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Full Length Research Paper

Alternating current (AC) electroosmotic flow of generalized Maxwell fluids through a circular microtube

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An analytical solution for the time periodic electroosmotic flow (EOF) of the generalized Maxwell model through a circular microtube is presented by solving the linearized Poisson-Boltzmann equation, together with Cauchy momentum equation and General Maxwell constitutive equation. By numerical calculations, we find that for lower Ω and smaller De, classical plug-like profile of Newtonian fluids is reduced. At lower frequency, the flow field can extend to the whole microtube. At higher frequency, however, the velocity amplitude variations away from electric double layer (EDL) almost decrease to zero. In addition, larger De leads to rapid variations of EOF velocity profiles with increased amplitude. The velocity amplitude of microtube is larger than that of plate microchannel by comparison.

Key words: Circular microtube, alternating current (AC) electroosmotic flow, generalized Maxwell fluid, normalized oscillation frequency, relaxation time.

INTRODUCTION

The microfluidics field has drawn increasingly more attention in both academia and industry because of its feasibility and efficiency for controlling flows through microscale devices (Stone et al., 2004). Most substances acquire surface electric charges when in contact with a polar medium. The rearrangement of the charges on the solid surface results in the formation of the electric double layer (EDL) (Bayraktar and Pidugu, 2006). When an electric field is applied tangentially along the charged surface, it will exert a body force on the ions within the EDL. The migration of the mobile ions will carry the adjacent and bulk liquid phase by viscosity, resulting in an electroosmotic flow (EOF). The growing importance to the EOF is due to their operational advantages, like plug flow type flow behaviour, absence of mechanical pumping equipments and better flow control.

Both theoretical and experimental investigations of direct current (DC) steady EOF have been well studied in various micro-capillaries geometric domains (Hunter, 1981). However, such steady EOFs are likely to necessitate relatively larger voltages and field strengths. Recently, time-dependent EOF has been attracting growing attention as an alternative mechanism of microfluidic transport. Keh and Tseng (2001), Chang (2009, 2010, 2012) and Kang et al. (2002) studied the DC transient EOF. Dutta and Beskok (2001), Wang et al. (2007), Chakraborty and Ray (2008) and Jian et al. (2010) studied the AC EOF.

All the above mentioned studies therein are concerned with Newtonian fluids. However, microfluidic devices are usually used to analyze non-Newtonian biofluids. The more general Cauchy momentum equation, instead of the Navier-Stokes equation should be used to describe the constitutive equations of non-Newtonian fluids.

Das and Chakraborty (2006), Chakraborty (2007), Zhao et al. (2008), Vasu and De (2010), Zhao and Yang (2010), Tang et al. (2009) and Deng et al. (2012) studied EOF of power-law fluids. They obtained analytical or numerical solutions of the velocity profiles which depend on the power law index *n*. Recently, the extension to viscoelastic fluids was done by Park and Lee (2008). Afonso et al. (2009) derived an analytical solution for the combined electroosmosis and Poiseuille flow of Phan-Thien and

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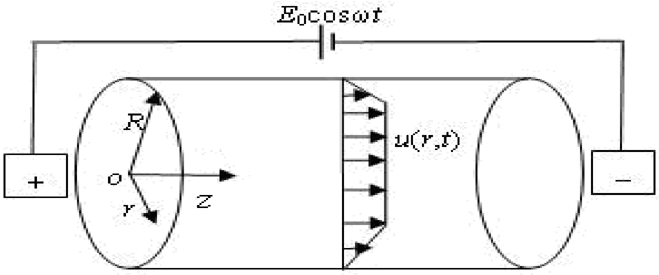

Figure 1. Schematic of AC EOF of the generalized Maxwell fluids in a circular microtube.

Tanner (PTT) and finitely extensible non-linear elastic-Peterlin (FENE-P) models (Bird et al., 1980) in a two-dimensional channel. Liu et al. (2011) and Jian et al. (2011) obtained exact solutions associated with the alternating current (AC) EOF of General Maxwell fluids in a slit microchannel and a two-dimensional rectangular microchannel, respectively. So, this paper extended our previous works to the circular microtube. The rest of this article is organized as follows. The physical description of the problem and the analytical solution to the equations governing the time periodic EOF of the generalized Maxwell model, discussion on the numerical results of the study and the conclusion.

