International Journal of the Physical Sciences Vol. 6(25), pp. 5906-5911, 23 October, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.741

Full Length Research Paper

Suboptimal sparse channel estimation for multicarrier underwater acoustic communications

Nina Wang[1]*, Guan Gui[2,3], Zhi Zhang[1] and Ping Zhang[1]

[1]Key Laboratory of Universal Wireless Communications, Ministry of Education, Beijing University of Posts and Telecommunications, Beijing, China.
[2]Department of Electrical Engineering, University of Electrical Science and Technology of China, Chengdu, China.
[3]Department of Electrical and Communication Engineering, Tohoku University, Sendai, Japan.

Accepted 26 September, 2011

Multipath channels often exhibit sparse structures in multicarrier underwater acoustic (MC-UWA) communication systems. Conventional linear channel estimators, such as least squares (LS) and minimum mean square error (MMSE) are considered as optimal solutions under the assumption of rich multipath. However, they often result in low spectral efficiency due to neglect of the sparsity in multipath channels. In this paper, we propose a novel sparse channel estimation method with compressive sensing. The proposed estimator has better estimation performance and low complexity when compared with the LS estimator. Simulations verified the proposed method with respect to the mean square error (MSE) and the computational complexity.

Key words: Underwater acoustic (UWA), orthogonal frequency division multiplexing (OFDM), sparse multipath, compressive sensing (CS), sparse channel estimator.

INTRODUCTION

Underwater acoustic (UWA) communications have been extensively investigated in recent years, because of its wide applications on military, commerce and environment protection (Stojanovic, 1996). The UWA channel is a highly time-varying and complicated Doppler-effected multipath channel due to internal waves, platform and sea-surface motion (Eggen et al., 2000). The long delay spread leads to serious inter-symbol interference (ISI) (Kilfoyle and Baggeroer, 2000). The Doppler effects destroy the orthogonality of the subcarrier and lead to inter-carrier interference (ICI) for multicarrier UWA systems (Stojanovic and Preisig, 2000). Accurate channel state information (CSI) is indispensable on coherent receiving and channel equalization at the receiver, thus, channel estimation becomes a huge challenge under such execrable multicarrier underwater acoustic (MC-UWA) channel conditions.

In recent years, numerous channel measurements have shown that the MC-UWA multipath channels tend to exhibit sparse structures. That means the majority of the channel taps are either zero or below the noise floor (Li and Preisig, 2007; Stojanovic, 2008; Gui et al., 2011). However, the conventional training-based linear methods, such as least squares (LS), are incapable of exploiting the inherent sparsity of the MC-UWA multipath channels which lead to the overutilization of the limited resources, such as energy and bandwidth. In other word, exploiting the channel sparsity will improve the spectral and energy efficiency by using some effective channel estimation technique. Kang and Iltis (2008) proposed variants of matching pursuit (MP) algorithms (Mallat and Zhang, 1993) for MC-UWA systems without a compressive sensing theoretical background. As the development of compressive sensing (CS) (Berger et al., 2010a,b) continues, some compressive estimators provide for sparse multipath MC-UWA systems using orthogonal matching pursuit (OMP) (Tropp and Gilbert, 2007) and basic pursuit (BP) algorithm (Candes et al., 2006) which outperform the traditional subspace algorithms (root-MUSIC and ESPRIT) (Berger et al., 2010a,b). The performance of BP algorithm is theoretically guaranteed, while the OMP lacks (Taubock et al., 2010). Researchers have verified that BP has a slight edge over OMP but computationally complex (Berger et al., 2010a, b).

*Corresponding author. E-mail: ninawnn@gmail.com.

