

Full length Research Paper

Evaluation of the risk of drug addiction with the help of fuzzy sets

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The primary focus of this paper is to present a general view of the current applications of fuzzy logic in medical analogy of consumption of drugs. The paper also deals with the origin, structure and composition of fuzzy sets. The authors particularly review the medical literature using fuzzy logic. Fuzzy set theory can be considered as a suitable formalism to deal with the imprecision intrinsic to many real world problems. Fuzzy set theory provides an appropriate framework for the representation of vague medical concepts and imprecise modes of reasoning. The authors present two concrete illustrations to investigate the impact of the risk related to drug addictions, like smoking and alcohol drinking and thereby highlighting the social problem related to health.

Key words: Estimation, classical set, validation, diagnostic test.

INTRODUCTION

Logic studies the notions of consequence; it deals with propositions, set of propositions and the relation of consequence among them. Formal logic represents this by means of well-defined logical calculi. Logical calculus has two notions of consequence; they are syntactical, based on a notion of proof and semantically which are based on notions of truth. Fuzzy propositions use linguistic variables such as age; with values- young, very old and old. The truth of a fuzzy proposition is a matter of degree. Aristotle in his, laws of thought and law of the excluded middle described every proposition must either be True or False. Plato laid the foundation, for what would become fuzzy logic, indicating that there was a third region beyond True and False. Lukasiewicz, who first proposed a systematic alternative to the bi-valued logic of Aristotle described the 3-valued logic; where the third value is possible. He showed that, it is possible to derive infinite-valued logic. In 1965, Lotfi Zadeh published his seminal work on "Fuzzy Sets" describing the mathematics of fuzzy set theory.

In 1973, Lotfi Zadeh proposed his theory of fuzzy logic where the membership function operates over the range of real numbers $[0, 1]$. New operations for the calculus of logic were proposed, and shown to be in principle at least a generalization of classic logic. Lotfi Zadeh is regarded as the father of fuzzy theory. He was born in Iran and graduated from the University of Tehran. In the United States, he received his doctoral degree at Columbia University. He worked at Princeton University and then became a professor at the University of California in 1959. He was brilliant researcher in control theory and system theory before he proposed a theory whose objects, that is, fuzzy sets are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of a degree. The word fuzzy means indistinct, imprecise, obscure, blurred, vague, ambiguous etc. Hence, one may be tempted to interpret fuzzy set as a vague set or ambiguous set but this would be a wrong interpretation. In fact, 'fuzzy set' is a well defined concept in mathematics. The driving force behind this change is the realization that classical set theory is inadequate for dealing with imprecision, uncertainty and the complexity of the real world. This motivates the evolution of fuzzy set theory. The example

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theory. The example of age with different values; young, very old, old, containing adjective, verb and adverb may not form sets in the usual mathematical sense, the fact remains that such imprecisely defined classes play an important role in human thinking, particularly in the domain of pattern recognition, digital communication and Information technology. Zadeh describes fuzzy logic as formalization of approximate reasoning.

Although, "fuzzy logic" may seem to imply imprecision, it's based on a reliable and rigorous discipline. Fuzzy logic lets us accurately describe control systems in words instead of complicated math. Fuzzy logic, based on fuzzy set theory, allows us to express the operational and control laws of a system linguistically in words. Although, such an approach might seem inadequate, it can actually be superior to and much easier than a more mathematical approach. The main strength of fuzzy set theory, a generalization of classical set theory, that excels in dealing with imprecision. In classical set theory, an item is either a part of a set or not. There is no in-between, there are no partial members. Fuzzy set theory recognizes that very few crisp sets actually exists Herrera et al., 2004. There is a paradigm shift from crisp set to fuzzy set. The paradigm shift is necessitated by "the need to bridge the gap between mathematical models (biology, medicine, social science) and experience". This paradigm allows expressing observation and measurement uncertainties, managing complexity and capturing human common-sense reasoning and decision making.

