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Topology optimization method for materials zoning of a concrete face rockfill dam

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For finding a reasonable layout of the fill materials in the dam body of a concrete face rockfill dam (CFRD), a new algorithm is proposed based on continuum topology optimization. In the present approach, the materials in dam are classified into several materials with different moduli and each material has a material number (an integer) according to the value of modulus. Material with higher modulus has a bigger material number. The finite element method (FEM) is adopted to find the deformation of structure. In design domain, the material number of a finite element is considered as a design variable in optimization. Hence, zoning optimization of fill materials in a CFRD is a typical multiple material layout optimization with discrete design variables. To improve the efficiency of optimization process, a criterion is used for update of material numbers. It says that the stiffer materials are layout in the areas with higher strain energy density (SED). According to the criterion, materials for those elements with low SEDs are replaced with softer materials. The optimization is completed if all the volumes of materials reach their critical values. Two numerical examples are considered to show the feasibility and efficiency of the method. In particular, the material zoning of a CFRD is studied. The deflection of face slab of dam is less than the value reported. The result implies the present material layout is a stiffer design.

Key words: Layout optimization, concrete face rockfill dam (CFRD), multiple materials, strain energy density (SED), finite element.

INTRODUCTION

Concrete face rockfill dam (CFRD) is very popular in modern dam engineering applications due to their advantages over other dam types: (1) the cost is relatively low because of the use of local materials; (2) CFRD has a wide adaptability to foundation conditions. It is especially true in a project where high hydrostatic uplift is necessary when the foundation conditions of geology are poor; (3) it can be constructed rapidly even in bad weather condition; and (4) sound leakage control can make dam be in normal service in which conditions such as the seepage in dam is serious. Historically, CFRD was firstly built in 1940s and developed rapidly since the introduction of vibrating rollers in the 1970s. Significant advances in the design and construction of dams have been achieved since then. Currently, the highest CFRD in the world is, so far, the Shuibuya CFRD in China, whose height reaches 233 m. During recent years, more and more CFRDs with the height above 200 m have been built around the world (Li, 2007), for example, Nam Ngum 3 Dam (220 m, Laos), Campos Novos Dam (200 m, Brazil), Bakun Dam (205 m, Malaysia), Agbulu Dam (234 m, Philippine), and Morro de Arica Dam (215 m, Peru).

However, in recent years, some of the dams are not able to work well because of the serious seepage (even piping) for various reasons (Southcott et al., 2003). For example, Gollias Dam (125 m, Columbia) is no longer in service due to the failure of peripheral joints. The face slab of Xibeikou Dam in China (95 m) has obvious cracks. Tianshengqiao No. 1 Dam (178 m, China) and Xingo Dam (150 m, Brazil) have broken cushion layer. *Corresponding author. E-mail: wangzz0910@163.com.
Serious leakage with poor zoning of materials in rockfill even leads to the failure of Gouhou Dam of 78 m in China (Chen, 1993) and Zhushuqiao Dam of 78 m in China (Xu, 2005).

It is well-known that seepage/piping is the major cause leading to the failure of dams. To avoid the failure of dam caused by leakage, two major methods are considered in practice. One is the use of anti-seepage control technique to insert effective filters in dam for releasing water uplift pressures and avoid soil particles being taken away by piping (Terzaghi et al., 1996). Another is to optimize the layout of zones in rockfill to avoid the crack on concrete face slab through reducing the difference between the deformations of face slab and rockfill or opening of joints (Mori, 1999; Foster et al., 2000). Thus, achieving optimal layout of rockfill zones is important for reducing the chance of failure of a CFRD.