PROBLEM FORMULATION AND ANALITICAL SOLUTIONS

The unsteady EOF of the incompressible generalized Maxwell fluids through a circular microtube of radius r with the z-axis being in the axial direction is shown in Figure 1. The chemical interaction of electrolyte liquid and solid wall generates an EDL, a very thin charged liquid layer at the solid-liquid interface. The EOF is pumped by an axial AC electric field with strength E_0, the liquid inside the EDL sets in motion along z-direction due to electroosmosis. It is assumed that the flow is fully developed in space, only the axial velocity component exists and the radical velocity is zero.

Traditionally, large-sized channels flow is often driven by pressure usually generated by mechanical pumps. In microchannels, however, it becomes increasingly difficult to utilize pressure-driven flow mode as the channel size shrinks, especially down to micro and submicrometer ranges. Provided that the pressure gradient along z-direction is ignored (the two ends are open), the Cauchy momentum equation of laminar flow can be expressed as follows:

$$\rho\frac{\partial u(r,t)}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \rho_e(r)E_z(t) \tag{1}$$

Where $u(r, t)$ is the axial velocity, which is along positive z-direction, ρ is the fluid density, t is time, τ_{yx} is the stress tensor, $\rho_e(r)$ is volume charge density and $E_z(t)$ is AC electric field. Here, the applied electric field strength is greatly smaller than 10^5 V/m, the flow system cannot become chaotic. Because the time scale related to electromigration in the EDL, which is of order 10^{-8}~10^{-7} s (Hsu et al., 1997), is at least two-orders smaller than that associated with the evolution of EOF, which is of order 10^{-5} ~ 10^{-3} s, the transient effect of the EDL can then be neglected. Therefore, we have assumed that the time-dependent EOF does not affect the charge distribution in the EDL. For generalized Maxwell fluids, constitutive equation satisfies (Bird et al., 2001):

$$\tau_{rz} = -\int_{-\infty}^{t}\{\frac{\eta_0}{\lambda_1}\exp[-\frac{(t-t')}{\lambda_1}]\frac{\partial u(t-t')}{\partial r}\}dt' \tag{2}$$

Where λ_1 is the relaxation time, η_0 is the zero shear rate viscosity. In Equation 2, the stress at time t depends on the velocity gradients at all past times t'. However, because of the exponentials in the integrand, greatest weight is given to times t' that are near t. Thus, the fluid "memory" is better for recent times than for more remote times in the past. This phenomenon is called "fading memory" (Bird et al., 2001). Substituting Equation 2 into Equation 1 yields:

$$\rho\frac{\partial u(r,t)}{\partial t} = \int_{-\infty}^{t}\{\frac{\eta_0}{\lambda_1}\exp[-\frac{(t-t')}{\lambda_1}]\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u(r,t')}{\partial r})\}dt' + \rho_e(r)E_z(t) \tag{3}$$

AC electric field and velocity of periodic EOF can be written in complex forms as:

$$u(r,t) = \Re\{u_0(r)e^{i\omega t}\}, \quad E_z(t) = \Re\{E_0e^{i\omega t}\} \tag{4}$$

Where the $\Re\{\}$ denotes the real part of the function, ω is imposed AC electric field oscillating angular frequency. After substitution of Equation 4 into Equation 3, we can write:

$$\rho\Re\{i\omega u_0(r)e^{i\omega t}\} = \Re\{\frac{1}{r}\frac{d}{dr}[r\frac{du_0(r)}{dr}]\int_{-\infty}^{t}\frac{\eta_0}{\lambda_1}\exp[-\frac{(t-t')}{\lambda_1}]e^{i\omega t'}dt'\} + \rho_e(r)\Re\{E_0e^{i\omega t}\} \tag{5}$$

We make the change of variable $s = t - t'$, Equation 5 becomes:

$$\rho\Re\{i\omega u_0(r)e^{i\omega t}\} = \Re\{\frac{1}{r}\frac{d}{dr}[r\frac{du_0(r)}{dr}]e^{i\omega t}\int_{0}^{\infty}\frac{\eta_0}{\lambda_1}e^{-\frac{s}{\lambda_1}}e^{-i\omega s}ds\} + \rho_e(r)\Re\{E_0e^{i\omega t}\} \tag{6}$$

