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Detection of hydrological impacts of climate change in Benin by a multifractal approach

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This study analyzed the hydrological impacts of climate change over Bénin synoptic stations from 1951 to 2010 using multifractal approach. This method is based on the study of temporal evolution of fractal dimension (D_f), multifractality index (α), co-dimension (C_1) and probable maximum singularity (γ_s). The comparison of the average values of yearly fractal dimension ($\overline{D_f}$) obtained over the sub-period [1951-1970], [1971-1990] and [1991-2010] has shown and confirmed that [1971-1990] is a drought period in the region. However, the sub-period [1991-2010] is not the end of the drought over all the synoptic stations as known before, in West Africa. During the period from 1951 to 2010 over all synoptic stations, except Natitingou, there is a slight increase in γ_s exponent, controlling the extremes values of rainfall. Thus, the evolution of the extremes rainfall over this period has a slight increasing tendency. These results confirm those obtained by some studies over the Ouémé River Basin (Benin) using the climate index method. Thus, the multifractal approach is an excellent tool to evaluate the hydrological impacts of climate change using only three parameters.

Key words: Climate change, multifractal, hydrological impacts, extreme rainfall, Bénin.

INTRODUCTION

West African zone has experienced a sharp drop in annual rainfall since 1970 (Bigot et al., 2005; Goula et al., 2006). However, this climatic context has coincided with an increase in flood damage, both in urban and rural areas (Panthou, 2013). According to Panthou (2013), extreme rainfall is an indicator of climate change and particularly of the potential for intensifying the hydrological cycle. According to Hoang et al. (2014), extreme rainfall study is very useful for the management of water resources. Generally, in the literature, the evolution of the extremes rainfalls is made through the study of annual climate indices (Zhang et al., 2011) and especially by the standardized index of rainfall anomaly (Lamb, 1982). Based on these climate indices in sub-Saharan of Africa, Ozer and Ozer (2005) found an increase in extreme rainfall events, in western Niger

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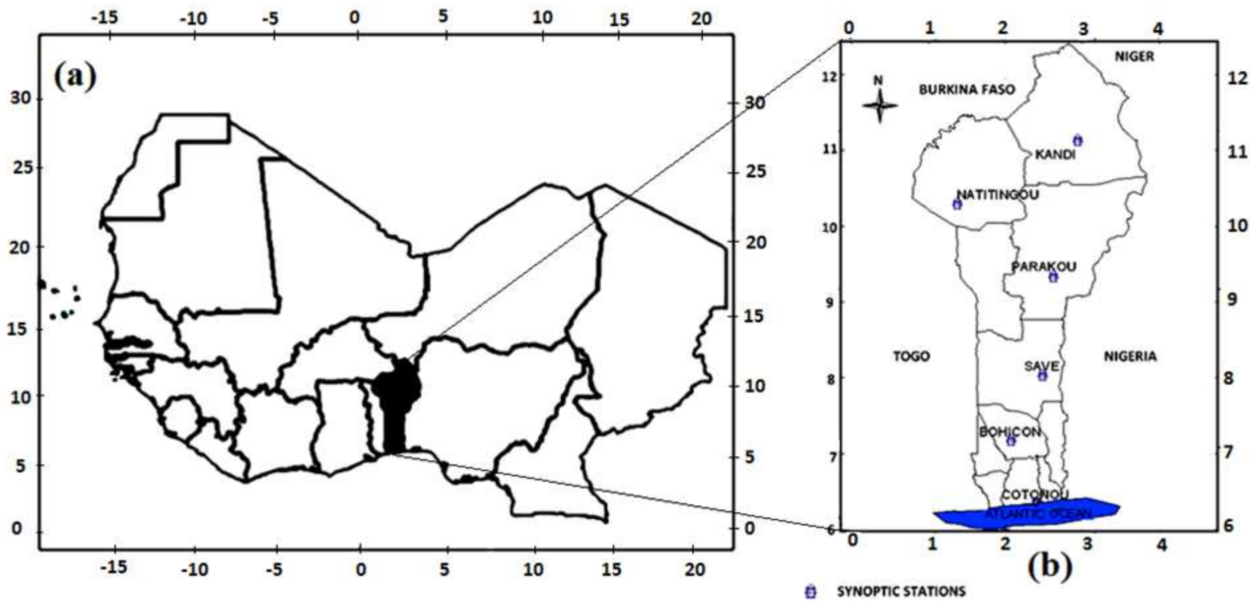

Figure 1. Study sites location: (a) Bénin location in West Africa, (b) Synoptic stations location in Benin Republic.

Republic. But, others studies, for example (Oguntundé et al., 2011; Easterling et al., 2000), (Aguilar et al., 2009; Ozer et al., 2009; Soro et al., 2016; Mason et al., 1999) have indicated a decrease in respectively Nigeria, Guinea Conakry, in eastern Niger, Ivory-Coast and South Africa in the last few decades, using climate indices.

In Benin Republic, Hountondji et al. (2011) have shown significant decreasing trends for the total precipitation, for the annual total rainfall and for the maximum rainfall in the period 1960-2000 from 21 rainfall stations. Hounkpè et al. (2016) have shown an increase in heavy historical precipitation for the period 1960-2012 over the Ouémé River Basin (Benin). Recently, N'Tcha M'Po et al. (2017) analyzed the trends of extreme daily rainfall indices over the Ouémé basin using the observed data from 1950 to 2014. They detected significant decline in the number of heavy and very heavy rainfall days, heavy and extremely heavy rainfall, consecutive wet days and annual wet-day rainfall total using climate indices method. All these methods are not a 'physical-based method'. They are based on the annual climate indices using a lot of parameters (for example RX*n*day, Maximum *n-days* precipitation, PRCPTOT: Annual total wet-day precipitation). Their great weakness is that they do not consider the characteristics of rainfall (Variability, intermittence, scaling regimes etc).

