Journal of Biophysics and Structural Biology Vol. 2(3), pp. 28-34, April 2010
Available online at http://www.academicjournals.org/jbsb
ISSN 2141 - 2200 © 2010 Academic Journals

Full Length Research Paper

BPES analyses of a new diffusion-advection equation for fluid flow in blood vessels under different bio-physico-geometrical conditions

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Accepted 25 January, 2010

In human physiological and pathological flow systems, it is not possible to rule out diffusion in all advective processes because perfusion goes hand in hand with diffusion processes. It is the perfusion throughout the capillary bed and then the diffusion of fluids throughout the tissue that is the subject of most magnetic resonance functional imaging procedures. It is observed from literature that basic theory of perfusion is mostly based on experimental observation which makes it entirely computational with quite a lot of data fitting. Therefore, it is quite rigorous and has many phenomena that seem not to have a common background. It is very important to attempt developing a theory that would take most issues (if not all) into consideration under a common phenomenon. In this study, based on the Bloch NMR flow equations along with the Boubaker polynomials expansion scheme (*BPES)*, we describe analytically the dynamics of perfusion processes by an equation which combines both diffusive and advective properties.

Key words: Bloch NMR flow equations, diffusion-advection equation, blood vessels, BPES scheme.

INTRODUCTION

Distribution of oxygen to every corner of the body is accomplished by the cardiovascular system, with the help of the most important fluid in the body: the Blood, the stream of life. Life depends so much on blood such that its importance cannot be over emphasized. It has been investigated that any obstacle to the normal flow of blood causes a malfunctioning in the body system that leads to cardiovascular related diseases.

Functional magnetic resonance imaging (Martinez et al., 2002; Valfouskaya and Adler, 2005; Segnorile et al., 2006) consists of several different imaging methods that are used to visualize and, in some cases, quantify blood and fluid movement beyond the general vascular system. It is the perfusion through out the capillary bed and the diffusion of fluid throughout the tissue that is the subject of most magnetic resonance functional procedures (Sprawls, 2000).

Most perfusion processes within the human body are always changing from time to time with regards to a lot of body conditions. These processes take place in tube - like vessels (for example, the blood vessel) which are always under some sort of pressure and since they are elastic in nature, we need to characterize the flow velocity and the diffusion coefficient from point to point. If we take for example, the case of a sudden rush of blood to a part of the body tissue, the blood vessel carrying blood to the part of the tissue would suddenly become larger because of increased pressure and at that point, the flow velocity and diffusion coefficient changes. If the cause of the sudden demand for more blood is removed, the vessel goes back to its normal shape. Therefore, it would be very crucial to account for the velocity and diffusion (Awojoyogbe, 2004) coefficient at all points for an accurate description of the process under investigation.

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Hence, based on the Bloch NMR flow equations (Zoppou and Knight, 1997; Awojoyogbe et al., 2010; Awojoyogbe, 2007; Awojoyogbe, 2003; Awojoyogbe, 2002; Awojoyogbe, 2008), we must use the diffusion-advection equation with spatially varying diffusion coefficients as proposed in this study.

MATHEMATICAL METHOD

In this study, a mathematical (analytical) technique in the form of a plane wave is applied to transform the time dependent Bloch NMR flow equation to diffusion-advection equation for the qualitative analysis of nuclear magnetization. We consider the perfusion (or transport) of any specific blood component as one dimensional since blood flow within the vessels is directional and, even in bifurcations, flow has a resultant direction of fluid flow. Therefore, for any NMR sensitive substance of interest, the perfusion process is given by the NMR advection - diffusion equation derived from the Bloch NMR flow equations (Awojoyogbe, 2004)

$$v^2\frac{\partial^2 M_y}{\partial x^2}+2v\frac{\partial^2 M_y}{\partial x\partial t}+vT_o\frac{\partial M_y}{\partial x}+T_o\frac{\partial M_y}{\partial t}+\frac{\partial^2 M_y}{\partial t^2}+\Omega M_y=F_o\gamma B_1(x,t)$$
(1a)

