state of spin I_1 to I_2 can carry an angular momentum L between $I_1 + I_2$ and $|I_1 - I_2|$. In the rotation-vibration model, pioneered by Bohr and Mottelson (Bohr and Mottelson, 1969), the low-lying, even-parity states of even-even nuclei are ascribed to the collective quadrupole motion of the nucleus as a whole.

The M1-E2 mixing parameter δ is defined as:

$$\delta = \pm \frac{T(E2)}{T(M1)} = \pm \frac{\sqrt{3}}{10} \frac{w}{c} \sqrt{\frac{B(E2|I \rightarrow I')}{B(M1|I \rightarrow I')}} \quad (6)$$

Where the \pm sign must be chosen depending on the relative sign of the reduced matrix element (Helmer, 1994). The electric quadrupole and magnetic dipole transition probabilities T(E2) and T(M1) are, respectively,

Table 3. Calculated and experimental mixing ratios for $^{122-128}\text{Te}$ isotopes.

Isotopes	Spin Parity $I_i^\pi \rightarrow I_f^\pi$	Theory $ \delta(E2/M1) $	Experiment (ENSDF); Küçükburşa and Yörük, 1999. $\delta(E2/M1)$
^{122}Te	$2_2^+ \rightarrow 2_1^+$	1.52	-3.48
	$4_2^+ \rightarrow 4_1^+$	0.37	-0.57
	$2_3^+ \rightarrow 2_1^+$	0.94	----
	$2_3^+ \rightarrow 2_2^+$	1.38	-0.3 < δ < 0.0
	$4_3^+ \rightarrow 4_1^+$	1.27	$1.3^{+0.3}_{-0.4}$
^{124}Te	$2_2^+ \rightarrow 2_1^+$	3.58	-3.55, -3.40
	$4_2^+ \rightarrow 4_1^+$	0.23	-0.18 -0.26
	$2_3^+ \rightarrow 2_1^+$	1.47	$1.5^{+0.6}_{-0.3}$ 0.52
	$2_3^+ \rightarrow 2_2^+$	0.54	0.10 0.23
^{126}Te	$2_2^+ \rightarrow 2_1^+$	3.80	-4.8, $-4.25^{+0.15}_{-0.01}$
	$4_2^+ \rightarrow 4_1^+$	1.63	$0.09 < \delta < 1.8^{+0.7}_{-0.4}$
^{128}Te	$2_2^+ \rightarrow 2_1^+$	3.23	$4.6^{+1.6}_{-1.0}$
	$2_3^+ \rightarrow 2_1^+$	3.18	$4.2^{+2.0}_{-1.0}$
	$3_3^+ \rightarrow 4_1^+$	1.24	1.4
	$3_3^+ \rightarrow 2_2^+$	3.84	$0.45, 4.2^{+2.5}_{-1.2}$

$$T(E2|I \rightarrow I') = \frac{4\pi}{75} \frac{1}{\eta} \left(\frac{w}{c}\right)^5 B(E2|I \rightarrow I')$$

$$T(M1|I \rightarrow I') = \frac{16\pi}{9} \frac{1}{\eta} \left(\frac{w}{c}\right)^3 B(M1|I \rightarrow I') \quad (7)$$

The δ -mixing ratios for some selected transitions in $^{122-128}\text{Te}$ isotopes are calculated from the useful equations as above and with the help of B(EI) and B(MI) values which are obtained from NPBEM (computer code which is subroutine of NPBOS package program) (Otsuka and Yoshida, 1985); the results are given in Table 3. In general, the calculated electromagnetic properties of the tellurium isotopes do not differ significantly from those calculated in experimental and previous theoretical work (Rikovska et al., 1987; Evaluated Nuclear Structure Data File (ENSDF) <http://www.nndc.bnl.gov/ensdf/>)

