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Full Length Research Paper

Development, utilization and statistical evaluation of Hardy Cross pipe network analysis softwares

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Pipe network analysis is an essential aspect in the design of water distribution networks. The Hardy Cross method is one of the methods commonly employed in pipe network analysis. Two softwares are developed using the Hardy Cross method for analysis of flow in pipe networks based on FORTRAN and Visual BASIC (VB) programming languages. HARDY CROSS1 (FORTRAN-based) and PNET Expert (VB-based) are proposed. They differ only in the algorithm for computing the friction factor for head loss computation within pipeline; the former utilizes the Moody's formulation while the later uses the Barr's equation. Previous study proved that Moody's equation has lower total errors than Barr's equation, thus accounting for the differences in their algorithm implementation and results. Evaluation and analysis of these softwares using statistical methods (total error, coefficient of determination, coefficient of correlation, and reliability) are presented with particular attention to accuracy, validity and good fitness of each of the software. Two flow rate problems are used to demonstrate the applicability of the softwares. The study showed that the overall total errors (for the two Problems) for the flow rates are 1.33 and 0.60, for PNET Expert and HARDY CROSS1 respectively. Coefficients of determination values are 0.999976 for HARDY CROSS1 and 0.999948 PNET Expert. The values of coefficient of correlation were 0.999988 and 0.999974 for HARDY CROSS1 and PNET Expert respectively. It is then concluded that, the HARDY CROSS1 is the best option, because it gave the highest coefficient of determination, lowest total errors, and lowest reliability value. Its better performance is related to the algorithm utilized for head loss computation.

Key words: Hardy Cross, pipe network analysis, flow rate, statistical evaluation, head loss, friction factor.

INTRODUCTION

Pipe network analysis, as described by Featherstone and Nalluri (1997) involves the determination of pipe flow rates and pressure heads, which satisfy continuity and energy conservation equations. Water distribution network analysis provides the basis for the design of new systems and the extension of existing ones. Design criteria such as specified minimum flow rates and pressure distributions across a network are affected by the arrangement and sizes of the pipes and distribution of the outflows. Since a change in diameter in one pipe length will affect the flow and pressure distribution everywhere, network design is not an explicit process.

Common challenges encountered in the distribution system are those due to pressure and discharge in certain areas of a city where the elevation is relatively high or the place is far away from the water treatment plant. Webber (1971) posited that the network of distribution mains is nearly always the most expensive item of equipment in a water undertaking. Also, the cost of its upkeep generally represents a large proportion of the annual maintenance budget. It is therefore incumbent upon the water engineer to devote some considerable care to the design and simulation of the most efficient distribution system and this entails an accurate prediction of the flows and pressures in the various pipe

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components. Distribution networks should be designed with sole aim of ensuring safe working pressures without compromising the supply efficiency, resilience of system components and avoiding unnecessary cost implications.

According to Chadwick and Morfett (1998), all these limiting values of flows and pressure heads can only be achieved through proper network analysis. Meeting this challenge requires a comprehensive network modelling capability that includes the rapid and accurate calculation of various designs and operating parameters.

The most commonly used computational techniques for pipe network are: Linear theory, Hardy Cross and Newton-Raphson methods (Oke, 2007; Jeppson, 1976; Featherstone and Nalluri, 1997; Wood and Rayes, 1981; Wood and Charles, 1972). Other methods of pipe network analysis include equivalent diameter method, electric analyzer, gradient method and analysis using optimization technique (Brkic, 2011; Brkic, 2009; Ormsbee, 2006; Morley et al., 2001). A brief review of the theoretical framework of each of these methods along with critique of the relative advantages and/or limitations of each method was presented by Ormsbee (2006), Basha and Kassab, (1996), Waheed (1992) and Lee (1983).

Details of the principles of the Hardy Cross method can be found in standard texts and articles (Jeppson, 1976; Brkic, 2009; Ormsbee, 2006; Cross, 1936; Brkic, 2011; Babatola et al., 2008; Featherstone and Nalluri, 1997). There exists a lot of softwares developed using the linear theory and Newton-Raphson techniques, but rarely do one finds Hardy Cross based software. This may be attributed to the easier convergence of linear theory and Newton-Raphson techniques over the Hardy Cross method which could sometimes take long period of time to converge to a solution and in some cases, may fail to converge at all for complex distribution networks. The rate of convergence of the solution depends on the assumed flow initialization (Ormsbee, 2006; Basha and Kassab, 1996; Gay and Middleton, 1971). Jeppson (1976) developed a FORTRAN program for the analysis of pipe networks using the Hardy Cross method. This program has enjoyed so much reputation among researchers. But when Jeppson's program was studied closely, the following drawbacks were eminent and manifested.

i. There was no provision for calculating the Reynolds number and friction factor for each loop continuously as the flows within each pipe is corrected.

ii. The program calculates the Reynolds number and the friction factor of each pipe using the initial guess of the flow rate and maintains the results throughout the execution of the program. There was an omission of a negative sign in finding the error in the flows that is, ΔQ, which is given by

$$\Delta Q = -\frac{\sum_{i=1}^{n}(h_i)}{2\sum_{i=1}^{n}\left(\frac{h_i}{Q_i}\right)} \quad (1)$$

where h is the head loss in a pipe based in the estimated flow Q.

iii. The assignment statement for the correction of the assumed flows is wrong.