Next, we perform the integration over s:

$$\rho\Re\{i\omega u_0(r)e^{i\omega t}\} = \Re\{\frac{1}{r}\frac{d}{dr}[r\frac{du_0(r)}{dr}](\frac{\eta_0}{1+i\lambda_1\omega})e^{i\omega t}\} + \rho_e(r)\Re\{E_0e^{i\omega t}\} \tag{7}$$

For small values of electrical potential ψ of the EDL, the Debye-Hückel linearization approximation can be applied, which means physically that the electrical potential is small compared with the thermal energy of the charged species. By solving the Poisson-Boltzmann equation subjected to proper boundary conditions, the well-known net charge density distribution for a circular microtube finally can be written as (Li, 2004):

$$\rho_e(r) = -\varepsilon k^2\psi_0\frac{I_0(kr)}{I_0(kR)}, \text{ and } k = (\frac{2n_0{z_v}^2e^2}{\varepsilon k_bT})^{1/2} \tag{8}$$

Where ε is the dielectric constant of the electrolyte liquid, ψ_0 is the zeta potentials on the wall, I_0 are the modified Bessel functions of first kind of order zero, n_0 is the ion density of bulk liquid, z_v is the valence, e is the electron charge, k_b is the Boltzmann constant, T is the absolute temperature and $1/k = \delta$ denotes the EDL thickness.

Substituting Equation 8 into Equation 7, we may now remove the real-operator sign from both sides, as well as the common multiplier

$e^{i\omega t}$, to get:

$$\frac{1}{r}\frac{d}{dr}[r\frac{du_0(r)}{dr}]-[\frac{i\rho\omega(1+i\lambda_1\omega)}{\eta_0}]u_0(r)=\varepsilon k^2\psi_0 E_0\frac{I_0(kr)}{I_0(kR)}\frac{(1+i\lambda_1\omega)}{\eta_0} \tag{9}$$

This equation is subjected to the following boundary conditions:

$$u_0(r)|_{r=R}=0 \tag{10}$$

$$\frac{du_0(r)}{dr}|_{r=0}=0 \tag{11}$$

Equations 9 to 11 govern the EOF of generalized Maxwell fluids inside the EDL near a charged circular wall surface. Introducing the following dimensionless groups:

$$\bar{r}=\frac{r}{R},\ K=kR,\ \bar{u}_0(\bar{r})=\frac{u_0(r)}{U_{eo}},\ U_{eo}=-\frac{\varepsilon\psi_0 E_0}{\eta_0},\ De=\lambda_1\omega,\ \Omega=\frac{\rho\omega R^2}{\eta_0} \tag{12}$$

Where K is called the normalized reciprocal thickness of the EDL denoting the ratio of the radius of microtube to Debye length, U_{eo} denotes steady EOF velocity of Newtonian fluids in a slit microchannel, De is normalized relaxation time and Ω means normalized oscillation frequency. The physical implication of Ω represents the ratio of the diffusion time scale ($t_{\mathrm{diff}}=\rho R^2/\eta_0$) to the period of the applied electric field ($t_E=1/\omega$), based on the kinematic viscosity and the excitation frequency.

Using Equation 12, Equation 9 and corresponding boundary conditions of Equation 10 to 11 are normalized as:

$$\frac{d^2\bar{u}_0(\bar{r})}{d\bar{r}^2}+\frac{1}{\bar{r}}\frac{d\bar{u}_0(\bar{r})}{d\bar{r}}-[i\Omega(1+iDe)]\bar{u}_0(\bar{r})=-K^2\frac{I_0(K\bar{r})}{I_0(K)}(1+iDe) \tag{13}$$

$$\bar{u}_0(\bar{r})|_{\bar{r}=1}=0 \tag{14}$$

$$\frac{d\bar{u}_0(\bar{r})}{d\bar{r}}|_{\bar{r}=0}=0 \tag{15}$$

Equation 13 is a second-order inhomogeneous ordinary differential equation for the complex function $\bar{u}_0(\bar{r})$, and its general solution can be expressed as:

$$\bar{u}_0(\bar{r})=\bar{u}_{0h}(\bar{r})+\bar{u}_{0s}(\bar{r}), \tag{16}$$

Where $\bar{u}_{0h}(\bar{r})$ and $\bar{u}_{0s}(\bar{r})$ are solutions of corresponding homogeneous equation and a special solution of Equation 13, respectively. The homogeneous form of Equation 13 is written as:

$$\frac{d^2\bar{u}_{0h}(\bar{r})}{d\bar{r}^2}+\frac{1}{\bar{r}}\frac{d\bar{u}_{0h}(\bar{r})}{d\bar{r}}-[i\Omega(1+iDe)]\bar{u}_{0h}(\bar{r})=0 \tag{17}$$

and its general solution is:

$$\bar{u}_{0h}(\bar{r})=CI_0[\sqrt{i\Omega(1+iDe)}\bar{r}]+DK_0[\sqrt{i\Omega(1+iDe)}\bar{r}] \tag{18}$$

