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Full Length Research Paper

An estimation of time-varying signal frequency using recent finite observations

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For estimating a time-varying signal frequency, an alternative estimator with a finite memory structure is proposed. The proposed estimator is developed under a maximum likelihood criterion using only the most recent finite observations on the window. The proposed estimator is first represented in a batch form, and then in a recursive form for computational advantage. The proposed estimator is shown to have good inherent properties, such as unbiasedness and deadbeat. Finally, via computer simulation and comparison, the proposed approach is shown to outperform remarkably the variable forgetting factor (VFF) Kalman filtering approach.

Key words: Estimating signal frequency, finite memory structure, unbiasedness, deadbeat.

INTRODUCTION

The problem of estimating the time-varying frequency of a signal has been addressed in signal processing areas (Lim and Oppenheim, 1979; Wong and Jin, 1990; Boashash, 2003; Mack and Jain, 1983; Lee et al., 1999). Among them, the performance of the conventional Kalman filtering approach (Lim and Oppenheim, 1979) for estimating a time-varying signal frequency could be degraded since it has an infinite memory structure that uses all the past observations accomplished by equal weighting, and tends to accumulate during its implementation. Therefore, in the time-weighted-error (TWE) Kalman filtering (Mack and Jain, 1983) and the variable forgetting factor (VFF) Kalman filtering (Lee et al., 1999), the forgetting factor was introduced to decrease the influence of the older observation by assigning less weight to older observations. However, in these existing approaches, the estimate of the signal frequency is not optimal since it is not a solution of the optimization problem. Moreover, strictly speaking, they still have infinite memory structures.

Therefore, in this paper, an alternative estimator with a finite memory structure is proposed for estimating a time-varying signal frequency. The proposed estimator is developed under a maximum likelihood criterion using only the most recent finite observations on the window. The proposed estimator will be first represented in a batch form, then in a recursive form for computational advantage. It has been shown that the proposed estimator has good inherent properties such as unbiasedness and deadbeat, which cannot be obtained by the existing Kalman filtering based approaches (Lim and Oppenheim, 1979; Mack and Jain, 1983; Lee et al., 1999). In addition, due to the finite memory structure, the proposed estimator can have inherent properties, such as a bounded input/bounded output (BIBO) stability and robustness against temporary modeling uncertainties and round-off errors according to Jazwinski (1968) and Bruckstein and Kailath (1985). Finally, through computer simulation and comparison, the proposed approach is shown to outperform remarkably the VFF Kalman filtering approach for the estimate of the signal frequency which varies relatively quickly.

ESTIMATION OF TIME-VARYING SIGNAL FREQUENCY USING RECENT FINITE OBSERVATIONS

The system for signal frequency estimation can be written by a discrete time-varying state-space model (Mack and Jain, 1983; Lee et al., 1999):

$$a_{i+1} = \Phi_i a_i, \quad s_i = H_i a_i + v_i \tag{1}$$

where $a_i \in R^n$ is the unknown parameter vector, Φ_i is a time-varying parameter transition matrix, and H_i is *n*-dimensional observation represented by $[s_{i-1} \quad s_{i-2} \quad \cdots \quad s_{i-n}]$. The observation noise v_i are zero-mean white Gaussian with covariance R_i. On the most recent window [$i-M(\equiv i_M), i$], the finite number of observations in equation is expressed as follows:

$$S_{i-1} = L_{i-1} a_i + V_{i-1} \tag{2}$$

where $L_{i-1} = \begin{bmatrix} H_{i-M}\Psi_{i,M}^{-1} \\ H_{i-M+1}\Psi_{i,M-1}^{-1} \\ \vdots \\ H_{i-1}\Psi_{i,1}^{-1} \end{bmatrix}$, $\Psi_{i,j} \equiv \Phi_{i-1}\Phi_{i-2}\cdots\Phi_{i-j}$, $S_{i-1} \equiv \left[s_{i_M} \ s_{i_M+1} \cdots s_{i-1}\right]^T$ and $V_{i-1} \equiv \left[v_{i_M} \ v_{i_M+1} \cdots v_{i-1}\right]^T$. It is noted in Equation 2 that the noise term V_{i-1} is a zero-mean with covariance Π_{i-1} given by $\Pi_{i-1} \equiv \left[diag(R_{i_M} \quad R_{i_M+1} \quad \cdots \quad R_i)\right]$.