Furthermore, for the sake of environmental protection and cost reduction in a CFRD project, rockfill should be zoned such that the materials from required excavation and borrow areas with the shortest haul distance are used as much as possible. Before construction, therefore, it is very important to obtain a reasonable layout of the materials (including soils, (weathered) rock, gravels, and concrete) in a CFRD. In the literature, materials zoning optimization is commonly employed to solve the problem of CFRD design. For example, Guo et al. (1998) developed an optimal design of a CFRD on alluvial deposit. Cai (2005) proposed an interval analysis method to optimize materials zones in rockfill. Recently, Cai et al. (2008) presented an optimal design for a cemented CFRD. Wang and Gao (2009) introduced an optimal design on cross section of a CFRD in China. In these size or shape optimization models, initial designs are made according to national specifications and only few parameters of dam are considered as design variables to determine the layout of rockfill zones. However, they adjusted only the interface between major and secondary rock-fill zones in a CFRD (Figure 1), in which initial design is commonly dependent on experience and engineering judgment of researchers (Cooke, 1984). It is known that the experiences are obtained from the design and construction of lower CFRD. As the height of a CFRD is over 200 m, the feasibility of the rockfill zoning design according to the old experiences cannot be ensured. To give a safe design of rockfill zoning based on solid theory, the model (material zoning optimization) developed here can, in fact, be viewed as a typical multi-phase layout problem in topology optimization (Figure 2).

It is noted that topology optimization (that is, layout optimization) of continuums becomes popular in recent years owing to significant advancement achieved in computer technology and the method of topology optimization in the last 25 years. The topology optimization here includes homogenization method (Bendsøe and Kikuchi, 1988), SIMP method (Rozvany et al., 1992), ESO method (Xie and Steven, 1993), and Level-set method (Wang et al., 2003). Both of the homogenization method and the SIMP method are material approach, in which the design variables are the size(s) of the patterns of a unit cell in each finite element (supposing the FEM is adopted to solve deformation of structure). ESO and level-set methods are geometry method, in which the topology of the design domain changes directly (without considering the inner patterns of unit cell in each finite element). Nowadays, topology optimization has been successfully adopted for engineering design in various fields, for example, automotive industry, aerospace, and civil engineering. In most of the publications on topology optimization, the structure considered has only one or two solid materials. But in practical engineering, lots of structures contain multiple materials. For instance, in the design of a CFRD, designers have to handle multi-phase layout problem which cannot be solved directly by the existing methods. Recently, much attention has been paid to the topology

To the best of our knowledge, the optimization for the layout of rockfill in CFRD is still an open question. This is the motivation of this work.

In this paper, an algorithm based on ESO method (Xie and Steven, 1993) is presented for solving optimal layout problems of multiple material systems. It proves to be particularly suitable for material characteristics of a CFRD. It begins with classifying the materials in rockfill into several materials. These materials are then identified with different identity (ID) according to the moduli of materials. For example, if we have three different materials in total in a CFRD, the materials' ID can be labeled as 1, 2, and 3, respectively. The material with ID of No. 1 is usually the softest material and 3 is stiffest material. Having identified the materials, a finite element simulation is carried out to find the deformation field and the strain energy densities (SEDs) of elements in design domain. Finally, the ascending sort of the SEDs is obtained based on the finite element analysis above. By way of the ascending sort obtained, the material in those elements with lower SEDs will be replaced with softer materials, for example, material marked as 3 being replaced with the material marked as 2 in those elements with lower SED. The optimal layout of rockfill in a CFRD is obtained when the volumes of all materials with different moduli reach their critical values.

**METHODOLOGY**

**Basic equations for a CFRD**

In this work, only small deformation of a linear elastic structure is considered. The reasons why we do not consider plastic deformation of materials are as follows. Firstly, layout optimization is an approach to give an initial design scheme for a practical engineering. The final material distributions obtained by numerical methods may be revised by considering other design rules/criteria or even convenience of construction. Secondly, the computational cost for nonlinear structural analysis (Qin and He, 2005) is much higher than that for linear elastic analysis. Moreover, the linear elastic deformation is a dominant part of the total deformation in CFRD and thus it can provide acceptable results for an initial design. Finally, the materials in a CFRD are usually compacted as hard as possible during construction. As a consequence, the mechanical properties of the materials used in the present model are under pre-stress states. It is, thus, reasonable to treat the mechanical properties as linear elastic in the proposed model. As such, the basic equations and boundary conditions for a CFRD are given as:

(a) **Geometry equation**

\[ \varepsilon_{ij} = (v_{i,j} + v_{j,i})/2 \]  

(b) **Constitutive equation**

\[ \sigma_{ij} = D_{ijkl} \cdot \varepsilon_{kl} \]  