Where;

$\Omega=T_g+\gamma^2 B_1{}^2(x,t)$; $F_o=\frac{M_o}{T_1}$; $T_g=\frac{1}{T_1T_2}$ and $T_0=\frac{1}{T_1}+\frac{1}{T_2}$,

γ is the gyromagnetic ratio, *D* is the diffusion coefficient, v is the fluid velocity, T_1 is the spin lattice relaxation time, T_2 is the spin relaxation time, M_o is the equilibrium magnetization, $B_1(x,t)$ is the applied magnetic field and M_y is the transverse magnetization.

Solutions to Equation (1a) have been discussed by a number of analytical methods (Awojoyogbe, 2008; Oyodum et al., 2009), and for the present purpose it is sufficient to design the NMR system in such a way that the transverse magnetization M_y, takes the form of a plane wave,

$$M_y(x,t)=Ae^{mx+nt} \tag{1b}$$

Subject to the following theoretical conditions:

$$\frac{1}{T_1T_2}>>\gamma^2 B_1{}^2(x,t) \tag{2}$$

$$n=-2vm\pm\frac{\sqrt{4(v^2m^2-T_g)}}{2} \tag{3}$$

Where m and n are dependent on the NMR flow parameters and B_1 is independent of x and t. based on equations (1b, 2 and 3), we can write equation (1a) in the form of diffusion-advection equation for the nuclear magnetization.

$$\frac{\partial}{\partial x}\left(v(x)M_y\right)+\frac{\partial M_y}{\partial t}=\frac{\partial}{\partial x}\left(D(x)\frac{\partial M_y}{\partial x}\right)+\frac{F_o}{T_o}\gamma B_1(x,t) \tag{4}$$

Where D(x) is the variable diffusion coefficient. Equation (4) is a generalize equation of motion for the NMR flow system with a spatially variable velocity and diffusion coefficient. The behavior of the transverse magnetization or signal is depicted by the solution to equation (4). If we make the following assumption:

$$x=\exp(u_0X) \tag{5}$$

$$\left.\begin{aligned}D(x)&=D_0u_0^2x^2\\ v(x)&=u_0x\end{aligned}\right\} \tag{6}$$

Equation (4) becomes

$$\frac{\partial M_y}{\partial t}=D_0\frac{\partial^2 M_y}{\partial X^2}-(1-2D_0u_0)\frac{\partial M_y}{\partial X} \tag{7}$$

Where

$$D_o=\frac{v^2}{T_o} \tag{8a}$$

$$\frac{F_o}{T_o}\gamma B_1(x,t)=u_0M_y \tag{8b}$$

Analytical solution to equation (7) is similar to those of the diffusion equation of variable diffusion coefficient (Zoppou and Knight, 1997). Hence, the solution could be written as

$$M_y(X,t;X_0)=\frac{A_{BPES}}{2\sqrt{D_0u_0^2\pi t}}\exp\left(\frac{-\left[X-X_0-(1-2D_0u_0)t\right]^2}{4D_0t}\right) \tag{9}$$

The value of the constant A_{BPES} is determined using the Boubaker Polynomials Expansion Scheme (*BPES)* (Awojoyogbe, 2008; Zhao et al., 2008). The calculation protocol takes into account conjointly the properties of the *BPES* along with the already noticed (Awojoyogbe, 2008; Oyodum et al., 2009) similarity between equation (7) and the characteristic differential equation of the Boubaker polynomials.