Consequent upon the aforementioned problems inherent in Jeppson's program, there was the need to develop another program using FORTRAN language that will eradicate the above drawbacks. The Hardy Cross method is widely used in comparison with the other methods, because there is less number of loops than nodes in a water distribution system, and consequently, lesser computations are required to balance the system (Waheed, 1992). Developers of most new pipe network analysis algorithms, use Hardy Cross method as a benchmark for validation of their new algorithms (Brkic, 2011; Sarbu, 2011; Brkic, 2009; Gay and Middleton, 1971). These and many more, give the motivation for using Hardy Cross method in this study. The objectives of this study are to develop softwares using FORTRAN and Visual BASIC (VB) programming languages based on the Hardy Cross method of analysis of pipe networks that can handle small and large distribution systems to solve flow rates and head losses in pipes; and to use the developed softwares to solve two problems of distribution networks and compare the results obtained based on statistical data evaluation criteria. These problems consist of two and three loops pipe network as shown in Figures 1 and 2, respectively.

MATERIALS AND METHODS

The head balance criterion is that the algebraic sum of the head losses around any closed loop is zero; the sign convention that clockwise flows (and the associated head losses) are positive is adopted. Details of the steps for the Hardy Cross method may be found in standard texts and articles (Brkic, 2011; Brkic, 2009; El-Bahrawy, 1997; Featherstone and Nalluri, 1997).

Flow rate problems employed

To evaluate the softwares developed in this study, two-loop (Figure 1) and three-loop (Figure 2) pipe networks are adopted from the literature (Featherstone and Nalluri, 1997), and pipe and loop numbers (IDs) were assigned. Outflow at each node and assumed flow directions were drawn as illustrated in Figs. 1 and 2. Analysis of the pipe networks was carried out manually and the results serve as the expected values of each fitting procedure. Analysis results (final discharges and head losses) were then used for statistical evaluations using total error, coefficient of correlation, coefficient of determination and reliability tests.

Description of the softwares

Visual FORTRAN 90 compiler and Visual BASIC 6.0 are used for creating, running and compiling ***HARDY CROSS1*** and PNET Expert respectively. Codes for ***HARDY CROSS1*** are developed according to an earlier study by Lukman, 2004. ***HARDY CROSS1*** has a default input data file name as INPUT PROJECT.SI and an

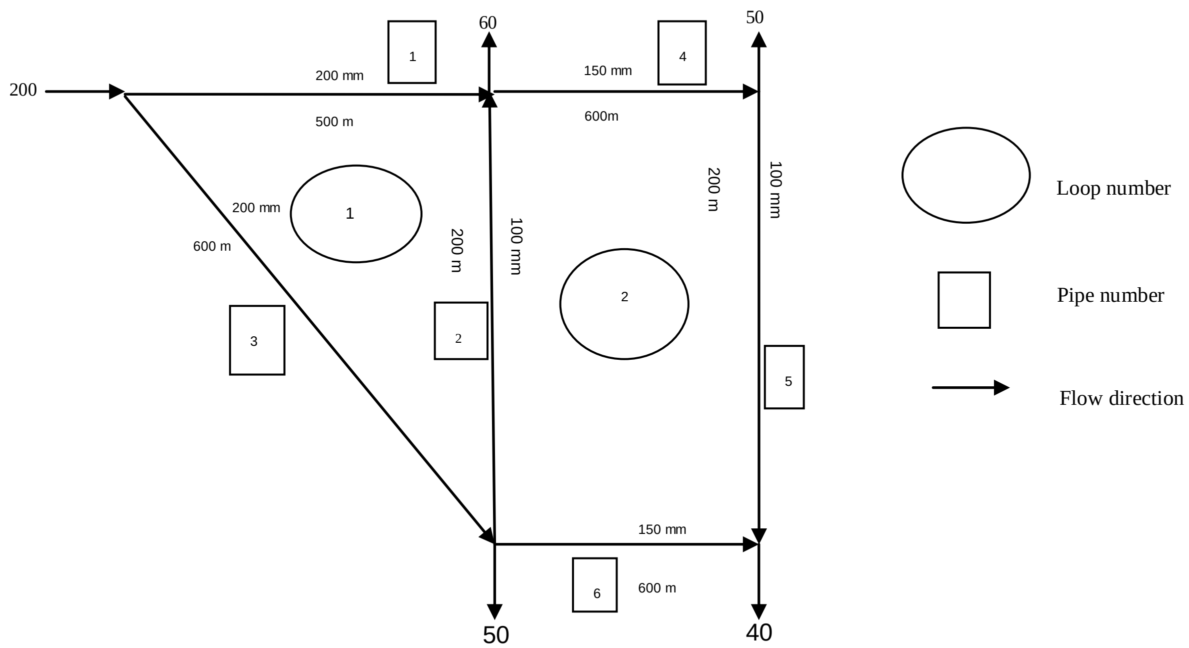

Figure 1. Pipe characteristics: lengths (m), diameters (mm), nodal demands (l/s) and assumed flow directions for Problem 1.