(c) **Equilibrium equation**

\[ \sigma_{ij,j} + f_j = 0 \]  

(d) **Force boundary**

\[ \sigma_{ij} \cdot n_j = F^* \text{ on } \Gamma^* \]
(e) Displacement boundary

\[ v_i = v_i^* \quad \text{on } \Gamma_v \]  

(5)

where \( \sigma_{ij} \) is the stress tensor, \( \varepsilon_{ij} \) strain tensor, \( v_i \) displacement vector, \( f_i \) body force vector, \( F_i^* \) traction on the boundary \( \Gamma_o \) of the solution domain \( \Omega \), \( n_j \) the components of outward normal vector to the boundary \( \partial \Omega = \Gamma_o + \Gamma_v \), and \( v_i^* \) prescribed displacement on the boundary \( \Gamma_v \). In this work, the finite element method (FEM) (the commercial software ANSYS of version 12.0, 2010) is employed to solve the boundary-value problem (1-5).

\( D_{ijkl} \) is the material stiffness tensor. If the stress and strain tensors are written as \( [\sigma_x \sigma_y \sigma_z \tau_{xy} \tau_{yz} \tau_{zx}]^T \) and \( [e_x e_y e_z e_{xy} e_{yz} e_{zx}]^T \), respectively, the matrix \( D_{ijkl} \) can be expressed in the form

\[
D^{(m)} = \begin{bmatrix}
\frac{1}{Z} & -\frac{v_{y}v_{z}v_{y}E_{x}}{Z} & -\frac{v_{y}v_{z}v_{z}E_{y}}{Z} & 0 & 0 & 0 \\
-\frac{v_{x}v_{z}v_{y}E_{x}}{Z} & \frac{1}{Z} & -\frac{v_{x}v_{z}v_{z}E_{y}}{Z} & 0 & 0 & 0 \\
-\frac{v_{x}v_{y}v_{y}E_{x}}{Z} & -\frac{v_{x}v_{y}v_{z}E_{y}}{Z} & \frac{1}{Z} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2G_{ij}}{Z} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{2G_{ij}}{Z} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2G_{ij}}{Z}
\end{bmatrix}
\]

(6)

For an orthotropic material, say material \( m \), where \( Z = 1 - v_{xy}v_{xy} - v_{xz}v_{xz} - v_{yz}v_{yz} - 2v_{xy}v_{yz}v_{xz} \). The symmetry of the matrix \( D^{(m)} \) requires that:

\[
\begin{align*}
\frac{V_{xy} + V_{yz}V_{yz}}{Z} & = \frac{V_{xy} + V_{yz}V_{yz}}{Z} E_x \\
\frac{V_{yz} + V_{yz}V_{yz}}{Z} & = \frac{V_{yz} + V_{yz}V_{yz}}{Z} E_y \\
\frac{V_{xz} + V_{yz}V_{xz}}{Z} & = \frac{V_{xz} + V_{yz}V_{xz}}{Z} E_z
\end{align*}
\]

(7)

For an isotropic material, \( V_{ij} = V \), \( E_x = E \cdot G_y = G = E/2(1 + \nu) \) (i.e., \( x, y, z \) in which \( E \) and \( V \) are respectively the elastic modulus and Poisson’s ratio).

The strain energy density (SED) \( U_e \) in an element, say element \( e \), is defined as:

\[
U_e = \frac{1}{2} \sum_{i=1}^{3} \sigma_i \cdot \varepsilon_i
\]

(8)

where \( \sigma_i \) and \( \varepsilon_i \) (i=1, 2, 3) are the \( i \)-th principal stress and principal strain, respectively.