However, multifractal approach could be a prospect to characterize the impact of climate change. For example, Hoang et al. (2012) have shown that this approach is the most appropriate for analyzing and characterizing the strong spatio-temporal variability observable over a wide range of rainfall fields. In addition, Hoang et al. (2014) suggested the possibility of detecting the hydrological impacts of climate change from the evolution of multifractal parameters. The strength of these methods is that they use less parameters with physical interpretation. In West Africa and especially in Benin, no study has analyzed the impacts of climate change through a multifractal approach. This paper aims to study, for the first time in Benin, the hydrological impacts of climate change through a multifractal approach.

MATERIALS AND METHODS

Site description

The study covers all the synoptic stations of Benin Republic (Figure 1) whose geographical positions are presented in Table 1. Benin Republic is characterized from the South to North by three climatic zones in which the synoptic stations are located (Boko, 1988): (1) Cotonou and Bohicon are located in sub-equatorial region where March is the hottest month (~ 26°C), while August is the coldest month (~ 24°C). The daily and annual thermal amplitudes are respectively ~ 10 and ~ 5°C. The relative humidity ranges between 70 and 95% because of the proximity to the Atlantic Ocean. The sub-equatorial climate has four seasons: a long rainy season (April to July) followed by a short dry season (August-September) and a short rainy season (October-November) followed by long dry season (December to March) in the year. However, (2) the stations of Kandi and Natitingou are located in Sudanian region in the northern part of the country. The daily mean of air temperatures in Natitingou and Kandi are respectively ~25 and ~35°C. (3) Savè and Parakou are located in the transition area between the two kinds of climatic zone. The daily mean of air temperatures in Savè and

Table 1. Benin’s synoptic stations geographical coordinates.

Synoptic stations	Latitude °N	Longitude °E	Altitude (m)
Cotonou	6.21	2.23	3.9
Bohicon	7.10	2.03	166.00
Save	8.02	2.26	198.51
Parakou	9.21	2.37	392.00
Natitingou	10.19	2.52	289.75
Kandi	11.08	1.23	460

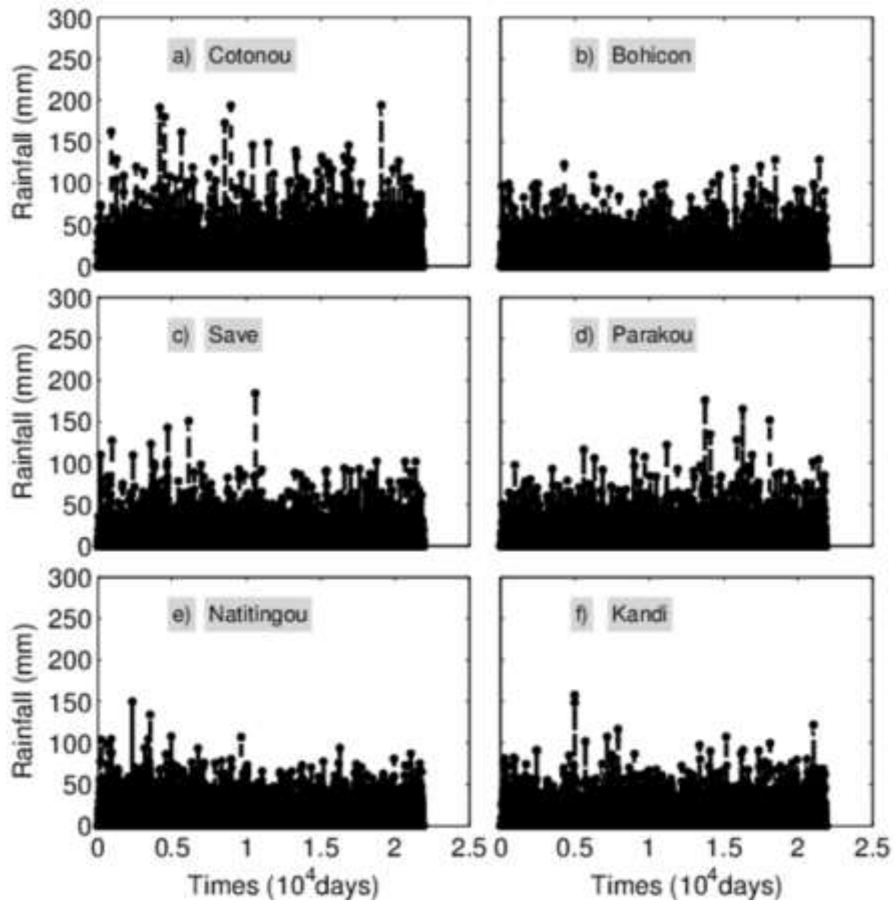

Figure 2. Temporal variations of rainfall time series collected from 1951 to 2010: (a) Cotonou station, (b) Bohicon station, (c) Savè station, (d) Parakou station, (e) Natitingou station and (f) Kandi station.

Parakou is ~27°C (Table 1).

Data records

Data were provided by the Agency for the Aerial Navigation’s Security in Africa and in Madagascar (ASECNA). Daily rainfall data from Bénin synoptic stations are used from 1951 to 2010. Rainfall variation during the study period is presented in Figure 2 for each synoptic station. Figure 2 shows the daily rainfall strong variability throughout all the synoptic stations.