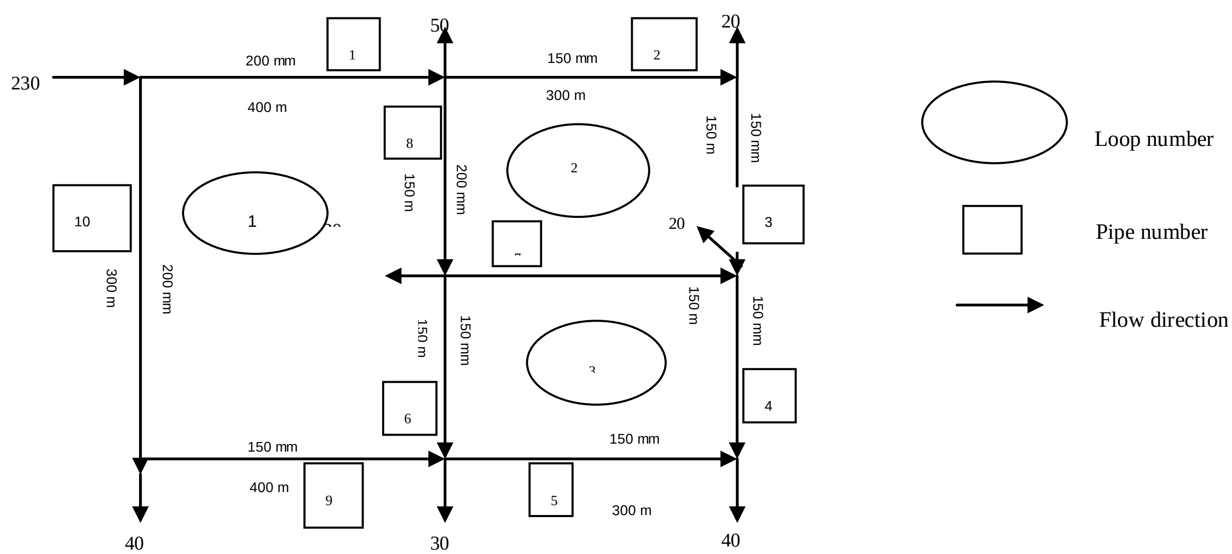

Figure 2. Pipe lengths (m), diameters (mm), nodal demands (l/s) and assumed flow directions (Problem 2).

output data file name as OUTPUT PROJECT.SI. ***HARDY CROSS1*** displays only the final values of flows and head losses in pipes. The input data file is to be created by the user, while the output data file is to be created by the compiler when it is running. The two file names can be changed as the user wishes.

Pseudocode for ***HARDY CROSS1*** is presented as follows:

i. Create input and output data and result files respectively.
ii. Enter the number of pipes, number of loops, maximum number of iterations, fluid kinematic viscosity, stopping criterion error term, twice the value of acceleration due to gravity.
iii. Enter diameters, lengths, roughness coefficient and assumed flow for each pipe in the network.
iv. Compute relative roughness for all pipes
v. Enter loop ID no (identification number, starting with the first loop).
vi. Print loop ID (starting with the first loop).
vii. Enter the number of pipes in that loop together with the pipe IDs positive or negative, depending on the flow direction.
viii. Take the absolute value of the assumed flows.

