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Numerical simulation of groundwater recharge from an injection well

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This paper presents the numerical simulation of groundwater recharge from a point source, that is, injection well using Explicit Finite Difference Model (FDFLOW) and Galerkin Finite Element Model (FEFLOW). The proposed model aims at simulation of groundwater flow in two-dimensional, transient, unconfined aquifer for a chosen synthetic Test Case. These models are validated with reported analytical solutions for a test run period of 210 days. It is found that the FEFLOW model performed better than FDFLOW model in terms of conservation of mass and oscillations in numerical solutions. For simulation of recharge from an injection well test run period of 1500 days is considered. The accretion in groundwater volume from an injection well is analyzed. Further the effect of injection rate of a well and aquifer parameter is analyzed on model results. It is found that both the model solutions are highly sensitive to injection rate and moderately sensitive to transmissivity whereas the specific yield has negligible effect on numerical solutions.

Key words: Explicit Finite Difference Model (FDFLOW), Finite Element Model (FEFLOW), model validation, mass balance, courant number, sensitivity of models to recharge rate, transmissivity and specific yield.

INTRODUCTION

Due to global warming it has become the need of the hour that ever depleting groundwater resources are to be continuously replenished using modern artificial recharging techniques. This has drawn the attention of researchers worldwide to device and use variants of numerical models for simulation of the hydrologic process of groundwater recharge.

It is found that Explicit Finite Difference Model (FDFLOW) and Finite Element Model (FEFLOW) models provide meaningful simulations of recharge from injection well to aquifers than available physical and electric-analog models; moreover, these models provide ease in simulating complex aquifer geometry and varying aquifer parameters. Because these are quite so, numerical models of groundwater flow are properly conceptualized version of a complex aquifer system which approximates the flow phenomenon.

The approximations in the numerical models are effected through the set of assumptions pertaining to the geometry of the domain, ways the heterogeneities are smoothed out, the nature of the porous medium, properties of the fluid and the type of the flow regime.

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The complex aquifer system is treated as a continuum, which implies that the fluid and solid matrix variables are continuously defined at every point in the aquifer domain. The continuum is viewed as a network of several representative elementary volumes, each representing a portion of the entire volume of an aquifer with average fluid and solid properties taken over it and assigned to the nodes of superimposed grid used for the spatial discretization of the domain. Anderson and Woessner (1992) discussed that numerical models also help in synthesizing field information and handling the large amount of input data in regional scale problems.

Wang and Anderson (1982) discussed that numerical models are applied either in an interpretive sense to gain insight into controlling the aquifer parameters in a site-specific setting or in generic sense to formulate regional regulatory guidelines and act as screening tools to identify regions suitable for artificial recharge by various methods.

Tseng and Ragan (1973) analyzed the dynamic response of the two-dimensional unconfined aquifers subject to localized recharge in both fully and partially penetrated aquifer systems by finite difference method. This method treats the nonlinear free surface boundary as an initial condition and the overall flow region has been solved as boundary value problem. The variations of free surface profiles with respect to time are also analyzed.

Sucharit and Parot (2001) conducted groundwater movement study in north part of lower central plain of Thailand by MODFLOW model. Both the steady and transient state models were calibrated with the observed data. The model also provided flow water balance. From sensitivity analysis of model results, it was found that the groundwater flow simulations are more sensitive to hydraulic conductivity.

Thomson et al. (1984) presented a Galerkin finite element model using Picard and Newton-Raphson algorithms designed for solution of non-linear groundwater flow equation. The model uses both triangular and rectangular finite elements for aquifer discretization. The influence area coefficient technique is used instead of conventional numerical integration scheme to obtain element matrices. It is found that this new technique is successful in reduction of computational cost.

Fagherazzi et al. (2004) developed the discontinuous Galerkin method which uses a finite-element discretization of the groundwater flow domain. Their model used an interpolation function of an arbitrary order for each element of the domain. The independent choice of an interpolation function in each element permits discontinuities in transmissivity in the flow domain. This formulation is shown to be of high order accuracy and particularly suitable for accurately calculating the flow field in porous media.

