Predicting sediment concentration upstream of sandbags in gullies: Modeling and analysis

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The study was aimed at optimization of the use of sandbags in controlling gully erosion through the application of principles of hydrodynamics. Field studies were carried out in an artificial channel excavated on the catchment of Otamiri River at Federal University of Technology, Owerri. Artificial runoff was simulated into the channel by pumping water from Otamiri River. Sandbags of various patterns and heights were placed across the channel to reduce the erosive effect of runoff and encourage siltation. Flow characteristics including sediment concentration and sediment accumulation rates upstream were recorded. A mathematical model relating the flow parameters with sediment accumulation rate and concentration upstream was developed based on the principle of mass balance. Model verification indicated a high level of correlation between measured and predicted values of the variables. The coefficient of correlation ($R^2$) was computed as 0.964, while the standard error of estimates ($S_e$) was 0.00384. The mathematical relationships obtained in the study will be useful in the design and sustainable use of sandbags for erosion control.

Key words: Erosion, gully, sandbags, sediments, optimization and model.

INTRODUCTION

Erosion is a three phase process consisting of detachment of individual soil particles from the soil mass and their transportation by erosion agents (example, wind and water) with subsequent deposition of the related sediments into land depressions, as influenced by natural (geologic soil erosion) or human (accelerated soil erosion) activities (Hudson, 1981). A gully is a relatively deep, vertical walled channel, recently formed within a valley where no well-defined channel previously existed (Bettis, 1983). Gully erosion is a major ecological problem in various parts of the World. The incidence has become a major natural disaster in Nigeria especially in the South East, due to high rainfall intensity, nature of soil, topography and high population density (Osuagwu and Agunwamba, 2011). The numerous gully sites are some of the hazard features that characterize this zone as well as other zones that adjoin them (Ofomata, 1985). Asiabaka and Boers (1988) had estimated that over 1970 gully sites exist in Imo and Abia states of South Eastern Nigeria. This estimated figure has not abated for the last twenty-five years but rather is on the increase in the affected areas.

Conventional concrete works used in most erosion control works have failed to yield optimum results due to poor design and construction coupled with inadequate maintenance. Besides, the use of heavy concrete structures for erosion control is very expensive. There is therefore the need to adopt alternative methods that are cost effective. Among these methods include the use of sand bags, which are basically units of bags filled with sand and placed across flow directions in natural channels to reduce runoff velocity and encourage siltation upstream. Functioning as Checkdams, they trap soil particles, silt and other sediments and after a period the gully is expected to fill up to the new level set by the dam height (CASQA, 2003). Sandbags are usually made from various materials but the most common is woven propylene. Their measurements are about 14” (35.35 cm) wide and 24 to 26” (60.96 to 66.04 cm) long (Hellevang, 2011).

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Sandbags are perfect for flood control, erosion control, reinforcing of existing dyke structures, levee construction and extra weight for traffic/road signs (USACE, 2011).

Sandbags are randomly adopted by rural inhabitants in Nigeria to reduce the erosive effect of runoff. There is however absence of standards and specifications for its optimal application. It is therefore necessary to develop analytical concepts based on principles of hydrodynamics that would optimize the use of these materials. The objective of the study was therefore to develop a mathematical model that relates flow parameters with sediment accumulation rate and concentration of sediments upstream of sandbag location in a gully.

MATERIALS AND METHODS

Data for analysis were collected through field and laboratory investigations. Specifically, the following data were collected: depth of flow (m), rainfall amounts (mm) and duration (hrs), flow rate of runoff (m³/s), velocity of flow (m/s), rate of accumulation of sediments (m³/s) and silt concentration. An experimental channel was constructed on the slope of Otamiri River at Federal University of Technology, Owerri. The channel has a cross section of 1.2 × 0.6 m with a length of 15 m. Artificial runoff simulated by pumping water from Otamiri River was directed into the channel (Figure 1).

Polypropylene bags of 0.9 m length and 0.45 m width were filled with sand to an average thickness of 0.25 m and placed across the gullies (Figures 2a and 2b). The heights and patterns of placement (transverse and longitudinal) were varied for different flow conditions. The rate of accumulation of sediments in (m³/s) was determined by measurement of the average heights of deposits left behind the sandbags over given time intervals. The depths were multiplied with the area of deposits to get the volume of silts accumulated.

