

Full Length Research Paper

Stability analysis of a reinforced sand beam

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A reinforced sand beam was tested at the laboratory, in order to understand the behaviour of the sand during the failure mechanism. A series of structural tests on the sand and on the reinforcement material were conducted first, and then a reinforced sand beam was constructed in the laboratory. The results were checked theoretically and numerical simulations were also carried out, to examine the moment stiffness of the reinforced structure. In order to generate a general result, elasticity moduli of the different conditions were calculated and compared to each other, with the changing circumstances. It was clearly proved that, reinforced sand beams have a strong bending resistance. The increasing effect of reinforcement was calculated using the experiment results.

Key words: Reinforcement, stability analysis, soil structure, geosynthetics, sand.

INTRODUCTION

This study aims to investigate the ability of sand to keep its stability (equilibrium) with sand behavior, due to collapse and its stabilization by means of reinforcement elements. According to Ohta et al. (1997), resistance supplied by reinforcement and displacements with the reinforcement element were also investigated, as a matter of great interest.

The sand does not have any resistance as a beam, as it cannot keep its form by itself. Geosynthetics help the sand to hold itself as a beam. In that sense, the behaviour of the sand is examined with the reinforcement element, which is referred to as the composite material's behaviour.

An experimental procedure for the whole mechanical response of the composite material was considered. From this process, empirical results were obtained. These results were used to form an applicable equation to estimate the equivalent elastic moduli of, geosynthetic

reinforced sand. This mechanism could then be applied to the field trial, in order to validate it in the future.

Using the interaction between the material elements, a parametric study was conducted after the experimental process. The "finite element method" (FEM.) was used to simulate the behaviour of the model and field. The stiffness and the failure, give an idea for a parametric study of stress-strain relationship that will guide the constitutive equation of the sand. These results were compared with the monitored data of the displacement, in order to find the nearest behaviour so that, a suitable constitutive equation for sand could be developed.

METHODOLOGY

Laboratory model test

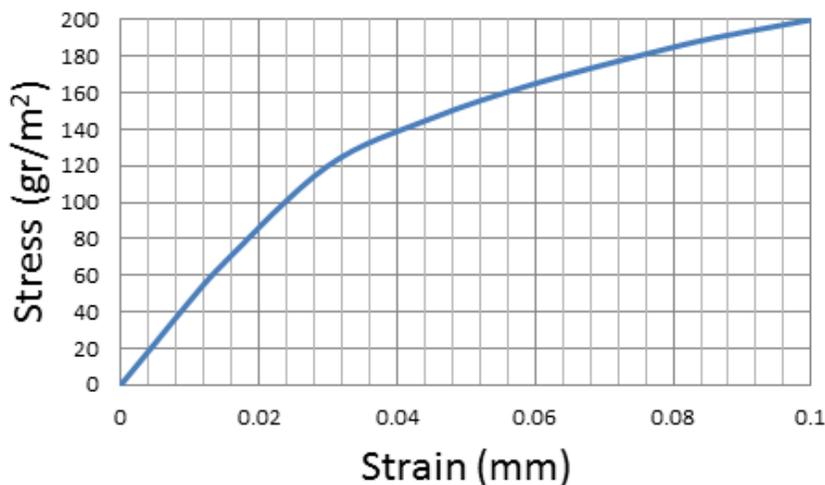
A series of experiments in the laboratory were performed, to

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Table 1. The properties of Toyoura sand.

Properties		Value
Average void ratio	$e_{ave} =$	0.930
Water content	$w =$	0.159%
Unit weight	$\rho_t =$	1.365 gf/cm ³

**Figure 1.** Stress-strain relationship of tissue paper.

investigate the efficiency of using geosynthetics. By using these results, the aim of a laboratory model test was to gain a better understanding of the mechanical interaction, between sand and reinforcement.

The sand itself does not have any moment stiffness, however, when it is designed as a beam-shaped sand structure reinforced by geosynthetics, moment stiffness is created. If this mechanism is used successfully, it could be possible to use sand at a structure such as embankment, footing or slope. In order to make it possible, the following areas must be understood:

1. How the reinforcement material should be used
2. How much stiffness can we expect, when using geosynthetics
3. How stiffness, which comes from sand and geosynthetics can be increased (in which condition or composite system of the stiffness can be maximized)
4. How can the real embankment be built successfully as experienced in the laboratory.

All these items were examined and tested carefully in the laboratory, by means of calculations. Toyoura Sand which is very common is used in laboratory experiments. Table 1 shows the physical properties of Toyoura Sand (Nishiura et al., 1993).

