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Full Length Research Paper

Five-dimensional φ^4 field theory at finite temperature in the symmetric and broken phases

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We examine the behavior of φ^4 theory in five dimensions. We provide the effective potential for the symmetric and broken symmetry phase. Our results suggests that due to the presence of an infinite flat extra dimension, the transition from the broken phase to the symmetric case can be delayed with respect to the four dimensional case. We also find that the phase transition at one-loop cannot be of the first-order.

Key words: Extra dimension, scalar fields, phase transition.

INTRODUCTION

Extra dimension is an important subject in the realm of theoretical physics and they are main ingredients of unified theories, like string theory. Large and infinite extra dimensions can lower the scale of grand unification theories (Arkani-Hamed et al., 1999; Rubakov, 2001). And if we are lucky, we may find evidence of such extra dimensions in Large Hadron Collider. So it is useful to study field theories in higher dimensions.

In a seminal paper, Dolan and Jackiw (1974) presented a comprehensive account of temperature effects in variety of four dimensional field theories. In Dienes et al. (1999), an authoritative account of the subject matter is given in higher dimensions with emphasis on compact dimensions and applications to string theory. In a more recent work (Ansari and Suresh, 2007) a simple scalar theory has been formulated in five dimensions.

But there are several issues that deserve further investigation. First of all in the treatment (Dienes et al., 1999) of the effect of an extra dimension which is compactified on a circle is to contribute a Kaluza-Klein tower of particles. And for the φ^4 theory, their effective potential is the sum of a four dimensional one loop Coleman–Weinberg effective-potential and the correction due to Kaluza–Klein modes. Hence, they consider the effective potential as observed from our familiar four dimensional world.

The second issue is the discussion of symmetry restoration at high temperature. Some authors (Dolan and Jackiw, 1974; Ansari and Suresh, 2007) practically start from a symmetric phase. But some others (Kirzhnits and Linde, 1976; Linde, 1979) start the discussion from a broken symmetry phase. We provide arguments in favor of the latter approach.

The third issue is the correct computation of the effective potential in a fully five-dimensional theory and its implications. In Ansari and Suresh (2007), they consider the φ^4 theory in five dimensions, with signature of the metric (+,–,–,–,–). Even though the treatment of the subject matter in this paper is fully five dimensional, but their expressions for the zero temperature correction of the effective potential as well as their finite temperature correction of the effective potential are not correct. And finally, we are not aware of any study of φ^4 theory in the low temperature limit in higher dimensions.

The plan of this paper is as follows: the effective potential at zero temperature for a φ^4 theory was discussed in five dimensions. At one loop level, we utilized different methods to obtain the effective potential for the broken symmetry phase and the symmetric phase, after which we considered the effective potential at finite

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temperature. In this study, it was observed that at high temperature, the effective potential V_{eff} (φ_c) has no odd power of φ_c, though the critical temperature for the phase transition was obtained from a broken symmetry phase to the symmetric phase. We found that in a fully five dimensional theory, the presence of extra dimension can delay this phase transition. Also, due to lack of the presence of odd powers of φ_c in the effective potential, the first order phase transition is prohibited at one-loop level. Finally, this study's conclusions are presented and technical details are given in the appendices.

EFFECTIVE POTENTIAL AT ZERO TEMPERATURE

In the symmetric phase, the potential at tree level is:

$$V = \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!M}\varphi^4, \quad (1)$$

where m^2 and λ are positive. The coupling constant acquires dimension in 5D, hence it is divided by M to make it dimensionless. In the effective potential formalism, a classical field φ_c is defined as the vacuum expectation value of the field operator in the presence of a source (Peskin and Schroder, 1995; Ansari and Suresh, 2007). The effective potential V_{eff} (φ_c) is the expectation value of the energy density in a certain state for which the expectation value of the field is φ_c (Coleman, 1985). The effective potential at one loop level is:

$$V_{eff} = V_{eff}^0 + \hbar V_{eff}^1, \quad (2)$$

where

$$V_{eff}^0(\varphi_c) = \frac{1}{2}m^2\varphi_c^2 + \frac{\lambda}{4!M}\varphi_c^4, \quad (3)$$

and

$$V_{eff}^1(\varphi_c) = \frac{1}{2}\int\frac{d^5k_E}{2\pi^5}\ln(k_E^2 + m^2 + \frac{\lambda}{2M}\varphi_c^2). \quad (4)$$

