

Full Length Research Paper

The effect of top quark mass on three jet rate and measurement of strong coupling constant

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We present the calculation of top quark mass corrections to three-jet rates in electron-positron annihilation at center of mass energies of 60 and 206 GeV. Furthermore, we also calculate the effects of quark-antiquark interactions on QCD model for three-jet rates. We extract the strong coupling constant, α_s , through our results from the massive three-jet rate and the massless one. We observe that the effect of "t" quark on three jet rate does not contribute significantly to the calculations of strong coupling constant and we can ignore this effect in higher energies.

Key words: Transverse momenta, three-jet rate, top quark, strong coupling constant.

INTRODUCTION

The importance of the corrections due to the mass of heavy quark mass in the jet productions in electron-positron collision at Next Leading Order (NLO), has been taken to consideration in measurement of strong coupling constant. In our recent paper, we have calculated the coupling constant from event shape observables at Next to Next Leading Order (NNLO) [Sepehri et al., 2009]. In this paper, we extend our analysis to higher energies to include the effect of top quark at 60 GeV.

Important subject is the effect of top quark on three jet rate. A typical strategy to determine the mass of, say, the top-quark at the centre-of-mass (c.m.) energy of the collider is to compare the ratio of three-jet production cross sections for heavy and light quarks and calculate strong coupling constant, α_s , for each of them [Sepehri et al., 2009; D0 Collaboration, 2009; Langenfeld et al., 2009; ALEPH Collaboration, 2000; OPAL Collaboration, 2001]. Of course, some quark anti quark interactions can destroy our result and should be regard in QCD model.

In this paper, we will discuss some aspects of the Next-to-Next-Leading order (NNLO), calculations of $e^-e^+ \rightarrow 3$ jets with massive quarks, (top quark), and we are going to extract the strong coupling constant, α_s , from the massive three-jet rate and the massless one at the center

of energies of 60 and 206 GeV.

JADE Algorithm

We separate two and three jet events by employing the jet clustering algorithm introduced by the JADE and OPAL Collaboration [2000]. In this algorithm, the scaled

mass spread defined as $Y_{ij} = \frac{m_{ij}^2}{E_{vis}^2}$ with

$m_{ij}^2 = 2E_i E_j (1 - \cos \theta_{ij})$ is calculated for each pair of particles in the event where E_i , E_j denote the energies, θ_{ij} is the angle between two objects i, j under consideration and E_{vis}^2 is the squared invariant mass of the hadronic final state. If the smallest of the Y_{ij} values is less than a parameter Y_{cut} , the corresponding pair of particles is combined into a cluster by summing the four momenta. This process is repeated, using all combinations of clusters and remaining particles, until all the Y_{ij} values exceed Y_{cut} . The clusters remaining at this stage are defined as the jets.

The distribution of jet multiplicities obtained by these clustering algorithms depends on the jet defining parameter Y_{cut} . For small Y_{cut} , many jets are found because of the hadronization of fluctuation process, whereas for large Y_{cut} , mostly two jet events are found

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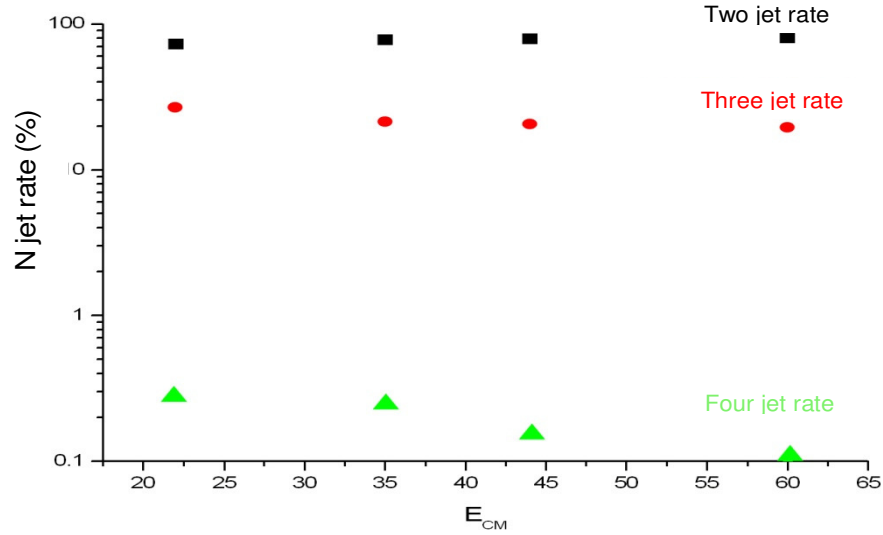