NON FUZZY (CLASSICAL) SETS TO FUZZY SETS

One of the important tools in modern mathematics is the theory of sets. Every branch of mathematics can be considered as a study of sets of objects of one kind or another. For instance geometry is the study of set of points. Algebra is concerned with the set of numbers and operations on those sets Singh, 1986. Analysis mainly deals with set of function. The study of sets and their use in the foundation of mathematics began in the latter part of the nineteenth century by German mathematician George Cantor (1845 - 1918). According to Cantor, "A set is a collection into a whole of definite and distinct objects of the intuition or thought and the objects are called 'elements' of the set ". About the turn of the twentieth century paradoxes of various kinds namely Russell's paradox (1901); Cantor's paradox (1932); Burali-Forti's Paradox (1897), were discovered which directly or indirectly originated from the notion of the set and which shook the foundations of mathematics in general and set theory in particular. Frege indeed admitted that Russell's paradox undermines the foundations of his life work to construct arithmetic on the basis of the set theory. There are three principal philosophies of mathematics each of which has a sizable group of adherents. These are logistic school, of which Frege, Russell and White - Head are the main expositors, the intuitionist school was led by the Dutch mathematician L.E.J. Brouwer and Heyting and the formalist school developed principally by David Hilbert. These schools of mathematics were greatly influenced by the appearance of paradoxes. The mathematicians of these schools approached the problem posed by the paradoxes according to the philosophy of mathematics they held. The basic assumption that has been made in any axiomatic set theory as well as in Cantor's intuitive set theory is that given to any set A and any object x of the universe of

discourse X, it can be decided whether $x \in A$ holds or not. Hence, corresponding to any subset A of X, they can construct a unique real valued function $f_A(x)$, called characteristic function defined over the universe of discourse X such that:

$f_A : X \rightarrow [0, 1]$ such that

$f_A(x) = 1$ if $x \in A$
A is true

$= 0$ if $x \in A$ if false

However, in real physical world, the above assumption ($x \in A$ or $x \notin A$) may not be true. In order to exemplify, let us consider the "Class of new cars", "Class of short men", "Class of new, high buildings", "Class of all real numbers which are much greater than 1", "Class of expensive bike", "Class of highly contagious diseases", "Class of beautiful flowers", "Class of the boy who resembles his father", the class of real number "approximately equal to 3", "Class of sunny days". We observe that due to the presence of the terms 'new', 'short', 'high', 'much', 'expensive', 'highly', 'beautiful', 'sunny', 'approximately', in the formation of above classes some kind of imprecision or ambiguity or vagueness arises in deciding whether an individual element in the context is an element of the class or not. Indeed, classes of such types do not constitute a set in usual mathematical sense Wilder, 2006. Sets of this type that is very often involve some adjectives, verbs and adverbs or some combination thereof which are not sharply defined in their descriptions. Numerous other examples may be found in every branch of science as well as in writings and daily conversations. In fact, most of the classes of objects encountered in the real physical world are this 'fuzzy' not sharply defined type. They do not have precisely defined criteria of membership. In such classes, an object need not necessarily either belong to or not belong to a class; there may be intermediate grades of membership. In other words, they can say the transition from member to non-member appears gradual rather than abrupt. Thus, the fuzzy set introduces vagueness with the aim of reducing complexity by eliminating the sharp boundary and dividing members of the class from non-members.