There are several ways to carry out the calculations. One of such method is by differentiating the divergent integral with respect to the external momentum (Yang and Ni, 1995). This method can be extended if we differentiate with respect to some mass parameter. Let,

$$M^2 = m^2 + \frac{\lambda}{2M}\varphi_c^2. \quad (5)$$

Then, we differentiate V_{eff}^1 with respect to M^2 three times. The result is:

$$\frac{\partial^3 V_{eff}^1}{\partial(M^2)^3} = \int\frac{d^5k_E}{(2\pi)^5}\frac{1}{(k_E^2+M^2)^3} = \frac{1}{64\pi^2 M} \quad (6)$$

Next, we integrate the aforementioned result with respect to M^2 three times and we obtain:

$$V_{eff}^1 = \frac{1}{120\pi^2}M^5 + \frac{c_1}{2}M^4 + c_2M^2 + c_3 \quad (7)$$

Hence, the effective potential at one loop level is:

$$V_{eff} = \frac{1}{2}m^2\varphi_c^2 + \frac{\lambda}{4!M}\varphi_c^4 + \frac{1}{120\pi}M^5 + \frac{C_1}{2}M^4 + C_2M^2 + C_3 \quad (8)$$

The constants C_1, C_2 and C_3 are determined by imposing some appropriate normalization conditions (Linde, 1979). For the symmetric phase by requiring:

$$V_{eff}\big|_{\eta=0} = 0, \quad \frac{d^2V_{eff}}{d\varphi_c^2}\Big|_{\eta=0} = m^2, \quad \frac{d^4V_{eff}}{d\varphi_c^4}\Big|_{\eta=0} = \frac{\lambda}{M}, \quad where\ \eta = \varphi_c. \quad (9)$$

We find

$$C_1 = -\frac{m}{32\pi^2}, \qquad C_2 = \frac{m^2C_1}{3}, \qquad C_3 = \frac{m^4C_1}{10}. \quad (10)$$

Therefore, the renormalized effective potential, at zero temperature for the symmetric phase is:

$$V_{eff}(\varphi_c) = \frac{1}{2}m^2\varphi_c^2 + \frac{\lambda}{4!M}\varphi_c^4 + \frac{\hbar}{24\pi^2}[\frac{M^5}{5} - \frac{3mM^4}{8} + \frac{m^3M^2}{4} - \frac{3m^5}{40}] \quad (11)$$

In Appendix I, we utilize another method and we obtain the same expression for the effective potential of the symmetric case. In the broken symmetry phase, the potential at tree level is:

$$V = -\frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!M}\varphi^4, \quad (12)$$

where again m^2 and λ are positive. This potential has a local maxima at φ = 0 and two minima at $\varphi^2 = \frac{6m^2M}{\lambda} = \sigma^2$. The computation of effective potential for the broken symmetry phase is similar to that of symmetric phase. However, for the broken phase $M^2 = -m^2 + \frac{\lambda}{2M}\varphi_c^2$. The effective potential for this phase at one loop level is:

$$V_{eff} = -\frac{1}{2}m^2\varphi_c^2 + \frac{\lambda}{4!M}\varphi_c^4 + \frac{1}{120\pi^2}M^5 + \frac{E_1}{2}M^4 + E_2M^2 + E_3. \qquad (13)$$

The constants E_1, E_2 and E_3 are determined by imposing some appropriate normalization conditions (Linde, 1979). For the broken symmetry phase, we require:

$$V_{eff}\Big|_{\eta=\sigma} = -\frac{3m^4M}{\lambda}, \qquad \frac{dV_{eff}}{d\varphi_c}\Big|_{\eta=\sigma} = 0, \qquad \frac{d^2V_{eff}}{d\varphi_c^2}\Big|_{\eta=\sigma} = 2m^2. \qquad (14)$$