Low computational complexity is crucial since the MC-UWA channels have to be estimated real time. In this paper, we introduce a compressive channel estimation method with compressive sampling matching pursuit (CoSaMP) (Needell and Tropp, 2009) for sparse multipath MC-UWA channels. The proposed method allows an even faster implementation than OMP. Simulations show that the running time is only less than a quarter of that of OMP, while the estimation performance is only slightly poorer than BP. Therefore, the CoSaMP algorithm combines the high performance of accuracy and the low computational complexity for MC-UWA systems.

The rest of the paper is organized as follows. Subsequently, this study gives an overview of compressive sensing and introduces the system model adopted, after which it gives a detailed description of the proposed approach. This is followed by a comparison of the performance of the CoSaMP sparse channel estimation with the traditional methods. Finally, the study is concluded.

COMPRESSIVE SENSING AND SYSTEM MODEL

Overview of compressive sensing

Compressive sensing theory aims to recover high dimension sparse signals with considerably fewer linear measurements. Consider that a signal $\bar{\mathbf{h}} \in \mathbb{C}^N$ can be represented in a basis $\{\boldsymbol{\psi}_k\}_{k=1}^N$ with the coefficients $h_k (h_k = \langle \bar{\mathbf{h}}, \boldsymbol{\psi}_k \rangle)$. The relationship can be expressed as:

$$\bar{\mathbf{h}} = \sum_{k=1}^{N} h_k \boldsymbol{\psi}_k = \boldsymbol{\Psi}\mathbf{h}, \quad (1)$$

where $\boldsymbol{\Psi} = [\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \cdots, \boldsymbol{\psi}_N]$ is a $N \times N$ full-rank matrix; $\mathbf{h} = [h_1, h_2, \cdots, h_N]^T$ is a $N \times 1$ coefficient vector. Obviously, $\bar{\mathbf{h}}$ and $\mathbf{h}$ are two different forms of the same signal. In certain cases, $\mathbf{h}$ can be a finite length discrete time signal, while $\bar{\mathbf{h}}$ is the corresponding frequency-domain expression in a limited bandwidth and the matrix $\boldsymbol{\Psi}$ is the discrete Fourier transform (DFT) matrix. If the coefficient vector $\mathbf{h}$ is $S-$ sparse, then, there are at most S ($S \ll N$) non-zero coefficients and $\bar{\mathbf{h}}$ is compressible.

If $\bar{\mathbf{h}}$ is compressible in an orthogonal basis $\boldsymbol{\Psi}$, then the real application of compressive sensing is to recover the signal $\bar{\mathbf{h}}$ from $M (M \ll N)$ measurements which can be acquired by linear transformation. That is:

$$\mathbf{y} = \mathbf{X}\bar{\mathbf{h}} = \mathbf{X}\boldsymbol{\Psi}\mathbf{h} = \mathbf{Z}\mathbf{h}, \quad (2)$$

where $\mathbf{y}$ is a $M \times 1$ measurement vector; $\mathbf{X}$ is a $M \times N$ measurement matrix which is uncorrelated with $\boldsymbol{\Psi}$; $\mathbf{Z} = \mathbf{X}\boldsymbol{\Psi}$ is a $M \times N$ matrix; $M \geq cL\log(N/S)$, c is a constant introduced in Baraniuk (2007) work. As the number of the entities in signal $\bar{\mathbf{h}}$ is more than that of the equations set, the solution of $\bar{\mathbf{h}}$ is not unique. Certain criteria must be met in order to get the optimal solution.

One way to solve this problem is to consider l_0 norm minimization which aims to find the sparsest solution in the feasible solution set:

$$\hat{\mathbf{h}} = \arg\min \|\mathbf{h}\|_0 \quad \text{s.t.} \quad \mathbf{Z}\mathbf{h} = \mathbf{X}\boldsymbol{\Psi}\mathbf{h} = \mathbf{y}, \quad (3)$$