Chen and Chau (2006) have discussed the use of knowledge-based system technology along with the heuristic knowledge of model for manipulation of models for hydrologic system. Further they employed expert system shell in prototype knowledge-based system for modeling hydrologic processes.

Muttil and Chau (2006) applied machine learning (ML) techniques and genetic programming (GP) for modeling and prediction of ecological parameters in flow system. It is found that these techniques are proved to be useful for prediction of long term trends in flow system.

Chau (2007) discussed the integration of numerical simulation of flow with ontology based knowledge management (KM), artificial intelligence technology with the conventional hydraulic algorithmic models in order to assist novice application users in selection and manipulation of various mathematical tools and a Java/XML-based scheme for automatically generating knowledge search components.

The objectives of the present study are: development of FDFLOW and FEFLOW models for simulation of recharge from an injection well; validation of developed FDFLOW and FEFLOW models; evaluation of the performance of the models based on mass balance and stability criteria; analysis of the sensitivity of the model solutions to aquifer parameters viz. transmissivity and specific yield and injection rate.

MODEL DEVELOPMENT

Groundwater flow equation

The governing equation of two-dimensional, horizontal, and transient groundwater flow in homogeneous, isotropic and unconfined aquifer is given by Illangasekare and Doll (1989),

$$S_y \frac{\partial h}{\partial t} = T_{xx}\frac{\partial^2 h}{\partial x^2} + T_{yy}\frac{\partial^2 h}{\partial y^2} + \sum_{i=1}^{n_w} Q_i\,\delta(x_o - x_i, y_o - y_i) \qquad (1)$$

where S_y is the specific yield, [dimensionless]; h is the hydraulic head averaged over vertical, [L]; t is the time, [T]; T_{xx} and T_{yy} are components of the transmissivity tensor, [L^2/T] which are approximated as $T_{xx} \approx K_{xx}\,h$ and $T_{yy} \approx K_{yy}\,h$, provided the change in the head in unconfined aquifer is negligible as compared to its saturated thickness; K_{xx} and K_{yy} are components of the hydraulic conductivity tensor, [L/T]; x and y are spatial coordinates, [L]; Q_i is the injection rate at i th injection well, [L^3/T]; n_w is the number of injection wells in the domain; $\delta(x_o - x_i, y_o - y_i)$ is the Dirac delta function; x_o and y_o are the Cartesian coordinates of the origin , [L]; x_i and y_i are the coordinates of i th injection well, [L]. Equation (1) is subject to the following initial condition which is given as:

$$h(x,y,0) = h_0 \qquad (x,y) \in \Omega \qquad (2)$$

Where h_0 is the initial head over the entire flow domain, [L] and Ω is the flow domain, [L^2]. Equation (1) is subject to the Dirichlet type of boundary condition which is given as:

$$h(x,y,t) = h_1 \qquad (x,y) \in \Gamma_1; \quad t \geq 0 \tag{3}$$

Where h_1 is the prescribed head over aquifer domain boundary Γ_1, [L].

The Neumann boundary condition with zero groundwater flux can be given as:

$$\left[\{q_b(x,y,t)\} - [T]\nabla h(x,y,t)\right]'.\{n\} = 0 \qquad (x,y) \in \Gamma_2; \ t \geq 0 \tag{4}$$

Where q_b is the specified groundwater flux across boundary Γ_2, [$L\ /T$]; $[T]\nabla h$ is the groundwater flux across the boundary Γ_2, [$L\ /T$] and n is normal unit vector in outward direction.

FDFLOW model

The explicit finite difference method is employed in FDFLOW model to solve Equation (1). In this model the unknown nodal head at next time level is explicitly computed from the four neighboring nodes with known heads at the previous time level. The explicit finite difference scheme is adopted because, it is computationally efficient than the alternating direction implicit finite difference scheme, but it has the restriction of the size of time step used in simulation. However, the stability of the solutions can be ensured by constraining the length of a time step. In this model the entire aquifer domain is discretized into rectangular computational cells by superimposing the mesh centered finite difference grid over the domain.

The computational cells are formed around the intersection points of grid column and row lines which are referred to as a node. Thus each node with grid column index i and grid row index j represents a computational cell. The size of each rectangular computational cell is Δx and Δy in x- and y- directions respectively.