Where C and D are constants, which can be determined from boundary conditions of Equations 14 and 15. Considering the formation of the right hand side of Equation 13, the special solution can be supposed having the following form:

$$\bar{u}_{0s}(\bar{r})=EI_0(K\bar{r}) \tag{19}$$

Where E is constant. Substituting Equation 19 into Equation 13 yields

$$E\{\frac{d^2I_0(K\bar{r})}{d\bar{r}^2}+\frac{1}{\bar{r}}\frac{dI_0(K\bar{r})}{d\bar{r}}-[i\Omega(1+iDe)]I_0(K\bar{r})\}=-K^2(1+iDe)\frac{I_0(K\bar{r})}{I_0(K)} \tag{20}$$

From the electrical potential equation

$$\frac{d^2\psi(\bar{r})}{d\bar{r}^2}+\frac{1}{\bar{r}}\frac{d\psi(\bar{r})}{d\bar{r}}=K^2\psi(\bar{r})$$

of the EDL, we can obtain easily its general solution

$$\psi(\bar{r})=C_1I_0(K\bar{r})+C_2K_0(K\bar{r}).$$

Therefore, the $I_0(K\bar{r})$ satisfies:

$$\frac{d^2I_0(K\bar{r})}{d\bar{r}^2}+\frac{1}{\bar{r}}\frac{dI_0(K\bar{r})}{d\bar{r}}=K^2I_0(K\bar{r}) \tag{21}$$

Substituting Equation 21 into Equation 20, and equalizing the coefficients in front of the modified Bessel functions I_0 at the two sides of the equation, we have:

$$E=\frac{-K^2(1+iDe)}{\{K^2-[i\Omega(1+iDe)]\}I_0(K)} \tag{22}$$

Inserting Equations 18, 19 and 22 into Equation 16, the normalized complex velocity amplitude of the EOF can be written as:

$$\bar{u}_0(\bar{r})=CI_0[\sqrt{i\Omega(1+iDe)}\bar{r}]+DK_0[\sqrt{i\Omega(1+iDe)}\bar{r}]-\frac{K^2(1+iDe)I_0(K\bar{r})}{\{K^2-[i\Omega(1+iDe)]\}I_0(K)}. \tag{23}$$

Using the boundary conditions of Equations 14 and 15, the constants C and D can be determined as:

$$C=\frac{K^2(1+iDe)}{\{K^2-[i\Omega(1+iDe)]\}I_0[\sqrt{i\Omega(1+iDe)}]},\ D=0 \tag{24}$$

Inserting Equation 24 into Equation 23, the final normalized complex velocity amplitude can be obtained which takes the form

$$\bar{u}_0(\bar{r}) = \frac{K^2(1+iDe)}{\{K^2-[i\Omega(1+iDe)]\}}\{\frac{I_0[\sqrt{i\Omega(1+iDe)}\bar{r}]}{I_0[\sqrt{i\Omega(1+iDe)}]} - \frac{I_0(K\bar{r})}{I_0(K)}\} \quad (25)$$

RESULTS AND DISCUSSION

In the previous section, analytical solutions were derived for the periodic EOF of generalized Maxwell fluids through a circular microtube. They depend greatly on the normalized reciprocal thickness K of the EDL, the normalized oscillation frequency Ω and Deborah number De. This paper has adopted as rheological constitutive equation the generalized linear Maxwell model, which is limited to flows with infinitesimally small displacement gradients. However, it is possible to use this model outside this limitation provided there is very fast fading memory in order to remember the large strain events when $\lambda_{\max}\dot{\gamma} << 1$. That is to say, some limit $\lambda U_{eo}/\delta << 1$ must apply.