where $\hat{\mathbf{h}}$ is the estimation vector of $\mathbf{h}$. $\|\mathbf{h}\|_0$ is l_0 norm which counts the number of non-zero taps of $\mathbf{h}$ and $\mathbf{y}$, $\boldsymbol{\Psi}$ and $\mathbf{X}$ are known. However, Equation 3 is non-deterministic polynomial (NP) hard problem and computationally infeasible. It is then natural to consider l_1 norm minimization (Donoho, 2006). In this context,

$$\hat{\mathbf{h}} = \arg\min \|\mathbf{h}\|_1 \quad \text{s.t.} \quad \mathbf{Z}\mathbf{h} = \mathbf{X}\boldsymbol{\Psi}\mathbf{h} = \mathbf{y}, \quad (4)$$

where the l_1 norm is defined as $\|\mathbf{h}\|_1 = \sum_{i=1}^{N} |h_i|$. In case of the measurement vector contamination with noise, $\mathbf{y} = \mathbf{X}\bar{\mathbf{h}} + \mathbf{n} = \mathbf{X}\boldsymbol{\Psi}\mathbf{h} + \mathbf{n} = \mathbf{Z}\mathbf{h} + \mathbf{n}$, some error tolerance $\varepsilon \geq 0$ is allowed.

$$\hat{\mathbf{h}} = \arg\min \|\mathbf{h}\|_1 \quad \text{s.t.} \quad \|\mathbf{X}\boldsymbol{\Psi}\mathbf{h} - \mathbf{y}\|_2 \leq \varepsilon. \quad (5)$$

This is also known as basic pursuit (BP) algorithm. It has been proved that reliable recovery of a sparse signal still depends on the matrix $\mathbf{Z}$ which should be designed to satisfy the restricted isometry property (RIP) (Candes, 2008). While it is a complicated issue to identify whether the matrix $\mathbf{Z}$ satisfies the RIP, an alternative method by constructing a suitable measurement matrix $\mathbf{X}$ is adopted. As shown by the study of Baraniuk (2007), if the measurement matrix $\mathbf{X}$ is uncorrelated to the basis, $\boldsymbol{\Psi}$ obeys the RIP with high probability, such as random Gaussian, Bernoulli and partial Fourier matrices. Also, it is observed that the matrix $\mathbf{Z}$ satisfies the RIP with high probability.

System model

In this paper, we consider the MC-UWA system in form of OFDM structure and estimate the channel for every OFDM symbol. Assume that there are N subcarriers. The modulated data stream is splited into the parallel streams by serial-to-parallel conversion. The new parallel signals ($X_i, 0 \leq i \leq N-1$) are transformed from frequency

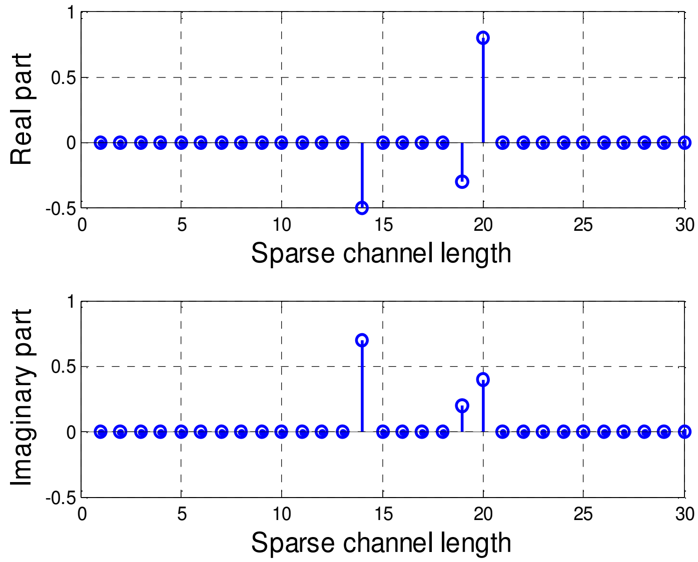

Figure 1. An example of sparse multipath UWA channel model sample. Channel length is 30 with 3 nonzero taps.