Our final result for the effective potential at zero temperature for the broken symmetry phase is:

$$V_{eff}(\varphi_c) = -\frac{1}{2}m^2\varphi_c^2 + \frac{\lambda}{4!M}\varphi_c^4 + \frac{\hbar}{24\pi^2}\left[\frac{M^5}{5} - \frac{3\sqrt{2}mM^4}{8} + \frac{\sqrt{2}m^3M^2}{4} - \frac{21\sqrt{2}m^5}{20}\right] \qquad (15)$$

EFFECTIVE POTENTIAL AT FINITE TEMPERATURE

A common method to study the temperature effect in quantum field theory is imaginary time formalism (Quiros, 1999). Essentially, it amounts to replacing a fifth Euclidian momenta k_5 by a discrete w_n and integration by summation. This method has been extended to five dimension in Dienes et al. (1999). One loop correction of the effective potential in this formalism in five dimension is (Ansari and Suresh, 2007):

$$V_{eff}(\varphi_c) = \int \frac{d^4k}{(2\pi)^4}\left[\omega + \frac{1}{\beta}\ln(1-\exp(-\beta\omega))\right], \qquad (16)$$

$$\beta = \frac{1}{k_BT}, and$$

$$\omega^2 = K^2 + M^2, \qquad K^2 = k_1^2 + k_2^2 + k_3^2 + k_4^2 \qquad (17)$$

The first term of Equation 16 corresponds to the zero temperature one-loop correction and has been computed in effective potential at zero temperature. We denote the second term by $V_\beta^1(\varphi_c)$ which is:

$$V_\beta^1(\varphi_c) = \frac{1}{\beta}\int\frac{d^4k}{(2\pi)^4}\ln(1-\exp(-\beta\omega)), \qquad (18)$$

Effective potential at high temperature

We evaluate $V_\beta^1(\varphi_c)$ in the high temperature limit $M\beta <<$ 1, the result is (Appendix II):

$$V_\beta^1(\varphi_c) = -\frac{3\varsigma(5)}{4\pi^2\beta^5} + \frac{\varsigma(3)M^2}{8\pi^2\beta^3} - \frac{3M^4}{128\pi^2\beta} + \frac{M^4\ln(M\beta)}{32\pi^2\beta} - \frac{M^5}{120\pi^2} + \frac{M^6\beta}{2304\pi^2} + O(M^8\beta^3). \qquad (19)$$

Now by the addition of Equation 11 and Equation 19, we will have the complete one-loop effective potential for the symmetric case and by addition of Equations 15 and 19, we will have the complete one-loop effective potential for the broken symmetry case. At this high temperature limit, we see that the effective potential does not have any odd power of M.

Effective potential at low temperature

The general expression for the effective potential from Equation 40 of Appendix II is:

$$V_\beta^1(\varphi_c) = \frac{1}{8\pi^2\beta}\int_M^\infty(\omega^3 - \omega M^2)d\omega\ln[1-\exp(-\beta\omega)]. \qquad (20)$$

At the low temperature limit, namely when $M\beta >>$ 1, we have:

$$\ln[1-\exp(-\beta\omega)] = \sum_{n=1}^\infty \frac{\exp(-\beta\omega n)}{n}. \qquad (21)$$

Upon substituting the above expression in Equation 20, we find:

$$V_\beta^1(\varphi_c) = \frac{1}{8\pi^2\beta^5}\sum_{n=1}^\infty\left[\frac{\Gamma(4, n\beta M)}{n^5} - \frac{M^2\beta^2\Gamma(2, n\beta M)}{n^3}\right], \qquad (22)$$

where the incomplete Gamma function Γ(a, x) is defined by:

$$\Gamma(a,x) = \int_x^\infty t^{a-1}\exp(-t)dt \qquad (23)$$

By using the asymptotic expansion of the incomplete gamma function (Abramowitz and Steugun, 1972) for very large values of x, namely:

$$\Gamma(a,x) \sim x^{a-1)} \exp(-x)[1+\frac{a-1}{x}+\frac{(a-1)(a-2)}{x^2}+.....] \quad (24)$$

The leading contribution to the temperature dependent effective potential for very large values of $M\beta$ is:

$$V_\beta^1(\varphi_c) = \frac{M^2}{4\pi^2\beta^3}\exp(-\beta M). \quad (25)$$

This expression shows that in this region of very low temperature and for the broken symmetry phase, the temperature correction of the effective potential is exponentially suppressed.

