

Full Length Research Paper

Groundwater management using a new coupled model of flow analytical solution and particle swarm optimization

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In this research, a steady state analytical solution is suggested and derived for the groundwater flow equation in a homogeneous unconfined aquifer. The solution can be used for calculating the hydraulic heads in a complex flow field caused by multi injection-pumping wells having different rates in a domain subjected to a uniform recharge. The analytical method, used for flow simulation, is coupled with the Particle Swarm Optimization (PSO) method, used for optimization, to solve the groundwater management problem. A new model called Analytic-PSO, which consists of the two methods, is originally developed. The performance of the proposed model was tested on a popular hypothetical example to maximize the total pumping rate from located well system at steady state condition. The results show the superiority of the proposed model to obtain the maximum pumping rate compared with other methods of previous work. The application was extended to use the Analytic-PSO model for determination of both the optimal location and maximum pumping rate for each well for the pre-specified number of wells in the same hypothetical example. Obtained results of the hypothetical example illustrate the ability of the Analytic-PSO model to solve efficiently the groundwater management problem in the real field aquifers.

Key words: Groundwater management, analytical solution, particle swarm optimization, optimization methods, Fourier series.

INTRODUCTION

Groundwater is considered an important source of freshwater especially in arid semi-arid zones which is used for several life purposes such as drinking, domestic, industrial, and irrigation uses. Indiscriminate exploitation of this source causes environmental hazards including decline of groundwater level and well interference. Consequently, sustainable management strategies have to be developed by decision makers to optimally utilize the groundwater resources. Groundwater management problems are typically solved by researchers using the simulation-optimization approach. In the simulation-optimization approach, a coupled optimization and groundwater flow model is used to obtain the optimum

strategy.

During the past two decades, several computer codes have been established to deal with groundwater management problems by linking groundwater flow and optimization models (McKinney and Lin, 1994; Wang and Zheng, 1998; Wu et al., 1999; Wu and Zhu, 2006; Zhu et al., 2006; Ayvas, 2009; and Gaur et al., 2011a). These codes differ in the used numerical model to simulate the groundwater flow system, the type of groundwater management problems and the approaches used to solve these management problems (Gaur et al., 2011b). In most of the previous groundwater management studies, the flow models were based on the finite difference method (FDM) or finite element method (FEM). These two methods, which are used to predict the hydraulic heads for the whole flow domain, have several limitations such as domain discretization error, selection of appropriate boundary conditions, numerical stability, and

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approximate location of well over the cell. Analytic element method (AEM) is considered one of the analytical methods used to simulate the groundwater flow. AEM can give an exact analytic solution for groundwater flow problems and is capable of simulating streams, lakes, and complex boundary conditions (Strack, 1989).

Optimization techniques are categorized into two types. The first one is deterministic optimization technique including Linear Programming (LP), Non-Linear Programming (NLP), and Dynamic Programming (DP). The second type is the stochastic optimization including Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Shuffled Complex Evolution developed at University of Arizona (SCE-UA), Simulating Annealing (SA),...etc. Groundwater management problems are usually nonlinear and non-convex mathematical programming problems (McKinney and Lin, 1994). For such problems, using deterministic optimization techniques may result in some unexpected situations. These techniques usually require good initial solutions to produce an optimal solution. Also, they rely on local gradients of the objective function to determine the search direction, and thus, may converge to local optimal solutions (Ayvas, 2009). Therefore, use of the stochastic optimization techniques is usually preferred due to their ability of finding solutions without requiring gradients and initial solutions.

There are several studies dealing with the solution of groundwater management problems using stochastic optimization methods. One of the first applications was performed by McKinney and Lin (1994). In that study, the GA-based groundwater simulation optimization models were developed to solve three groundwater management problems. They found that genetic algorithms could effectively and efficiently be used to obtain globally (or, at least near globally) optimal solutions to these groundwater management problems. Wang and Zheng (1998) compared the performance of GA and SA for maximization of pumping and minimization of the cost.

Their results showed that both methods yield nearly identical and better solutions than various other programming methods. Wu et al. (1999) developed a GA based SA penalty function approach (GASAPF) to solve a groundwater management model. Their results showed that GASAPF model can effectively solve the groundwater management model. Wu and Zhu (2006) applied (SCE-UA) to solve groundwater management models. Using the developed solution algorithm, two management models were developed for an unconfined aquifer: linear model of maximum pumping and nonlinear model of minimum pumping cost. In a later study, Zhu et al. (2006) compared the performances of SCE-UA and GA methods in the solution of a management model for deep groundwater resources of the Yangtze Delta, which is a multi-aquifer system with large area and complicated geology conditions. Their results showed that the SCE-UA is more effective than GA in the solution of the management model. Ayvas (2009) tested Harmony

Search (HS) algorithm on three separate groundwater management problems. Their results showed that the HS yields nearly same or better solutions than the previous solution methods and may be used to solve management problems in groundwater modeling. Gaur et al. (2011) developed two models for the solution of groundwater management problem. The first one consists of a linkage between analytical element and particle swarm optimization methods (AEM-PSO) whereas the second model consists of finite difference and particle swarm optimization methods (FDM-PSO). The comparative analysis was performed between AEM and FDM, and the abilities of the AEM method to solve groundwater management problems were investigated. Also Gaur et al. (2011) applied (AEM-PSO) model to the Dore river basin, France, to solve two groundwater hydraulic management problems. They examined the effect of piping length in the total developed cost for new wells. They also used the (AEM-PSO) model to determine optimal locations, discharges and optimum number of wells.