domain to time domain via $N-$ point inverse discrete Fourier Transform (IDFT). The last L_{CP} samples of the IDFT output are appended as cyclic prefix to form one OFDM symbol to avoid intersymbol interference (ISI). After the parallel-to-serial conversion, the serial data stream is passed through the frequency selective fading channel with the channel impulse response:

$$\mathbf{h}(n)=\sum_{l=0}^{L-1}h_l\delta(n-\tau_l), \tag{6}$$

where h_l is the complex amplitude gain; τ_l is the delay in the lth multipath and L is the number of multipath. For sparse multipath model, only very few elements of h_l is non-zero as shown in Figure 1.

At the receiver, after the serial-to-parallel conversion and cyclic prefix (CP) removal, the new parallel data streams transform to the frequency domain signals ($Y_i, 0\le i\le N-1$) by $N-$point discrete Fourier transform (DFT). The OFDM system can be expressed as:

$$Y_i = X_i H_i + n_i, \quad 0\le i\le N-1, \tag{7}$$

where H_i is the element of $\mathbf{H}=\mathrm{DFT_N}(\mathbf{h})=[H_0,H_1,\cdots H_{N-1}]$; n_i is the additive white Gaussian noise (AWGN), with zero mean and variance δ_n^2.

Considering all the subcarriers, then Equation 7 can be written as the matrix form:

$$\mathbf{Y}=\mathbf{XH}+\mathbf{n}=\mathbf{XWh}+\mathbf{n}=\mathbf{Zh}+\mathbf{n}, \tag{8}$$

where $\mathbf{Y}=[Y_0,Y_1,\cdots Y_{N-1}]^T$ is the $N-$ point DFT of the receive signals after CP removal. $\mathbf{X}=diag(X_0,X_1,\cdots X_{N-1})$ is a matrix with X_i on its diagonal. $\mathbf{h}=[h(0),h(1),\cdots h(L-1)]^T$, $\mathbf{n}\sim N(0,\delta^2\mathbf{I_N})$, $\mathbf{Z}=\mathbf{XW}$; $\mathbf{W}$ is the DFT matrix which only includes the first L columns.

$$\mathbf{W}=\begin{bmatrix} W^{00} & \cdots & W^{0(L-1)} \\ \vdots & \ddots & \vdots \\ W^{(N-1)0} & \cdots & W^{(N-1)(L-1)} \end{bmatrix} \tag{9}$$

$$W^{nl}=e^{-j2\pi nl/N}, 0\le n\le N-1, 0\le l\le L-1 \tag{10}$$

COMPRESSIVE CHANNEL ESTIMATION

In the MC-UWA systems, sparse channels are probed by sending known data (pilots) on the pilot subcarriers. Assume that the number of pilot subcarriers is N_p and the matrix for selected pilot subcarriers is $\mathbf{S}(N_P\times N)$. We can write Equation 8 as:

$$\mathbf{Y}_P=\mathbf{X}_P\mathbf{H}+\mathbf{n}_P=\mathbf{X}_P\mathbf{W}_P\mathbf{h}+\mathbf{n}_P=\mathbf{Z}_P\mathbf{h}+\mathbf{n}_P, \tag{11}$$

where $\mathbf{Y}_P=\mathbf{S}\square\mathbf{Y}$; $\mathbf{X}_P=\mathbf{S}\square\mathbf{X}\square\mathbf{S}^T$; $\mathbf{W}_P=\mathbf{S}\square\mathbf{W}$; $\mathbf{n}_P=\mathbf{S}\square\mathbf{n}$.

Table 1. Sparse channel estimation with CoSaMP algorithm.

Input: $\mathbf{Y}_P$, $\mathbf{Z}_P$, K .

Output: channel estimator $\hat{\mathbf{h}}_{CoSaMP}$.

Initialize: The initial index set of non-zero taps: $\Omega^0 = \varnothing$.