In a laboratory model test, a variety of reinforcement materials were tested. At the beginning, three different types of very thin non-woven fabrics were chosen as reinforcement materials. These geosynthetics are made from polyester fibres.

The basic principle of reinforcement with geogrid is the mobilization of a high tensile force, at low strain within the soil structure. This is achieved by an interlocking bond between sand and grid.

According to Kasahara (1992), after performing several

experiments it was realized that, the real-geosynthetics give a higher strength than expected in such a small experimental model. For this reason, it was decided to use tissue papers as a reinforcement material in the laboratory, as there was no stress-strain data for the tissue paper from the manufacturer, several extension tests were performed in the laboratory. From these tests, a stress-strain relationship was obtained as shown in Figure 1.

The box used in the experiment was a rectangular parallelepiped box, 30 cm high, 50 cm wide and 84 cm deep. This was made from transparent acrylic plates as shown in Figure 2. The front and rear sides of the box were equipped with movable gates which can slide vertically, to let the sand out during the experiment. The rectangular lumbers 4 x 4 cm, are placed at the bottom as a support.

The vertical displacements of the reinforced sand structure are measured by dial gauges, installed at the upper part of the box while the sand is removed from the span and beam start, to have displacement. Dry Toyoura sand ($\rho_t = 1.37 \text{ g/cm}^3$) was used in the experiments. Tissue paper is the thinnest appropriate sheet acceptable as a reinforcing material. The whole sheet 1.50 m width was ordered from the manufacturer.

The shape of the beam is shown in Figure 3. Both sides of the sheets are folded back as in Figure 3b, to prevent the sand flow and to hold the beam as explained in the figure. The height of the beam was changed in the series of experiments, and the numbers of reinforced sand layers were also changed every time, to understand the effect on the stiffness of the beam.

During the experiment, a funnel shown in Figure 4 was fixed at a certain height, through which sand was placed homogeneously with a constant speed. The sand was put into the funnel by a small scoop (about 930 g of sand per scoop). Each layer was controlled, based on the number of scoops. Void ratio of sand is about 0.930. The tissue paper was folded by about 200 mm at both ends

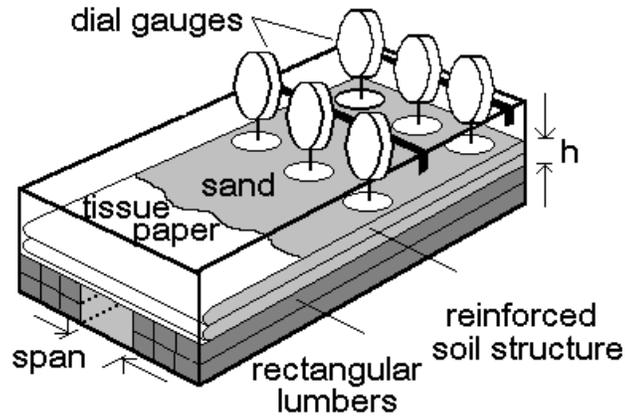


Figure 2. Test equipment.

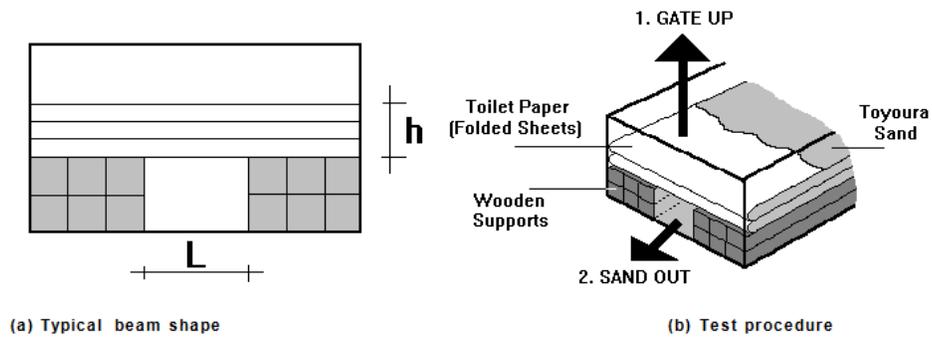


Figure 3. Model beam of reinforced soil.