There are two main objectives of this study; the first one is to derive a new analytical solution for the groundwater flow equation to predict the hydraulic heads in the unconfined aquifer, and the other objective is to develop a groundwater resources management model that combines a new analytical and particle swarm optimization methods (Analytic-PSO). The proposed management model is tested on the most popular hypothetical example to obtain the maximum pumping rate and a comparison is carried out with the corresponding ones given by other previous studies. Also (Analytic-PSO) model is used to determine both the optimum locations and discharges of wells for the pre-specified number of wells.

PROBLEM FORMULATION

In the design of ground water management systems, there are usually two sets of variables: decision variables and state variables. In the considered model, the decision variables include the well locations and pumping rates. These are the variables that can be specified, managed, or controlled by the designer. The purpose of the design process is to identify the best combination of these decision variables. On the other hand, the state variable is the hydraulic head, which is the dependent variable in the groundwater flow equation (Wang and Zheng 1998). The governing equation describing the three dimensional movement of ground water is as follows (Bear, 1979):

$$\frac{\partial}{\partial x} \left(K_{xx} \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_{yy} \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_{zz} \frac{\partial h}{\partial z} \right) - f = S_s \frac{\partial h}{\partial t} \quad (1)$$

in which, K_{xx} , K_{yy} , K_{zz} : the principal components of hydraulic conductivity aligned along the x, y, and z

coordinate axes, respectively; h : the hydraulic head; f : a flux term that incorporates pumping, recharge, or other sources or sinks; S_s : the specific storage; and t : the time.

In this study, two separate groundwater management problems are solved. The objective function in the two problem is to maximize the total pumping rate from an aquifer. For the first management problem, it is assumed that the numbers and the locations of the wells are known whereas in the second management problem the number of wells is only known. Consequently, decision variables are only the pumping rates in the first management problem and both the pumping rates and locations of the wells in the second management problem. The two management problems are subjected to constraints on lower and upper bounds of pumping rates. Also, there is an additional constraint on hydraulic heads at well locations such that they must be greater than a specified lower bound. The management model can mathematically be stated as follows (Ayvas, 2009):

$$Obj. = \max \left[\sum_{i=1}^{N_w} Q_i - P(h) \right] \quad (2)$$

Subjected to:

$$h_i \geq h_{i, \min}, \quad i = 1, 2, 3, \dots, N_w \quad (3)$$

$$Q_{i, \min} \leq Q_i \leq Q_{i, \max}, \quad i = 1, 2, 3, \dots, N_w \quad (4)$$

$$P(h) = \begin{cases} h_{i, \min} - h_i & \text{if } h_i < h_{i, \min} \\ 0 & \text{if } h_i \geq h_{i, \min} \end{cases} \quad i = 1, 2, 3, \dots, N_w \quad (5)$$

in which, Q_i : the pumping rate of well i ; N_w : the number of wells; $P(h)$: penalty term; $h_{i, \min}$: the minimum hydraulic head value at well i ; and $Q_{i, \min}$ and $Q_{i, \max}$: the minimum and maximum bounds of the pumping rates at well i , respectively.

ANALYTICAL SOLUTION

There are several previous studies that suggested different analytical solutions for the groundwater flow equation in confined and unconfined aquifers (Grubb, 1993; Shan, 1999; Kim and Ann, 2001; Yeo and Lee, 2003; Ma et al., 2009). Lee and Yeo (2003) suggested an analytical steady state solution using double Fourier transformation to deal with arbitrarily located multi-injection/pumping wells in anisotropic homogeneous confined aquifer. This methodology is adopted in the present study with the following modifications:

1. The analytical solution is carried out for the groundwater flow equation that describes unconfined aquifer.

2. The effect of uniform recharge on the studied domain is taken into consideration.

3. Different types of boundary conditions (BCs) are considered in the studied domain (two Dirichlet BC and two Neumann BCs).

4. The suitable number of Fourier coefficients is determined.

5. Application of the sigma-approximation technique, given by Jerri (1998), to reduce the effect of Gibbs phenomenon.

For the unconfined aquifer shown in Figure 1, the following assumptions are taken into consideration for the derivation of analytical solution: (1) the Dupuit's hydraulic assumption is employed to vertically integrate the flow equation, reducing it from three dimensional geometry to two dimensional, (2) the aquifer specific storage is ignored such that the governing equation becomes time independent, (3) the wells fully penetrate the aquifer thickness, (4) the impervious bed of the aquifer is considered horizontal, (5) hydraulic conductivity is assumed constant throughout the studied domain, and (6) two Dirichlet boundary conditions are assumed having the same value.

According to the previous assumptions, Equation 1 can be written as follows:

$$\frac{\partial}{\partial x} \left(Kh \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(Kh \frac{\partial h}{\partial y} \right) = f(x, y) \quad (6)$$

on the rectangular domain $(x, y) \in \Omega = [0, L] \times [0, H]$ subject to the boundary conditions:

$$h(x, 0) = h(x, H) = h_o \quad (7)$$

$$\frac{\partial h}{\partial x}(0, y) = \frac{\partial h}{\partial x}(L, y) = 0 \quad (8)$$

in which, L : length of the studied domain in the direction of x -axis, H : length of the studied domain in the direction of y -axis, and h_o : boundary condition constant head.