RESULTS

Development of mathematical formulations

Mathematical relationship among the following parameters: flow rate (Q), sediment concentration (C), rate of accumulation of sediments or storage (S) and volume of sediments deposited upstream of sandbags (V).

The principle of material balance (Figure 3) was adopted in the formulations. Differential equations that relate the above parameters were derived with following basic assumptions:

(i) That one dimensional flow occurs in the channel.
(ii) That the Principle of Material balance is applicable.
(iii) That the rate of sediment accumulation is constant.

Therefore, initial conditions are stated thus:

At t=0, C=C₀. For all values of x>0

\[ C₁(0) = C₀ \]

C₀, indicates concentration of the runoff before being discharged into the gully. Thereafter, C increases with time due to erosion of the bed by the runoff.

Based on the Principle of Material balance, we state the following equation:

\[ \text{Rate of accumulation of sediments} = \text{inflow} - \text{outflow} \]

From which following differential equations are derived:

\[ \frac{\partial(QC)}{\partial t} = Q(C₀ - C) - k₀ u \frac{\partial C}{\partial x} \]

\[ V \frac{\partial C}{\partial t} = Q(C₀ - C) - k₀ u \frac{\partial C}{\partial x} \]

\[ \frac{VdC₁}{dt} = Q(C₀ - C₁) - SC₁ \]

where:

\[ Q = \text{flow rate, } Q \ (m³/s) \]
\[ u = \text{velocity of flow, } (m/s) \]
\[ C = \text{Sediment Concentration of runoff.} \]
\[ C₀ = \text{Initial Sediment Concentration} \]
\[ C₁ = \text{Concentration at time, } t \ (s) \]
\[ S = \text{Rate of Accumulation of sediments (storage), } (m³/s) \]
\[ V = \text{Volume of sediments deposited upstream of sandbags, } (m³) \]

\( k₀ \) is a dimensional coefficient in \( L³ \), while, \( \frac{\partial C}{\partial x} \) and \( \frac{\partial C}{\partial t} \) refer to change in concentration per length and change in concentration with time respectively.

From Equation 4, Using Integrating factor Method:

\[ \frac{dC₁}{dt} + \left( \frac{Q+S}{V} \right) C₁ = \frac{QC₀}{V} \]

Integrating factor (I. F.) = \( e^\left(\frac{(Q+S)/V}{V}\right)dt \)

I. F. = \( e^\left(\frac{(Q+S)/V}{V}\right)t \)

\[ C₁ - \frac{QC₀}{Q+S} = ke^{-\left(\frac{Q+S}{V}\right)t} \]

\[ \ln \left[ C₁ - \frac{QC₀}{Q+S} \right] = \ln k - \left(\frac{Q+S}{V}\right)t \]
Let,

\[ (C_1 - \frac{QC_0}{Q+S}) C_1 = A \quad (10) \]

Therefore, equation 9 becomes

\[ \ln A = \ln k - \left(\frac{Q+S}{V}\right)t \quad (11) \]

It can be seen that equation 11 is linear. Thus, a plot of \( \ln A \) against \( t \) would give a straight line with \( \ln k \) as the intercept and \( (Q+S)/V \) as the slope.

Values of \( \ln (A) \) for different time intervals were computed from three sets of \( Q, C_0, \) and \( S, C_1 \) values measured at different time intervals. The plots of \( \ln (A) \) against time \( (t) \) are presented in Figures 4 to 6 with the resultant fitting line for each set of data labeled ‘1’ ‘2’ and ‘3’ accordingly. In each case, the \( k \) value for the line that has highest correlation coefficient \( (R^2) \) was chosen as optimum. Results from the graphs indicate that values of \( k \) vary from 0.048 to 0.144. The relationship between \( k \) and \( C_0 \) is presented graphically in Figure 7. The fitting equation is therefore:

\[ k = 7.38C_0^{1.608} \quad (12) \]

Thus, \( k \) could be estimated easily from \( C_0 \).