Approach

This part of the paper describes respectively the different methods applied to study the hydrological impacts of climate change through a multifractal approach. To achieve our goals, the following methods are applied: (i) The yearly intermittency degree of rainfall is characterised through the yearly fractal dimension (D_f) computed with the method of box counting. (ii) The character of the rain processes is studied by the spectral analysis method. (iii) The universal multifractal parameters α and C_1 are directly determined by Trace Moment method (TM). (iv) Finally, the probable maximum singularity (γ_s) is deduced from the universal multifractal parameters α and C_1.The details of these methods are presented as follows.

Fractal dimension

The fractal dimension is computed by using the method of box counting (Mandelbrot, 1982; Lovejoy et al., 1987; Hubert and Carbonnel, 1989). The total observation time *T* is divided into *n* contiguous intervals of length *λ* taken as successive powers of 2. The total number of occupied intervals *N(λ)*, in which at least one rainy day has been observed, is then counted. To be more precise *N(λ)* if the number of boxes where it rains on the scale λ; if the data forms a one-dimension fractal, then:

$$N(\lambda) \propto \lambda^{-D_f} \tag{1}$$

$$\log\big(N(\lambda)\big) = -D_f \, \log(\lambda) + K \tag{2}$$

Where, K is a constant, *Df* is defined as the fractal dimension. When plotting *log N(λ)* as a function of *log(λ)*, a straight line with slope (*-Df*) is obtained. The maximum value of the fractal dimension will be equal to 1 (Hubert and Carbonnel, 1989; Biaou, 2004) (*0< Df <1*).

Spectrum analysis

Spectrum analysis on Fourier transform translates the time domain characteristics of data into frequency domain. The scaling regime of rainfall time series can be estimated from their power spectrum *S(f)* (Lovejoy et al 2010). If a time series spectrum obeys a power law, the relation between *S(f)* and the frequency *f* is given by Equation 3:

$$S(f) = f^{-\beta} \tag{3}$$

The spectral exponent β is identified as a slope of the spectrum straight-lined zone, plotted in a double logarithmic diagram (Fraedrich and Larnder, 1993; Olsson et al., 1993; Cheng et al., 2013). The value of *β* is often calculated by linear regression. According to Schertzer and Lovejoy (1987); Calif et al. (2014), if β>1 in the power spectrum then the process is non- conservative. Otherwise if β<1, the process is conservative.

Trace moment method (TM)

According to Schertzer and Lovejoy (1987), the fundamental equation of the multifractal formalism in terms of statistical moments is written as:

$$\langle \phi_\lambda^q \rangle \approx \lambda^{K(q)} \tag{4}$$

where ϕ_λ is the rainfall field seen at resolution λ, and $\langle . \rangle$ is the averaging operator, q is the order of the statistical moment. $\lambda = T/_d$ (where *T* is the largest time scale of the scaling regime, *d* is the observation duration). $K(q)$ is a function characterizing the scale invariance of the statistical moments of order *q.* $K(q)$ is deduced as the slope of the diagram obtained when plotting $\log(\langle \phi_\lambda^q \rangle)$ as a function of $\log(\lambda)$, (Schertzer and Lovejoy, 1987a, 1991b; Hoang et al., 2011). In this study, *q* ranges from 0.1 to 3.5 with step equal to 0.1. According to Schertzer and Lovejoy (1987), in the context of universal multifractals, when the study field is conservative (that is, when β<1), $K(q)$ is expressed as:

$$K(q) = \frac{C_1}{\alpha - 1}(q^\alpha - q) \tag{5}$$

Where the parameter *α* is the multifractality index (*0 ≤ α ≤2*), which measures the variability of the rain intermittency (*α* = 0 if it rains always, rain the same way or if it does not rain). C_1 is the co-dimension of the average rainfall, which measures the average intermittency of the rain ($C_1 = 0$ if the rain is homogeneous or if it rains all the time). According to Hoang et al. (2011) in the Trace Moment method (TM) universal multifractal parameters *α* and C_1 are directly determined by the first two derivatives of the function $K(q)$ when $q = 1$ as :

$$C_1 = \left.\frac{dK(q)}{dq}\right|_{q=1} \tag{6}$$

$$\alpha = \left(\left.\frac{d^2K(q)}{dq^2}\right|_{q=1}\right) \Big/ C_1 \tag{7}$$

If C_1 and *α* increase, then the extremes (or the extreme intensities of the rain) increase and conversely if they decrease both. When C_1 and α vary in opposite directions, the response of the extremes will depend on which of the two variations dominates. In this case, the probable maximum singularity method is more appropriate to analyse extreme fluctuations (Royer et al., 2008; Hoang et al., 2014).

Probable maximum singularity method

According to Schertzer and Lovejoy (1987), the probable maximum singularity (γ_s) is defined as:

$$\gamma_s = C_1 \frac{\alpha}{\alpha - 1}\left(C_1^{\frac{1-\alpha}{\alpha}} - \frac{1}{\alpha}\right) \tag{8}$$

The slope of the linear regression of (γ_s) with respect to time allows one to deduce the (linear) trend of the temporal evolution of (γ_s). A positive value of this slope will represent a tendency towards increasing extremes, and conversely to a tendency to decrease them for a negative slope.

From the literature, some studies in West Africa (Balme, 2004; Balme et al., 2006; Ozer et al., 2003) have divided the period of [1951-2010] as follows: [1951-1970] is considered as the wet period, (1971-1990) is the drought period and (1991-2010) is the end of the drought. In order to verify this division of [1951-2010] period and to assess the risk of hydrological impacts of climate change by multifractal approach, the following computations are done:

1) The yearly fractal dimension for each year of the historical period from 1951 to 2010, then, the average of the yearly fractal dimension obtained over the sub-period [1951-1970], [1971-1990] and [1991-2010] are calculated and compared. The mean value of the fractal dimension in each sub-period is noted ($\overline{D_f}$). It is important to keep in mind that a low fractal dimension D_f value corresponds to a high intermittency of rainfall. In addition, high intermittency corresponds to a large percentage of zeros rainfall (the percentage of rainfall zeros is the fraction of time steps for which no rainfall has been recorded). In this study we considered as zero precipitation when the rainfall is less than 6 mm.