Equation 6 can be linearized by the substitution $\phi = h^2/2$ and the equation then takes the form:

$$K \frac{\partial^2 \phi}{\partial x^2} + K \frac{\partial^2 \phi}{\partial y^2} = f(x, y) \quad (9)$$

With the boundary conditions:

$$\phi(x, 0) = \phi(x, H) = \frac{h_o^2}{2} \quad (\text{Dirichlet BCs}) \quad (10)$$

$$\frac{\partial \phi}{\partial x}(0, y) = \frac{\partial \phi}{\partial x}(L, y) = 0 \quad (\text{Neumann BCs}) \quad (11)$$

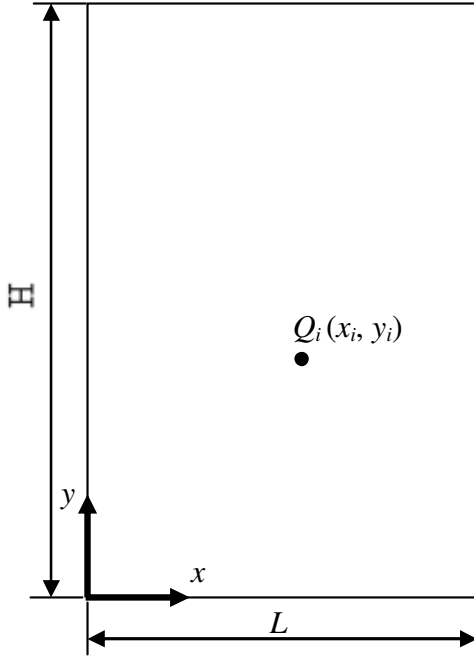


Figure 1. Definition sketch of the studied domain.

The analytical solution of Equation 9 can be written as a double Fourier series of the form:

$$\phi(x, y) = c + \left[\frac{\tilde{a}_0}{2} + \sum_{n=1}^{\infty} \tilde{a}_n \cos \frac{n\pi x}{L} + \tilde{b}_n \sin \frac{n\pi x}{L} \right] \left[\frac{\tilde{c}_0}{2} + \sum_{m=1}^{\infty} \tilde{c}_m \cos \frac{m\pi y}{H} + \tilde{d}_m \sin \frac{m\pi y}{H} \right] \quad (12)$$

where the constant c is obtained from the boundary conditions to be:

$$c = \frac{h_o^2}{2} \quad (13)$$

From condition given in Equation 11.:

$$\left[\sum_{n=1}^{\infty} \frac{n\pi}{L} \tilde{b}_n \right] \left[\frac{\tilde{c}_0}{2} + \sum_{m=1}^{\infty} \tilde{c}_m \cos \frac{m\pi y}{H} + \tilde{d}_m \sin \frac{m\pi y}{H} \right] = 0 \quad (14)$$

and

$$\left[\sum_{n=1}^{\infty} \frac{n\pi}{L} (-1)^n \tilde{b}_n \right] \left[\frac{\tilde{c}_0}{2} + \sum_{m=1}^{\infty} \tilde{c}_m \cos \frac{m\pi y}{H} + \tilde{d}_m \sin \frac{m\pi y}{H} \right] = 0 \quad (15)$$

which implies that $\tilde{b}_n = 0$. From condition given in Equation 10:

$$\left[\frac{\tilde{a}_0}{2} + \sum_{n=1}^{\infty} \tilde{a}_n \cos \frac{n\pi x}{L} \right] \left[\frac{\tilde{c}_0}{2} + \sum_{m=1}^{\infty} \tilde{c}_m \right] = 0 \quad (16)$$

and

$$\left[\frac{\tilde{a}_0}{2} + \sum_{n=1}^{\infty} \tilde{a}_n \cos \frac{n\pi x}{L} \right] \left[\frac{\tilde{c}_0}{2} + \sum_{m=1}^{\infty} (-1)^m \tilde{c}_m \right] = 0 \quad (17)$$

which implies that $\tilde{c}_0 = \tilde{c}_m = 0$. According to these conditions, the analytical solution of Equation 9 can be written as double cosine-sine Fourier series in the form:

$$\phi(x, y) = \frac{h_o^2}{2} + \sum_{m=1}^{\infty} \left[\frac{c_{0,m}}{2} + \sum_{n=1}^{\infty} c_{n,m} \cos \frac{n\pi x}{L} \right] \sin \frac{m\pi y}{H} \quad (18)$$

To evaluate the Fourier coefficients $c_{n, m}$, the solution given in Equation 18 is substituted in Equation 9 to obtain the following form:

$$\sum_{m=1}^{\infty} \left[\frac{b_{0,m}}{2} + \sum_{n=1}^{\infty} b_{n,m} \cos \frac{n\pi x}{L} \right] \sin \frac{m\pi y}{H} = f(x, y) \quad (19)$$

where:

$$b_{n,m} = -K\pi^2 \left(\frac{n^2 H^2 + m^2 L^2}{H^2 L^2} \right) c_{n,m} \quad (20)$$

in which, $b_{n,m}$ represents the coefficient of the double cosine-sine Fourier series of the function $f(x, y)$ and can be obtained by the canonical form:

$$b_{n,m} = \frac{4}{HL} \int_0^L \int_0^H f(x, y) \cos \frac{n\pi x}{L} \sin \frac{m\pi y}{H} dy dx \quad (21)$$

hence, $c_{n,m}$ is given by:

$$c_{n,m} = \frac{-4HL}{K\pi^2} \int_0^L \int_0^H \frac{f(x, y)}{n^2 H^2 + m^2 L^2} \cos \frac{n\pi x}{L} \sin \frac{m\pi y}{H} dy dx \quad (22)$$