Figure 1. Jet fraction for different centre of mass energies. The data given form AMY Collaboration (1993), DELPHI Collaboration (1996), ALEPH Collaboration (2000) and TASSO Collaboration (1984).

and the $q\bar{q}g$ -events are not resolved. However, Monte Carlo studies show that there is a range of cluster parameters, for which QCD effects can be resolved and the fragmentation effects are sufficiently small. In the following, the parameter $Y_{cut}=0.04$ is used which is found to be a reasonable cut [JADE Collaboration, 1986].

In Figure 1, we show jet fractions for different centre of mass energies. Our results are completely consistent with the results obtained by the QCD model [JADE and OPAL Collaboration, 2000; JADE Collaboration, 1986; Rodrigo and Krauss, 2004].

THE EFFECT OF TOP QUARK PRODUCTION ON THREE JET RATE

A typical strategy to determine the mass of, say, the top-quark at the centre-of-mass (c.m.) energy of the collider is to compare the ratio of three-jet production cross sections for heavy and light quarks and calculate strong coupling constant, α_s , for each of them [Sepehri et al., 2009; D0 Collaboration, 2009; Langenfeld et al., 2009; ALEPH Collaboration, 2000; OPAL Collaboration, 2001]. The Drell–Yan process is the production of a lepton pair of large invariant mass M in hadron-hadron collisions by the mechanism of quark–antiquark annihilation [Drell and Yan, 1971]. This mechanism advice us that some quark-antiquarks interact with each other after production at collider and electron positron are produced instead. The cross section for this interaction is derived as following [Drell and Yan, 1971]:

$$\frac{\sigma_{q\bar{q} \rightarrow e^+e^-}}{\sigma_0} = \frac{4\pi\alpha_s^2}{3\omega^2} \frac{1}{N} Q_q^2 \quad (1)$$

with $Q_q = -1/3, +2/3 \leftrightarrow q$ charge

N is due to the fact that only when the colour of the quark matches with the colour of the antiquark can annihilation into a colour–singlet final state takes place and ω is related to kinetic energy of quark and antiquark.

We should sum over all interaction cross sections with different kinetic energy.

$$\frac{\sigma_{int}^1}{\sigma_0} = \int_{2m_q}^{M_Z} d\omega \frac{4\pi\alpha_s^2}{3\omega^2} \frac{1}{N} Q_q^2 = \frac{4\pi\alpha_s^2 Q_q^2}{3N} \left(\frac{M_Z - 2m_q}{2m_b M_Z} \right) \quad (2)$$

where m_q and M_Z are related to quark mass and central mass of energy respectively.

It is not the entire story. Maybe some quark-antiquark interacts with each other and produces new quark-antiquark. We have:

$$\frac{\sigma_{int}^2}{\sigma_0} = \int_{2m_q}^{M_Z} d\omega \sigma_{QCD} \quad (3)$$

$$= \int_{2m_q}^{M_Z} d\omega \frac{8\alpha_s^3}{A^2} \ln\left(\frac{2\omega}{A}\right) = \frac{8\alpha_s^3}{A^2} \left[\frac{(-4m_q \ln(\frac{4m_q}{A}) + 2M_Z \ln(\frac{2M_Z}{A}))}{A} \right]$$

where σ_{QCD} is the cross section for gluon bremsstrahlung in a quark-antiquark collision [Anchordoqui and Goldberg, 2003], α_s is the QCD

coupling constant, and Λ is an infrared cutoff related to the off-shell momentum of the exchanged gluon. These interactions should be regarded to QCD models for three jet rates:

