Short Communication

On improving the optimization process of a manufacturing run time problem with defective and machine breakdown

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This research note improves the optimization process of a manufacturing run time problem studied by Chiu et al. (2011) by presenting a direct proof of convexity of the long-run average cost function of such a problem. It can be used to replace Theorem 1 on conditional convexity given by Chiu et al. enhance quality of optimization process, and eliminate the computational efforts in verifying the conditional convexity.

Key words: Optimization process, convexity, manufacturing, replenishment run time, breakdown, production control.

INTRODUCTION

Manufacturing run time problem considering random machine breakdown and/or scrap rate have been studied extensively (Groenevelt et al., 1992; Makis and Fung, 1998; Law and Wee, 2006; Chiu et al., 2007; Lin et al., 2008; Chen et al., 2010; Chiu et al., 2010; Cheng and Ting, 2010; Chiu and Ting, 2010; Pai and Ting, 2011; Ting et al., 2011). In a recent paper, Chiu et al. (2011) studied a manufacturing run time problem with defective rate and random machine breakdown. They derived the long-run average cost per unit time, and proposed a theorem on conditional convexity of the cost function in their optimization process. For the purpose of improving quality of their optimization process, we reexamine and present a direct proof of convexity of cost function for such a specific manufacturing run time problem. To ease readability, the following notation used in this note is the same as that in Chiu et al. (2011).

P = production rate (items per unit time),
λ = demand rate (items per unit time),

x = a random defective rate, x is a random variable with known probability density function,
β = number of breakdowns per year, a random variable that follows the Poisson distribution,
M = cost for repairing and restoring the machine,
K = setup cost for each production run,
C = production cost per item ($/item, inspection cost per item is included),
C_s = disposal cost for each scrap item ($/item),
H = holding cost per item per unit time ($/item/unit time),
t = production time before a random breakdown occurs,
t_1 = the optimal production run time (that is, production uptime) to be determined,
g = a constant time needed to repair and restore the machine.
T = cycle length whether a machine breaks down or not,
$TCU(t_1)$ = the total production-inventory costs per unit time whether a breakdown takes place or not,
$E[TCU(t_1)]$ = the expected total inventory costs per unit time whether a breakdown takes place or not.

Recall numerical expressions for $E[TCU(t_1)]$ and $w(t_1)$ as follows (from Equations (20) and (21) in Chiu et al. (2011), respectively).

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$$E[TCU(t_1)] = \left[\frac{1}{1-E[x]}\right]\left[\frac{K\lambda\beta}{P(1-e^{-\beta t_1})}+\frac{M\lambda\beta}{P}+C\lambda+C_sE[x]\lambda\right]+\left[\frac{-t_1e^{-\beta t_1}-\frac{1}{\beta}e^{-\beta t_1}+\frac{1}{\beta}}{[1-E[x]](1-e^{-\beta t_1})}\right][\delta]$$

where $\delta = h[(P-\lambda)[1-2E[x]]+PE[x^2]+\lambda E[x]g\beta]$

(1)

$$w(t_1) = \left[\frac{K\lambda\beta}{P\delta}+\frac{2(1-e^{-\beta t_1})}{\beta(1+e^{-\beta t_1})}\right]$$

(2)

MATERIALS AND METHODS

Proof of convexity of the long-run cost function

First, one obtains the first and second derivatives of E[$TCU(t_1)$] as follows (from Equation (1)).

$$\frac{d\,E[TCU(t_1)]}{dt_1} = \frac{e^{-\beta t_1}}{[1-E[x]](1-e^{-\beta t_1})^2}\left[\frac{K\lambda(-\beta^2)}{P}+[\beta t_1+e^{-\beta t_1}-1]\cdot\delta\right]$$

(3)

$$\frac{d^2\,E[TCU(t_1)]}{dt_1^2} = \frac{\beta e^{-\beta t_1}(1+e^{-\beta t_1})}{[1-E(x)](1-e^{-\beta t_1})^3}\left\{\left[\frac{K\lambda\beta^2}{P}+\frac{2\delta(1-e^{-\beta t_1})}{(1+e^{-\beta t_1})}\right]-\beta\delta t_1\right\}$$

(4)

In Equation (4), because the mean time between breakdowns $1/\beta$ is exponentially distributed with density function $f(t) = \beta e^{-\beta t_1}$ and cumulative density function $F(t) = 1-e^{-\beta t_1}$; as β and $t_1 > 0$, the first term of Equation (4) is greater than zero. This is

$$\frac{\beta e^{-\beta t_1}(1+e^{-\beta t_1})}{[1-E(x)](1-e^{-\beta t_1})^3} > 0$$

(5)

Therefore, for the second derivative of E[$TCU(t_1)$] to be greater than zero, the second term of Equation (4) must be greater than zero. That is

if $\left[\frac{K\lambda\beta^2}{P}+\frac{2\delta(1-e^{-\beta t_1})}{(1+e^{-\beta t_1})}\right]-\beta\delta t_1 > 0$