A fuzzy set can be defined mathematically by assigning each individual in the universe of discourse value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which individual is "similar" or "compatible" with the concept represented by the fuzzy set Kosko, 1993. Thus, individuals may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. These membership grades are very often represented by real-number values ranging in the closed interval between 0 and 1. This concept of a fuzzy set, which is a "class" with a continuum of grades of membership or they can say that a fuzzy set on a set X is sorted of generalized "Characteristic function" on X, whose degree of membership may be more than "yes" or "no". Thus, a fuzzy set representing the concept of sunny days might assign a degree of membership 1 to a cloud of 0%, 0.8 to a cloud cover of 20%, 0.4 to a cloud cover of 30% and 0 to a cloud cover of 75%. These grades signify the degree to which each percentage of cloud cover approximates the subjective concept of sunny days and the set itself models the semantic flexibility inherent in such a common linguistic term. Because full membership and full non-membership in the fuzzy set can still be indicated by the values of 1 and 0, they can consider the "crisp" set to be restricted in case of the more general fuzzy set for which only these two grades of membership are allowed. The notion of fuzzy sets can be represented; Let X denotes a universal set which is also refereed as a field of reference or universe of discourse. Then, the membership function f_A by which a fuzzy set A is usually defined has the form;

$f_A : X \rightarrow [0, 1]$ where $[0, 1]$

denotes the interval of real numbers from 0 - 1, inclusive. The value of f_A at x , $f_A(x)$ is 1 or 0 according as x belongs or does not belong to A . When A is a fuzzy set, then the nearer the value of $f_A(x)$ to 0, the more tenuous is the membership of x in A , with the "degree of belonging" increasing with increase in $f_A(x)$.

MEDICAL LITERATURE

Sources of uncertainty

The complexity of medical practice makes traditional quantitative approaches of analysis inappropriate. In medicine, the lack of information and its imprecision, and many times, contradictory nature are common facts Nieto, 2004. The sources of uncertainty can be classified as follows.

Information about the patient

Medical history of the patient, which is usually, supplied by the patient and/or his/her family. This is usually highly subjective and imprecise.

Physical examination

The physician usually obtains objective data, but in some cases the boundary between normal and pathological status is not sharp.

RESULTS

Results of laboratory and other diagnostic tests are also subject to some mistakes and even to improper behaviour of the patient prior to the examination.

Symptoms

The patient may include simulated, exaggerated, understated symptoms, or may even fail to mention some of them.

Classification

The authors stress the paradox of the growing number of mental disorders versus the absence of a natural classification. The classification in critical (that is, borderline) cases is difficult, particularly when a categorical system of diagnosis is considered.

Fuzzy logic and medicine

Fuzzy logic plays an important role in medicine. Some examples showing that fuzzy logic crosses many disease groups are the following:

(i) Fuzzy information granulation of medical images. Blood vessel extraction from 3-D MRA images.

(ii) Awareness monitoring and decision-making for general anaesthesia.

Acquisition of fuzzy association rules from medical data.

(iii) Fuzzy logic in a decision support system in the domain of Coronary heart disease risk assessment.

(iv) A model-based temporal abductive diagnosis model for an intensive coronary care unit.

(v) A Fuzzy Model for Pattern Recognition in the Evolution of Patients

(vi) To predict the response to treatment with citalopram in alcohol dependence.

(vii) To analyze diabetic neuropathy and to detect early diabetic retinopathy.

(viii) To calculate volumes of brain tissue from magnetic resonance imaging (MRI), and to analyze functional MRI data.

(ix) To assist the diagnosis of central nervous systems tumors (astrocytic tumors).

(x) To discriminate benign skin lesions from malignant melanomas

(xi) To improve decision-making in radiation therapy and to visualize nerve fibers in the human brain.

(xii) To represent quantitative estimates of drug use.

Many other areas of application, to mention a few, are to study fuzzy epidemics, to make decisions in nursing, to overcome electro acupuncture accommodation. The diagnosis of disease involves several levels of uncertainty and imprecision and it is inherent to medicine. A single disease may manifest itself quite differently, depending on the patient, and with different intensities. A single symptom may correspond to different diseases Papageorgious et al., 2003. On the other hand, several diseases present in a patient may interact and interfere with the usual description of any of the diseases. The best and most precise description of disease entities uses linguistic terms that are also imprecise and vague. Moreover, the classical concepts of health and disease are mutually exclusive and opposite. However, some recent approaches consider both concepts as complementary processes in the same continuum. According to the definition issued by the World Health Organization (WHO), "health is a state of complete physical, mental and social well-being and not merely the absence of disease or infirmity". The loss of health can be seen in its three forms, disease, illness and sickness.