A new estimator is developed for the current a_i under a maximum likelihood criterion and it is denoted by $\hat{a}_i$, and has a finite memory structure that processes linearly the finite observations S_{i-1} of Equation 2 on [i_M, i]. The noise term V_{i-1} has the following multivariate Gaussian density function:

$$f(V_{i-1}) = \frac{1}{\sqrt{(2\pi)^M \, |\Pi_{i-1}|}} \exp\left[-\frac{1}{2} V_{i-1}^T \Pi_{i-1}^{-1} V_{i-1}\right].$$

It has been noted that linear transformation on, and linear combinations of, Gaussian random processes are themselves Gaussian random processes. Thus, the multivariate Gaussian density function of S_{i-1} is derived from a shifted version of $f(V_{i-1})$ as follows:

$$\begin{aligned} f(S_{i-1} \mid a_i) &= f(S_{i-1} - L_{i-1}a_i) \\ &= \frac{1}{\sqrt{(2\pi)^M \, |\Pi_{i-1}|}} \exp\left[-\frac{1}{2}(S_{i-1} - L_{i-1}a_i)^T \Pi_{i-1}^{-1} (S_{i-1} - L_{i-1}a_i)\right] \end{aligned}$$

called the likelihood function. The maximum likelihood filter is obtained from maximizing the likelihood function. To maximize $f(S_{i-1} \mid a_i)$ with respect to a_i, equivalently, minimize,

$$J = \frac{1}{2}(S_{i-1} - L_{i-1}a_i)^T \Pi_{i-1}^{-1} (S_{i-1} - L_{i-1}a_i). \tag{3}$$

Differentiating both sides of Equation 3 gives:

$$\frac{\partial J}{\partial a_i} = L_{i-1}^T \Pi_{i-1}^{-1} (S_{i-1} - L_{i-1}a_i) = 0 \tag{4}$$

called the likelihood equation. Assume that $\{\Phi_i, H_i\}$ is observable and $M \geq n$, the estimate $\hat{a}_i$ of the time-varying signal frequency is then given by the solution of Equation 4 as follows:

$$\hat{a}_i = (L_{i-1}^T \Pi_{i-1}^{-1} L_{i-1})^{-1} L_{i-1}^T \Pi_{i-1}^{-1} S_{i-1}. \tag{5}$$

However, in the gain matrix $(L_{i-1}^T \Pi_{i-1}^{-1} L_{i-1})^{-1} L_{i-1}^T \Pi_{i-1}^{-1}$ in Equation 5, the inversion computation of matrices Π_{i-1} and $L_{i-1}^T \Pi_{i-1}^{-1} L_{i-1}$ is required. The dimension of these matrices becomes large as the window length M increases. In this case, the computation amount for the filter gain matrix increases. Therefore, for time varying systems, this computational load might be very burdensome since the filter gain must be computed, newly for every windows. Therefore, the estimate $\hat{a}_i$ of Equation 5 with a batch form is represented in a recursive form on the window for computational advantage. Define,

$$\Sigma_{i_M+j+1} \equiv L_{i_M+j}^T \Pi_{i_M+j}^{-1} L_{i_M+j} \tag{6}$$

then it can be represented in

$$\begin{aligned} \Sigma_{i_M+j+1} &= \Phi_{i_M+j}^{-T} \begin{bmatrix} L_{i_M+j-1} \\ H_{i_M+j} \end{bmatrix}^T \begin{bmatrix} \Pi_{i_M+j-1} & 0 \\ 0 & R_{i_M+j} \end{bmatrix}^{-1} \begin{bmatrix} L_{i_M+j-1} \\ H_{i_M+j} \end{bmatrix} \Phi_{i_M+j} \\ &= \Phi_{i_M+j}^{-T} \left(\Sigma_{i_M+j+1} + H_{i_M+j}^T R_{i_M+j}^{-1} H_{i_M+j}\right) \Phi_{i_M+j} \end{aligned} \tag{7}$$

with $\Sigma_{i_M} = 0$. Note that $\Sigma_{i_M} = 0$ should be satisfied to obtain the same Σ_{i_M+1} in Equations 6 and 7. Using Equation 6, the estimate $\hat{a}_i$ in Equation 5 can be written as $\Sigma_i \hat{a}_i \equiv L_{i-1}^T \Pi_{i-1}^{-1} S_{i-1}$ which can be obtained from the following subsidiary estimate defined by,

$$\hat{\theta}_{i_M+j} \equiv \Sigma_{i_M+j} \hat{a}_{i_M+j} \equiv L_{i_M+j-1}^T \Pi_{i_M+j-1}^{-1} S_{i_M+j-1}.$$