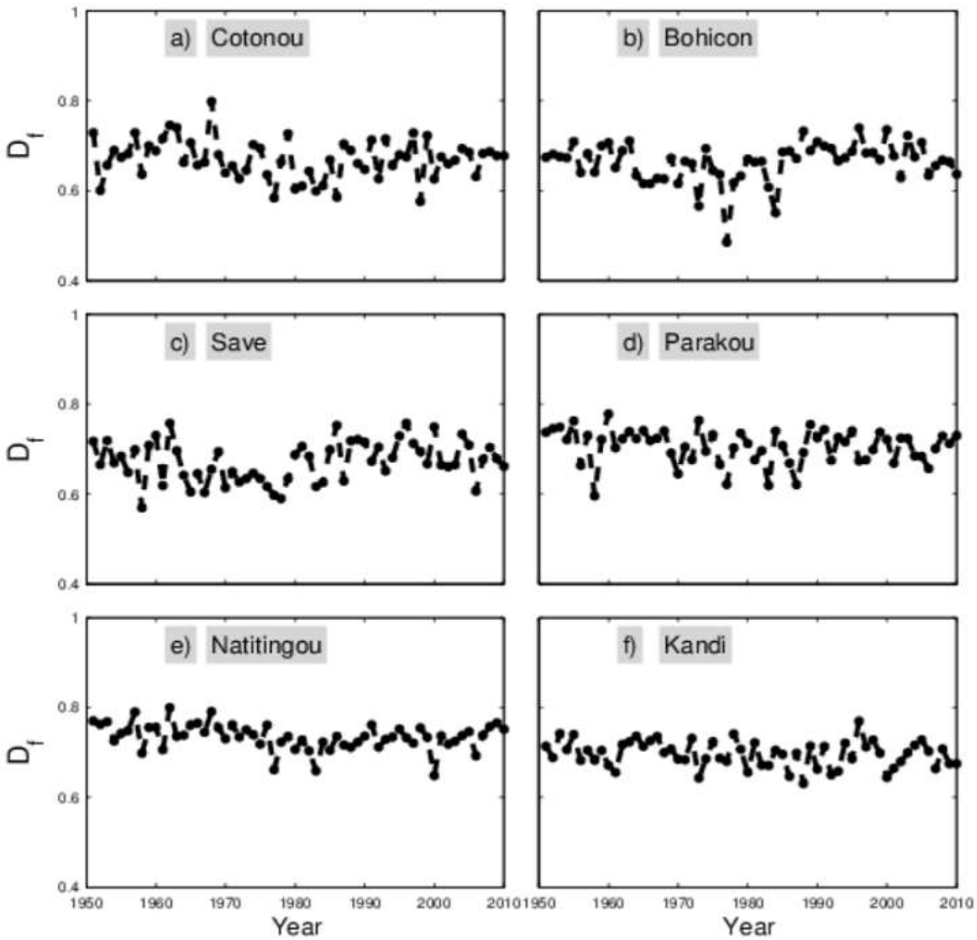

Figure 3. Temporal evolution of fractal dimension, (a) Cotonou Station, (b) Bohicon Station, (c) Savè Station, (d) Parakou Station, (e) Natitingou Station and (f) Kandi Station.

2) The multifractal parameters C_1 and α and the probable maximum singularity (γ_s) for each year of the historical period from 1951 to 2010 are calculated. Their evolution has been studied during 1951 to 2010 and during three separate sub-period: the first is [1951 - 1970]; the second is [1971-1990] and the third is [1991-2010].
3) The (linear) trend of the temporal evolution of (γ_s) over [1951-2010]; [1951 -1970]; [1971-1990] and [1991-2010] is deduced. A positive value of trend represents a tendency towards increasing extremes, and conversely to a tendency to decrease them for a negative slope.
4) The statistical significance of trend is evaluated by Mann-Kendall test. We presented all the trends obtained over period and sub-period, but only interpret those that are statistically significant.

RESULTS AND DISCUSSION

Temporal evolution of fractal dimension and spectral exponent

Figure 3 presents the fractal dimension for each year of the historical period from 1951 to 2010. This figure shows that the fractal dimension varies year to year. Thus, rainfall intermittency varies with the year and the geographical position of the studied station.

The average of the yearly fractal dimension obtained over the sub-period [1951-1970] (black line), [1971-1990] (red line) and [1991-2010] (blue line) is presented in Figure 4. Whatever the synoptic station considered the fractal dimension mean ($\overline{D_f}$) obtained over [1971-1990] is systematically lower than those of [1951-1970] and [1991-2010]. This low value of ($\overline{D_f}$) means that in [1971-1990] rainfall times series present a high intermittency comparing to [1951-1970] and [1991-2010]. This intermittency corresponds to the large percentage of zeros in rainfall times series (Here the zeros are defined as rainfall heights less than 6 mm). This result agrees with the work which found that [1971-1990] is a drought period in West Africa (Balme, 2004; Balme et al., 2006; Ozer et al., 2003). Except for the synoptic stations of Save and Bohicon the values of ($\overline{D_f}$) obtained in the sub-period [1991-2010] are systematically smaller than those of [1951-1970]. This result means that in [1991-2010] rainfall times series present a high intermittency comparing to [1951-1970] over all synoptic station,