In FDFLOW model, the spatial derivatives and temporal derivative in Equation (1) are approximated by central finite difference and forward difference schemes respectively which will result into the following equation

$$\frac{S_{y_{i,j}}(h_{i,j}^{t+\Delta t} - h_{i,j}^{t})}{\Delta t} = \frac{(T_{xx_{i,j}}(h_{i+1,j}^{t} - 2h_{i,j}^{t} + h_{i-1,j}^{t}))}{\Delta x^2} + \frac{(T_{yy_{i,j}}(h_{i,j+1}^{t} - 2h_{i,j}^{t} + h_{i-1,j}^{t}))}{\Delta y^2} + \frac{Q_{i,j}}{\Delta x\,\Delta y} + q_{i,j} \tag{5}$$

Equation (5) can be expressed in matrix form as:

$$\begin{Bmatrix} h_1^{t+\Delta t} \\ h_2^{t+\Delta t} \\ . \\ . \\ h_N^{t+\Delta t} \end{Bmatrix} = \begin{bmatrix} \left(\frac{T_{xx_1}\Delta t}{S_{y_1}\Delta x^2}\right) & 0 & 0 & 0 \\ 0 & \left(\frac{T_{xx_2}\Delta t}{S_{y_2}\Delta x^2}\right) & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & \left(\frac{T_{xx_N}\Delta t}{S_{y_N}\Delta x^2}\right) \end{bmatrix} + \begin{bmatrix} \left(\frac{T_{yy_1}\Delta t}{S_{y_1}\Delta y^2}\right) & 0 & 0 & 0 \\ 0 & \left(\frac{T_{yy_2}\Delta t}{S_{y_2}\Delta y^2}\right) & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & \left(\frac{T_{yy_N}\Delta t}{S_{y_N}\Delta y^2}\right) \end{bmatrix} \begin{Bmatrix} h_1^{t} \\ h_2^{t} \\ . \\ . \\ h_N^{t} \end{Bmatrix} \tag{6}$$

Thus the unknown nodal head vector in Equation (6) is solved using the direct matrix inversion technique available in MATLAB environment.

FEFLOW model

This model employs the Galerkin finite element technique for computing the head distribution in aquifer. In this method the trial solution of the head is substituted into Equation (1) which results into the residual. By using the weighting functions as the shape functions, the weighted residual is integrated and forced to zero to yield the system of linear equations. The set of the linear equations is solved to get the nodal head distribution. The groundwater flow domain is discretized into finite number of nodes using a triangular finite element mesh.

Each node of the domain is identified by an index L and the finite element by an index e. The three nodes of a finite element are labeled as i, j and k in either clockwise or anticlockwise manner. The time domain is discretized into finite number of discrete time steps. The size of each time step is Δt. The trial solution of the groundwater head to be used in finite element formulation is given as:

$$\hat{h}(x,y,t) = \sum_{L=1}^{N} h_L(t)\, N_L(x,y) \tag{7}$$

Where $\hat{h}$ is the trial solution of groundwater head, [L]; N is the total number of nodes in the flow domain; h_L is the nodal groundwater head at any time t, [L]; N_L is the linear shape function at any point (x, y) in the aquifer domain. The shape function is defined piecewise but in continuous manner over entire flow domain which ranges from 0 to 1. The trial solution is substituted for unknown nodal head h in Equation (1) which results into the residual of groundwater head which is expressed as:

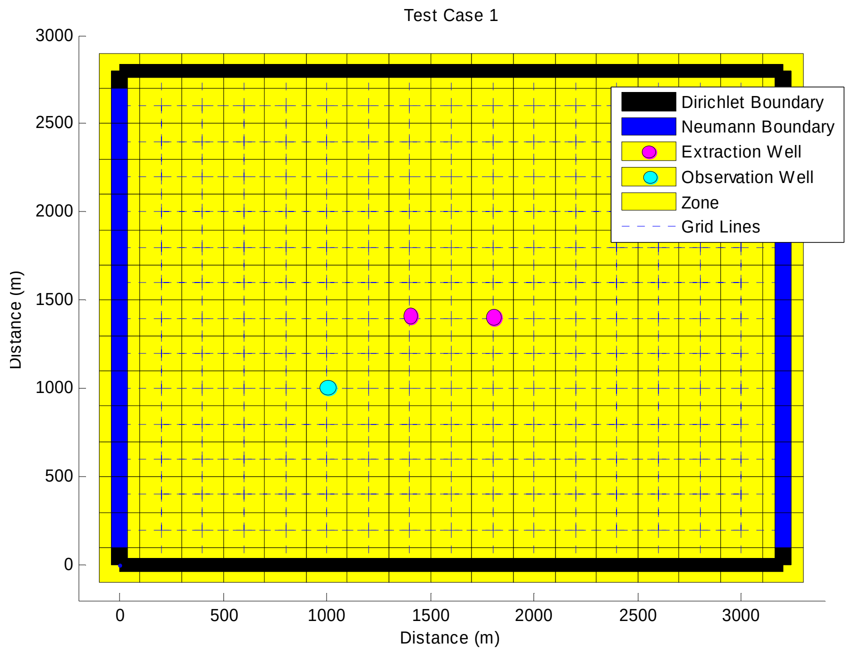

Figure 1. Schematic of aquifer modeled in chosen synthetic Test Case 1 for the validation of FDFLOW and FEFLOW models.

$$\varepsilon^h(x,y,t)=T_{xx}\frac{\partial^2 \hat{h}}{\partial x^2}+T_{yy}\frac{\partial^2 \hat{h}}{\partial y^2}+\sum_{i=1}^{n_w} Q_i\delta\left(x_o-x_i,y_o-y_i\right)-S_y\frac{\partial \hat{h}}{\partial t} \tag{8}$$

Where ε^h is the residual of groundwater head at any point (x, y) and at time t; $[L]$. The residual of the head is weighted and integrated over entire flow domain to obtain the nodal head distribution. The integral of the weighted head residual is forced to zero to yield the system of algebraic equations and the same is given as:

$$\iint_{\Omega}\left(T_{xx}\frac{\partial^2 \hat{h}}{\partial x^2}+T_{yy}\frac{\partial^2 \hat{h}}{\partial y^2}+\sum_{i=1}^{n_w} Q_i\delta\left(x_o-x_i,y_o-y_i\right)-S_y\frac{\partial \hat{h}}{\partial t}\right)W_L\,dx\,dy=0 \tag{9}$$

Where W_L is the weighting function at a node L. Applying the numerical integration for the various terms of Equation (7) the following system of linear equations is obtained and the same can be written as:

$$\left([\boldsymbol{G}]+\frac{1}{\Delta t}[\boldsymbol{P}]\right)\left\{h_{i,j}^{t+\Delta t}\right\}=\left(\frac{1}{\Delta t}[\boldsymbol{P}]\right)\left\{h_{i,j}^{t}\right\}+\{\boldsymbol{B}\}+\{\boldsymbol{f}\} \tag{10}$$

Where $[\boldsymbol{G}]$ is the global conductance matrix, $[\boldsymbol{P}]$ is the global storage matrix, $\{\boldsymbol{B}\}$ is the global load vector and $\{\boldsymbol{f}\}$ is the global boundary flux vector. From the known head distribution at previous time level the unknown head distribution at the next time level is obtained by recursively solving the set of algebraic equations given in Equation (10).

RESULTS AND DISCUSSION

Validation of numerical models

The chosen synthetic Test Case 1 is aimed at validating the FDFLOW and FEFLOW models. The validation of groundwater flow models is accomplished by comparing model simulations with the reported analytical solutions Illangasekare and Doll (1989). For synthetic Test Case 1 a rectangular, homogeneous, and isotropic unconfined aquifer is chosen as shown in Figure 1. The rectangular aquifer is selected mainly to satisfy the shape constraints imposed for the analytical solution. The aquifer system is