To deal with imprecision and uncertainty, the authors have at the disposal fuzzy logic. Fuzzy logic introduces partial truth values, between true and false. According to Aristotelian logic, for a given proposition or state, they only have two logical values, true-false, black-white, 1 - 0. In real life, things are not either black or white, but most of the times are grey. Thus, in many practical situations, it is convenient to consider intermediate logical values. Let us show this with a very simple medical example. Consider the statement "you are healthy". Is it true if you have only a broken nail? Is it false if you have a terminal

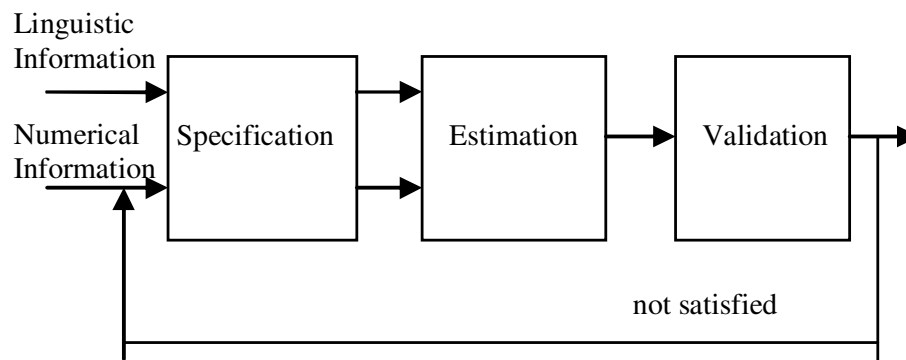


Figure 1. A system identification.

cancer? Everybody is healthy to some degree h and ill to some degree i Szczepaniak et al., 2005. If you are totally healthy, then of course $h = 1$, $i = 0$. Usually, everybody has some minor health problems and $h < 1$, but $h + i = 1$. In the other extreme situation, $h = 0$, and $i = 1$, so that, you are not healthy at all (you are dead). In the case, you have only a broken nail, they may write $h = 0.999$, $i = 0.001$; if you have a painful gastric ulcer, $i = 0.6$, $h = 0.4$, but in the case you have a terminal cancer, probably $i = 0.95$, $h = 0.05$. Uncertainty is now considered essential to science and fuzzy logic is a way to model and deal with it using natural language. The authors can say that, fuzzy logic is a qualitative computational approach. Since uncertainty is inherent in fields such as medicine and fuzzy logic takes into account such uncertainty, fuzzy set theory can be considered as a suitable formalism to deal with the imprecision intrinsic to many biomedical and bioinformatics problems. Fuzzy logic is a method to render precise what is imprecise in the world of medicine.

FUZZY SYSTEM IDENTIFICATION

The concept of a mathematical model is fundamental to system analysis and design which require the representation of systems phenomenon as a functional dependence between interacting input and output variables. Mathematical models are essential for prediction and control purposes Rosen (1982). Conventionally, a mathematical model for a system is constructed by analysing input-output measurements from the system. These numerical measurements are important because they represent the behaviour of the system in a quantitative fashion. Very often, there exists another important information source for many engineering systems, knowledge from human experts. This knowledge known as linguistic information, provides qualitative instructions and descriptions about the system and is especially useful when the input and output measurements are difficult to obtain. Fuzzy models are capable of doing this kind of information naturally and conveniently, while

conventional mathematical models usually fail to do so. Moreover, it is interesting to note that fuzzy models have the same ability to process numerical information as conventional models, if such information is available. Being able to deal with linguistic information and numerical information is one of the important properties of fuzzy models. The second important property of fuzzy models is their ability to handle non linearity. It is well known fact that most engineering systems are non linear to some extent. Interpretability is another salient feature of fuzzy models. A fuzzy model has a transparent model structure. Each rule in the model acts like a “local model” in the sense that it only covers a local region of the input-output space, and its contribution to the whole output of the model is easily understood. Fuzzy system identification consists of three basic sub problems, specification, estimation and validation. This is depicted in the Figure 1.