SYMMETRY RESTORATION AT HIGH TEMPERATURE

Having discussed the effective potential at various cases, we are in a position to discuss symmetry restoration at high temperature. The main point is that one has to start from a broken symmetry phase. Some authors (Dolan and Jackiw, 1974; Ansari and Suresh, 2007) start from a symmetric phase and they carry the calculation effective potential for zero temperature at φ_c = 0. And then use an imaginary mass to find the critical temperature or the thermal mass. The shortcoming of this approach is that they are faced by meaningless mathematical expressions. For instance, in the work of Ansari and Suresh (2007), their zero temperature one-loop correction becomes imaginary and hence meaningless. However, in the work of Dienes et al. (1999) Kirzhnitz and Linde (1976) and Linde (1979), they discuss the symmetry restoration in a proper framework. Now, we start from the broken phase of our theory, at zero temperature we have two distinct minima and by choosing one of them as the vacuum, the symmetry is spontaneously broken. At high temperature, we see that the magnitude of coefficient of the quadratic term in the effective potential is a decreasing function of the temperature. At the critical temperature this coefficient vanishes.

In the leading order of the temperature we have:

$$-m^2 + \frac{\varsigma(3)\lambda}{8\pi^2\beta_c^3 M} = 0. \quad (26)$$

From this expression, one can obtain the value of critical temperature T_c. One can calculate the position of the minima of the effective potential before phase transition in the leading order form $\frac{dV_{eff}}{d\varphi_c} = 0$. We obtain:

$$[-m^2 + \frac{\varsigma(3)\lambda}{8\pi^2\beta^3 M}] + \frac{\lambda}{6M}\varphi_c^2 = 0. \quad (27)$$

So, at high temperature, the two minima of the broken phase get closer to the origin. The broken phase has three extremum points, two minima and one local maxima at the origin. At the critical temperature, these three points coincides. But as the potential is bounded, from below the resultant point is a global minima, and symmetry is restored.

To compare with the result of the four dimensional world from Dolan and Jackiw (1974), we have:

$$(T_c)_{D=5} = \sqrt{\frac{24m^2}{\lambda}}. \quad (28)$$

And from Equation 26, we have:

$$(T_c)_{D=5} = [\frac{8\pi^2 m^2 M}{\varsigma(3)\lambda}]^{1/3} \quad (29)$$

Hence, $(T_c)_{D=5}$ is dependent on fundamental mass scale. Furthermore, assuming the parameter λ and the value of the parameter m to be the same in both cases we found:

$$\frac{(T_c)_{D=5}}{(T_c)_{D=4}} = 0.82[\frac{\sqrt{\lambda M}}{m}]^{1/3} \quad (30)$$

Therefore, for large values of M, one can delay the phase transition. Similar result in the context of compact extra dimension has been obtained by Dienes et al. (1999).

In four dimension, the presence of a term proportional to $\frac{M^3}{\beta}$ creates a barrier which separates the local maxima of the scalar potential from a local minima at the origin.

The phase transition to the symmetric phase thus proceed by bubble nucleation and it is of first order (Dolan and Jackiw, 1974).

Our calculation of effective potential shows that there is no odd power of M in the effective potential as the M^5 terms from the zero temperature correction and from V_β^1 exactly cancel. Therefore, the presence of an infinite extra dimension prevents the occurrence of a first order phase transition. In fact in the work of Dienes et al. (1999), it was shown that for any dimension D > 4 the phase transition is not first order.