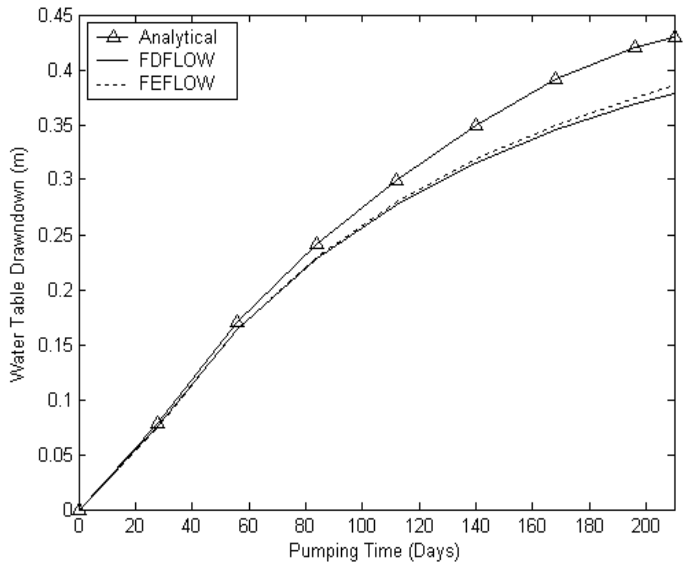

Figure 2. Validation of FDFLOW and FEFLOW models for synthetic Test Case 1.

3,200 m long and 2,800 m wide. The head at the top and bottom sides of the aquifer boundary is considered to have constant value of 100 m throughout the simulation, and left and right sides of the aquifer boundary is considered to have zero groundwater flux. Two pumping wells are placed at a location of (1400 m, 1400 m) and (1800 m, 1400 m) from the origin, as shown in Figure 1. Initially the groundwater head is assumed static with a value of 100 m at all nodes in the aquifer domain. The water table drawdown caused by pumping are observed at an observation well situated at a location of (1000 m, 1000 m) from the origin.

The aquifer parameters used in the simulation include aquifer thickness (b =30 m), constant pumping rates for the two pumping wells (Q_{p1} & Q_{p2} = 1142.85 and 1428.57 m^3/d), effective porosity (θ =0.30), aquifer transmissivty (T =885.71 m^2/d) and specific yield (S_y =0.15) respectively. For FDFLOW model, the aquifer is discretized using mesh centered finite difference grid which results into 255 computational cells with uniform nodal spacing of 200 m in both x- and y- directions. For FEFLOW model, the aquifer is discretized using the triangular finite element mesh with 448 elements. The size of the square finite difference cell and isosceles triangular element is 200 m. Total nodes with Dirichlet boundary condition and Neumann boundary conditions are 32 and 28, respectively.

The water table drawdown values at an observation well due to the pumping by pair of wells for 210 days are computed by FDFLOW and FEFLOW models. The time-drawdown curves obtained by FDFLOW and FEFLOW models and reported analytical solution Illangasekare and Doll (1989) are compared as shown in Figure 2. The drawdown values are computed as 0.38 and 0.39 m by FDFLOW and FEFLOW models respectively which are quite comparable with the reported drawdown of 0.42 m by analytical solution implying the validity of the developed flow models.

The mass balance error analysis for the flow models used in numerical experiments for Test Case 1 showed that both the FDFLOW and FEFLOW conserves the mass satisfactorily and the average mass balance error in both the models is well within the limit i.e. up to 0.69%.The FDFLOW and FEFLOW model solutions are found to be stable for the Courant number of 0.14 for the chosen time step of 1 day.

Simulation of recharge from injection well

This Test Case 2 (Illangasekare and Doll, 1989) is aimed at simulating the groundwater flow behavior under the condition of recharge from injection well. The aquifer and flow parameters, initial and boundary conditions and spatial discretization used in this Test Case are same as that of synthetic Test Case 1 except that of different specific yield of 0.10 and test period of 1500 days (Figure 3). An injection well situated at the location of (2200 m, 1800 m) recharges the aquifer at the constant rate of 8214.28 m^3/d.