The residual: $\mathbf{r}^0 = \mathbf{Y}_P$.

The iteration index: $i = 1$.

Repeat

Channel identification: $\mathbf{S}_i = \mathbf{Z}_P{}^* \mathbf{r}^{i-1}$, $\Omega^i_{\mathbf{S}_i} = \mathrm{supp}(|\mathbf{S}_i|, 2K)$.

Update the channel dominant taps: $\Omega^i = \Omega^{i-1} \cup \Omega^i_{\mathbf{S}_i}$.

Channel estimation with LS: $\mathbf{h}^i |_{\Omega^i} = \mathbf{Z}_P |_{\Omega^i}{}^{\dagger}\ \mathbf{Y}_P$.

Find the dominant channel coefficients. Set the non-dominant channel coefficients to be zero: $\Omega^i = \mathrm{supp}(|\mathbf{h}^i |_{\Omega^i}|, K)$, $\mathbf{h}^i |_{(\Omega^i)^c} = \mathbf{0}$.

Update the estimation residual: $\mathbf{r}^i = \mathbf{Y}_P - \mathbf{Z}_P |_{\Omega^i} \cdot \mathbf{h}^i |_{\Omega^i}$.

Until stopping criterion is true

$\Omega^i_{\mathbf{S}_i} = \mathrm{supp}(|\mathbf{S}_i|, 2K)$ denotes that $\Omega^i_{\mathbf{S}_i}$ is the index set of the maximum $2K$ channel coefficients in $|\mathbf{S}_i|$. $(\Omega^i)^c$ is the complementary set of Ω^i . $\mathbf{h}^i |_{\Omega^i}$ is a vector consisted of the elements in the index set Ω^i of $\mathbf{h}^i$.

Conventional linear estimator with LS

The LS channel estimation method is widely adopted, because it is easy to implement. As Equation 11 shows, the estimated channel impulse response can be obtained from:

$$\hat{\mathbf{h}}_{\mathbf{LS}} = (\mathbf{W}_p^{\mathbf{H}} \mathbf{X}_p^{\mathbf{H}} \mathbf{X}_p \mathbf{W}_p)^{-1} \mathbf{W}_p^{\mathbf{H}} \mathbf{X}_p^{\mathbf{H}} \mathbf{Y}_P. \tag{12}$$

Equation 12 shows that the LS algorithm does not utilize the channel sparsity of MC-UWA systems. Referring to the recent research of Bajwa et al. (2010), an increase in the number of degrees of freedom (DoF) available for communication can lead to significant gains in the rate and reliability, that is, if the sparsity is utilized effectively, we can improve the spectral efficiency and estimation accuracy greatly for practical MC-UWA systems.

Sparse channel estimation with CoSaMP

Here, we apply the compressive sensing to sparse channel estimation with CoSaMP algorithm in the MC-UWA communication systems.

The CoSaMP algorithm is an iterative reconstruction algorithm which is an effective method. In contrast with the aforementioned LS algorithm, the CoSaMP algorithm needs the priori information of channel sparsity K . The accurate channel estimator is obtained by refining the support set in each iteration until the halting criteria $\{i : i \geq 4K \mid \|\mathbf{h}^i - \mathbf{h}\|_2^2 \leq 10^{-3}\}$ is satisfied. The CoSaMP algorithm can be described as shown in Table 1 with the channel model in Equation 11.

SIMULATIONS

Here, in order to evaluate the channel estimation performance of the proposed CoSaMP algorithm, we adopt mean square error (MSE) to quantize the channel estimation error. The MSE performances of LS, BP and OMP will also be evaluated as references. The MSE is expressed as:

$$\mathrm{MSE} = \frac{1}{\mathrm{M}} \sum_{m=1}^{\mathrm{M}} \left\| \mathbf{h} - \hat{\mathbf{h}}_m \right\|_2^2, \tag{13}$$

where M is the number of Monte Carlo simulation runs.