We have presented the important results with pertinent dimensionless parameters. However, in practical engineering problems, we also need to mention some typical values of the corresponding dimensional parameters. Some parametric ranges must be determined before we carry out numerical computations. Moreover, the relaxation time should be smaller than the oscillation period (observation time), that is, $\lambda_1<2\pi/\omega$ or De<2π must be satisfied. Ordinarily, the value of EDL thickness δ has the scale of 10^{-7} to 5×10^{-7}m in room temperature. Additionally, the validity of the linearized Poisson-Boltzmann equation is that the wall zeta potential is less than 25 mV. So the scale of the EOF velocity U_{eo} for Newtonian fluid is about 10^{-5} to 2.5×10^{-4} ms^{-1}. From the above condition of $\lambda U_{eo}/\delta << 1$, the valid region of the relaxation time λ_1 is 4×10^{-4} to 5×10^{-2} s, which safely fall into the region of the relaxation time λ_1 [it changes from 10^{-4} to 10^3s (Bird et al., 2001; 1987). Moreover, from the relation $\lambda_1<2\pi/\omega$, we can evaluate the scope of external electric field angular frequency, which changes from 40 to 5 π × 10^3 rad.s^{-1}. In the following calculations, typical parameters can be taken as follows (Goswami and Chakraborty, 2009): $\rho \approx 10^3$ kg.m^{-3}, $\eta_0 \approx 2 \times 10^{-3}$ kg.m^{-1}s^{-1}, R ≈ 100 µm. Thus, the normalized oscillation frequency Ω can be evaluated from 0.2 to 25 π and the normalized reciprocal thickness K of the EDL changes from 10 to 100, which coincides with the assumption of thin EDL.

When K = 20, R = 100 µm, $\eta_0/\rho = 2 \times 10^{-6}$ m^{2}s^{-1}, Figure 2 shows normalized EOF velocity amplitudes of generalized Maxwell fluids across a circular microtube with several De (0.2, 0.5, 0.8, 1.0 and 1.5) for different Ω = 0.2 π, π, 5 π, 10 π, 15 π, 25 π (those correspond to their dimensional counterparts f = 20 Hz, 100 Hz, 500 Hz, 1 kHz, 1.5 kHz, 2.5 kHz), respectively. It can be noted from Figure 2 that as expected, for lower Ω and smaller De, classical plug-like Helmholtz-Smoluchowski EOF velocity is reduced (Figure 2a and b). Moreover, the velocity magnitude is almost the same with that of Helmholtz-Smoluchowski EOF velocity.

For prescribed normalized relaxation time De, increasing frequency Ω leads to wave-like EOF velocity profiles no matter whether the frequency is large or not. At the same time, the amplitudes of the EOF velocity decrease gradually. At lower frequencies, the flow field can extend to the whole microtube due to the same scale of the diffusion time and the oscillation time period (Figure 2c to d). At higher frequencies, however, the EOF velocity amplitude variations are restricted only within a thin layer near the solid surface and the velocity away from the EDL almost decreases to zero (Figure 2e to f). The reason is that the diffusion time scale is much greater than the oscillation time period. Therefore, there is no sufficient time for the flow momentum to diffuse far into the center of the microtube.

From Figure 2, it can be observed that for fixed frequency Ω, the velocity amplitude increases with normalized relaxation time De whenever frequency Ω is large or small. Physically, the reason is that if the De number is large, polymer molecules that are distorted by the flow will not have time to relax during the time scale of the oscillation. Further increasing De will lead to the flow process happens so fast that the polymer molecules have no time to change configuration, and the fluid behaves more and more as a Hookean elastic solid.

Figure 3 compares the normalized EOF velocity amplitudes of generalized Maxwell fluids between the plate microchannel and the microtube for fixed De and Ω numbers (K = 20). From Figure 3, we found that the velocity amplitude of microtube is larger than that of plate microchannel.

Conclusions

An analytical solution of the time periodic EOF of the general Maxwell fluids through a circular microtube under the Debye-Hückel approximation is presented in this work. The computational results show that the velocity profiles of the General Maxwell fluids depend greatly on the normalized oscillation frequency Ω and Deborah number De. For lower Ω and smaller De, classical plug-like Helmholtz-Smoluchowski EOF velocity is reduced. For given De, increasing frequency Ω leads to wave-like EOF velocity profiles with decreased amplitude. At lower frequencies, the flow field can extend to the whole microtube. At higher frequencies, however, the EOF velocity amplitude variations are restricted only within EDL. It still can be observed that for fixed frequency Ω, the velocity amplitude increases with normalized relaxation time De. In addition, the velocity amplitude of microtube is larger than that of plate microchannel.