Consider the case where:

$$f(x, y) = W - \sum_{i=1}^{N_W} Q_i \delta(x - x_i) \delta(y - y_i) \quad (23)$$

in which, W : the uniform rainfall or uniform evaporation (W takes minus sign in case of uniform evaporation), Q_i : the injection or pumping rate of the i^{th} well (Q_i takes minus sign in case of injection), δ : the Dirac delta function,

N_w : number of wells. Then, from Equation 22 and for integer $m \geq 1$:

$$c_{0,m} = \frac{-4H}{K\pi^2 m^2 L} \left\{ [(-1)^m - 1] \frac{WLH}{m\pi} + \sum_{i=1}^{N_w} Q_i \sin \frac{m\pi y_i}{H} \right\} \quad (24)$$

$$c_{n,m} = \frac{-4HL}{K\pi^2 (n^2 H^2 + m^2 L^2)} \sum_{i=1}^{N_w} Q_i \cos \frac{n\pi x_i}{L} \sin \frac{m\pi y_i}{H}, \quad (25)$$

$n \neq 0$

In actual computations, a truncated Fourier series is used. To achieve a certain degree of accuracy in computations, the number of Fourier coefficients is chosen such that it ensures that $|c_{n,m}| < \varepsilon$ for a sufficiently small positive constant ε . From Equation 22:

$$|c_{n,m}| = \left| \frac{4HL}{K\pi^2 (n^2 H^2 + m^2 L^2)} \sum_{i=1}^{N_w} Q_i \cos \frac{n\pi x_i}{L} \sin \frac{m\pi y_i}{H} \right| \quad (26)$$

$$\leq \frac{4HLN_w}{K\pi^2 (n^2 H^2 + m^2 L^2)} \max_{1 \leq i \leq N_w} |Q_i|$$

and if equal number of Fourier coefficients is set for both variables x and y and denote it by M , then the following inequality holds:

$$M > \sqrt{\frac{4HLN_w}{K\pi^2 \varepsilon (H^2 + L^2)}} \max_{1 \leq i \leq N_w} |Q_i| \quad (27)$$

Finally, since the function $f(x, y)$ considered in the proposed model contains the Dirac delta function, Equation 18, the obtained Fourier series is affected by the Gibbs phenomenon. This phenomenon describes that the partial sum of the Fourier series has large oscillations near the jump, which might increase the maximum of the partial sum above that of the function itself. Several methods are available to reduce the Gibbs effect such as Fejer summation (Carslaw, 1921) and sigma approximation (Jerri, 1998). The sigma-approximation technique is used and the truncated series solution then takes the form:

$$\phi(x, y) = c + \sum_{m=1}^M \frac{\sin\left(\frac{m\pi}{M+1}\right)}{\left(\frac{m\pi}{M+1}\right)} \left[\frac{c_{0,m}}{2} + \sum_{n=1}^N c_{n,m} \frac{\sin\left(\frac{n\pi}{N+1}\right)}{\left(\frac{n\pi}{N+1}\right)} \cos \frac{n\pi x}{L} \right] \sin \frac{m\pi y}{H} \quad (28)$$

The hydraulic head can be computed using the following equation:

$$h(x, y) = \sqrt{2\phi(x, y)} \quad (29)$$

PARTICLE SWARM OPTIMIZATION

PSO was developed by Kennedy and Eberhart (1995). The PSO is inspired by the social behavior of a flock of migrating birds trying to reach an unknown destination. In PSO, each solution is a 'bird' in the flock and is referred to as a 'particle'. The birds in the population only evolve their social behavior and accordingly their movement towards a destination (Shi and Eberhart, 1998).

Physically, this mimics a flock of birds that communicate together as they fly. Each bird looks in a specific direction, and then when communicating together, they identify the bird that is in the best location. Accordingly, each bird speeds towards the best bird using a velocity that depends on its current position. Each bird, then, investigates the search space from its new local position, and the process is repeated until the flock reaches a desired destination. It is important to note that the process involves both social interaction and intelligence so that birds learn from their own experience (local search) and also from the experience of others around them (global search).

The process is initialized with a group of random particles (solutions). The i^{th} particle is represented by its position as a point in a S -dimensional space, where S is the number of variables. Throughout the process, each particle i monitors three values: its current position (X_i); the best position it reached in previous cycles (P_i); its flying velocity (V_i). These three values are represented as follows (Elbeltagi et al. 2005):

Current position $X_i = (x_{i1}, x_{i2}, \dots, x_{iS})$

Best previous position $P_i = (p_{i1}, p_{i2}, \dots, p_{iS})$

Flying velocity $V_i = (v_{i1}, v_{i2}, \dots, v_{iS})$ (30)

In each time interval (cycle), the position (P_g) of the best particle (g) is calculated as the best fitness of all particles. Accordingly, each particle updates its velocity V_i to catch up with the best particle g , as follows (Shi and Eberhart 1998):

$$\text{New } V_i = \omega \times \text{current } V_i + c_1 \times RN_1 \times (P_i - X_i) + c_2 \times RN_2 \times (P_g - X_i) \quad (31)$$

using the new velocity V_i , the particles update their position as follows:

$$\text{New position } X_i = \text{current position } X_i + \text{New } V_i; V_{\max} \geq V_i \geq -V_{\max} \quad (32)$$

in which, c_1 and c_2 : two positive constants named learning factors ($c_1 = c_2 = 2$); RN_1 and RN_2 : two random functions in the range $[0, 1]$; V_{\max} : upper limit on the maximum change of particle velocity (Kennedy and Eberhart, 1995); and ω : an inertia weight employed as an improvement proposed by Shi and Eberhart (1998) to