The rise in water table caused by recharge to the aquifer from injection well is observed at an observation well situated at a location of (1800 m, 1000 m). The head distribution obtained by both the models is compared. The sensitivity of time-accretion curve obtained by

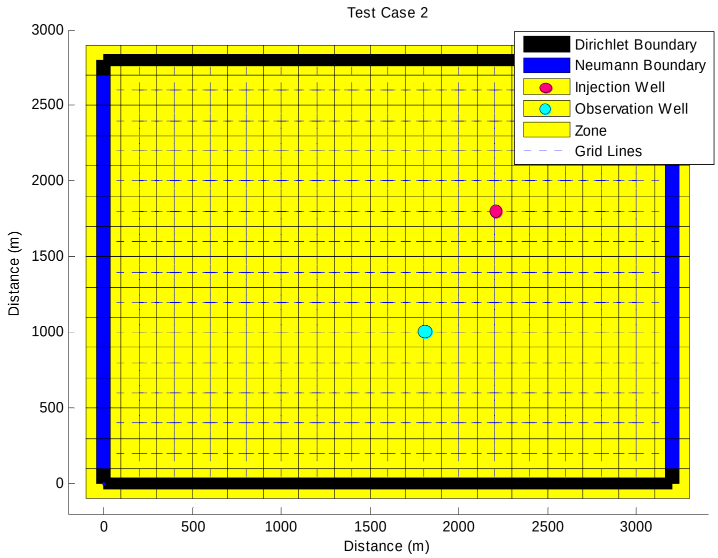

Figure 3. Schematic of aquifer modeled in synthetic Test Case 2 for the simulation of two-dimensional transient groundwater flow in unconfined aquifer under the recharge from an injection well.

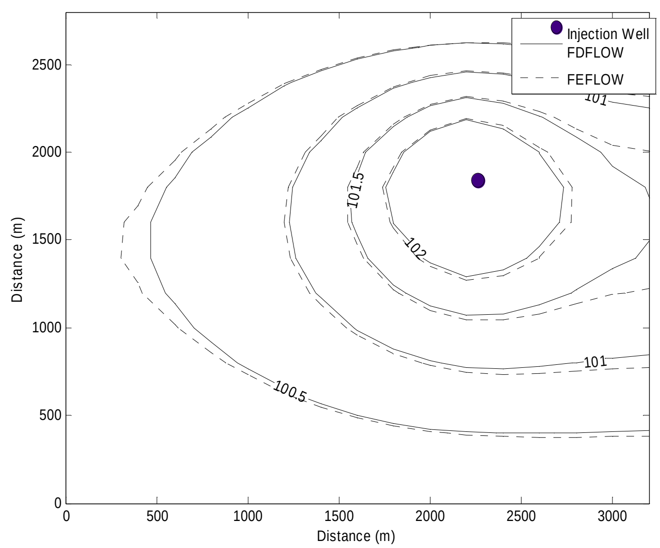

Figure 4. Comparison of groundwater head distribution by FDFLOW and FEFLOW models under the injection well conditions for Test Case.

FDFLOW and FEFLOW models to the variation in transmissivity, specific yield and injection rate have also been studied.

Comparison of head distribution by FDFLOW and FEFLOW models under the recharge from an injection well

The comparison of the groundwater head distribution simulated by FDFLOW and FEFLOW model is shown in the Figure 4. It is found from the results that the maximum rise in water table is noted at the node situated at location (1800 m, 2200 m), which is in close proximity of the injection point is 105.30 and 105.36 m, respectively by FDFLOW and FEFLOW model due to recharge of water from an injection well. The contour of head value of 100.5 m obtained by FEFLOW model has experienced some numerical dispersion due to the effect of zero groundwater flux boundary. The deviation of