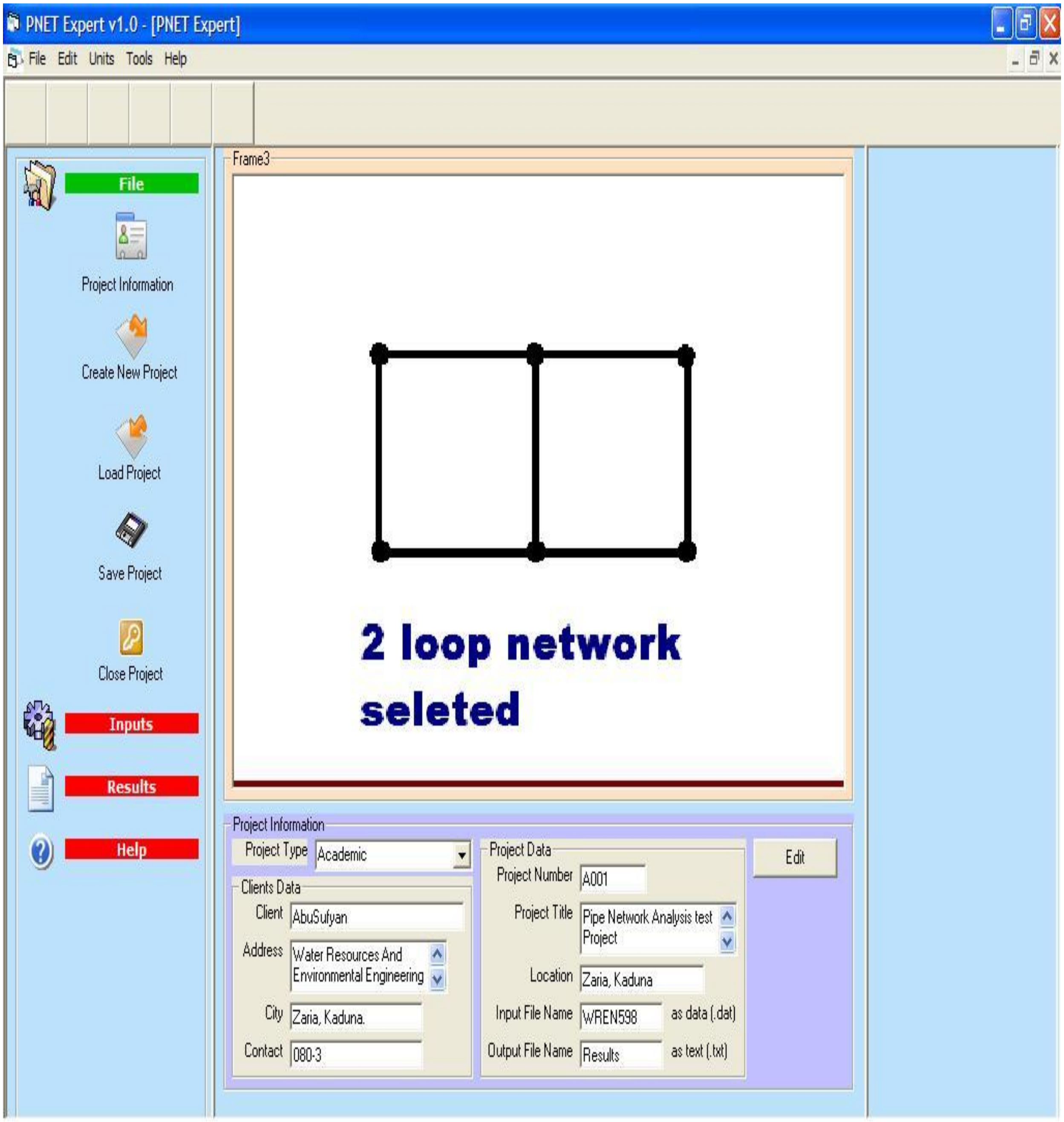

Figure 3. PNET expert home page.

ix. Compute Reynolds number, friction factor (using Moody's equation), coefficient of head loss and head loss.
x. Compute the numerator of equation (1).
xi. Compute the denominator of equation (1).
xii. Repeat steps (e)-(k) for the next pipe.
xiii. Compute equation (1).
xiv. Correct the flows
xv. Repeat steps (f)-(n) for all the loops in the pipe network.
xvi. Has the target stopping criterion been met?
xvii. If no, then repeat steps (f)-(n), that is, next iteration.
xviii. If yes, then print the final flow rates in pipes.
xix. Using the final flow rates, compute and print the head losses for all pipes.
xx. Stop.

Pipe Network Expert (PNET Expert) Version 1.0 is windows-based application software that is designed purposely for pipe network analysis (Figure 3). The software is developed using VB language by one of the authors according to Abusufyan (2009). Figure 4 is used to input data relating to the total number of pipes to be used in the network, fluid type, viscosity and number of loops. Figure 5 shows where all pipe details should be entered, edited and saved. After data have been processed, users have the options of viewing individual loop results, final results or printing or saving the results in notepad application. The Pseudocode for PNET Expert is similar to that of ***HARDY CROSS1*** except that steps (d) and (f) are

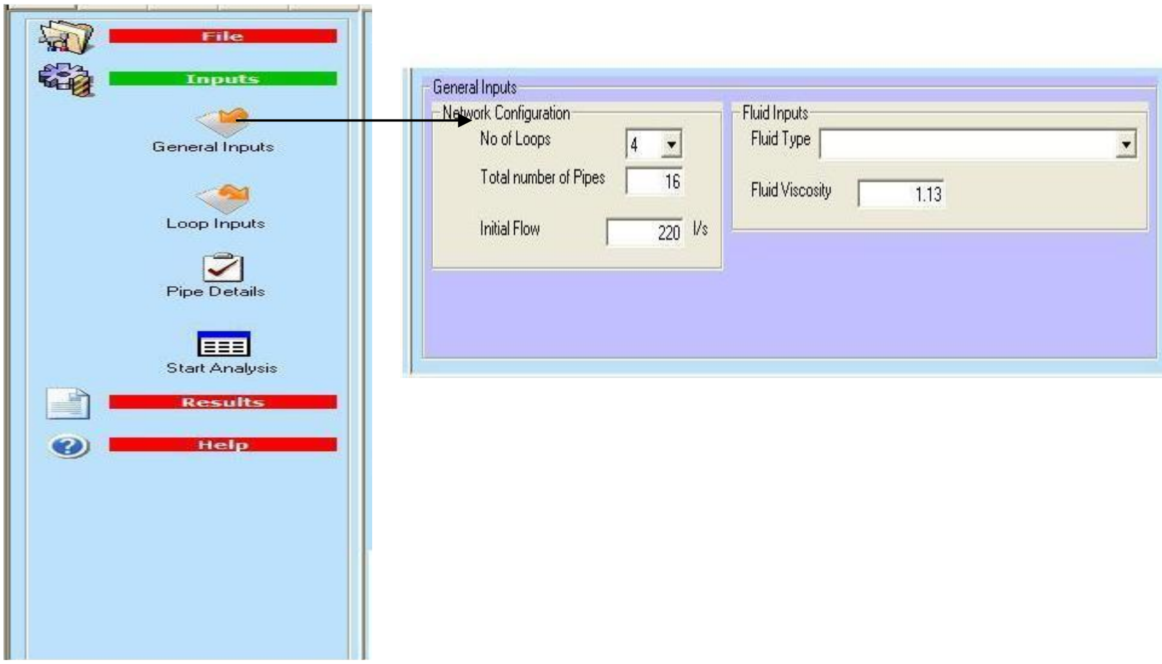

Figure 4. General input screen.