For a component of the blood in the unit of magnetic moment being transported across the blood vessel, the value of the constant A_{BPES} is

$$A_{BPES}=\frac{1}{x_0\exp\left[(u_0t-D_0u_0^2t)\right]}=\frac{\exp\left[(D_0u_0^2t-u_0t)\right]}{x_0} \tag{10}$$

Subject to the following constraint:

$$\int_0^\infty M_y(x,t)\,dx=1 \qquad \forall t \tag{11}$$

The NMR transverse magnetization for the instantaneous release can therefore be written as:

$$M_y(x,t)=\frac{1}{2\sqrt{D_0u_0^2\pi t}}\left(\frac{1}{x}\right)\left(\frac{x}{x_0}\right)^{1/2D_0u_0}\exp\left(-\frac{\left[P_{02}^2+t^2\right]}{4D_0t}\right) \tag{12}$$

This one-dimensional solution is valid quite well for the upstream and downstream of the bifurcation. For a continuous source of unit magnetic moment (Zoppou and Knight, 1997), the behavior of the NMR signal is obtained by integrating equation (12) with respect to time.

$$M_y(x,t)=\left(\frac{1}{x}\right)\left(\frac{x}{x_0}\right)^{1/2D_0u_0}\int_{-\infty}^{\infty}\frac{1}{2\sqrt{D_0u_0^2\pi t}}\exp\left(-\frac{\left[P_{02}^2+t^2\right]}{4D_0t}\right)dt \tag{13}$$

Where

$P_{02}^2=\left(\frac{1}{u_0}\ln\left(\frac{x}{x_0}\right)\right)^2$ is the one directional perfusion function and u_o is a constant. The expression in equations (13) gives the behavior of the NMR signal $M_y(x,t)$ at all points for a material or substance which is being transported (in perfusion). However, perfusing particles behave differently in different geometries. This requires that we applied some additional experimental conditions to appropriately describe equation (13) in different geometries.

THE MULTIDIMENSIONAL PERFUSION PROCESS – CYLINDRICAL GEOMETRY

Although perfusion in multi-dimension is quite rare, we may need to discuss this situation because such process can be applicable in the analysis of complex flow in regions of bifurcations. In turbulent flow for example, particles are transported in a way that is very difficult to specify the direction of the flowing particles or the direction of the resultant velocity. Hence, there is a need for point to point characterization of the fluid velocity, the diffusion coefficient and the NMR signal.

Since perfusing substances obey the advection equation, the appropriate equation to accurately describe a flow process in a cylindrical geometry based on equation (7) derived as;

$$\frac{\partial}{\partial r}\left(v_rM_y\right)+\frac{1}{r}\frac{\partial}{\partial\phi}\left(v_\phi M_y\right)+\frac{\partial}{\partial z}\left(v_zM_y\right)+\frac{\partial M_y}{\partial t}=$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(D_{r3}\frac{\partial M_y}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\phi}\left(D_\phi\frac{\partial M_y}{\partial\phi}\right)+\frac{\partial}{\partial z}\left(D_z\frac{\partial M_y}{\partial z}\right)+\frac{F_o}{T_o}\gamma B_1(\vec{r},t) \tag{14a}$$

Where

$$D_{r3}=D_rr \tag{14b}$$

Making the following assumptions (Sprawls, 2000):

$$\begin{aligned} r &= \exp\left(u_{03}R\right)\\ \phi &= \exp\left(v_{03}\Phi\right)\\ z &= \exp\left(w_{03}Z\right)\end{aligned} \tag{14c}$$

We can write

$$\left.\begin{aligned}\partial r &= u_{03}\exp\left(u_{03}R\right)\partial R = u_{03}r\partial R\\ \partial\phi &= v_{03}\exp\left(v_{03}\Phi\right)\partial\Phi = v_{03}\phi\partial\Phi\\ \partial z &= w_{03}\exp\left(w_{03}Z\right)\partial Z = w_{03}\,\theta\partial Z\end{aligned}\right\} \tag{14d}$$

$$\begin{aligned}\partial r^2 &= {u_{03}}^2r^2\partial R^2\\ \partial\phi^2 &= {v_{03}}^2\phi^2\partial\Phi^2\\ \partial z^2 &= {w_{03}}^2z^2\partial Z^2\end{aligned}$$