Specification involves finding the important input variables from all possible input variables, specifying membership functions, partitioning the input space, and determining the number of fuzzy rules comprising the underlying model. Parameter estimation involves the determination of unknown parameters in the model using some optimization method based on both linguistic information obtained from human experts and numerical data obtained from the actual system to be modelled. Specification and estimation are interwoven, and neither of them can be independently identified without resort to other. Validation involves testing the model based on some performance criterion. If the model cannot pass the test, we must modify the model, the model structure and re-estimate the model parameter. It may be necessary to repeat this process many times before a satisfactory model is found. The specification of a fuzzy model involves selection of input variables. The different techniques for selecting input variables are forward selection, backward selection, and best subset procedure. The choice of membership functions affect how well a fuzzy model behaves. Empirical evaluation of different membership functions can be useful in guiding the choice of membership functions. Fuzzy models are constructed

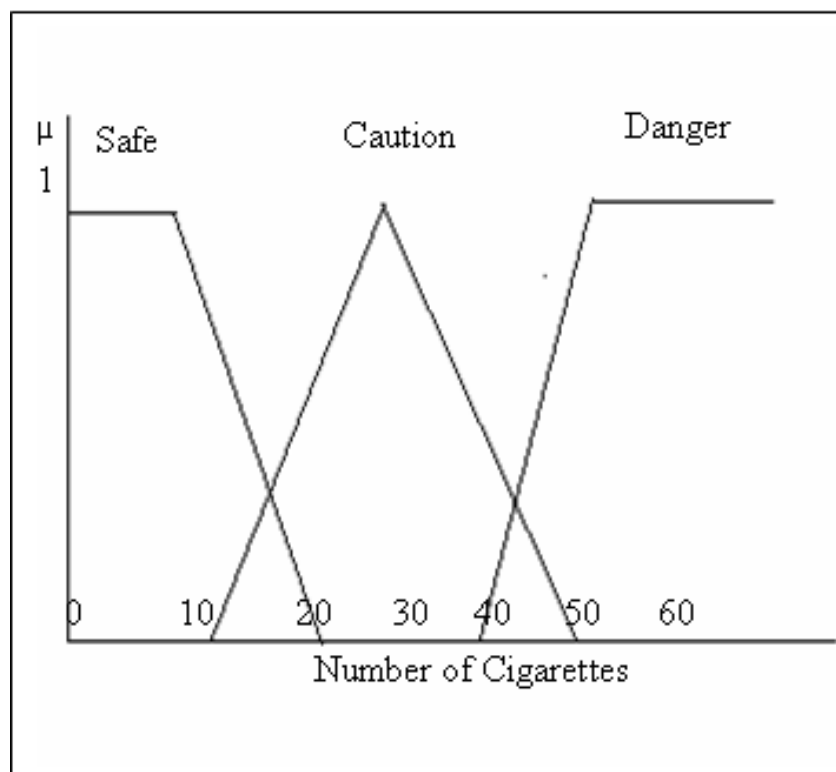


Figure 2. Membership functions of risk of smoking.

using triangular, bell shaped and Gaussian membership function. Triangular membership function is inferior to bell shaped and Gaussian membership function. These membership functions are compared with other membership functions especially, with the sin function $\sin(x)/x$, based on how closely the resultant fuzzy models approximate the real systems. More extensive empirical investigation is needed in this area. Parameter estimation problems, in general, involve the optimization of antecedent membership parameters and the consequent parameters. In evaluating fuzzy models cross validation, residual analysis, and information-theoretic criteria are employed.