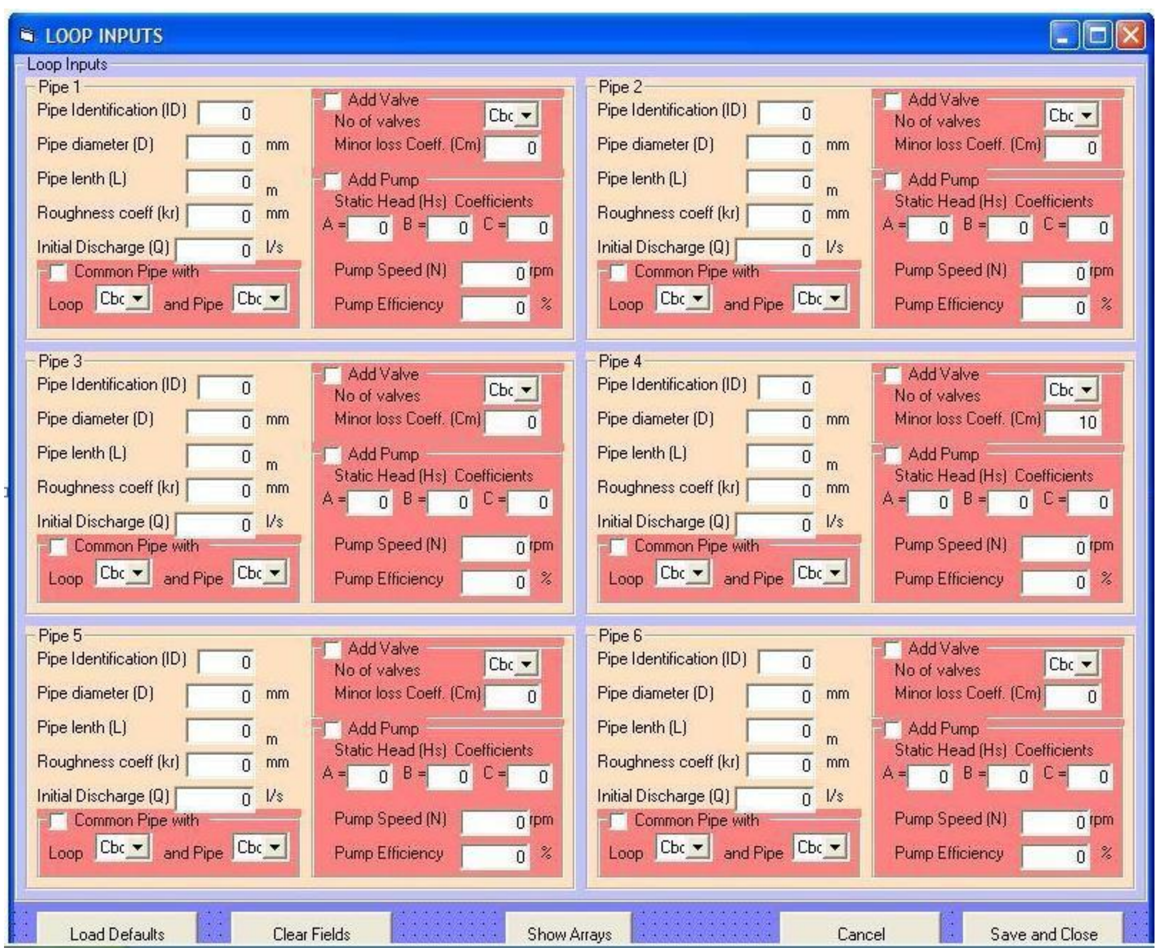

Figure 5. Loop inputs screen.

eliminated. In addition, Barr's equation was used in step (i) as against Moody's equation. PNET Expert and ***HARDY CROSS1*** are applied to solve Problems 1 and 2 whose diagrams are contained in Figures 1 and 2. A sample loop input screen is shown in Figure 5. A simplified flowchart for PNET Expert and ***HARDY CROSS1*** is shown in Figure 6. Where the two softwares will differ is basically in the method for computing the friction factor.

RESULTS AND DISCUSSION

Using ***HARDY CROSS1***, PNET Expert and manual calculation to analyse the networks in Figure 1 and 2, the results in Tables 1 and 2 are obtained for Problems 1 and 2 respectively. Problem 1 consists of 2 loops, 6 pipes and 5 nodes, while Problem 2 is made up of 3 loops, 10 pipes and 8 nodes. From the analysis results in Tables 1 and 2, ***HARDY CROSS1*** and PNET Expert are able to reproduce the expected flow rate and headloss values within certain range of accuracy which is to be thoroughly evaluated using statistical evaluation tools.

Statistical evaluation

Total error

Total error, TE, represents the overall or total error that may occur in a test result due to both the imprecision (random error) and inaccuracy (systematic error) of the measurement procedure (Oke, 2007). The total error, which is the sum of the squares of the errors between the obtained values and the predicted values, can be interpreted as a measure of variation in the values predicted unexplained by the values in obtained data (Oke and Akindahunsi, 2005). The lower the value of total error, the higher is the accuracy, validity, and good fitness of the software (Oke, 2007; Douglas et al., 2010). Total error (Err^2) can be computed using Equation 2:

$$Err^2 = \sum_{i=1}^{n}(Y_{obs\,i} - Y_{cal\,i}) \qquad (2)$$

Observed values are those obtained from the softwares whereas calculated values are computed manually from the benchmark problems. Table 3 shows the computation of total error for each of the software. The total errors are 0.6024 and 1.3325 for ***HARDY CROSS1*** and PNET Expert respectively. These results indicate that ***HARDY CROSS1*** has the least error, followed by PNET Expert. The larger error in the PNET Expert software may be attributed to the fact that the two softwares were developed using different friction factor formulae. PNET Expert uses Barr's equation while ***HARDY CROSS1*** uses Moody's equation. Babatola et al. (2008) proved that Moody's equation has lower total errors than Barr's equation. The former should be preferred over the later because of its explicit case and its availability as a chart.