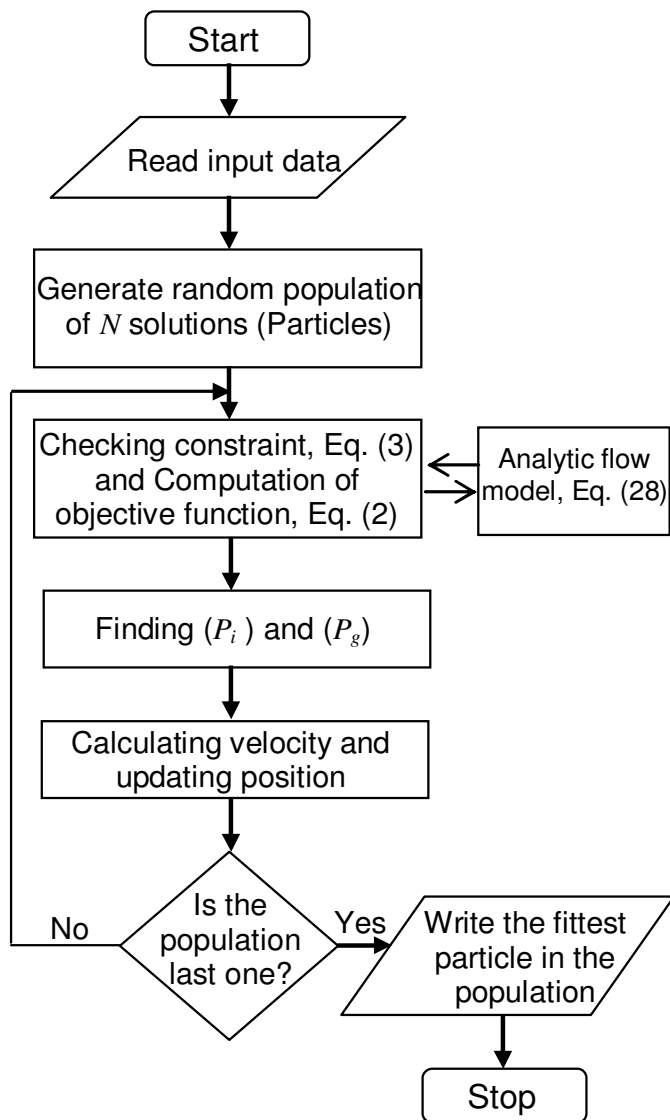


Figure 2. Flow chart for coupled Analytic-PSO model.

control the impact of the previous history of velocities on the current velocity. The operator ω plays the role of balancing the global search and the local search; and was proposed to decrease linearly with time from a value of 1.4 to 0.5 (Shi and Eberhart, 1998). As such, global search starts with a large weight and then decreases with time to favor local search over global search (Eberhart and Shi, 1998).

It is noted that the second term in Equation 31 represents *cognition*, or the private thinking of the particle when comparing its current position to its own best. The third term in Equation 31, on the other hand, represents the *social* collaboration among the particles, which compares a particle's current position to that of the best particle (Kennedy 1997). Also, to control the change of particles' velocities, upper and lower bounds for velocity change is limited to a user-specified value of V_{max} . Once

the new position of a particle is calculated using Equation 32, the particle then flies towards it (Shi and Eberhart 1998). The main parameters used in the PSO technique are the population size (number of birds), number of cycles, the maximum change of a particle velocity V_{max} , and the inertia weight ω .

SIMULATION-OPTIMIZATION MODEL

After developing the simulation and optimization models, mentioned in the two previous sections, both models are coupled to solve groundwater management problems. Figure 2 shows the flow chart of the Analytic-PSO model. The coupled Analytic-PSO model is particularly developed to apply the principles of simulation-optimization approach, where the optimization model repeatedly calls the simulation model to find the optimum solution of the problem. The optimization model calls simulation model to predict the state variables (hydraulic heads at well locations). The values of those state variables are used to check the constraints and then penalty value is considered if constraint violations occurred. The whole solution procedure is successively repeated to generate new solution (well discharges in the first application and set of co-ordinates and discharge of the wells in the second application, which excessively increases the computational burden) until the global (or near global) solution is obtained.

NUMERICAL APPLICATION

To investigate the performance of applying Analytic-PSO model to solve groundwater management problems, two problems are used as examples. The first one concerns with determining the maximum pumping from pre-specified well system, whereas the second problem includes determination of both coordinates and maximum pumping from a pre-specified number of wells. The two applications are performed in unconfined aquifer system given by McKinney and Lin (1994). Figure 3 shows the plan view and cross-sectional elevation of the studied aquifer. As can be seen from this figure, the aquifer having dimensions of $4500 \times 10000 \text{ m}^2$. The boundary conditions include the Dirichlet at the north (river) and south (swamp) sides ($h_0 = 20 \text{ m}$); and the no-flow at the east and west sides (mountains). The aquifer is composed from sand and gravel and it is assumed that porous medium is homogeneous and isotropic. The hydraulic conductivity and the areal recharge rate (K and W) are 50 m/day and 0.001 m/day , respectively. There are 10 pumping wells having locations listed in Table 1.