### Optimization Model of Rockfill Zoning of a concrete face rockfill dam (CFRD)

As mentioned above, the major task of this work is to optimize the layout of rockfills in a CFRD. In particular, the purpose is to maximize the stiffness of whole structure (or minimize the mean structural compliance). Before construction, the volume of each material in rockfill is confined. Therefore, the constraints in optimization model are volume constraints of the materials in rockfill of a CFRD. The optimization model for this problem can be expressed as:

\[
\begin{align*}
\text{Find } & \{ M_{e,m} \} \\
\text{min } & \{ c(M_{e,m}) \} \\
\text{s.t. } & \sum_{m} v_{e,m} \cdot \delta_{mi} - V_{cr,m} = 0, \quad (m = 1, 2, ..., N_m) \\
& K \cdot U = P
\end{align*}
\]

(9)

where \( M_{e,m} \) means that the material number of \( e \)-th element is \( m \), \( N_m \) the total number of free material types, \( \Omega_d \) the design domain, \( c \) the mean compliance of structure to be investigated, \( V_{cr,m} \) the critical volume of the material \( m \), and \( v_{e,m} \) the volume of \( e \)-th element with material \( l \). If \( m=l, \delta_{mi} = 1 \), else \( \delta_{mi} = 0 \). \( K \) is the global stiffness matrix of the structure in the finite element analysis. \( U \) and \( P \) are the global displacement vector and nodal force vector, respectively. \( K \) and \( P \) can be determined using the well-known finite element method.

### Update of material identities (IDs) of elements

In this part of the work, the procedure for updating the label of material number of an element is described. The purpose of updating element’s material number is to ensure all materials reach their critical loading values. Here a criterion is presented for updating the design variables in optimization. Concretely, a structural analysis is performed using FEM to provide the distribution of strain energy density (SED) in the solution domain and the ascending order of SEDs of elements is determined accordingly. Then the materials in those elements with lower SEDs are replaced with softer materials. It should be noted that, to keep the stability of algorithm, the number of elements updated in each iteration should be no more than 5% of the total number of elements. The process for different phases is repeated in the way mentioned above. The layout optimization of the structure is completed until all the volumes of the materials reach their critical values.

In Figure 3a, \( N_e \) is the total number of elements with Material \( m \) to be updated in the current iteration. To reduce computational cost in the iteration process, only the elements with material \( m \) in design domain are considered to be sorted according to their SEDs (Figure 3b). The total numbers of elements in Figure 3a and Figure 3b are the same, and the material ID of the first \( i \)-th elements in Figure 3b will change from \( m \) to \( m-1 \).

### Optimization procedure

The optimization process described above can also be written in the form of pseudo code for readers’ convenience:
Figure 3. Sorting of the SEDs of elements with Material \( m \) in design domain: (a) initial number sequence of elements with material \( m \) only and (b) ascending order of the values of SEDs of elements with either material \( m \) or \( m-1 \).

It should be noted that, in Algorithm 1, all of the elements are initially set to be the stiffest material. In the first Loop, the iteration will stop when the volume constraint on the last second material \( (m=2) \) is satisfied, because the volume constraint on the last material will be satisfied automatically. In the second Loop, the value of integer \( k(m) \) is initially specified according to critical values of the volume constraints \( (V_{mc}) \) and the volume ratio of the elements updated after each FE analysis. In the present study, the total volume of the elements with material updated is no higher than 5% of the volume of the whole design domain. The third Loop is used for the update of material property of elements.

SIMULATION AND DISCUSSION

In order to demonstrate the efficiency and applicability of the proposed algorithm, two examples are considered and their results are compared with those obtained from other approaches or experimental ones.

Example 1 - Validity of the present method

Figure 4 shows an initial design domain with the size of 1.2 m × 0.6 m × 0.01 m. The bottom of the rectangular is simply supported. A mesh of 120 × 60 quadrilateral plane stress elements is used in the analysis. Three concentrated forces are assumed to apply on the bottom surface, where \( P_1=30 \) N and \( P_2=2P_1 \). The structure is filled with three phases each of which has distinct elastic modulus. The elastic modulus of the stiffest solid material is 1.0 GPa. The volume ratios of three materials are set to be 0.2, 0.2, and 0.6, respectively. The objective is to find a layout of these materials in the structure whose compliance reaches its minimum. In iteration, 2% of the total elements are changed in each step. To examine effects of elastic moduli on the final material distributions, two cases of various elastic moduli are considered:

(a) \( E_3 : E_2 : E_1 = 10 : 5 : 1 \);

(b) \( E_3 : E_2 : E_1 = 200 : 100 : 1 \).