Substituting for \( k \) in equation 12, we obtain equation 13,

\[ \ln \left[C_1 - \frac{QC_0}{Q+S}\right] = \ln(7.38C_0^{1.608}) - \left(\frac{Q+S}{V}\right)t \quad (13) \]

**Model calibration**

The model was checked and adjusted using another set of experimental data (Table 1).

From Equation 13,

\[ C_1 = \frac{0.12 \times 0.08}{0.12 + 0.00012} + (7.38 \times 0.081608)e^{\left(\frac{(0.12+0.00012)}{0.144}\right)1000} = 0.21 = 0.127 + 0.1036e^{-105} \]

In order to have a feasible solution, a calibration factor \( \alpha \), was introduced;

\[ \alpha = 1.53 \times 10^{-3} \]

Equation 13 is thus adjusted as

\[ \ln \left[C_1 - \frac{QC_0}{Q+S}\right] = \ln(7.38C_0^{1.608}) - \left(\frac{Q+S}{V}\right)t \quad (14) \]

\[ C_1 = \frac{QC_0}{Q+S} + (7.38 \times C_0^{1.608})e^{-1.53\times10^{-3}\left((Q+S)/V\right)t} \quad (15) \]

**Model verification**

The set of data presented in Table 2 was used for model verification.
verification. Column ‘6’ of the Table shows measured values of $C_1$ at given time intervals for tests A to D.

Alternatively, equation 11 was used to predict values of $C_1$ using data from Table 2. A plot of measured values against predicted values is presented in Figure 8. The coefficient of correlation $R^2$ is 0.964 and standard error $= 3.841 \times 10^{-3}$. These values indicate closeness of the predicted values with the observed values; thus confirming the validity of the model developed.

**DISCUSSION**

Model verification indicated a high level of correlation between measured and predicted values of the variables. The coefficient of correlation ($R^2$) was computed as 0.964 while the standard error of estimates ($S_e$) was 0.00384. Since a different set of experimental data was used for the verification, the model can reliably be applied in the design and sustainable use of sandbags for erosion.
Figure 6. Plot of ln (A) against Time (t), (Case 3)

Figure 7. Variation of k with $C_o$.

Table 1. Experimental data for model calibration.

<table>
<thead>
<tr>
<th>Tests</th>
<th>$Q$ (m$^3$/s)</th>
<th>$C_0$</th>
<th>$S$ (m$^3$/s)</th>
<th>$t$ (s)</th>
<th>$C_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.12</td>
<td>0.08</td>
<td>0.00012</td>
<td>1200</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 2. Experimental data for model verification.

<table>
<thead>
<tr>
<th>Tests</th>
<th>$Q$ (m$^3$/s)</th>
<th>$C_0$</th>
<th>$S$ (m$^3$/s)</th>
<th>$t$ (min)</th>
<th>$C_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.12</td>
<td>0.024</td>
<td>0.00022</td>
<td>50</td>
<td>0.032</td>
</tr>
<tr>
<td>B</td>
<td>0.12</td>
<td>0.042</td>
<td>0.00028</td>
<td>60</td>
<td>0.065</td>
</tr>
<tr>
<td>C</td>
<td>0.12</td>
<td>0.033</td>
<td>0.00026</td>
<td>45</td>
<td>0.047</td>
</tr>
<tr>
<td>D</td>
<td>0.12</td>
<td>0.02</td>
<td>0.00019</td>
<td>40</td>
<td>0.025</td>
</tr>
</tbody>
</table>

control. The variation of sediment concentration upstream of sandbags with time could be predicted based on the following parameters; flow rate, sediment accumulation rate, and initial sediment concentration. The results obtained can be applicable to other types of erosion control check dams.

Conclusion

From the results of the studies, the following conclusions could be made:

(i) The use of sandbags could be optimized by application
and pattern of placement.

(ii) The mathematical model formulated from the principle of material balance could be used to predict the concentration of sediments in the runoff upstream of sandbags in a gully at any time interval.

Finally, we recommend further studies extending application of sandbags to deeper gullies.

REFERENCES