Conclusion

A relevant physical process pertinent to the earlier discussion is the cooling of the universe. Presumably, the universe starts at high temperature with high degree of symmetry. As the universe cools down, we enter the present low temperature universe which is in the broken symmetry phase. The vacuum expectation value of the field φ is zero and after the phase transition, this field acquires a non-zero vacuum expectation value. But the ordinary field theory is formulated at zero temperature. By using finite temperature field theory, we can find deviations with respect to the zero temperature case. So, we are forced to study the cosmological phase transition of the universe in the reverse order, namely, from the low temperature to the high temperature or from a broken symmetry phase to a symmetric case. Finite temperature field theory is a sophisticated subject. Dolan and Jackiw (1974) used advanced techniques, such as dimensional regularization to compute the effective potential in four dimensions.

However, we found that their method is only suitable for even dimensions. We found that in odd dimensions $D = 2N + 1$ with $N \geq 2$, the method of calculations of the effective potential is different from that of even dimensions. And we have demonstrated this fact in Appendix II for the case of $D = 5$.

On physical ground, our calculations predict a delayed cosmological phase transition, and the absence of odd powers of φ_c prevents this phase transition to be of the first-order. We know that as the universe cools down and after the phase transition, domain walls forms. It will be of interest to study the structure of these domain walls within the framework of higher dimensional scalar field theories. It is also desirable to include the effect of fermions. Another avenue for further work is to include the effect of gravity. These and other related issues are presently under considerations.

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APPENDIX

Appendix I: Regularization of five-dimensional φ^4 theory

Ansari and Suresh (2007) consider φ^4 theory in higher dimensions. But their expressions for the counter terms as well as their expression for the one loop correction of the effective potential is not correct. In addition, they introduce a cut off Λ on all loop momenta and they integrate Equation 4 and obtain:

$$V_{eff}^1(\varphi_c)=\frac{\hbar}{24\pi^3}[-\frac{2\Lambda^5}{25}+\frac{2}{15}\Lambda^3M^2-\frac{6}{15}\Lambda M^4+\frac{3}{15}iM^5\ln(\frac{M-i\Lambda}{M+i\Lambda})+\frac{3}{15}\Lambda^5\ln(M^2+\Lambda^2)]$$

But

$$\ln(\frac{M-i\Lambda}{M+i\Lambda})=\ln\frac{\rho e^{i\theta_1}}{\rho e^{i\theta_2}}=i(\theta_1-\theta_2). \tag{32}$$

In the polar representation of complex numbers (Churchil et al., 1974) the angles are restricted by $-\pi<\theta\leq\pi$. In the limit of $\Lambda\to\infty$ we have $\theta_1=-\frac{\pi}{2}$ and $\theta_2=\frac{\pi}{2}$. Therefore, the coefficient of the M^5 term is $\frac{\hbar}{120\pi^2}$. This result is consistent with Equation 7 of effective potential at zero temperature. It seems that Ansari and Suresh (2007) assumed that $\theta_1=\frac{3\pi}{2}$, hence, their coefficients of M^5 term is $-\frac{\hbar}{120\pi^2}$. This will also affect their renormalization procedure for this theory.

Now by keeping terms to order of $\frac{1}{\Lambda}$, we found:

$$V_{eff}^1(\varphi_c)=\frac{\hbar}{24\pi^3}[-\frac{\Lambda^5}{5}(-\frac{2}{5}+\ln(\Lambda^2))+\frac{1}{3}\Lambda^3M^2-\frac{1}{2}\Lambda M^4+\frac{M^5}{5}-\frac{1}{3}\frac{M^6}{\Lambda}]. \tag{33}$$

But this expression is divergent for $\Lambda\to\infty$. We introduce the counter term Lagrangian as:

$$V_{CT}=\frac{D_1}{2}\varphi_c^2+\frac{D_2}{4!M}\varphi_c^4 \tag{34}$$

By combining Equations 3, 33 and 34, the effective potential with counter terms is:

$$V_{eff}=V_{eff}^0+\hbar V_{eff}^1+V_{CT}. \tag{35}$$

Now, by imposing the same renormalization condition as stated in effective potential at zero temperature, we found:

$$D_1=\frac{\hbar}{24\pi^3}[-\frac{1}{3}\Lambda^3\frac{\lambda}{M}+\Lambda m^2\frac{\lambda}{M}-\frac{\pi}{2}m^3\frac{\lambda}{M}+\frac{m^4}{\Lambda}\frac{\lambda}{M}], \tag{36}$$

and

$$D_2=\frac{\hbar}{24\pi^3}[3\Lambda\frac{\lambda^2}{M^2}-\frac{9}{4}\pi\Lambda\frac{\lambda^2}{M^2}+\frac{6}{\Lambda}m^2\frac{\lambda^2}{M^2}]. \tag{37}$$