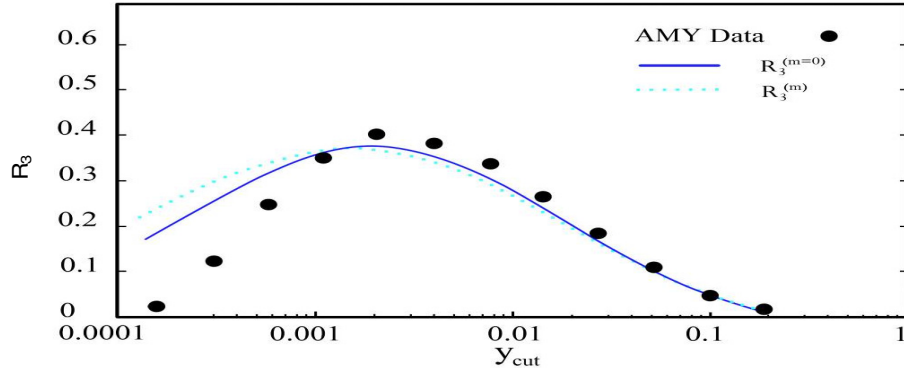


Figure 2. Effect of a t-mass (175 GeV), at three-jet rate at AMY energy as a function of the jet resolution parameter in the JADE scheme.

$$\frac{\sigma_{3-jet}^{exp}}{\sigma_0} = \frac{\sigma_{3-jet}^{theo} - \sigma_{int}^1 - \sigma_{int}^2}{\sigma_0} \quad (4)$$

$$C_{24}^3 = -\frac{1}{4} C_F^2 (L_y^2 - L_m^2) - \frac{1}{48} C_F C_A (L_y^4 - L_m^4) \quad (7)$$

Theoretically, three jet rates can be formally expressed as [Rodrigo and Krauss, 2004]:

where $\beta_n = \frac{11C_A - 2n}{12}$. Other coefficients are cited in Rodrigo and Krauss [2004].

$$\frac{\sigma_{3-jet}^{theo}}{\sigma_0} = \sum_{k=n-2}^{\infty} \left(\frac{\alpha_s(Q)}{\pi} \right)^k \sum_{l=0}^{2k} C_{kl}^3 \quad (5)$$

With t-mass of 175 GeV, the effect of the t-mass at the three-jet rates is depicted in Figure 2. The massive three-jet rate starts being larger than the massless one at values of the jet resolution parameters of the order of $Y_{cut} = 0.001$. The α_s values are given in Table 1.

where the coefficients C_{kl}^n are polynomials of order one in $L_y = \ln\left(\frac{1}{y_{cut}}\right)$ and $L_m = \ln\left(\frac{m^2}{Q_0^2}\right)$. The coefficients for the first order in α_s are given by [Rodrigo and Krauss, 2004]:

We observe that the effect of top quark on three jet rate does not contribute significantly to the calculations of strong coupling constant. In other word, the inclusion of heavy quark mass does not change the coupling constant considerably.

$$C_{11}^3 = -\frac{3}{2} C_F L_y - \frac{1}{2} C_F L_m \quad (6)$$

THREE JET RATE AT ALEPH COLLABORATION

$$C_{12}^3 = \frac{1}{2} C_F (L_y^2 - L_m^2)$$

Previously, we consider three jet rates at 60 GeV. With increasing energy, the effect of top quark mass on three jet rates is decreased. Figure 3 shows the parton-level theoretical predictions for the jet fractions compared to experimental hadron-level data from ALEPH Collaboration [2004].

where $C_F = \frac{4}{3}$, $C_A = 3$

By comparing Figures 2 and 3, we conclude that the effect of top quark mass for the three jet rates becomes systematically less as the amount of energy increases. With fitting these data with Equation (4), we derive strong coupling constant from QCD model. The α_s ' values are presented in Table 2.