To show the validity of the present algorithm, a 2-phase layout optimization is firstly considered (because the existing models can handle two-phase problem as mentioned before). Figure 5a shows the optimal layout of stiffer material obtained by SIMP method and the results in Figure 5b is obtained by the present method. They are in good agreement and the validity of the present method is verified.

Figure 6 presents the results of material distribution from the presented method for two cases. Figure 6a demonstrates the results of material distribution when \( E_3 : E_2 : E_1 = 10 : 5 : 1 \) (case (a)). Material 2 distributes compactly. The reason is that the ratio of \( E_3 \) to \( E_2 \) is lower than the ratio of \( E_2 \) to \( E_1 \). Figure 6b shows the optimal material distribution in the structure for the case (b), that is, \( E_3 : E_2 : E_1 = 200 : 100 : 1 \). It is found that the layout of Material 3 in Figure 6b is the same as that in Figure 6a because both cases have the same ratio of \( E_3 \) to \( E_2 \).

Example 2 - Materials zoning optimization for a concrete face rockfill dam (CFRD)

General description of Gongboxia project

Gongboxia hydropower project (Song, 2002) is located in the Yellow River. It is 25 km away from Xunhua County and 153 km from Xining City in Qinghai Province, China. The project was started in the year of 2000 and completed in 2006. The major purpose of the project is for hydroelectric generation, irrigation, and water supply. By considering the conditions of topography, geology condition, and the practical requirements for construction and operation, the type of reinforced concrete face rockfill dam was adopted in this project.

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Create FE model of structure, initiate parameters, initial material ID \( m = N_m \); \( k(m) \);

**First Loop** \((m = N_m; -1; 2)\)

\( V(m) = 0; \)

**Second Loop** \((k = 1; 1; k(m))\)

Find the SEDs of elements by FEM;

Find the ascending order of SEDs of the elements \((N_{el}(m)\) in total) with material \( m \);

\( V(m) = V(m) + dV; \)

**Third Loop** \((e = 1; 1; N_{el}(m))\)

\%\% new order ID of elements with material \( m \)

If \( dV < V_{mc} / k(m) \)

\( dV = dV + v(e); \)

\%\% \( v(e) \) the volume of element \( e \)

\( M(e) = m - 1; \)

End if

End loop \((e = 1; 1; N_{el}(m))\)

End loop \((k = 1; 1; k(m))\)

End loop \((m = N_m; -1; 2)\)

Algorithm 1. Pseudo code of the present algorithm.

![Algorithm 1](image)

**Figure 4.** Initial design domain of structure.

The normal, design, and check flood elevations for the dam are 2005.00, 2005.00, and 2008.28 m, respectively. The crest elevation is 2010.00 m. The maximum dam height is 132.20 m. The length and width of crest are 429.0 and 10.0 m, respectively. The upstream slope is 1:1.4, whereas the downstream slopes vary from 1:1.5 to 1:1.3. The overall slope at the downstream of the dam is 1:1.79 (Figure 1).

**Materials in dam**

In the original design, the main rockfill materials are the excavated materials (slightly/weakly weathered granite/schist) from borrow areas. The volume ratio of schist is no more than 30% and grains, whose size is less than 5 mm, is no more than 8%. It should be mentioned that *in-situ* compaction test may result in some excavated...
materials breaking down into fine grains after compaction. To meet the filter criteria and make full use of the excavated materials, the pervious strong zone filled with slightly weathered granite and schist is set at the upstream of the main rockfill zone. In the area, the volume ratio of schist is no more than 30%. Near the original ground line, the volume ratio of the grains, whose size is less than 500 mm, is no more than 20%. Almost 2/3 of the fill materials in 3B I zone are excavated materials (Figure 1).

From the description above, the rockfill materials are classified into 4 types in this study and the properties of material (after compaction) are shown in Table 1. As we cannot obtain the detailed data of materials properties in dam, only two cases are considered in the present work according to the layout of materials in Figure 1. In the first case, the volume ratio of stiffer materials (No. 3 and 4) is over 80% of the total fill materials. The amounts of Material 3 and Material 4 are approximately equal. In the second case, the amount of Material 4 is over 50% of total amount of fill materials.