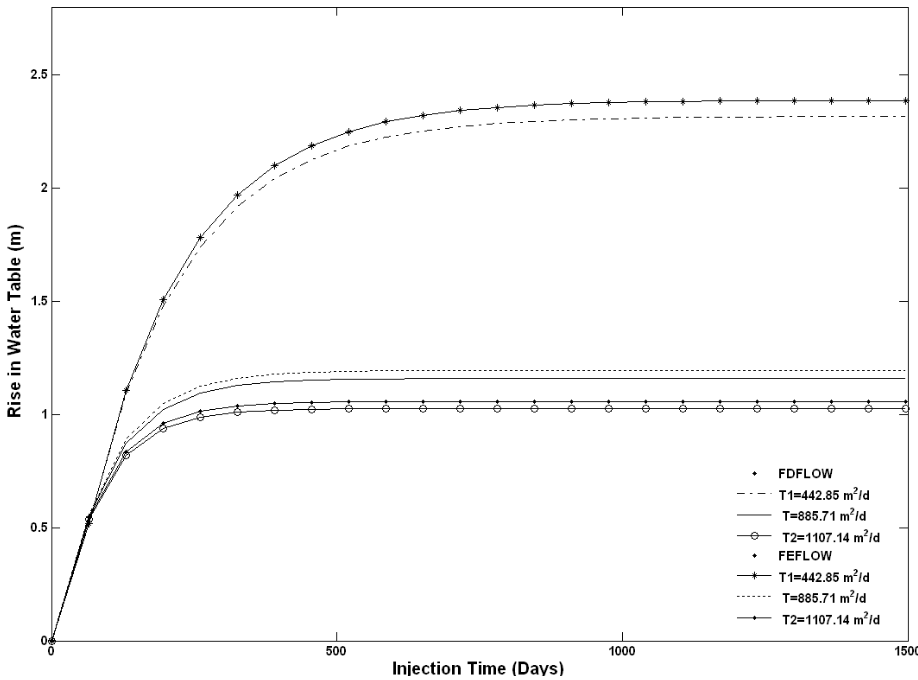

Figure 5. Effect of transmissivity on the rise in water table for synthetic Test Case 2.

FEFLOW computed solutions from analytical solutions in the later stages of simulation may be attributed to lower order interpolation function used in the FEFLOW model.

Effect of transmissivity on rise in water table

Figure 5 shows the plot of rise in water table against the injection time for different values of the transmissivity. The transmissivity values are varied over the range of 442.85 to 1107.14 m^2/d which is -50 to 25% of the base value of the transmissivity that is, 885.71 m^2/d. The increase in transmissivity by 25% results into 11.48% drop in water table rise whereas for the same period of pumping the decrease in transmissivity by 50% causes rise in water table by 100%.

It is seen from the results that the water table rise curves simulated by FEFLOW at the different values of transmissivity lie slightly above those simulated by FDFLOW model. These model solutions are moderately sensitive to the transmissivity of the aquifer as the transmissivity vector is approximated as the product of conductivity vector and saturated thickness and there are not spatial variation of hydraulic conductivity field.

Effect of specific yield on rise in water table

Figure 6 shows the curves of water table rise versus injection time for the different values of the specific yield. The magnitude of the specific yield is varied from 0.09 to 0.30 which is -25 to 200% of its base value which is 0.10. The results showed that the water table is continuously rising with the increase in injection time and after 400 days of injection it starts stabilizing and approaches to the constant value. The variation of the specific yield has lesser impact over the rise in water table for the chosen range of variation in the specific yield. The effect of the specific yield is found to be negligible over the numerical solutions because of constant and less magnitude of specific yield.

Effect of Injection rate on rise in water table

Figure 7 shows the influence of variation in injection rate of the well on the water table rise. The injection rate is varied from -50 to 50% of its base value which is 8214.28 m^3/d. The change in water table rise computed by FDFLOW model is observed to be -50% and +60% for the corresponding variation of injection rate and in case of FEFLOW simulation the change in water table has almost been same for the corresponding range of variation in injection rate because of an appropriate interpolation function used in finite element integration in FEFLOW model in the initial stages of simulation. But as the injection rate increases after 263 days the water table rise curve shifts leftward indicating the stabilization of head buildup at early stage of FDFLOW and FEFLOW solutions are more sensitive to injection rate because approximations in numerical formulations are not able to accurately simulate point source conditions.