Then, the subsidiary estimate $\hat{\theta}_{i_M+j}$ can be represented in the recursive form on the window as follows:

$$\begin{aligned} \hat{\theta}_{i_M+j+1} &= L_{i_M+j}^T \Pi_{i_M+j}^{-1} S_{i_M+j} \\ &= \Phi_{i_M+j}^{-T} \begin{bmatrix} L_{i_M+j-1} \\ H_{i_M+j} \end{bmatrix}^T \begin{bmatrix} \Pi_{i_M+j-1} & 0 \\ 0 & R_{i_M+j} \end{bmatrix}^{-1} \begin{bmatrix} S_{i_M+j-1} \\ s_{i_M+j} \end{bmatrix} \\ &= \Phi_{i_M+j}^{-T} \hat{\theta}_{i_M+j} + \Phi_{i_M+j}^{-T} H_{i_M+j}^T R_{i_M+j}^{-1} s_{i_M+j}, \quad 0 \leq j \leq M-1. \end{aligned} \tag{8}$$

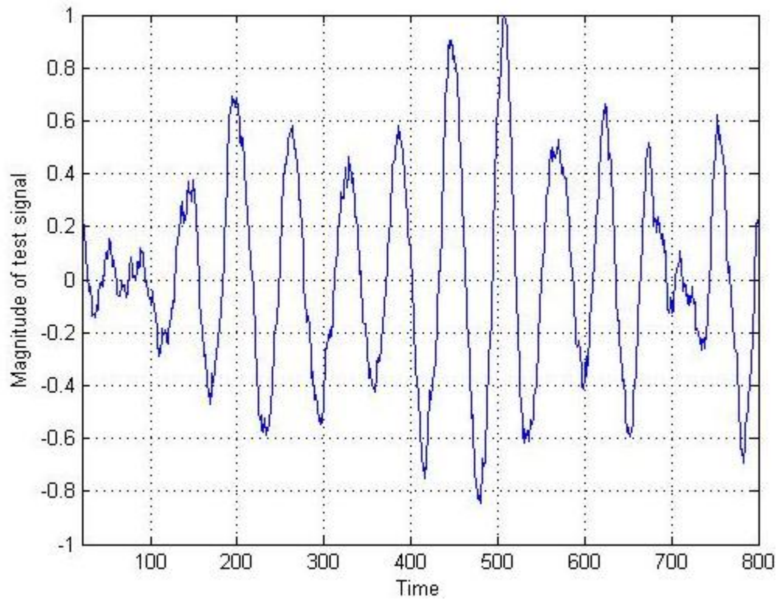

Figure 1. A quickly varying test signal.

Then, the estimate $\hat{a}_i$ of the time-varying signal frequency is obtained from the recursive form of Equation 8 on the window [i_M , i] as follows:

$$\hat{a}_i = \Sigma_i^{-1} \hat{\theta}_i \tag{9}$$

where $\Sigma_i = 0$ is given by Equation 7. Note that the recursive form in Equation 9 does not require the inversion computation of large dimensional matrices unlike the batch form. It will be shown in the following theorem that the estimate $\hat{a}_i$ of the time-varying signal frequency has an unbiasedness property when there are noises and a deadbeat property when there are no noises.

Theorem 1

The estimate $\hat{a}_i$ of the time-varying signal frequency is unbiased for noisy systems and exact for noise-free systems.