Assume that the system parameters are stable within an OFDM block. The parameters are as shown in Table 2.

Comparisons of the MSE performance

The parameters for the MC-UWA systems are as shown in **Table 2**. We compare the MSE performance of different channel estimators under signal-to-noise ratio (SNR) from 0 to 30 dB. As shown in Equation 13, the

Table 2. The common system parameters.

Linear channel estimation	LS algorithm
Compressive channel estimation	BP, OMP and CoSaMP (proposed)
The number of subcarriers	N = 1024
The number of pilot subcarriers	Np = 32
Channel length	L = 30
Channel sparsity	K = 5
Non-zero taps	Random Gaussian independent variable
SNR (dB)	0~30
M (trials)	1000

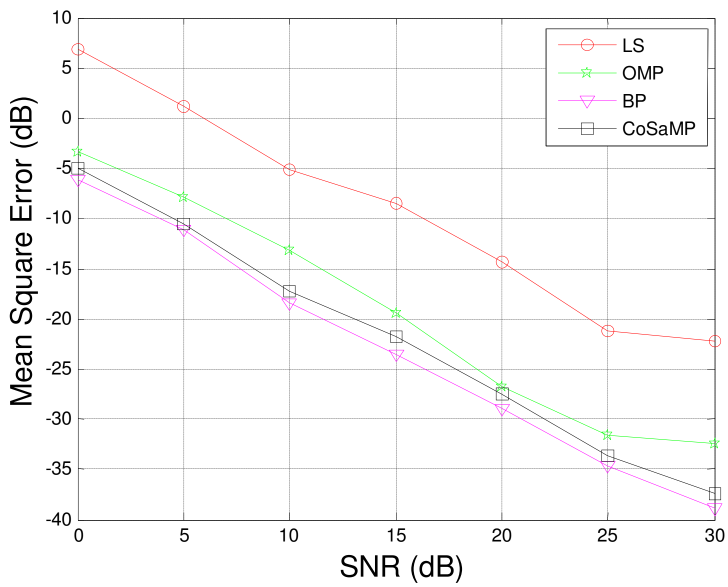

Figure 2. MSE performances of LS, BP, OMP and CoSaMP with SNR from 0 to 30 dB

smaller the MSE is, the better the channel estimator performs. The comparisons between LS, BP, OMP and CoSaMP algorithms are as shown in Figure 2. Clearly, the sparse channel estimators achieve better MSE performance than the conventional LS algorithm. Therefore, the sparse channel estimators can achieve the same MSE performance by using fewer pilots. In other words, the sparse channel estimators provide higher spectral efficiency. The proposed CoSaMP algorithm outperforms OMP, but is slightly worse than BP.

Comparison of the computational complexity

To study the computational complexity of the proposed method, we evaluate the CPU running time in seconds to compare the computational complexity of the different channel estimation methods in for SNR = 15 dB. It is worth mentioning that the CPU running time is not the exact measure of the computational complexity, while it can provide the rough estimation. Our simulations were performed under the condition of MATLAB 7.1 using 2.8 GHz Inter Core 2 processor with 1.96 G of memory. As shown in Table 3, the computational complexity of sparse channel estimators is higher than the conventional LS algorithm. The CPU time of CoSaMP is very lose to LS and is about three times to that of LS. However, BP and OMP are much more complex than CoSaMP; meaning that the proposed CoSaMP algorithm is also efficient.

Conclusions

In this paper, we propose a compressive channel estimator with CoSaMP algorithm for sparse multipath MC-UWA systems. Both theory analysis and computer

Table 3. Comparison of the computational complexity of LS, BP, OMP and CoSaMP.

Methods	LS	BP	OMP	CoSaMP
CPU time (s)	0.0156	0.5266	0.2344	0.0469
Ratio to LS	1	34	15	3

simulations confirm the proposed method and offer a good compromise between high spectral efficiency, good practical performance guarantee and low computational complexity. However, experiments are under the assumption that the channel sparsity is known. In further projects, we will consider more sophisticated CoSaMP channel estimation algorithm to sense the channel feature.