Groundwater management problem (1)

The first problem deals with the maximization of total

Pumping from an unconfined aquifer using a pre-specified system of wells, McKinney and Lin (1994). This typical problem was solved by various researchers (McKinney and Lin, 1994; Wang and Zheng, 1998; Wu et al., 1999; Wu and Zhu, 2006; Ayvas, 2009; Gaur et al., 2011) using different optimization methods such as LP, GA, SA, SCE-UA, HS, and PSO. The purpose of this section is to perform a comparison between results of Analytic-PSO model and the corresponding ones given by other previous researchers. The objective function for this problem was taken as Equation 2. Two constraints were considered, (1) hydraulic head should be above the aquifer bottom ($h_{i,min} = 0$), and (2) discharge range of the pumping wells should be within the limits of 0–7000 m³/day to prevent aquifer dewatering. The suitable number of Fourier coefficients is determined, in the worst case, by taking $\max|Q_i| = 7000$ m³/day, $N_w = 10$, $H = 10000$ m, $L = 4500$ m, $K = 50$ m/day, $\pi = (22/7)$, and $M = 60$, (Equation 27.) in the analytic flow model.

The sensitivity analysis is carried out to determine the PSO solution parameters as follows: number of particles = 40, number of cycles = 300, and $V_{max} = 200$.

The factor ω is also set as a linear function decreasing with the increase of number of generations where, at any generation i ,

$$\omega = 0.4 + 0.8 \times (\text{number of generations} - i) / (\text{number of generations} - 1) \tag{33}$$

such that $\omega = 1.2$ and 0.4 at the first and last generation, respectively.

After applying the model to the problem, it is found that the values of most of $|c_{n,m}|$ are of order 10^{-6} , and the maximum value in the last 60 term does not exceed 0.007.

The relationship between optimal total pumping rates and number of cycles is shown in Figure 4. From this figure, it can be shown that the Analytic-PSO model can achieve a fairly good solution for pumping rate (59000 m³/day) after only 71 cycles whereas, it converges to the optimal value of 59463.76 m³/day at the last cycle. Table 2 presented a comparison between the maximum discharge by 10 pumping wells in the present study and those given by other previous ones. As can be seen from the table, the Analytic-PSO model gives a higher value for total pumping rate (59463.76 m³/day) in comparison with other models. This result closely agrees with the PSO 2 solution (59425 m³/day) given by Gaur et al. (2011). Results of other previous solution methods are found to be 59300 m³/day in LP, 58000 m³/day and 59000 m³/day in GA, 59078 m³/day in GASAPF, 59266 m³/day in SCE-UA, 59400 m³/day in SA, and 59279 m³/day in HS. The hydraulic heads at wells and hydraulic head contours corresponding to the obtained total pumping rate are shown in Table 3 and Figure 5, respectively.

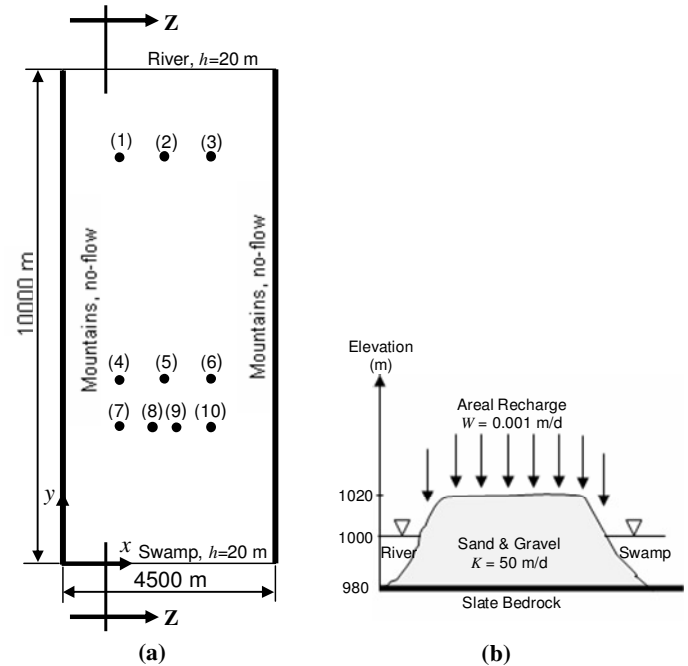


Figure 3. Unconfined aquifer model under consideration: (a) Plan view and (b) Sectional elevation Z-Z (After McKinney and Lin1994).

Table 1. Coordinates of pumping wells, shown in Figure 3 (McKinney and Lin, 1994).

Well number	x-coordinate (m)	y-coordinate (m)
1	1250	8500
2	2250	8500
3	3250	8500
4	1250	5000
5	2250	5000
6	3250	5000
7	1250	3500
8	2000	3500
9	2500	3500
10	3250	3500

The hydraulic heads at wells shown in Table 3 indicate that results from the proposed model satisfied constrains of the optimization model. Consequently, the Analytic-PSO model can effectively and efficiently be used to solve groundwater management problems.