If we define

$$\left.\begin{aligned}v_r &= u_{03}r\\ v_\phi &= v_{03}\phi r\\ v_z &= w_{03}z\end{aligned}\right\} \tag{14e}$$

and

$$\left.\begin{aligned}D_{r3} &= D_0u_{03}^2r^3\\ D_\phi &= D_0v_{03}^2\phi^2r^2\\ D_z &= D_0w_{03}^2z^2\end{aligned}\right\} \tag{14f}$$

Where and u_o, v_o, w_o, are constants. Equation (14a) becomes

$$\begin{aligned}&\frac{M_y}{u_{03}r}u_{03}^2r+\frac{u_{03}r}{u_{03}r}\frac{\partial M_y}{\partial R}+\frac{M_y}{v_{03}\phi r}v_{03}^2\phi r+\frac{v_{03}\phi r}{v_{03}\phi r}\frac{\partial M_y}{\partial\phi}+\frac{M_y}{w_{03}z}w_{03}^2z+\frac{w_{03}z}{w_{03}z}\frac{\partial M_y}{\partial z}+\frac{\partial M_y}{\partial t}\\ &=\frac{1}{u_{03}^2r^3}3D_0u_{03}^3r^3\frac{\partial M_y}{\partial R}+\frac{D_0u_{03}^2r^3}{u_{03}^2r^3}\frac{\partial^2M_y}{\partial R^2}+\frac{1}{v_{03}^2\phi^2r^2}2D_0v_{03}^3\phi^2r^2\frac{\partial M_y}{\partial\Phi}+\\ &\frac{D_0v_{03}^2\phi^2r^2}{v_{03}^2\phi^2r^2}\frac{\partial^2M_y}{\partial\Phi^2}+\frac{1}{w_{03}^2z^2}2D_0w_{03}^3z^2\frac{\partial M_y}{\partial Z}+\frac{D_0w_{03}^2z^2}{w_{03}^2z^2}\frac{\partial^2M_y}{\partial Z^2}+\frac{F_o}{T_o}\gamma B_1(\vec{r},t)\end{aligned} \tag{15}$$

Giving

$$\begin{aligned}&\left(u_{03}+v_{03}+w_{03}\right)M_y+\frac{\partial M_y}{\partial t}=D_0\left(\frac{\partial^2M_y}{\partial R^2}+\frac{\partial^2M_y}{\partial\Phi^2}+\frac{\partial^2M_y}{\partial Z^2}\right)-\\ &\left(1-3D_0u_{03}\right)\frac{\partial M_y}{\partial R}-\left(1-2D_0v_{03}\right)\frac{\partial M_y}{\partial\Phi}-\left(1-2D_0w_{03}\right)\frac{\partial M_y}{\partial Z}+\frac{F_o}{T_o}\gamma B_1(\vec{r},t)\end{aligned}$$

The equation of motion for NMR signals for a flow process in a cylindrical geometry can then be written as;

$$\begin{aligned}\frac{\partial M_y}{\partial t}&=D_0\left(\frac{\partial^2M_y}{\partial R^2}+\frac{\partial^2M_y}{\partial\Phi^2}+\frac{\partial^2M_y}{\partial Z^2}\right)-\left(1-3D_0u_{03}\right)\frac{\partial M_y}{\partial R}-\\ &\left(1-2D_0v_{03}\right)\frac{\partial M_y}{\partial\Phi}-\left(1-2D_0w_{03}\right)\frac{\partial M_y}{\partial Z}\end{aligned} \tag{16}$$

Provided that:

$$\frac{F_o}{T_o}\gamma B_1(\vec{r},t)=\left(u_{03}+v_{03}+w_{03}\right)M_y \tag{17}$$

We seek a solution to equation (16) for an instantaneous release in the form

$$M_y(R,\Phi,Z,t)=g_{31}(R,t;R_o)g_{32}(\Phi,t;\Phi_o)g_{33}(Z,t;Z_o) \tag{18}$$

Where, g_{31}, g_{32} and g_{33} (which are not tensors) are the solutions to the one-dimensional constant coefficient advective diffusion in the transformed space.