Proof

When there are noises on the window [i_M , i], since V_{i-1} is zero-mean in Equation 2, $E[S_{i-1}] = L_{i-1} E[a_i]$. Therefore, the following is true:

$$\begin{aligned} E[\hat{a}_i] &= (L_{i-1}^T \Pi_{i-1}^{-1} L_{i-1})^{-1} L_{i-1}^T \Pi_{i-1}^{-1} E[S_{i-1}] \\ &= (L_{i-1}^T \Pi_{i-1}^{-1} L_{i-1})^{-1} L_{i-1}^T \Pi_{i-1}^{-1} L_{i-1} E[a_i] \\ &= E[a_i] \end{aligned}$$

This completes the proof of the unbiasedness property. When there are no noises on the window [i_M , i] as $a_{i+1} = \Phi_i a_i$, $s_i = H_i a_i$, the observations S_{i-1} is determined by the current state a_i as $S_{i-1} = L_{i-1} a_i$. Therefore, the following is obtained directly from the proof of the unbiasedness property:

$$\hat{a}_i = a_i$$

This completes the proof of the deadbeat property.

Note that this deadbeat property indicates the finite convergence time and the fast estimation ability of the developed estimator. Above good inherent properties such as unbiasedness and deadbeat cannot be obtained by Kalman filtering approaches (Lim and Oppenheim, 1979; Mack and Jain, 1983; Lee et al., 1999).

SIMULATION RESULTS

As shown in the work of Lee et al. (1999) that the VFF Kalman filtering approach outperforms the conventional Kalman filtering approach of Mack and Jain (1983). Therefore, in this paper, the proposed approach was compared with the VFF Kalman filtering approach via computer simulations. In these simulations, the frequency varies with second order autoregression model which is updated at each sample. The transition matrix is $\Phi_i = I$. The test signal is assumed to vary relatively quickly as shown in Figure 1. To make a clearer comparison, Monte Carlo simulations of 50 runs are performed and each single simulation run lasts 800. The noise variance is

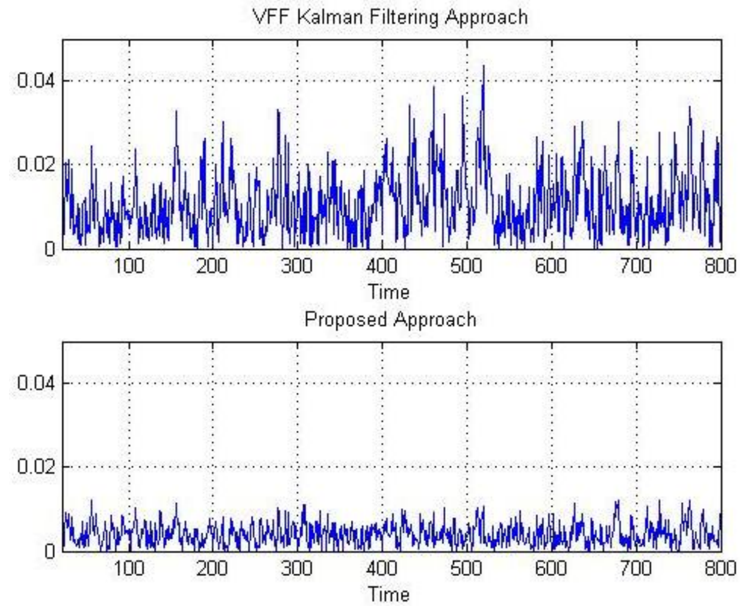

Figure 2. The mean of root-squared estimation errors.

taken as $R_i = 0.5^2$ and the window length is taken as $M = 20$. Figure 2 shows the mean of root-squared estimation errors of two approaches. For the estimate of the signal frequency which varies relatively quickly, the proposed approach outperforms remarkably the VFF Kalman filtering approach. This might result from the deadbeat property which means the finite convergence time and the fast estimation ability of the developed estimator. Therefore, it can be seen that, when the signal frequency corrupted by noises varies relatively quickly, the proposed approach gives a better estimate when compared with the existing Kalman filtering approaches (Mack and Jain, 1983; Lee et al., 1999).

Conclusion

This paper has proposed an alternative approach for estimating a time-varying signal frequency. The proposed estimator for time-varying signal frequency has been developed under a maximum likelihood criterion using only the most recent finite observations on the window. In the proposed approach, the estimate of the time-varying signal frequency has been represented in a batch form as well as recursive form for computational advantage. It has been shown that the estimate has good inherent properties such as unbiasedness, deadbeat and robustness. Via computer simulations, the proposed approach has been shown to give a better estimate when compared with the existing Kalman filtering approach when the signal frequency corrupted by noises varies relatively quickly.

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