Groundwater management problem (2)

The objective of the second problem is also maximizing the total pumping rate from the unconfined aquifer shown in Figure 3 as well as determining the best coordinates for the ten wells. The used objective function and hydraulic constrains are the same ones as in the problem

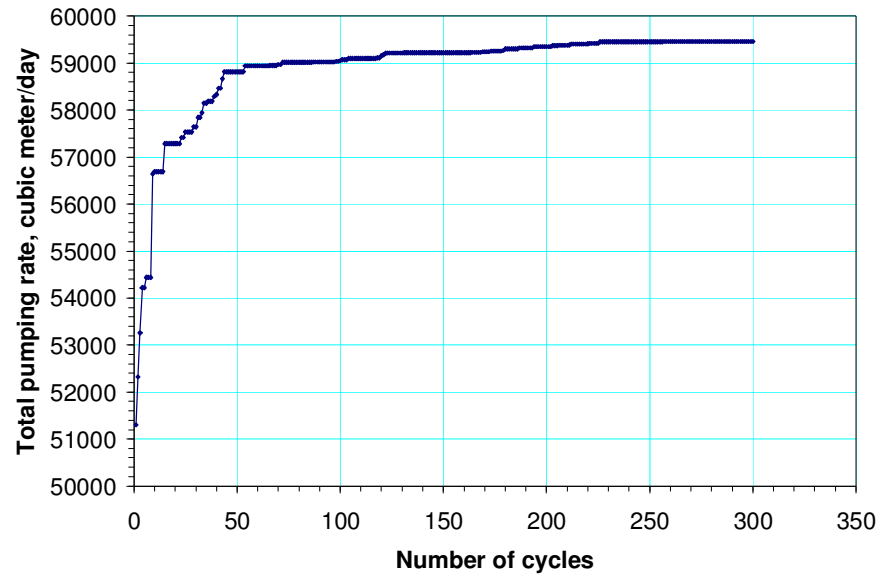


Figure 4. Relationship between optimal total pumping rates and number of cycles.

Table 2. Maximum discharge by 10 pumping wells using different optimization techniques (units: m³/day).

Well number	Optimizations techniques									Present study
	(LP) McKinney and Lin (1994)	(GA) McKinney and Lin (1994)	(GA) Wang and Zheng (1998)	(SA) Wang and Zheng (1998)	GASAPF Wu et al. (1999)	(SCE-UA) Wu and Zhu (2006)	(HS) Ayvaz (2009)	(PSO 1) Gaur (2011)	(PSO 2) Gaur (2011)	
1	7000	7000	7000	7000	7000	7000	7000	7000	7000	7000.00
2	7000	7000	7000	7000	7000	7000	7000	7000	7000	7000.00
3	7000	7000	7000	7000	7000	7000	7000	7000	7000	7000.00
4	6000	7000	5000	6200	6056	5987	5904	6300	6315	5960.43
5	4500	2000	5000	6700	4290	4477	4590	4600	4600	4503.70
6	6000	6000	6000	6200	6056	5986	5904	6150	6150	5949.65
7	6800	7000	7000	6650	6774	6814	6821	6500	6600	6729.79
8	4100	4000	4000	4000	4064	4094	4121	4100	4055	4282.12
9	4100	4000	4000	4000	4064	4094	4120	4100	4100	4230.97
10	6800	7000	7000	6650	6774	6814	6820	6600	6605	6807.09
Total pumping	59300	58000	59000	59400	59078	59266	59279	59350	59425	59463.76

Table 3. Hydraulic head at wells corresponding to the obtained total pumping in problem 1.

Well number	Head (m)
1	12.18
2	11.40
3	12.18
4	0.12
5	0.33
6	0.03
7	1.22
8	0.34
9	0.68
10	0.17

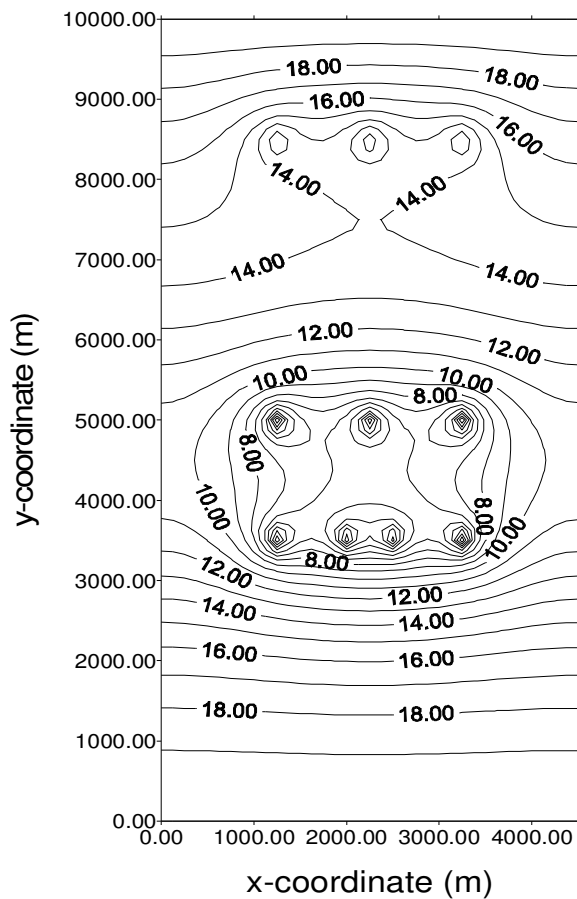


Figure 5. Contour heads corresponding to the obtained total pumping in problem 1.

1. The sensitivity analysis is performed to determine the Analytic-PSO model parameters as follows: $M = 60$, number of particles = 30, number of cycles = 200, $V_{max} = 350$ and the factor ω is considered as previously mentioned.