$$\left.\begin{aligned} g_{31}(R,t;R_o) &= \frac{A_{31}}{2u_{03}\sqrt{\pi D_o t}} \exp\left(\frac{-[(R-R_o)-(1-3D_o u_{03})t]^2}{4D_o t}\right) \\ g_{32}(\Phi,t;\Phi_o) &= \frac{A_{32}}{2v_{03}\sqrt{\pi D_o t}} \exp\left(\frac{-[(\Phi-\Phi_o)-(1-2D_o v_{03})t]^2}{4D_o t}\right) \\ g_{33}(Z,t;Z_o) &== \frac{A_{33}}{2w_{03}\sqrt{\pi D_o t}} \exp\left(\frac{-[(Z-Z_o)-(1-2D_o w_{03})t]^2}{4D_o t}\right) \end{aligned}\right\} \quad (19)$$

For a source of unit magnetic moment, we obtain

$$\int_0^\infty\int_0^\infty\int_0^\infty M_y(r,\phi,z,t)\,dr\,d\phi\,dz = 1 \qquad \forall t$$

The NMR transverse magnetization obtained for the instantaneous release after a long computation is;

$$M_y(r,\phi,z,t) = \frac{1}{8u_{03}v_{03}w_{03}(\pi D_o t)^{3/2}} \frac{1}{r_0\phi z}\left(\frac{r}{r_0}\right)^{-3/2}\left(\frac{r}{r_0}\right)^{1/2D_o u_{03}} \times\left(\frac{\phi}{\phi_0}\right)^{1/2D_o v_{03}}\left(\frac{z}{z_0}\right)^{1/2D_o w_{03}} \times\exp\left(\frac{-\left[P_{03}^2+(1-D_o u_{03})^2 t^2+2t^2\right]}{4D_o t}\right) \quad (20)$$

$$P_{03}^2 = \left(\frac{1}{u_{03}}\ln\left(\frac{r}{r_0}\right)\right)^2 + \left(\frac{1}{v_{03}}\ln\left(\frac{\phi}{\phi_0}\right)\right)^2 + \left(\frac{1}{w_{03}}\ln\left(\frac{z}{z_0}\right)\right)^2$$

For the case of a continuous release of an advected substance in cylindrical geometry, we shall integrate equation (20) with respect to time:

$$M_y = \frac{1}{8u_{03}v_{03}w_{03}} \frac{1}{r_0\phi z}\left(\frac{r}{r_0}\right)^{-3/2}\left(\frac{r}{r_0}\right)^{1/2D_o u_{03}}\left(\frac{\phi}{\phi_0}\right)^{1/2D_o v_{03}}\left(\frac{z}{z_0}\right)^{1/2D_o w_{03}} \int_{-\infty}^{\infty}\frac{1}{(\pi D_o t)^{3/2}}\exp\left(\frac{-\left[P_{03}^2+(1-D_o u_{03})^2 t^2+2t^2\right]}{4D_o t}\right)dt \quad (21)$$

The value of the constant A_{BPES} is determined using the Boubaker Polynomials Expansion Scheme *BPES* (Awojoyogbe, 2008; Zhao et al., 2008; Belhadj et al., 2009; Chaouachi et al., 2007; Fridjine et al., 2009; Fridjine and Amlouk, 2009; Fridjine et al., 2009; Ghanouchi et al., 2008;Gherib et al., 2008;Guezmir et al., 2009; Labiadh and Boubaker, 2007; Slama and Bessrour, 2009; Slama et al., 2009; Tabatabei et al., 2009)). The calculation protocol takes into account conjointly the properties of the *BPES* along with the already noticed (Awojoyogbe, 2008; Oyodum et al., 2009) similarity between Equation (13) and the characteristic differential equation of the Boubaker polynomials (Slama et al., 2008; Zhao et al., 2008).