**Simulation model of dam**

To simulate the layout of materials in dam, a finite element model is created. In the model, following data are used: The height of the dam is 120 m. The upstream slope is 1:1.4 and the downstream slope is 1:1.79. The width of dam top is 10 m. The depth of foundation is 120 m. The length of foundation at the upstream is 168 m and at the downstream is 216 m. The upstream water level is 120 m.

The material properties of face slab (subjected to gradient water pressure as shown in Figure 7) are the same as the concrete material listed in Table 1. The foundation material is the bedrock. In the remained area, that is, the rockfill zones, the materials No.1 ~ No. 4 listed in Table 1, are used for simulation. The volume fraction of a material is the volume of this material to the total volume of the structure (Figure 7). In simulation, the materials in face slab and in cushion zone are fixed. Cushion zone is filled with Material 1.

The structure shown in Figure 7 is discretized with 11770 plane strain elements with prestress by gravity, in which 9584 elements belong to dam body.

**Numerical results**

Figure 8 shows the material distributions in dam for the two cases in Table 1. Figure 8a gives the layout of materials in dam for Case 1 (Table 1). It demonstrates that the stiffest material (No. 4) is almost laid out at the
upstream of rockfill zone and the other weak materials are laid out both at the end and the top of the downstream to reduce the deformation of face slab. Similar conclusion can also be obtained from Figure 8b. The obvious differences in the two cases can be observed for the layouts of Materials 3 and 4. Clearly, the amount of Material 4 in Case 2 (Table 1) is greater than that in Case 1. Therefore, the “red triangular” in Figure 8b is greater than that in Figure 8a. On the other hand, the amount of Material 3 in Case 2 is less than that in Case 1, which leads to the continuity of the layout of Material 2. For the two cases, the layouts of Material 1 are nearly the same. The reason is that the difference among the moduli of four materials is very small.

From Figure 1, the shape of 3B zone (including 3B I and 3B II subzones) is nearly close to that of the stiffer materials in Figure 8b. However, the difference between the two figures is obvious. For example, Figure 8 implies that the upper part of dam should be laid out with softer materials, while in a real dam the stiffer materials are filled in the area.

Figure 9 shows the deformation of structure (dam and foundation) for two cases. It follows from the figure that the maximum deformation occurs at middle upstream. The values of maximum displacement for the two cases are 0.165 and 0.164 m on the face slab near the water level of 57.93 m (Figure 10). For both cases, the maximum normal deflections of face slab are the same, that is, 0.158 m at the site of 57.34 m. Two factors result in the maximum displacement appears within the water level of 57 m and of 58 m. They are the local high water pressure on the lower part of face slab and the high rigid displacement of upper part, respectively. In practical engineering, the maximum deflection of face slab is around 0.275 m (Wang and Wu, 2004).

Conclusions

Serious seepage or piping, usually caused by the cracking of face slabs significantly influences the safety of CFRDs. Zoning optimization of materials in dams is an approach to reduce the possibility of the cracking of face slab. In the present work, a topology optimization method is presented to obtain an optimal layout of materials (rockfill) in a CFRD. Two numerical examples are presented to assess the performance of the proposed method. The first example serves to demonstrate that stiffer materials should be laid out on the areas with higher SED.

The second example illustrates that the stiffest material should be laid out at the lower part of the upstream of a dam. The material with lowest modulus should be laid out at the two ends of downstream part. If the materials in a
Figure 8. Materials layout in dam after optimization.

Figure 9. Displacement fields in structure for two cases.
CFRD are classified more precisely, a better layout scheme for materials in the dam can be obtained. Results show that the maximum deformation (0.165 m) of face slab is less than that reported (0.275 m) by Wang and Wu (2004). The difference between the two results is mainly associated with different materials layout schemes used in modeling dam. It also implies that the structure from the present design is stiffer than the original structure. It is proved that the results from the present method can give a reliable initial rockfill zoning.

In the future work, the different constitutive laws of materials in dam are to be discussed to find their effects on the final material layout. Here we can give a prediction, that is, the influence of material constitutive on the optimal material distribution is very small.

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