Coefficient of determination and correlation coefficient

The coefficient of determination, r^2, is useful because it gives the proportion of the variance (fluctuation) of one variable that is predictable from the other variable. It is a measure that allows us to determine how certain one can be in making predictions from a certain model/graph. It represents the percent of the data that is the closest to the line of best fit. The coefficient of determination is such that $0 < r^2 < 1$, and denotes the strength of the linear association between two variables. It is mathematically expressed as the square of the correlation coefficient (Montgomery et al., 2010).

$$r^2 = \frac{\sum_{i=1}^{n}(Y_{obs\,i} - \bar{Y}_{cal\,i})^2 - \sum_{i=1}^{n}(Y_{obs\,i} - Y_{cal\,i})^2}{\sum_{i=1}^{n}(Y_{obs\,i} - \bar{Y}_{cal\,i})^2} \qquad (3)$$

Higher values of r^2 indicate higher accuracy, validity and good fitness of the software. Like total error, r^2 values are 0.9999765 and 0.9999480 for ***HARDY CROSS1*** and PNET Expert respectively (Table 4). These results indicate that ***HARDY CROSS1*** has a higher coefficient of determination than PNET Expert. The fact that the r^2 values are almost equal to 1 indicates how well the softwares can predict the pipe flows (Table 4). The coefficient of correlation (r) can be interpreted as the proportion of expected data variation that can be explained by the obtained data. Higher values of r indicate higher accuracy, validity and good fitness of the software. r can be expressed as follows:

$$r = \sqrt{r^2} = \sqrt{\frac{\sum_{i=1}^{n}(Y_{obs\,i} - \bar{Y}_{cal\,i})^2 - \sum_{i=1}^{n}(Y_{obs\,i} - Y_{cal\,i})^2}{\sum_{i=1}^{n}(Y_{obs\,i} - \bar{Y}_{cal\,i})^2}} \qquad (4)$$

A correlation greater than 0.8 is generally described as strong, whereas a correlation less than 0.5 is generally described as weak. These values can vary based upon the "type" of data being examined. A study utilizing scientific data may require a stronger correlation than a study using social science data (Montgomery et al., 2010). Like total error, r values are 0.9999882and 0.9999740 for ***HARDY CROSS1*** and PNET Expert respectively (Table 4). These results indicate that ***HARDY CROSS1*** has a higher correlation coefficient than PNET Expert. The fact that the r values are almost equal to 1 indicates how well the softwares can predict the pipe flows.