THE MULTIDIMENSIONAL PERFUSION PROCESS – SPHERICAL GEOMETRY

Within the bifurcation itself, we shall approximate the region to some spherical region (the shape actually varies). In such a spherical geometrical structure, the diffusion-advection equation describing the spatially variable perfusion process is given by,

$$\frac{\partial}{\partial r}(v_r M_y)+\frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta M_y)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(v_\phi M_y)+\frac{\partial M_y}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(D_{r4}\frac{\partial M_y}{\partial r}\right) +\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(D_{\theta 4}\frac{\partial M_y}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(D_\phi\frac{\partial M_y}{\partial\phi}\right)+\frac{F_o}{T_o}\gamma B_1(\vec{r},t) \quad (22)$$

Where:

$$D_{r4} = D_r r^2$$
$$D_{\theta 4} = D_\theta \sin\theta$$

Making the following assumptions [29]:

$$r = \exp(u_{04}R)$$
$$\phi = \exp(v_{04}\Phi) \quad (23)$$
$$\theta = \exp(w_{04}\Theta)$$

And

$$\left.\begin{aligned} v_r &= u_{04} r && \text{and} \quad D_{r4} = D_0 u_{04}^2 r^4 \\ v_\theta &= w_{04}\theta r && \text{and} \quad D_\phi = D_0 v_{04}^2\phi^2 r^2\sin^2\theta \\ v_\phi &= v_{04}\phi r\sin\theta && \text{and} \quad D_{\theta 4} = D_0 w_{04}^2\theta^2 r^2\sin\theta \end{aligned}\right\} \quad (24)$$

Equation (18) can be written as

$$(u_{04}+v_{04}+w_{04})M_y+\frac{\partial M_y}{\partial t}=D_0\left(\frac{\partial^2 M_y}{\partial R^2}+\frac{\partial^2 M_y}{\partial\Phi^2}+\frac{\partial^2 M_y}{\partial\Theta^2}\right) -(1-4D_0u_{04})\frac{\partial M_y}{\partial R}-(1-2D_0v_{04})\frac{\partial M_y}{\partial\Phi}-(1-2D_0w_{04})\frac{\partial M_y}{\partial\Theta}+\frac{F_o}{T_o}\gamma B_1(\vec{r},t) \quad (25)$$

Provided that

$$\frac{F_o}{T_o}\gamma B_1(\vec{r},t)=(u_{04}+v_{04}+w_{04})M_y \quad (26)$$

The equation of motion for NMR signals for a flow process within process within the bifurcation is given by:

$$\frac{\partial M_y}{\partial t} = D_0\left(\frac{\partial^2 M_y}{\partial R^2} + \frac{\partial^2 M_y}{\partial \Phi^2} + \frac{\partial^2 M_y}{\partial \Theta^2}\right) - (1-4D_0u_{04})\frac{\partial M_y}{\partial R}$$

$$-(1-2D_0v_{04})\frac{\partial M_y}{\partial \Phi} - (1-2D_0w_{04})\frac{\partial M_y}{\partial \Theta} \tag{27}$$

We seek solutions to the diffusion-advection equation for an instantaneous release in the form

$$M_y(R,\Phi,\Theta,t) = g_{41}(R,t;R_o)g_{42}(\Phi,t;\Phi_o)g_{43}(\Theta,t;\Theta_o) \tag{27a}$$

Where, g_{41}, g_{42} and g_{43} (which are not tensors) are the solutions to the one – dimensional constant coefficient advective diffusion in the transformed space

$$\left.\begin{aligned}
g_{41}(R,t;R_o) &= \frac{A_{41}}{2u_{04}\sqrt{\pi D_o t}}\exp\left(\frac{-[(R-R_o)-(1-4D_ou_{04})t]^2}{4D_ot}\right)\\
g_{42}(\Phi,t;\Phi_o) &= \frac{A_{42}}{2v_{04}\sqrt{\pi D_o t}}\exp\left(\frac{-[(\Phi-\Phi_o)-(1-2D_ov_{04})t]^2}{4D_ot}\right)\\
g_{43}(\Theta,t;\Theta_o) &== \frac{A_{43}}{2w_{04}\sqrt{\pi D_o t}}\exp\left(\frac{-[(\Theta-\Theta_o)-(1-2D_ow_{04})t]^2}{4D_ot}\right)
\end{aligned}\right\} \tag{28}$$