SUMMARY AND CONCLUSION

In this paper we have carried out an analysis for the even mass tellurium isotopes based on the IBM-2. The boson core parameters have been obtained and the main results for energy levels and quadrupole transition probabilities agree very well with experiment. In general, good agreement was obtained when compared with experiment. The boson-boson interaction parameters were fixed by the calculations on the boson core nuclei. The results indicate that the energy spectra of all different quasibands of the even-even Te isotopes can be reproduced quite well. It is noticed, however, that the results of B (E2) calculations for even-even tellurium nuclei were in better agreement with the existing experimental data. The best fit values for the Hamiltonian parameters for even-even tellurium isotopes are given in Table 1 and the calculated energy values which are compared with the experimental data (ENSDF) <http://www.nndc.bnl.gov/ensdf/> are given in Figures. 2(a)-(b)-(c)-(d) for $^{122-128}\text{Te}$ isotopes. The agreement is good for member of ground state, γ and β bands. The calculated values in this study show that the transitions connect the levels with the same parity and the E2 transitions are predominant.

A sensitive test of our projection is provided by comparing calculated B (E2) values with experimental predictions. The agreement between the values obtained in this analysis and the experimental results is good for ground state band and we hope that if the other parameters are normalized by means of this projection it can be considerably improved for γ -band and β -band for further work.

In this work, we have also examined the mixing ratio $\delta(E2/M1)$ of transitions linking the γ and ground state bands. The transitions which link low spin states and were obtained in the present work are in good agreement and show a little bit irregularities.

This work was supported by the Research Foundation of Kırıkkale University under grant number BAP-2006-06.

REFERENCES

- Bohr A, Mottelson B (1969). Nuclear Structure, Benjamin, New York, (1969 and 1975). Vol. 1 and 2.
- Küçükburca A, Yörük A (1999). IBM-2 calculations on the some even-even tellurium isotopes, Bull. Pure Appl. Sci. 18D: 177.
- Subber A, Hamilton WD, Park P, Kumar K (1987). An application of the dynamic deformation model to the tellurium isotopes, J. Phys. G: Nucl. Phys. 13: 161.
- Voight De, Dudek MJA (1983). High-spin phenomena in atomic nuclei Rev. Mod. Phys. 55: 949.
- Degriek E, Berghe GV (1974). Structure and electromagnetic properties of the doubly even Te isotopes. Nucl. Phys. A231: 141.
- Evaluated Nuclear Structure Data File (ENSDF) <http://www.nndc.bnl.gov/ensdf/>
- Iachello F, Scholten O (1979). Interacting Boson-Fermion Model of Collective States in Odd-A Nuclei. Phys. Rev. Lett. 43: 679.
- Iachello F, Van Isacker P (1991). The Interacting Boson-Fermion Model. Cambridge, Cambridge University Press.
- Yazar HR (2005). Using the Microscopic Background of the IBA-2 Model Electromagnetic Transitions in the Decay of Some Gadolinium Isotopes J. Korean Phys. Soc. October. 47(4): 592-596.
- Yazar HR, Uluer I, (2005). A Correspondence between IBA-1 and IBA-2 Model and Electromagnetic Transitions in Decay of Some Erbium Isotopes. Pramana J. Phys. 65(3): 393.
- Talmi I (1981). Interacting Bose-Fermi System in Nuclei. (ed) Iachello F, New York, Plenum Press pp. 329.
- Bardeen J, Cooper LN, Schrieffer JR (1957). Theory of Superconductivity. Phys. Rev. 108: 1175.
- Rikovska J, Stone NJ, Walkers WB (1987). Dynamical symmetries in even-even Te nuclides. Phys. Rev. C36: 2162-2165.
- Sambataro M (1982). A study of Cd and Te isotopes in the interacting boson approximation. Nucl. Phys. A380: 365.
- Scholten O (1980). PhD dissertation, University of Groningen.
- Scholten O (1985). The Interacting Boson-Fermion Model (ed) A Faessler, Prog. Part. Nucl. Phys. 14: 189.
- RG Helmer (1994). Nuclear Data Sheets 72: 83.
- Otsuka T, Yoshida N (1985). The IBM-2 computer code NPBOS, University of Tokyo.
- Lopac V (1970). Semi-microscopic description of even tellurium isotopes, Nucl. Phys. A155: 513.