Reliability

Reliability of any software depends on its accuracy and

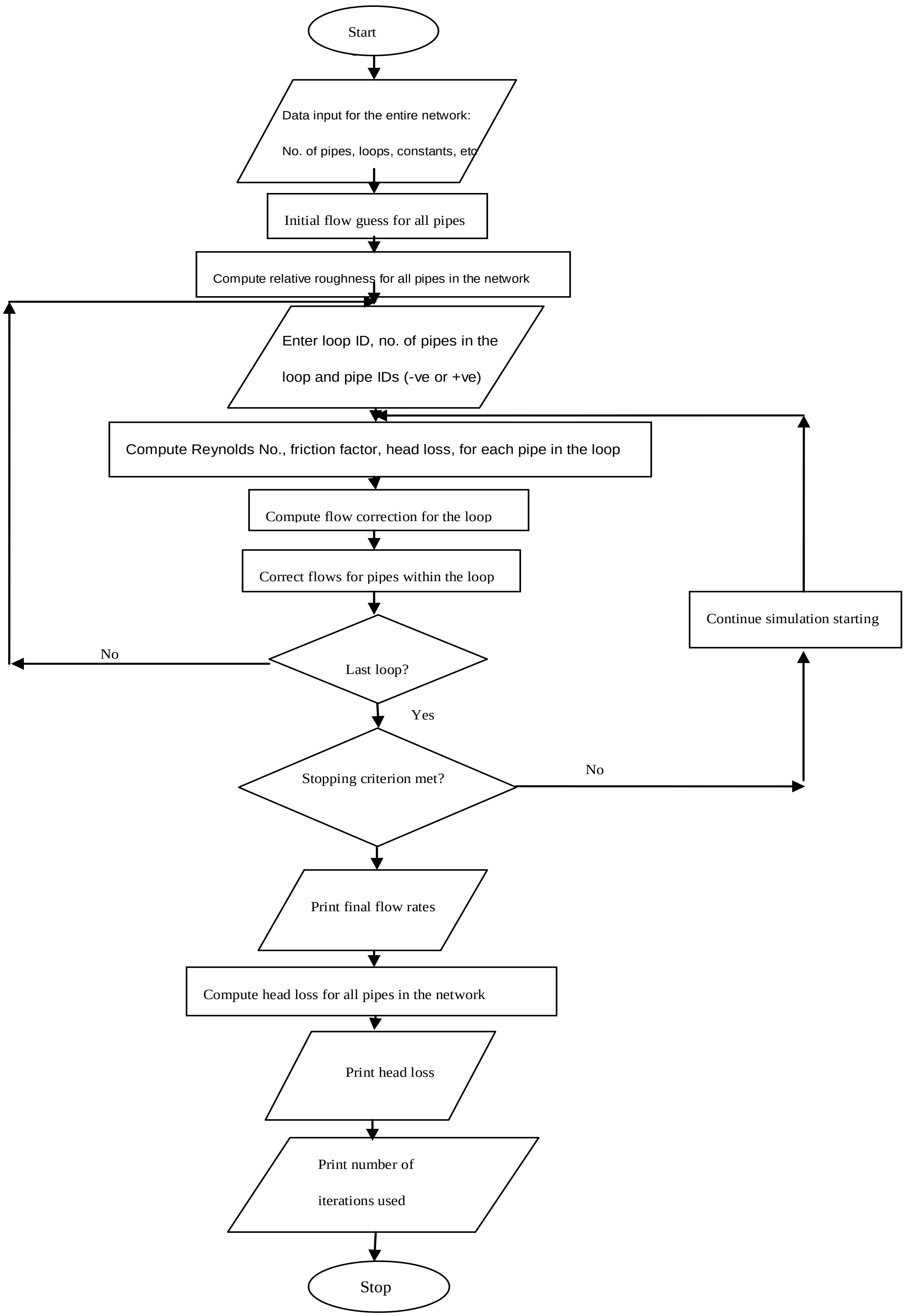

Figure 6. Flowchart for HARDY CROSS1 and PNET Expert.

Table 1. Summary of flows and head losses for Problem 1.

Pipe ID	HARDY CROSS1		PNET Expert		Expected Result	
	Observed flow (*l/s*)	Observed headloss (m)	Observed flow (*l/s*)	Observed headloss (m)	flow (*l/s*)	headloss (m)
1	104.74	20.84	104.57	18.30	104.80	20.84
2	0.63	0.02	0.75	0.01	0.65	0.00
3	95.25	20.80	95.43	18.28	95.20	20.84
4	45.06	21.54	45.33	18.425	45.40	21.22
5	4.54	0.76	4.67	0.54	4.42	0.74
6	44.94	20.45	44.67	17.89	44.42	21.11

Table 2. Summary of flows and head losses for Problem 2.

Pipe ID	HARDY CROSS1		PNET expert		Expected result	
	Observed flow (*l/s*)	Observed headloss (m)	Observed flow (*l/s*)	Observed headloss (m)	flow (*l/s*)	headloss (m)
1	136.33	31.05	136.02	28.54	136.50	31.05
2	56.52	3.52	56.22	3.52	56.50	3.52
3	9.81	0.03	9.80	0.01	10.00	0.03
4	2.45	5.30	2.06	4.61	2.50	5.30
5	53.67	0.62	53.99	0.51	53.50	0.62
6	93.67	4.12	93.99	3.03	93.50	4.12
7	29.81	2.10	29.80	1.83	30.00	2.10
8	24.07	0.33	24.15	0.25	24.00	0.33
9	13.88	22.00	13.95	20.16	14.00	22.00
10	26.12	11.19	26.05	10.22	26.00	11.19

Table 3. Values of total error for Problems 1 and 2.

HARDY CROSS1			PNET Expert		
Y_{obs} (l/s)	Y_{cal} (l/s)	$(Y_{obs} - Y_{cal})^2$	Y_{obs} (l/s)	Y_{cal} (l/s)	$(Y_{obs} - Y_{cal})^2$
104.74	104.8	0.0036	104.57	104.80	0.0511
0.63	0.65	0.0004	0.75	0.65	0.0104
95.25	95.20	0.0025	95.43	95.20	0.0511
45.06	45.40	0.1156	45.33	45.40	0.0055
4.54	4.42	0.0144	4.67	4.42	0.0645
44.94	44.42	0.2704	44.67	44.42	0.0645
136.33	136.50	0.0289	136.02	136.50	0.2343
56.52	56.50	0.0004	56.22	56.50	0.0795
9.81	10.00	0.0361	9.80	10.00	0.0412
2.45	2.50	0.0025	2.06	2.50	0.1901
53.67	53.50	0.0289	53.99	53.50	0.2352
93.67	93.50	0.0289	93.99	93.50	0.2352
29.81	30.00	0.0361	29.80	30.00	0.0412
24.07	24.00	0.0049	24.15	24.00	0.0237
13.88	14.00	0.0144	13.95	14.00	0.0025
26.12	26.00	0.0144	26.05	26.00	0.0025
Total error		0.6024	Total error		1.3325

Table 4. Computation of coefficient of determination and correlation coefficient for Problems 1 and 2.