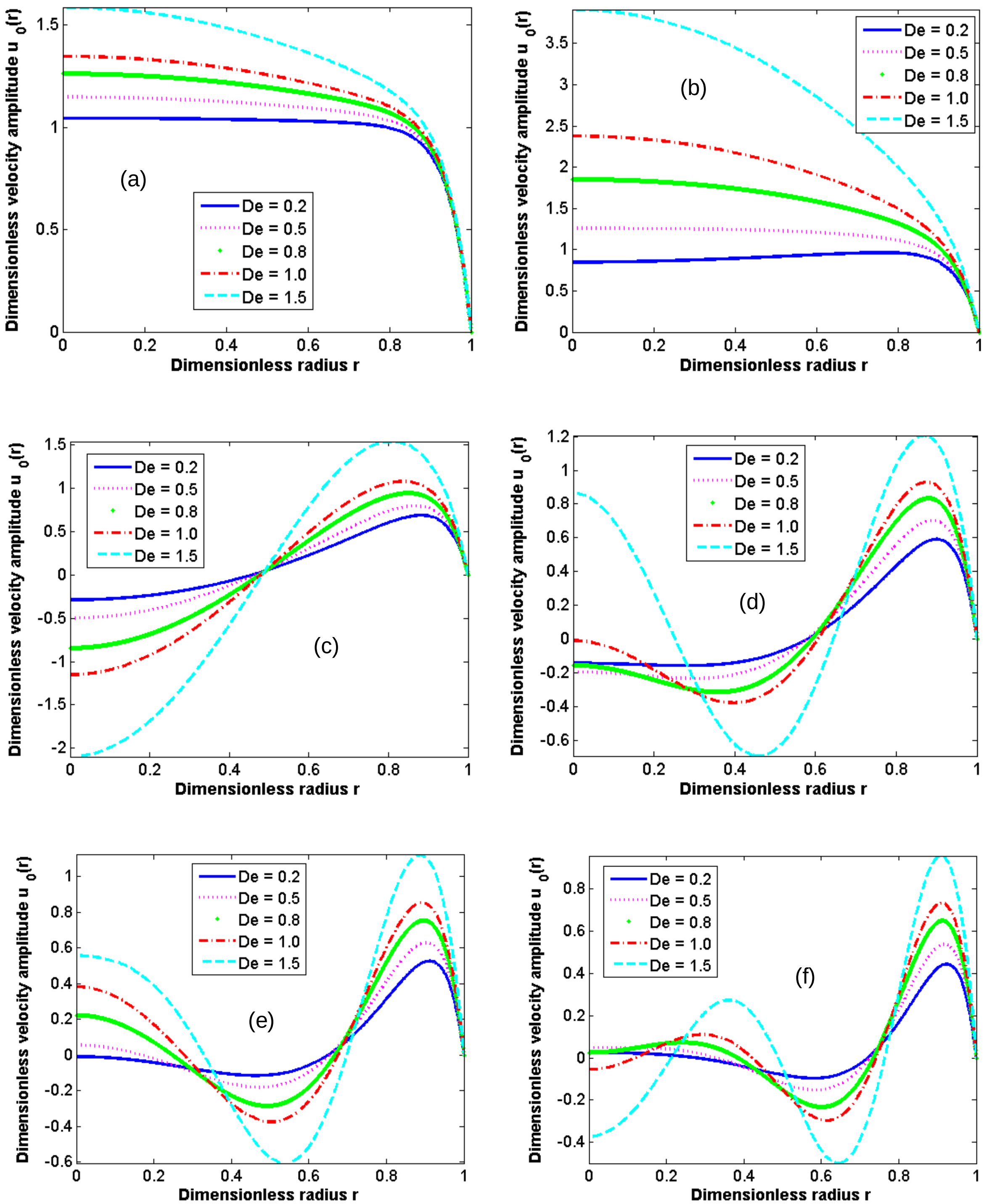

Figure 2. Normalized EOF velocity amplitudes of generalized Maxwell fluids across the microtube with several De numbers for different Ω (*K* = 20). (a) Ω = 0.2 π, (b) Ω = π, (c) Ω = 5 π, (d) Ω = 10 π, (e) Ω = 15 π, (f) Ω = 25 π.