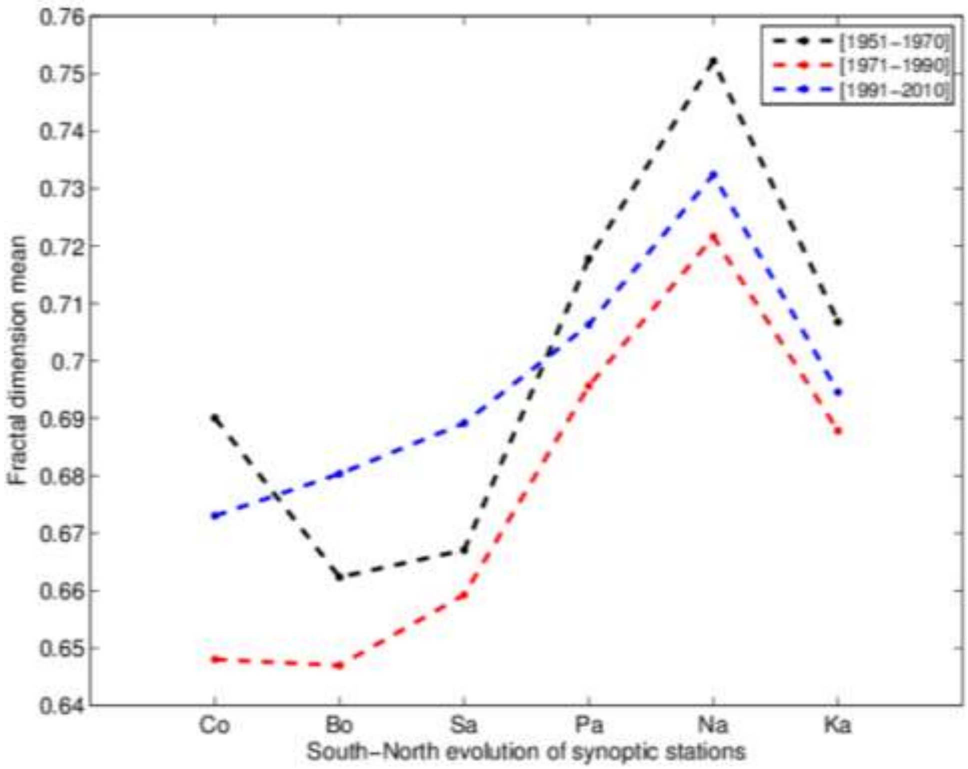

Figure 4. Spatio-temporal variation of the fractal dimension mean value, obtained over [1951-1970] (black line), [1971-1990] (red line), [1991-2010] (bleue line).

except Bohicon and Save. Thus, generally the degree of intermittency of the rainfall fields on the sub-period [1971-1990] is smaller than that of the sub-period [1991-2010], and that of [1991-2010] is lower than that of [1951-1970]. Therefore, [1991-2010] is not necessary the end of the drought over all synoptic stations. Thus, over the period [1991-2010], drought decreased slightly compared to [1971-1990] but cannot be systematically considered as a recovery period as mentioned in the literature by some studies (Balme, 2004; Balme et al., 2006; Ozer et al., 2003).

The spectral exponent β for each year of the historical period from 1951 to 2010 obtained in each Benin synoptic stations is presented in Figure 5. One can note that all the β values are systematically less than 1. Thus, whatever the year and the synoptic station, rainfall processes are conservative.

Temporal evolution of universal multifractal parameters

Figure 6 presents the temporal evolution of universal multifractal parameters α and C_1 obtained in each Benin synoptic stations. The results obtained in all the synoptic stations show that the evolutions of α are generally less stable and have larger sampling fluctuations than those of C_1. This means that the mean intermittency of the rainfall is rather stable whereas the multifractality of the rainfall is rather fluctuating with time. The green lines observed on each curve correspond to the trend of the temporal evolution of α and C_1. The values of the slope which characterize the trend of period [1951-2010] are seen in Table 2. The Mann-Kendall test results show that the trends are not simultaneous statistically significance for α and C_1 over all the stations. Moreover, the signs of α and C_1 trend are generally opposite; thus, the response of the extremes depends on which of the two variations dominates. Therefore, the study of probable maximum singularity is necessary to analyse extreme fluctuations in Benin synoptic stations.

Figure 7 presents the time evolution of the (γ_s) of the rainfall time series in synoptic stations. The red lines represent the trend of the temporal evolution of (γ_s) over each of the sub-period ([1951-1970]; [1971-1990]; [1991-2010]), whereas green line presents that of [1951-2010].

There is a tendency in the temporal evolution of the parameter γ_s. The results of the Mann-Kendall test (not shown) revealed that the trends obtained on the [1951-2010] period in all the synoptic stations are statistically significant, which is not necessarily the case for the sub-periods ([1951-1970]; [1971-1990] and [1991-2010]). This means that the probable maximum singularity method can not be rigorously used to study the hydrological