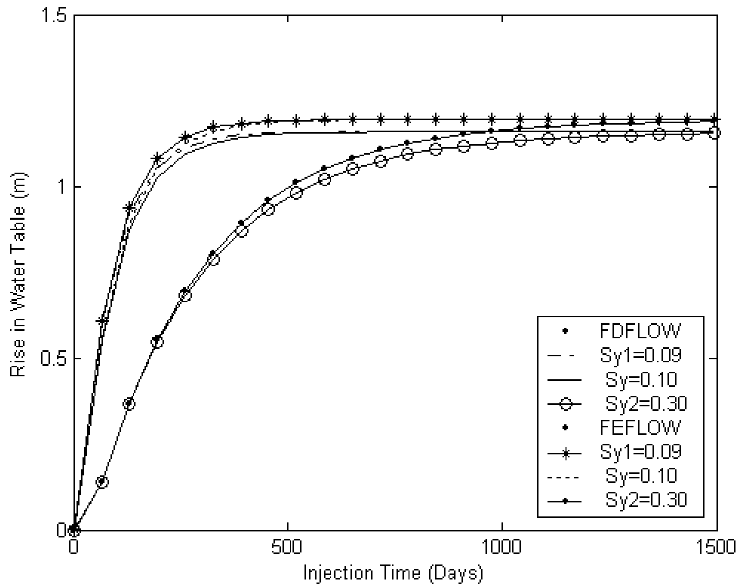

Figure 6. Effect of specific yield on the rise in water table for Test Case 2.

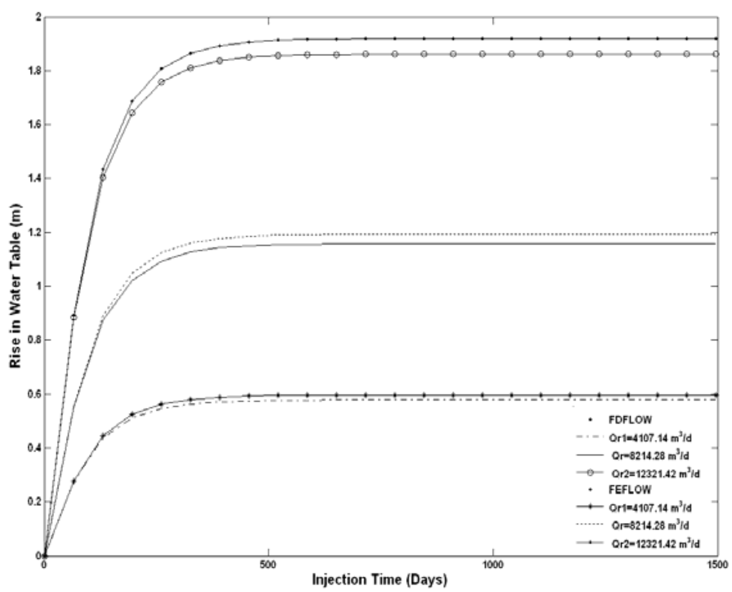

Figure 7. Effect of injection rate on the rise in water table for synthetic Test Case 2.

Conclusions

The following are the conclusions of the present study:

1). Validation of FDFLOW and FEFLOW model for Test Case 1 shows that there is a close agreement between computed and analytical solutions in the initial stages of the pumping, however, after 210 days of pumping the difference between drawdown obtained by FDFLOW and FEFLOW model and analytical solution are 10 and 7% respectively. Thus FEFLOW model has performed better than FDFLOW model.

2). The mass balance error analysis for the flow models used in numerical experiments for Test Case 1 showed that both the FDFLOW and FEFLOW conserves the mass satisfactorily and the average mass balance error

in both the models is well within the limit i.e. up to 0.69%.
3). For Test Case 1, the FDFLOW and FEFLOW model solutions are found to be stable for the Courant number 0.14 for the chosen time step of 1 day.
4). FEFLOW model simulates recharge conditions more accurately than FDFLOW model as evidenced by less numerical oscillations in FEFLOW model solutions.
5). FDFLOW and FEFLOW solutions are more sensitive to injection rate and moderately sensitive to the transmissivity of the aquifer and there is negligible effect of specific yield on model solutions.

The following are the limitations of the study:

1). For larger time step size, FDFLOW model solutions are highly oscillatory.
2). In FEFLOW model, use of linear interpolation function poorly simulates point source conditions.
The present work can be continued further as follows: Long time prediction of the hydraulic head can be made by conducting time series prediction of rainfall and irrigation return flow. Further large time steps can be accommodated in models to make these suitable for coupling with optimization models.

Conflict of Interest

The author have not declared any conflict of interest.

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