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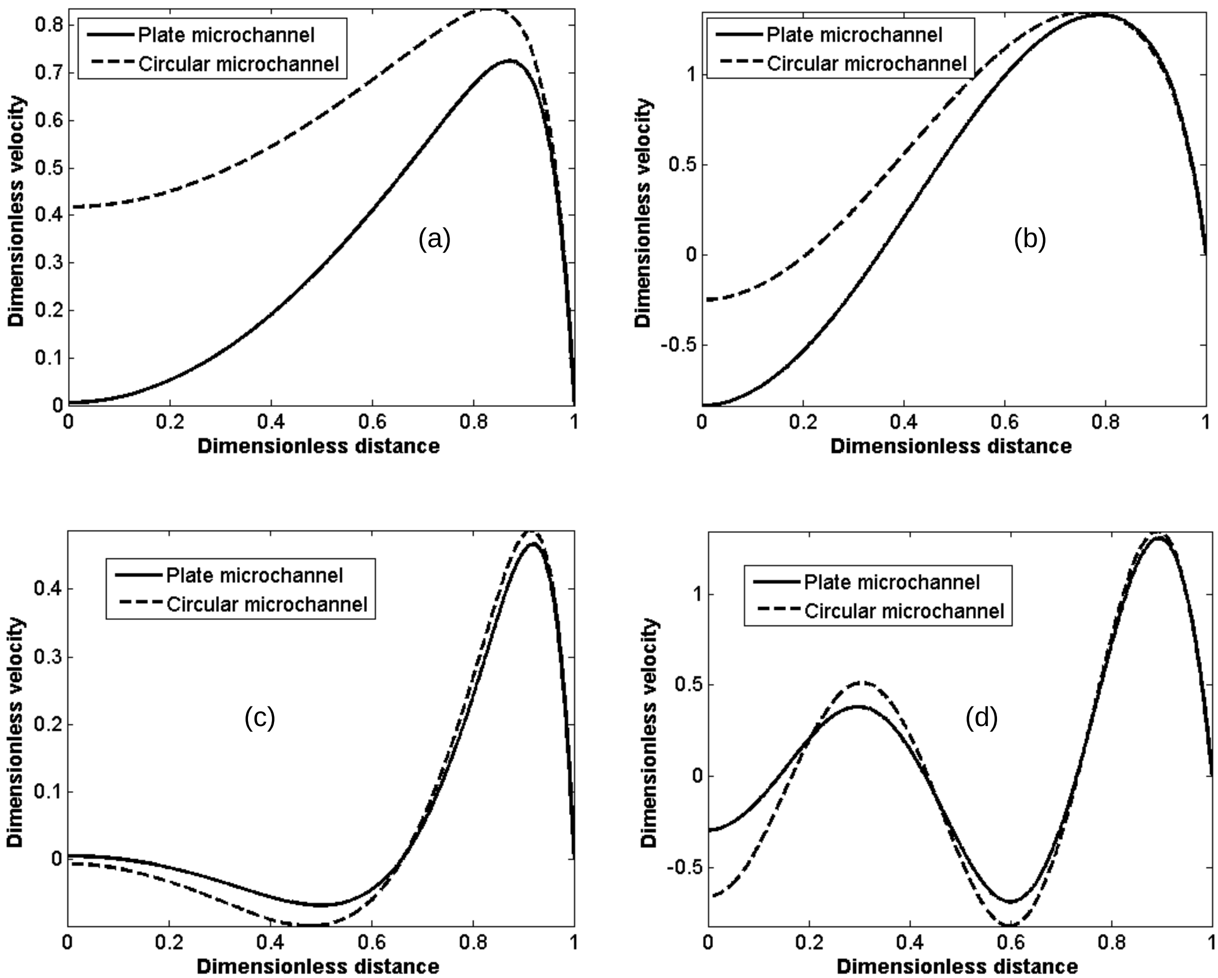

Figure 3. Comparison of the normalized EOF velocity amplitudes of generalized Maxwell fluids between the plate microchannel and the microtube (*K* = 20). (a) De = 0.1, Ω = 5, (b) De = 2, Ω = 5, (c) De = 0.1, Ω = 50, (d) De = 2, Ω = 50.

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