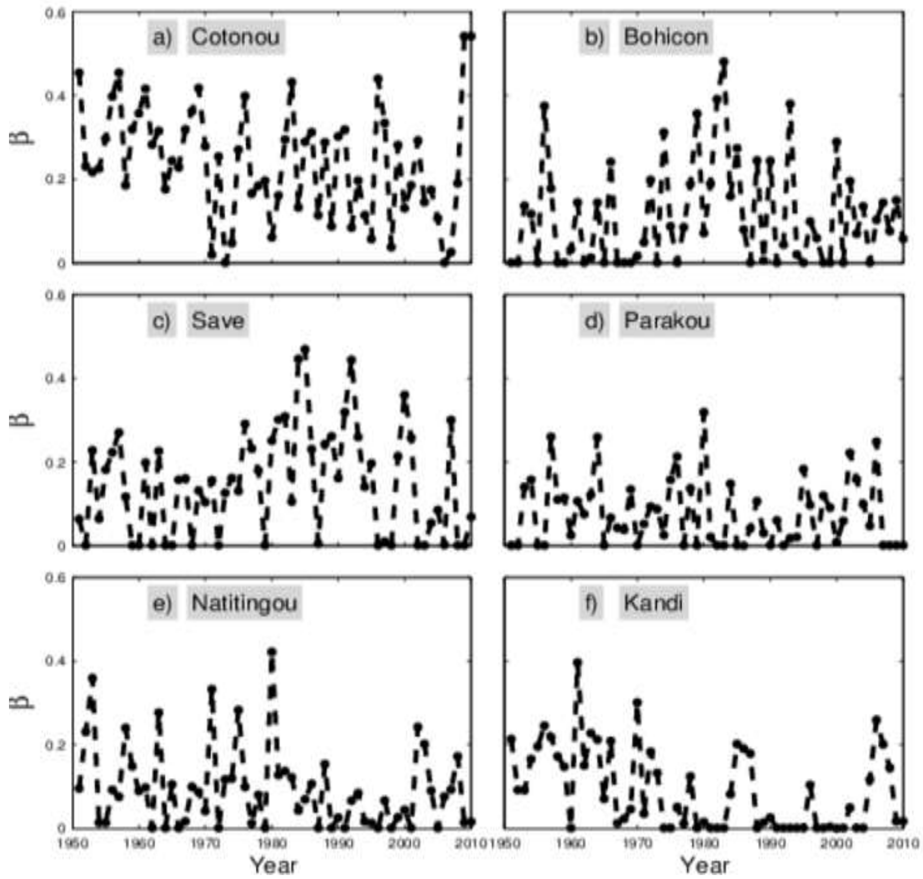

Figure 5. Temporal evolution of spectral exponent β, (a) Cotonou Station, (b) Bohicon Station, (c) Savè Station, (d) Parakou Station, (e) Natitingou Station and (f) Kandi Station.

Table 2. Trend of the temporal evolution of α and C_1 obtained on each synoptic station over [1951-2010] period.

Synoptic stations	α trend	C_1 trend
Cotonou	-0.0014	0.0007
Bohicon	-0.0028	0.0016
Save	-0.0051	0.0018
Parakou	0.0027	0.0004
Natitingou	-0.0015	0.0002
Kandi	-0.0009	0.0005

impact of climate change on these sub-periods.

The spatial evolution of the γ_s trend obtained over [1951-1970]; [1971-1990]; [1991-2010] and [1951-2010] is respectively presented by black line; red line; blue line and green line in Figure 8. Given that the tendency in the temporal evolution of the parameter γ_s on the sub-periods ([1951-1970]; [1971-1990] and [1991-2010]) is not statistically significant according to Mann-Kendall test as mentioned above, our analysis will be focused only on the curve which describes the spatial evolution of the γ_s trend obtained over [1951-2010] (green line).

The results obtained over the period from 1951 to 2010 revealed a positive trend in all synoptic stations except Natitingou. There is a slight increase in γ_s (except Natitingou). Thus, the evolution of the extremes rainfall over this period has a slight tendency to increase. These results agree with those obtained by Hounkpè et al. (2016) and N'Tcha M'Po et al. (2017) over the Ouémé River Basin (Benin) using the climate indices method. At Natitingou Station, there is a slight decrease in (γ_s)