$$C_{23}^3 = \frac{1}{2} C_F^2 (L_y^2 - L_m^2) (3L_y - L_m) - \frac{1}{24} C_F C_A (3L_y^3 - L_m^3) + \frac{1}{2} \beta_n C_F L_y (L_y^2 - L_m^2) + \frac{1}{6} (\beta_n - \beta_{n-1}) C_F L_m (L_y^2 - L_y L_m + 2L_m^2)$$

Conclusions

In this paper, the effect of top quark mass to three-jet rates in electron-positron annihilation at center of mass energies of 60 and 206 GeV, is brought under consideration. The strong coupling constant, α_s , has been

Table 1. α_s values for centre-of-mass energy of 60 GeV.

α_s	$m_t = 175 \text{ GeV}$	$m_q = 0$
	0.131 ± 0.006	0.132 ± 0.007

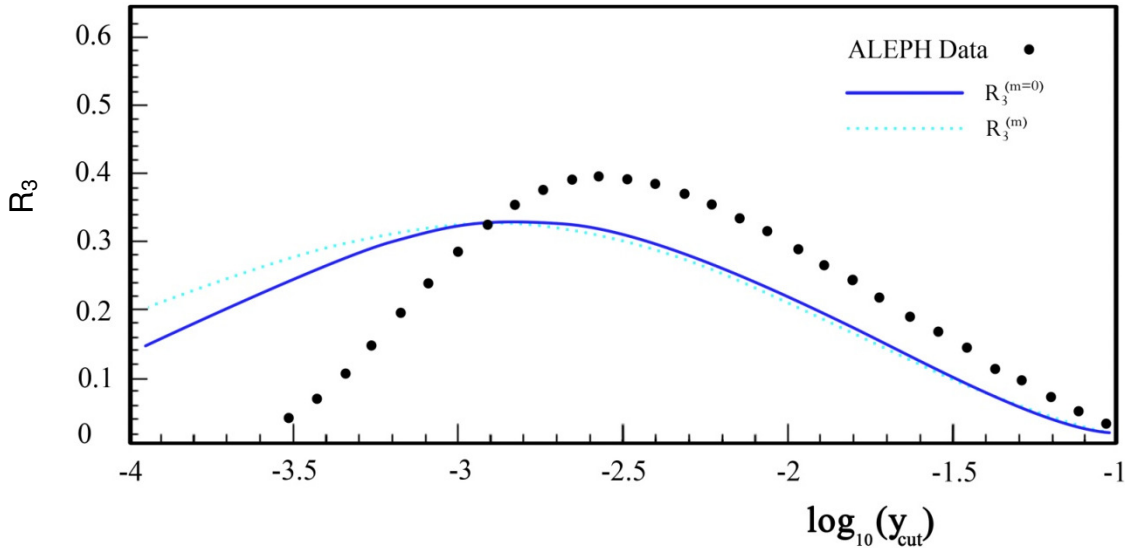


Figure 3. Jet rates compared to data from ALEPH ($Q = 206 \text{ GeV}$).

Table 2. α_s values for centre-of-mass energy of 206 GeV.

α_s	$m_t = 175 \text{ GeV}$	$m_q = 0$
	0.112 ± 0.004	0.113 ± 0.006

Table 3. α_s values for centre-of-mass energy of 60 and 206 GeV.

60 GeV	$m_t = 175 \text{ GeV}$	$m_q = 0$
α_s	0.131 ± 0.006	0.132 ± 0.007
206 GeV	$m_t = 175 \text{ GeV}$	$m_q = 0$
α_s	0.112 ± 0.004	0.113 ± 0.006

measured from massive three-jet rate and the massless one. It is observed that the effect of top quark mass on three-jet rates does not contribute significantly to the calculations of strong coupling constant (Table 3), and we can ignore this effect in higher energies.

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