For a source of unit magnetic moment,

$$\int_0^\infty\int_0^\infty\int_0^\infty M_y(r,\phi,\theta,t)\,dr\,d\phi\,d\theta = 1 \qquad \forall t$$

The coefficients A_{41}, A_{42} and A_{43} are constants. The integral gives

$$A_{41}\exp(u_{04}[R_o + (1-3D_ou_{04})t])A_{42}\exp(v_{04}[\Phi_o + (1-D_ov_{04})t])$$

$$A_{43}\exp(w_{04}[\Theta_o + (1-D_ow_{04})t]) = 1$$

$$A_{41}A_{42}A_{43}r_0\phi_0\theta_0\exp(u_{04}t-3D_ou_{04}^2t)\exp(v_{04}t-D_ov_{04}^2t)\exp(w_{04}t-D_ow_{04}^2t)=1$$

An obvious choice for A_{41}, A_{42} and A_{43} would be:

$$A_{41} = \frac{1}{r_0\exp(u_{04}t-3D_ou_{04}^2t)} = \frac{\exp(3D_ou_{04}^2t-u_{04}t)}{r_0},$$

$$A_{42} = \frac{1}{\phi_0\exp(v_{04}t-D_ov_{04}^2t)} = \frac{\exp(D_ov_{04}^2t-v_{04}t)}{\phi_0},$$

$$A_{43} = \frac{1}{\theta_0\exp(w_{04}t-D_ow_{04}^2t)} = \frac{\exp(D_ow_{04}^2t-w_{04}t)}{\theta_0}$$

$$M_y(r,\phi,\theta,t) = \frac{1}{8u_{04}v_{04}w_{04}(\pi D_ot)^{3/2}}\frac{1}{r_0\phi z}\left(\frac{r_0}{r}\right)^2\left(\frac{r}{r_0}\right)^{1/2D_ou_{04}}$$
$$\times\left(\frac{\phi}{\phi_0}\right)^{1/2D_ov_{04}}\left(\frac{\theta}{\theta_0}\right)^{1/2D_ow_{04}}\times\exp\left(\frac{-[P_{04}^2+(1-2D_ou_{04})^2t^2+2t^2]}{4D_ot}\right) \tag{29}$$

Where,

$$P_{04}^2 = \left(\frac{1}{u_{04}}\ln\left(\frac{r}{r_0}\right)\right)^2 + \left(\frac{1}{v_{04}}\ln\left(\frac{\phi}{\phi_0}\right)\right)^2 + \left(\frac{1}{w_{04}}\ln\left(\frac{\theta}{\theta_0}\right)\right)^2$$

For the case of a continuous release of an advected substance in spherical geometry, we shall integrate equation (29) with respect to time

$$M_y = \frac{1}{8u_{04}v_{04}w_{04}}\frac{1}{r_0\phi z}\left(\frac{r_0}{r}\right)^2\left(\frac{r}{r_0}\right)^{1/2D_ou_{04}}\left(\frac{\phi}{\phi_0}\right)^{1/2D_ov_{04}}\left(\frac{\theta}{\theta_0}\right)^{1/2D_ow_{04}}$$

$$\int_{-\infty}^{t_\infty}\frac{1}{(\pi D_ot)^{3/2}}\exp\left(\frac{-[P_{04}^2+(1-2D_ou_{04})^2t^2+2t^2]}{4D_ot}\right)dt \tag{30}$$