ACKNOWLEDGEMENT

This work was supported by the Major National Special, Science and Technology (S&T) Programs on designing and developing a new generation wireless broadband mobile communications network under grant No. 2009ZX03002-009-01.

REFERENCES

Bajwa WU, Haupt J, Sayeed AM, Nowak R (2010). Compressed channel sensing: A new approach to estimating sparse multipath channels. Proceeding of the IEEE, 98(6): 1058-1076.
Baraniuk R (2007). A lecture on compressive sensing. IEEE Signal Proces. Mag., 24(4): 118-121.
Berger CR, Wang Z, Huang J (2010a). Application of compressive sensing to sparse channel estimation. IEEE Commun. Mag., 48(11): 164-174.
Berger CR, Zhou S, Perisig JC, Willett P (2010b). Sparse channel estimation for multicarrier underwater acoustic communication: from subspace methods to compressed sensing. IEEE Trans. on Signal Proces., 58(3): 1708-1721.
Candes EJ (2008). The restricted isometry property and its implications for compressed sensing. C. R. Math. Acad. Sci., Serie I, 346: 589-592.
Candes EJ, Romberg J, Tao T (2006). Stable signal recovery from incomplete and inaccurate measurements. Commun. Pure Appl. Math., 59: 1207-1223.
Donoho DL (2006). For most large underdetermined systems of equations, the minimal l1 norm near-solution approximates the sparsest near-solution. Commun. Pure Appl. Math., 59(6): 907-934.
Eggen TH, Baggeroer AB, Preisig JC (2000). Communication over Doppler spread channels. Part I: Channel and receiver presentation. IEEE J. Ocean. Eng., 25(1): 62-71.
Gui G, Chen ZX, Wan Q, Huang AM, Adachi F (2011). Estimation of multipath time delays based on bi-sparse constraint. Int. J. Phys. Sci., 6(3): 402-406.
Kang T, Iltis RA (2008). Iterative carrier frequency offset and channel estimation for underwater acoustic OFDM systems. IEEE J. Sel. Areas Commun., 26(9): 1650–1661.
Kilfoyle DB, Baggeroer AB (2000). The state of the art in underwater acoustic telemetry. IEEE J. Ocean. Eng., 25(1): 4-27.
Li W, Preisig JC (2007). Estimation of rapidly time-varying sparse channels. IEEE J. Ocean. Eng., 32(4): 927-939.
Mallat SG, Zhang Z (1993). Matching pursuits and time-frequency dictionaries. IEEE Trans. on Signal Proces., 41(12): 3397-3415.
Needell, Tropp JA (2009). CoSaMP: Iterative signal recovery from incomplete and inaccurate samples. Appl. Comp. Harmonic Anal., 26(3): 301–321.
Stojanovic M (1996). Recent advances in high-speed underwater acoustic communications. IEEE J. Ocean. Eng., 21(2): 125-136.
Stojanovic M (2008). OFDM for underwater acoustic communications: Adaptive synchronization and sparse channel estimation. Presented at the Int. Conf. Acoustics, Speech, and Signal Processing, Las Vegas, NV.
Stojanovic M, Preisig J (2009). Underwater acoustic communication channels: Propagation models and statistical characterization. IEEE Commun. Mag., 47(1): 84-89.
Taubock G, Hlawatsch F, Eiwen D, Rauhut H (2010). Compressive estimation of doubly selective channels in multicarrier systems: leakage effects and sparsity-enhancing processing. IEEE J-STSP, 4(2): 255-271.
Tropp JA, Gilbert AC (2007). Signal recovery from random measurements via orthogonal matching pursuit. IEEE Trans. Inform. Theory, 53(12): 4655-4666.