By substituting these counter terms in Equation 35 and neglecting the field independent terms and terms that vanish in the limit of $\Lambda\to\infty$, we found:

$$V_{eff}(\varphi_c)=\frac{1}{2}m^2\varphi_c^2+\frac{\lambda}{4!M}\varphi_c^4+\frac{\hbar}{24\pi^2}[\frac{M^5}{5}-\frac{1}{2}\varphi_c^2(\frac{1}{2}\frac{\lambda}{M}m^3)-\frac{1}{4}\varphi_c^4(\frac{9\pi}{4}\frac{\lambda^2}{M^2}m)] \tag{38}$$

This result is equivalent to the effective potential of effective potential at zero temperature up to some field independent terms.

Appendix II: Evaluation of the integrals

To evaluate the temperature effective potential $V_\beta^1(\varphi_c)$ we notice that:

$$I=V_\beta^1(\varphi_c)=\frac{1}{\beta}\int\frac{d^4k}{(2\pi)^4}\ln[1-\exp(-\beta\omega)]=\frac{1}{8\pi^2\beta}\int k^3dk\ln[1-\exp(-\beta\omega)]. \tag{39}$$

But from Equation 17, we have $kdk=\omega d\omega$; therefore,

$$I=\frac{1}{8\pi^2\beta}\int_M^\infty(\omega^3-\omega M^2)d\omega\ln[1-\exp(-\beta\omega)]. \tag{40}$$

Integrating by parts the aforementioned expression, we obtain $I = I_1 + I_2$, where:

$$I_1=[\frac{1}{8\pi^2\beta}(\frac{\omega^4}{4}-\frac{\omega^2M^2}{2})\ln(1-\exp(-\beta\omega))]\Big|_M^\infty. \tag{41}$$

Hence,

$$I_1=\frac{M^4}{32\pi^2\beta}\ln[1-\exp(-\beta M)]. \tag{42}$$

We consider the series expansion of the logarithmic part and we obtain:

$$I_1 = \frac{M^4}{32\pi^2\beta}[\ln(\beta M) - \frac{\beta M}{2} + \frac{\beta^2 M^2}{24} - \frac{\beta^4 M^4}{2880} + O(\beta^6 M^6)] \quad (43)$$

And

$$I_2 = \frac{1}{8\pi^2\beta}\int_M^\infty (\frac{\omega^4}{4} - \frac{\omega^2 M^2}{2})\frac{\beta d\omega}{[\exp(\beta\omega) - 1]} \quad (44)$$

Next, we consider the decomposition $I_2 = I_3 + I_4$, where:

$$I_3 = \frac{1}{8\pi^2\beta}\int_0^\infty (\frac{\omega^4}{4} - \frac{\omega^2 M^2}{2})\frac{\beta d\omega}{[\exp(\beta\omega) - 1]} = -\frac{3\varsigma(5)}{4\pi^2\beta^5} + \frac{\varsigma(3)M^2}{8\pi^2\beta^3}, \quad (45)$$

and

$$I_4 = \frac{1}{8\pi^2\beta}\int_0^M (\frac{\omega^4}{4} - \frac{\omega^2 M^2}{2})\frac{\beta d\omega}{[\exp(\beta\omega) - 1]}. \quad (46)$$

To evaluate I_4 first, we perform a Taylor expansion of the integrand, then we integrate term by term, the result is:

$$I_4 = \frac{3M^4}{128\pi^2} + \frac{7M^5\beta}{960\pi^2} - \frac{M^6\beta^2}{1152\pi^2} + O(M^8\beta^3). \quad (47)$$

And from $V_\beta^1(\varphi_c)$ = $I_1 + I_3 + I_4$, we obtain the result stated in Equation 19 of effective potential at finite temperature.