HARDY CROSS1				PNET Expert			
Y_{obs} (l/s)	Y_{cal} (l/s)	$(Y_{obs}-Y_{cal})^2$	$(Y_{obs}-\bar{Y}_{cal})^2$	Y_{obs} (l/s)	Y_{cal} (l/s)	$(Y_{obs}-Y_{cal})^2$	$(Y_{obs}-\bar{Y}_{cal})^2$
104.74	104.80	0.0036	3410.93	104.57	104.80	0.0510	3391.56
0.63	0.65	0.0004	2089.12	0.75	0.65	0.0100	2077.98
95.25	95.20	0.0025	2392.50	95.43	95.20	0.0510	2409.74
45.06	45.40	0.1156	1.63	45.33	45.40	0.0050	1.02
4.54	4.42	0.0144	1746.98	4.67	4.42	0.0650	1735.80
44.94	44.42	0.2704	1.951	44.67	44.42	0.0650	2.77
136.33	136.50	0.0289	8098.76	136.02	136.50	0.2340	8042.35
56.52	56.50	0.0004	103.70	56.22	56.50	0.0800	97.64
9.81	10.00	0.0361	1334.21	9.80	10.00	0.0410	1335.16
2.45	2.50	0.0025	1926.06	2.06	2.50	0.1900	1960.09
53.67	53.50	0.0289	53.78	53.99	53.50	0.2350	58.49
93.67	93.50	0.0289	2240.43	93.99	93.50	0.2350	2270.34
29.81	30.00	0.0361	273.14	29.80	30.00	0.0410	273.57
24.07	24.00	0.0049	495.81	24.15	24.00	0.0240	492.08
13.88	14.00	0.0144	1053.45	13.95	14.00	0.0030	1048.91
26.12	26.00	0.0144	408.72	26.05	26.00	0.0030	411.56
$\bar{Y}_{cal}$		**46.34**		$\bar{Y}_{cal}$		**46.34**	
r^2		0.999976497		r^2		0.999947967	
r		0.999988249		r		0.999973983	

Table 5. Reliability values of the softwares when applied to Problem 1.

HARDY CROSS1			PNET Expert		
Y_{obs} (l/s)	Y_{cal} (l/s)	$(ln\ Y_{cal} - ln\ Y_{obs})*100$	Y_{obs} (l/s)	Y_{cal} (l/s)	$(ln\ Y_{cal} - ln\ Y_{obs})*100$
104.74	104.80	0.0573	104.574	104.80	0.2159
0.63	0.65	3.1253	0.752	0.65	-14.5764
95.25	95.20	-0.0525	95.426	95.20	-0.2371
45.06	45.40	0.7517	45.326	45.40	0.1631
4.54	4.42	-2.6787	4.674	4.42	-5.5876
44.94	44.42	-1.1638	44.674	44.42	-0.5702
RD		0.0392	RD		-20.5922

validity. The statistical approach developed to address reliability of any method is the testing of hypothesis that there is no difference between the expected results and the softwares (Oke, 2007). Sartory (2005) describes statistical relative difference between results obtained with the softwares and the expected results as follows:

$$RD = \left(lnA_q - lnB_q\right) x\ 100 \tag{5}$$

A lower RD value indicates that the software is reliable. Reliability values for the first and second Problems are 0.039, -20.592 and 4.273, 20.856 for ***HARDY CROSS1*** and PNET Expert respectively (Tables 5 and 6). These results indicate that ***HARDY CROSS1*** has a higher reliability than PNET Expert. Low reliability of the PNET Expert software may be attributed to the fact that the two softwares were developed using different friction factor formulae. PNET Expert uses Barr's equation (which has lower reliability than Moody's equation) while ***HARDY CROSS1*** uses Moody's equation.

Conclusion

The following conclusions can be drawn based on the study, that:

1. ***HARDY CROSS1*** and PNET Expert softwares presented herein are accurate, valid, reliable and have

Table 6. Reliability values of the softwares when applied to Problem 2.

HARDY CROSS1			PNET expert		
Y_{obs} (l/s)	Y_{cal} (l/s)	($ln\ Y_{cal}$ - $ln\ Y_{obs}$)*100	Y_{obs} (l/s)	Y_{cal} (l/s)	($ln\ Y_{cal}$ - $ln\ Y_{obs}$)*100
136.33	136.50	0.1246	136.02	136.50	0.3552
56.52	56.50	-0.0354	56.22	56.50	0.5004
9.81	10.00	1.9183	9.80	10.00	2.0509
2.45	2.50	2.0203	2.06	2.50	19.1645
53.67	53.50	-0.3173	53.99	53.50	-0.9025
93.67	93.50	-0.1817	93.99	93.50	-0.5174
29.81	30.00	0.6353	29.797	30.00	0.6790
24.07	24.00	-0.2912	24.154	24.00	-0.6396
13.88	14.00	0.8608	13.95	14.00	0.3578
26.12	26.00	-0.4605	26.05	26.00	-0.1921
RD		4.2733	RD		20.8561

good fitness for their application in pipe network simulation.
2. With particular reference to accuracy, ***HARDY CROSS1*** could be a better software of choice than PNET Expert.
3. PNET Expert software can be used as substitute to ***HARDY CROSS1*** when the need arises. This is because of its user-friendliness, use of graphical user interface (GUI) and users do not require knowledge of Visual BASIC prior to its use.
4. Differences in results from ***HARDY CROSS1*** and PNET Expert are attributed to the differences in the algorithm implementation for friction factor computation.

Nomenclature:

A_q: Expected discharges (*l*/s)
B_q: obtained discharges (*l*/s)
RD: reliability
***r^2*:** coefficient of determination
***r*:** coefficient of correlation
Y_{obsi}: obtained experimental values
Y_{cali}: expected values of each fitting procedure
Err^2: total error
n: number of data points
$\overline{Y_{cali}}$: average of expected values
Q: assumed flow
Δ Q: flow correction
K: coefficient of head loss

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