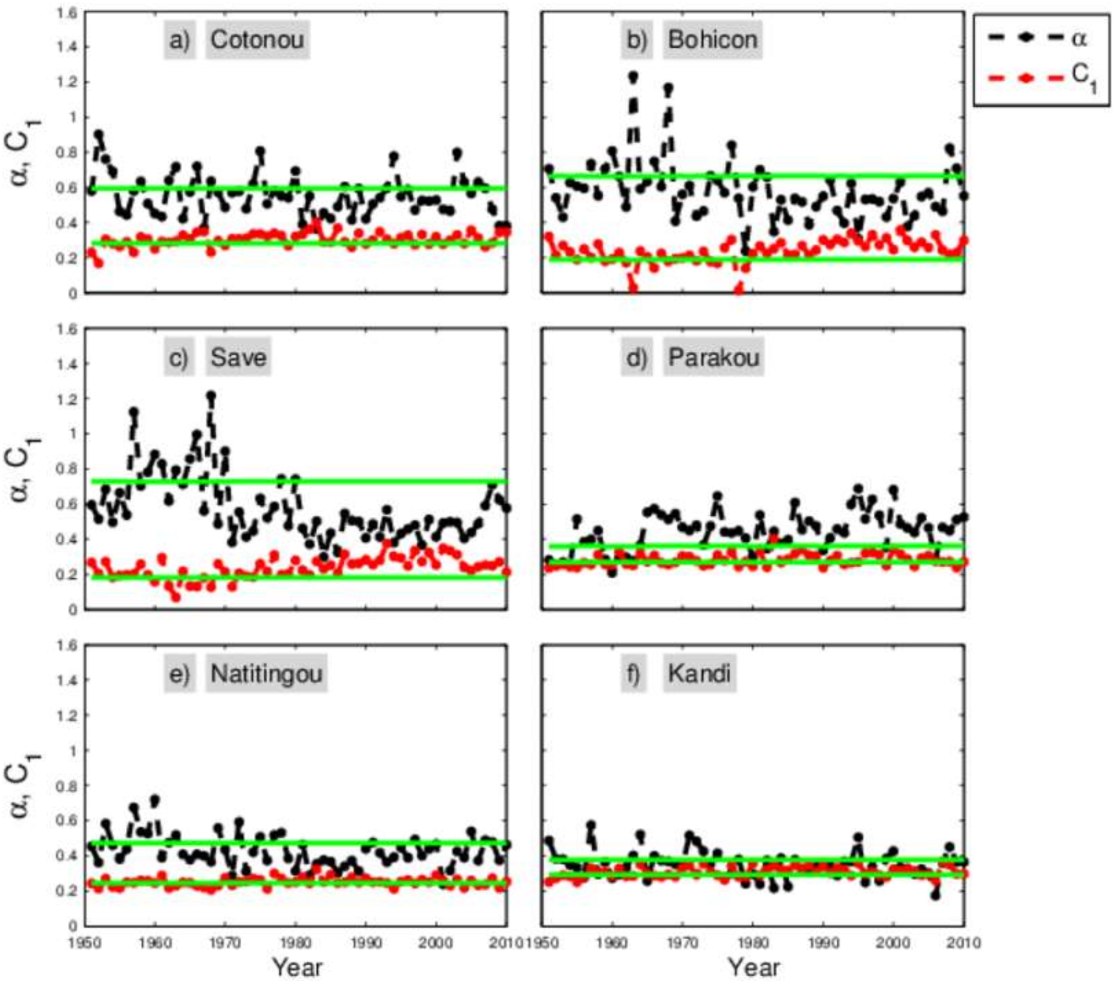

Figure 6. Temporal evolution of universal multifractal parameters α and C_1, (a) Cotonou station, (b) Bohicon station, (c) Savè station, (d) Parakou station, (e) Natitingou station and (f) Kandi station. The green line corresponds to the linear fit of the estimated values of α and C_1.

controlling the extreme values of rainfall, indicating a tendency to decrease extremes over the period from 1951 to 2010. This result could be explained by the fact that Natitingou is located in the mountainous region of Benin. The presence of mountains confers to this station the most watered of Benin. In summary, the reason behind the exception done at Natitingou could be linked to the fact that Natitingou is the rainiest region of Bénin because of the presence of mountains (~800 m of high). These different results show and confirm that the multifractal approach is an excellent tool to evaluate the hydrological impacts of climate change using only three parameters. Given that over the sub-period, the Mann-Kendall test revealed that trends are not statistically significant, we cannot use the results obtained to properly explain extreme rainfall evolution over these sub-periods. However, we must not neglect the consequences of a small increase in (γ_s), as it is the parameter that characterizes the extremes.

Conclusion

Generally, in the literature, the impacts of climate change are analysed through the study of annual climate indices and especially by the standardized index of rainfall anomaly. But the weakness of this method is that it does not consider the characteristics of the rains. In Benin, no study has analyzed the impacts of climate change through a multifractal approach. This paper aims to study for the first time in Benin the hydrological impacts of climate change through this approach. The main results obtained from the study are:

i) Whatever the synoptic station considered the yearly fractal dimension mean value ($\overline{D_f}$) obtained over [1971-1990] is systematically lower than those of [1951-1970] and [1991-2010]. This low value of ($\overline{D_f}$) means that in [1971-1990] rainfall times series present a high intermittency comparing to [1951-1970] and [1991-2010].