RESULTS AND DISCUSSION

Equations (12, 13, 20, 21, 29 and 30) are the NMR transverse magnetizations and signals for the instantaneous and continuous release of advected substances in Cartesian, cylindrical and spherical geometries respectively. These NMR signals are functions of diffusion coefficient D_o and their respective perfusion functions P_{02}^2, P_{03}^2 and P_{04}^2. The diffusion coefficient is related to the net displacement of molecules in a given time. The average distance, s, traveled relative to diffusion coefficient is given as:

$$s = \sqrt{2D_ot} \tag{31}$$

Based on equations (13, 21, 30, and 31) and applying some standard integral formulae, the NMR transverse magnetizations and signals for the continuous release of advected substances in Cartesian, cylindrical and spherical geometries can be written as:

$$\frac{M_y}{M_y(0)} = \exp(-2\beta) \tag{32a}$$

Where for example in spherical geometry,

$$M_y(0) = \frac{1}{8u_{04}v_{04}w_{04}}\frac{1}{r_0\phi z}\left(\frac{r_0}{r}\right)^2\left(\frac{r}{r_0}\right)^{1/2D_ou_{04}}\left(\frac{\phi}{\phi_0}\right)^{1/2D_ov_{04}}$$

$$\left(\frac{\theta}{\theta_0}\right)^{1/2D_ow_{04}}\frac{1}{s^3}\sqrt{\frac{\pi}{\delta^2}} \tag{32b}$$

$$\delta^2 = \frac{(1-2D_ou_{04})^2+2}{s^2} \tag{32c}$$

$$\beta = \frac{P_{04}^2}{s^2} = qP_{04}^2, \quad q = \frac{1}{s^2} \tag{32d}$$

In equation (32), the reduction in NMR signal $\frac{M_y}{M_y(0)}$ produced by the diffusion-perfusion process depends on the rate of diffusion expressed by the value of the diffusion coefficient, D_o, the perfusion function in the particular geometry and the perfusion sensitivity, q, which is determined by the average distance, s, traveled by a molecule in time t. The distance, s, depends on the diffusion coefficient for the specific tissue compartment within a voxel. A series of experiment to measure the perfusion function can be performed in which values of s, may be varied by varying n or m using equations (3, 8a and 31).

From equations (12, 20 and 29), the NMR signal intensity for the instantaneous release of advected substances in for example spherical geometry can be written as:

$$M_y = M_y(0)\exp(-\frac{P_{04}^2}{s^2}) \times \exp(-\frac{s^2\delta^2 t}{4D_o}) \tag{33}$$

By inspection of equation (33), it can be seen that the signal intensity is a product of signal attenuation due to perfusion and signal attenuation due to diffusion. Theoretically, a series of experiment can be performed in which either s, D_o or δ is varied by varying n or m using equations (3, 8a, 31, and 32c) while keeping t constant. The real experimental conditions under which the above description of equation (33) can be used to perform the diffusion and perfusion measurements will be considered in separate studies.

Conclusion

We have obtained basic analytical expressions for the transverse magnetizations (the NMR signals) for perfusion processes in different geometrical structures and biophysical conditions based on the Bloch NMR flow equations. These analytical results are quite interesting and promising in the context of some recent works on dynamical flows (Sprawls, 2000; Awojoyogbe et al., 2010; Hassell et al., 2008; Nicolis et al., 2002). The application of these fundamental results to solve real life flow problems in which NMR-sensitive materials are transported will be presented separately. It should be mentioned that acquisition of perfusion data requires fast imaging methods based on the appropriate choice of n or m in equation (3 and 8a) because images must be acquired every few seconds to properly measure the characteristics of the bolus passage. It should be noted that, in specific tissue, the diffusion rate might be different in different directions because of the orientation of certain tissue structures. This is a very important factor which must be taken into account when producing diffusion images. Hence, the results of diffusion-advection equation with spatially varying diffusion coefficients as discussed in this study, which is based on the fundamental Bloch NMR flow equations, can be invaluable mathematical tools to accurately understand the combined effect of diffusion and perfusion process in human physiological and pathological flow systems. The method presented in this study can have applications in functional magnetic resonance imaging (fMRI) with more accurate information. How the NMR parameters derived in the present model are linked to a practical measurement in terms of an fMRI sequence will be developed separately.

ACKNOWLEDGEMENT

The authors appreciate encouragement from Professor S.O.E Sadiku, Academic planning, Federal University of Technology, Minna, through the STEP B programme.

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