SUMMARY OF FINDINGS

Estimation of the risk of smoking

We know that fuzzy control work on fuzzy inference. The idea of fuzzy inference is applied to fuzzy expert system. It is now being used not only in engineering but in several fields of social sciences. In this light we will show the example based on consistencies of fuzzy sets. Let us consider the following situation of smoking which is one of the vital causes behind heart attack, cancer etc. among human beings:

(i) A doctor thinks that smoking less than 10 cigarettes a

day is not harmful for health (safe),

(ii) He presumes that smoking more than 50 smokes a day is definitely (absolutely) harmful (dangerous).

The doctor believes that smoking 20 - 30 smokes per day are potentially harmful (suspicious). The doctor understanding about smoking can be illustrated with the help of fuzzy sets as shown in the Figure 2. On the horizontal axis they is, on the x-axis, we express the number of smokes a day, and the membership function that expresses the risk of smoking has been indicated on the vertical axis i.e. on y axis between the real number 0 and 1. Let us suppose that Ajay's satisfaction of smoking is given by the fuzzy set A. That is he is satisfied if he has about 10 - 20 smokes a day. Also, they have Binod's fuzzy set B, which means Binod's requires about 40 - 50 smokes a day. The doctor's diagnosis (understanding) about smoking is given by the points of agreement of each of the fuzzy sets, safe, caution and danger. We know that the points of agreements are given by the highest value at the intersection of two fuzzy sets. The intersection of two fuzzy sets A and B is defined as,

$$(A \cap B)(X) = \min \{ A(X), B(X) \},$$

for all $x \in X$, where X is the universal set, $A(X)$ and $B(X)$ are the membership functions. They may infer that Ajay has the safety degree of 0.9, the degree being Caution of

0.9, and the danger degree of 0.0. Also, the authors have Arun Fuzzy set we infer that Arun's danger degree is 0.85 and his safety degree is almost zero. Here we note that the membership function is used to give the diagnosis that can be provided by the statistics or by the doctor's subjectivity. This example shows a way to give a subjective diagnosis, which might be too simple for real use. If the patient is required to be classified, we can take the label whose value is the highest. Here, they also observe that in this example, the horizontal axis gives continuous numbers.

Estimation of the risk of alcohol drinking

Alcohol drinking and Cigarette smoking during adolescence have been shown to be associated with a greater possibility of concurrent and future substance-related disorders. In order to report patterns of drug use and to describe factors associated with substance use in adolescents, a cross-sectional survey was carried out in a representative population sample of 3000 adolescents, aged 12 - 17 years, from Bangalore, silicon city of India to Bombay, a metro and commercial capital of India. The original survey covered the use of alcohol, tobacco, illicit drugs, and other psychoactive substances. For tobacco smoking and alcohol drinking, each subject of the population sample can be assigned a fuzzy degree of addiction (or risk use). With respect to the other fuzzy variable, if you drink no alcohol, the degree of this variable is 0. If you drink more than 75 cc of alcohol per day, the degree of alcoholism is 1. For 25 cc/d, the degree could be 0.4 and for 50 cc/d, 0.8. Suppose you correspond to the fuzzy set $\lambda = (1, 1)$, have recently had some health problems, and your physician has advised you to reduce your consumption of cigarettes and alcohol by half. The ideal situation for your health is, of course, the point $\mu = (0, 0)$, but it is possibly difficult to achieve. Since uncertainty is inherent in fields such as medicine, fuzzy logic takes into account such uncertainty to render precise, overcome imprecise in the world of medicine.

Conclusions

In this interesting paper the author presents a summary of the basic concepts and techniques underlying the application of fuzzy set theory to solve practical problems related to health. The study finds a number of examples relating to its use as a computational system for dealing with uncertainty and imprecision in the context of evaluation of risk associated with smoking and drinking habits. Health is a vital indicator of human development that will enable every individual to lead a social and economically productive life. Through the process of fuzzification, the authors gain greater generality and enhanced ability to model real world problems and higher expressive power to some extent for respective areas of non fuzzy mathematics.

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