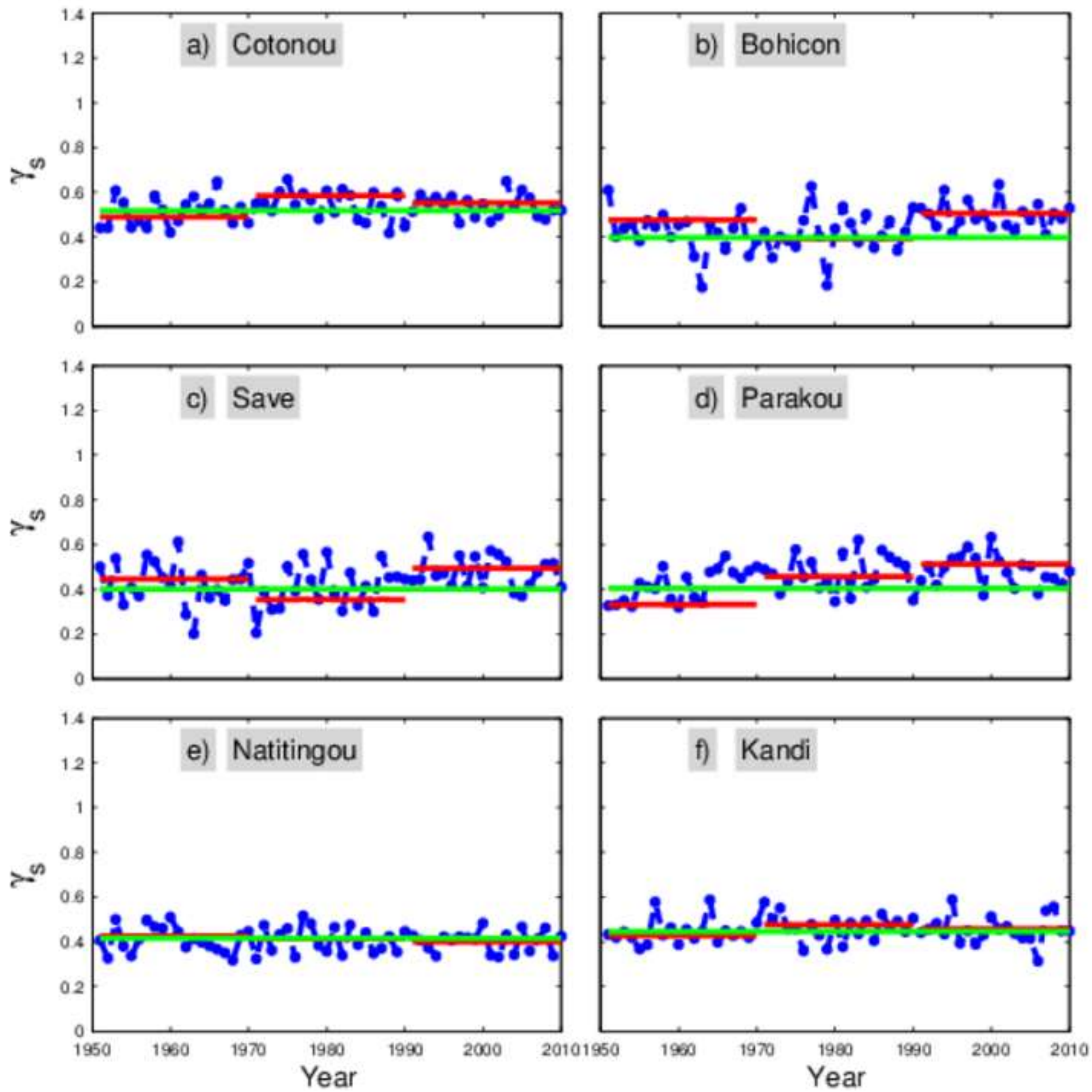

Figure 7. Time evolution of the (γ_s) of the rainfall time series in synoptic stations. The red and green line corresponds to the linear fit of the estimates of (γ_s) over respectively the sub-period ([1951-1970]; [1971-1990]; [1991-2010]) and [1951-2010].

Thus, [1971-1990] is systematically a drought period as found by some studies in literature;

ii) Except, Save and Bohicon, the values of $(\overline{D_f})$ obtained in the sub-period [1991-2010] are systematically smaller than those of [1951-1970]. This result means that in [1991-2010] rainfall times series present a high intermittency comparing to [1951-1970]. Therefore, [1991-2010] is not necessary the end of the drought over all these synoptic stations;

iii) The evolutions of α are generally less stable and have larger sampling fluctuations than those of C_1. This means that the mean intermittency of the rainfall is rather stable whereas the multifractality of the rainfall is rather fluctuating with time.

Therefore, the study of probable maximum singularity is necessary to analyse extreme fluctuations in Benin synoptic stations.

iv) The Mann-Kendall test revealed that γ_s trends obtained on the [1951-2010] period in all the synoptic stations are statistically significant, which is not necessarily the case for the sub-periods ([1951-1970]; [1971-1990]; [1991-2010]).

v) During the period from 1951 to 2010 over all synoptic stations except Natitingou, there is a slight increase in γ_s exponent controlling the extremes of rainfall. Thus, the evolution of the extremes rainfall over this period has a

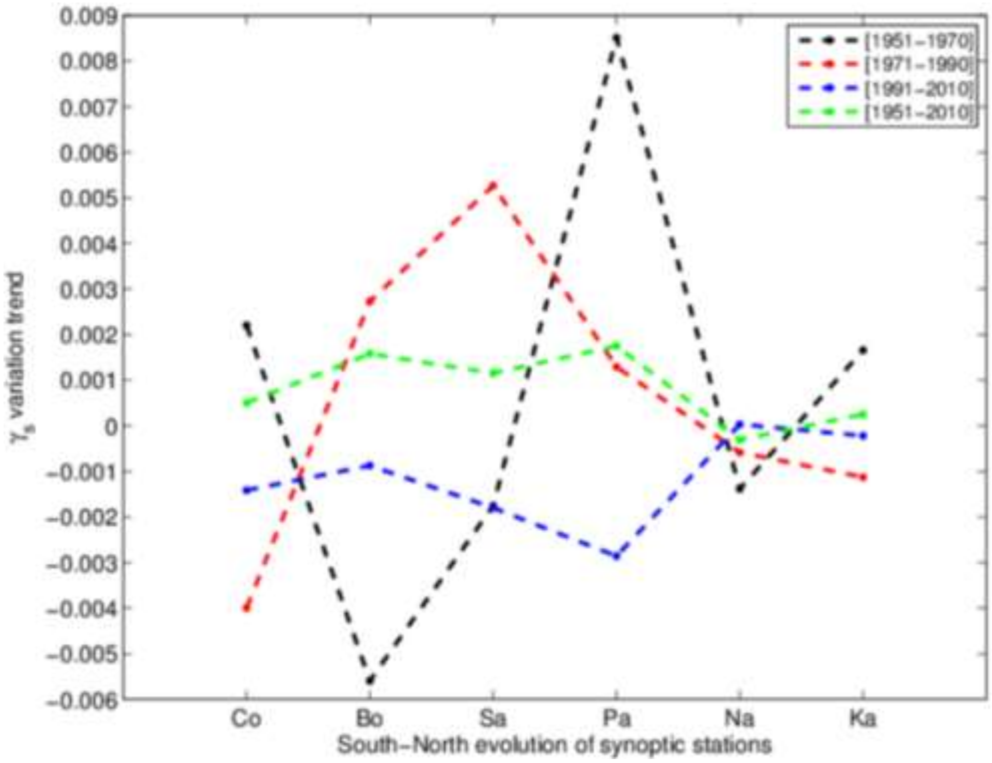

Figure 8. Spatial evolution of the γ_s trend obtained over [1951-1970] (black line); [1971-1990] (red line); [1991-2010] (blue line) and [1951-2010] (green line).

slight tendency to increase. These results agree with those obtained by some studies over the Ouémé River Basin (Benin) using the climate index method. Thus, the multifractal approach is an excellent tool to evaluate the hydrological impacts of climate change by only three parameters. We will probably, in our next works use this method to explain the future evolution of the extremes in Benin.

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CONFLICTS OF INTERESTS

The authors have not declared any conflict of interests.

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