(6)

or

if $\left[\frac{K\lambda\beta}{P\delta}+\frac{2(1-e^{-\beta t_1})}{\beta(1+e^{-\beta t_1})}\right]-t_1 > 0$

(7)

As β and $t_1 > 0$, $0 < e^{-\beta t_1} < 1$, so $1 < (1+e^{-\beta t_1}) < 2$, hence $\{2/(1+e^{-\beta t_1})\} > 1$; from Equation (7), one has

$$w(t_1) = \left[\frac{K\lambda\beta}{P\delta}+\frac{2(1-e^{-\beta t_1})}{\beta(1+e^{-\beta t_1})}\right] > \left[\frac{K\lambda\beta}{P\delta}+\frac{(1-e^{-\beta t_1})}{\beta}\right]$$

(8)

In Equation (8), let $y = K\lambda/P\delta$, one has

$$w(t_1) > \left[y\beta+\frac{(1-e^{-\beta t_1})}{\beta}\right]$$

(9)

For deriving the optimal run time, one would set the first derivative of E[$TCU(t_1)$] (Equation (3)) equal to zero (Hillier and Lieberman, 2001; Lin et al., 2008; Nahmias, 2009; Ting et al., 2011), and

because the first term of Equation (3) is greater than zero, this implies

$$\frac{K\lambda(-\beta^2)}{P}+[\beta t_1+e^{-\beta t_1}-1]\delta = 0$$

(10)

or

$$\beta t_1+e^{-\beta t_1} = 1+\frac{K\lambda\beta^2}{P\delta}$$

(11)

Substituting $y = K\lambda/P\delta$, one has

$$\beta t_1+e^{-\beta t_1} = 1+\beta^2 y$$

(12)

or

$$t_1 = \frac{(1-e^{-\beta t_1})}{\beta}+y\beta$$

(13)

Incorporating Equations (9) and (13), one has

$$w(t_1) > \left[y\beta+\frac{(1-e^{-\beta t_1})}{\beta}\right] = t_1$$

(14)

One verifies that $\beta t_1+e^{-\beta t_1}$ (refer to Equation (11)) is monotone with respect to $t_1 > 0$ and $w(t_1) > t_1$ (Equation (14)). Condition of Equation (7) holds, hence the second derivative of the E[$TCU(t_1)$] must be positive at the stationary point. That is

$$\frac{d^2\,E[TCU(t_1)]}{dt_1^2} = \frac{\beta e^{-\beta t_1}(1+e^{-\beta t_1})}{[1-E(x)](1-e^{-\beta t_1})^3}\left\{\left[\frac{K\lambda\beta^2}{P}+\frac{2\delta(1-e^{-\beta t_1})}{(1+e^{-\beta t_1})}\right]-\beta\delta t_1\right\} > 0$$

(15)

This concludes the proof of convexity of the long-run average cost function for such a specific production run time problem.

NUMERICAL EXAMPLE AND VERIFICATION

Here, the same example as in Chiu et al. (2011) was adopted to verify numerically the proposed proof. The following are values of related parameters in the example:

P = production rate, 10000 units per year,
λ = annual demand rate 4000 units,
x = defective rate which follows uniform distribution over the interval [0, 0.1],
β = number of breakdown that follows a Poisson distribution with mean $\beta = 0.5$ times per year,
M = repair cost $500 for each breakdown,
K = setup cost $450 for each production run,
C = $2 per item (inspection cost per item is included),
C_s = $0.3 disposal cost for each scrap item,
h = $0.6 per item per unit time,
g = 0.018 years, a constant time needed to repair and restore the machine.

To demonstrate the second derivative of $E[TCU(t_1)]$ is greater than zero, one uses the resulting $t_1^* = 0.3418$ (years) given by Chiu et al. (2011) to verify if Equation (14) holds. As a result, $[w(t_1^*) = 0.3686] > [t_1^*=0.3418]$. One finds that $[w(t_1^*) - t_1^*] > 0$, hence Equation (7) holds. With extra computational efforts, one can also show the following result from Equation (4):

$$\frac{d^2 E\left[TCU\left(t_1\right)\right]}{dt_1^2} = (200.3421)*(43.6475) = 8744.4312 > 0$$

Numerical verification is accomplished and the long-run average cost function $E[TCU(t_1)]$ is convex.

RESULT AND CONCLUSIONS

This research note improves the optimization process of a manufacturing run time problem studied by Chiu et al. (2011) by presenting a direct proof of convexity of the long-run average cost function of such a problem. For practitioners in the field who would like to adopt Chiu et al.'s solution procedure for solving the practical production run time problem, computational efforts in verifying conditional convexity (that is, Theorem 1 and Table 1 in Chiu et al. (2011)) can now be totally eliminated. This proof can be used to replace Theorem 1 given by Chiu et al. and enhance quality of the optimization process for such a realistic problem.

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