After applying the model it is found that each well can pump its maximum bound of the pumping rate (7000

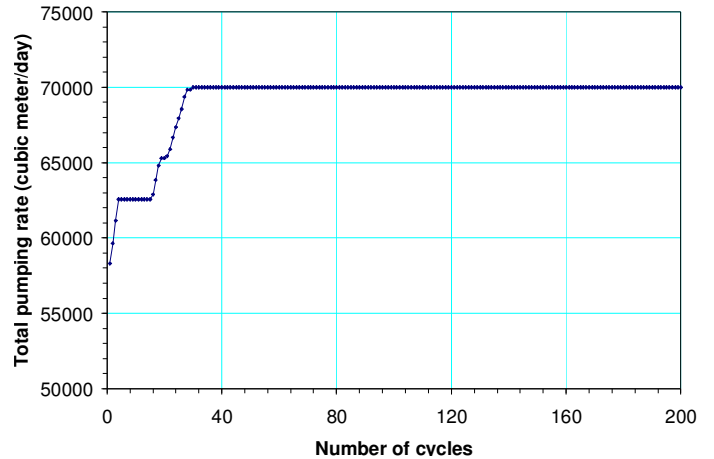


Figure 6. Relationship between optimal total pumping rates and number of cycles.

m³/day) without violation of the minimum hydraulic head value. Figure 6 demonstrates the relationship between optimal total pumping rates and number of cycles. As seen from this figure, the Analytic-PSO model can obtain an optimum solution for pumping rate (70000 m³/day) after only 30 cycles performing 900 simulations. Table 4 lists the obtained coordinates of wells and the hydraulic heads at well locations. Figure 7 shows the groundwater head contours generated by the Analytic-PSO model for the maximum pumping values.

Conclusions

A steady state analytical solution is suggested and derived for the groundwater flow equation in a homogeneous unconfined aquifer. The analytical solution is based on double Fourier series and is suitable to deal with any number of pumping or injection wells or combination of them. In addition, uniform rainfall or evaporation can be taken into consideration. The new analytical method is linked with the Particle Swarm Optimization method to solve the groundwater management problems. A new model called Analytic-PSO, which consists of the two methods, is originally developed. The Analytic-PSO model is verified on the most popular hypothetical example to obtain the maximum pumping rate and a comparison was carried out with the corresponding ones given by other previous studies. In addition, the model is used to determine both the optimum locations and discharges of wells for the pre-specified number of wells. The results showed that the Analytic-PSO model can effectively and efficiently be used to solve real groundwater management problems.

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Table 4. The obtained results for problem 2

Well no.	x-coordinate (m)	y-coordinate (m)	Head (m)
1	1538.81	553.1	16.15
2	4053.67	3398.12	4.18
3	885.40	2145.87	13.23
4	4238.68	8515.75	13.75
5	4154.02	1205.85	12.22
6	3339.77	3548.14	6.50
7	1369.39	6814.13	15.50
8	3442.24	9710.71	16.58
9	2565.95	4191.12	10.44
10	3521.94	9908.74	18.36

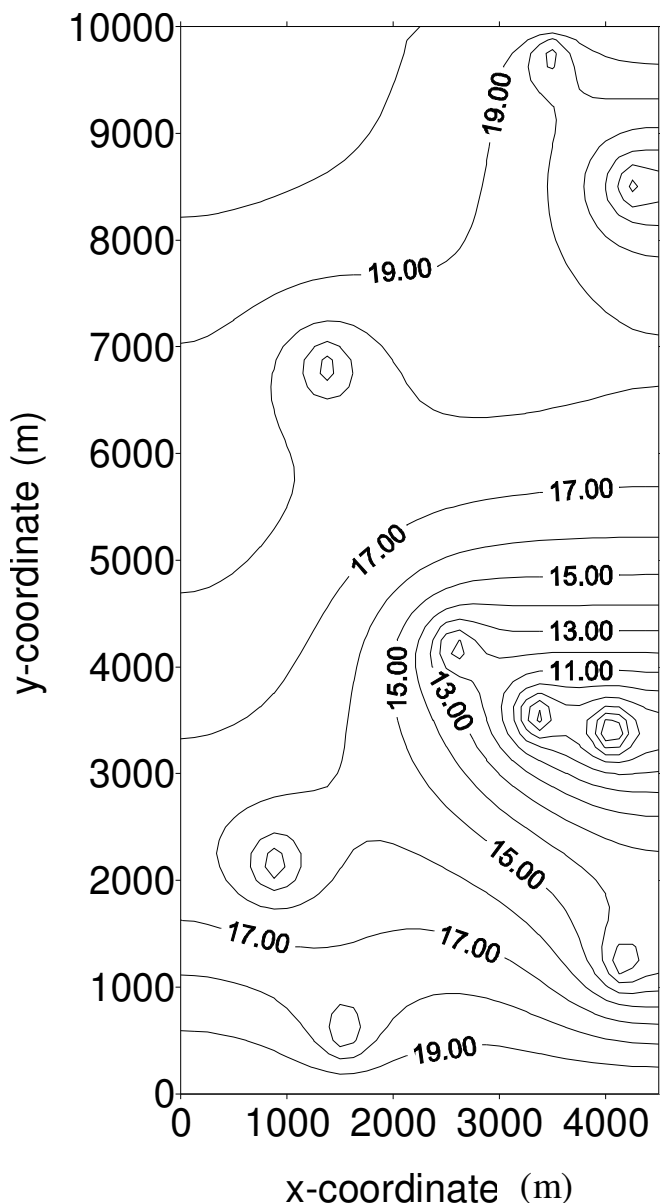


Figure 7. Contour heads corresponding to the obtained total pumping in problem 2.

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