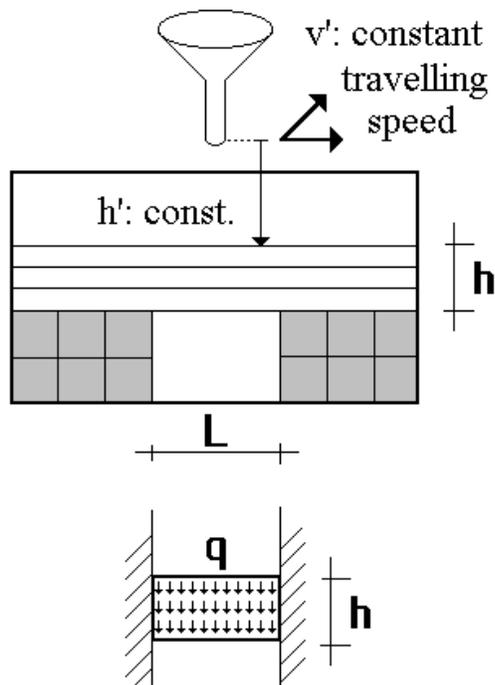


Figure 4. Loading system and the modeling.

Table 2. Summary of the test results.

Exp. No	Sand (g)	No. of sheets <i>n</i>	Span <i>L</i> (cm)	Beam height <i>h</i> (cm)	<i>n/h</i>	Settlement <i>Y_{max}</i> (cm)	<i>EI</i> (gfcm ²)	Young's Mod. <i>E</i> (gf/cm ²)
1	55.800	10	16	9.0	1.111	1.260	1670.02	27.49
2	55.800	10	24	9.0	1.111	4.700	2260.29	37.21
3	32.550	5	24	4.9	1.020	-	-	-
4	32.550	5	16	4.9	1.020	2.500	456.70	46.60
5	79.050	15	16	13.1	1.145	1.109	2751.60	14.69
6	79.050	15	24	13.1	1.145	3.600	4291.20	22.91
7	60.450	5	16	9.0	0.556	1.520	1379.93	22.71
8	73.935	28	16	12.6	2.222	0.863	3401.47	20.40
9	80.910	7	16	12.6	0.556	1.114	2635.07	15.81
10	53.475	20	16	9.0	2.222	1.146	1830.27	30.13
11	33.015	12	16	5.4	2.222	1.661	757.26	57.72
12	33.480	3	16	5.4	0.556	1.792	701.90	53.50
13	74.865	2	16	12.6	0.159	1.306	2247.68	13.48
14	33.480	1	16	5.4	0.185	1.950	645.03	49.16
15	76.260	14	16	12.6	1.111	1.092	2688.16	16.13
16	35.340	6	16	5.4	1.111	1.708	736.42	56.13
17	53.475	2	16	9.0	0.222	1.543	1359.36	22.38

as shown in Figure 3b so that, the sand could be prevented from flowing out when the gate was released. The model test procedure is described as follows:

1. Determine the span of beam with a wooden lath.
2. Put the sand in the span temporarily, to support the beam.
3. Lay the first layer of tissue paper.
4. Use a scoop and measure the necessary amount of sand for one layer.
5. Adjust the funnel height and evenly place the sand by letting it free fall.
6. Fold the ends of tissue paper on the second layer.
7. Place tissue paper on the second layer
8. Repeat the above steps 3) through 7) until the required number of layers is obtained.
9. Set the dial gauges to measure vertical displacements.

Up to here, this was the preparation stage for the experiment. There is also sand beneath the beam at the span part. This span part should be emptied in order to

start the experiment and monitor the displacement.

The front and back gates are opened to remove the sand at the supporting part. The sand working as a temporary support of the span is taken out slowly from the span. During this process the dial gauges are used to measure the average settlement of the beam-shaped structure. The largest amount of settlement usually occurs at the beginning. The amount of settlement is recorded for 24 h after taking out all the sand beneath the span.

After 24 h there is no change in the settlement of the beam-shaped structure. If reinforcement sheets are not folded back, sand would flow from the layers and big displacements would occur. For this reason, folding back the sheets is adopted as a practice for these laboratory tests.

RESULTS

In the tests, the number of the geotextiles was

changed from 1 to 28 layers, two lengths of span 240 mm and 160 mm, were tested. The height of the beam varied from 49 to 131 mm. The results and the different conditions are summarized in Table 2.

The stiffness of the beam is found from the settlement of the beam. In order to find the relationship between the maximum settlement *Y_{max}* and the beam height *h*, series of experiments were executed keeping the density of reinforcement constant. If the reinforced sand beam is approximated as an elastic beam loaded by a uniform load at *q*, corresponding to the weight of the reinforced sand, Equation (1) can be used:

$$Y_{\max} = \frac{qL^4}{384 \cdot EI} \Leftrightarrow EI = \frac{qL^4}{384 \cdot Y_{\max}} \quad (1)$$

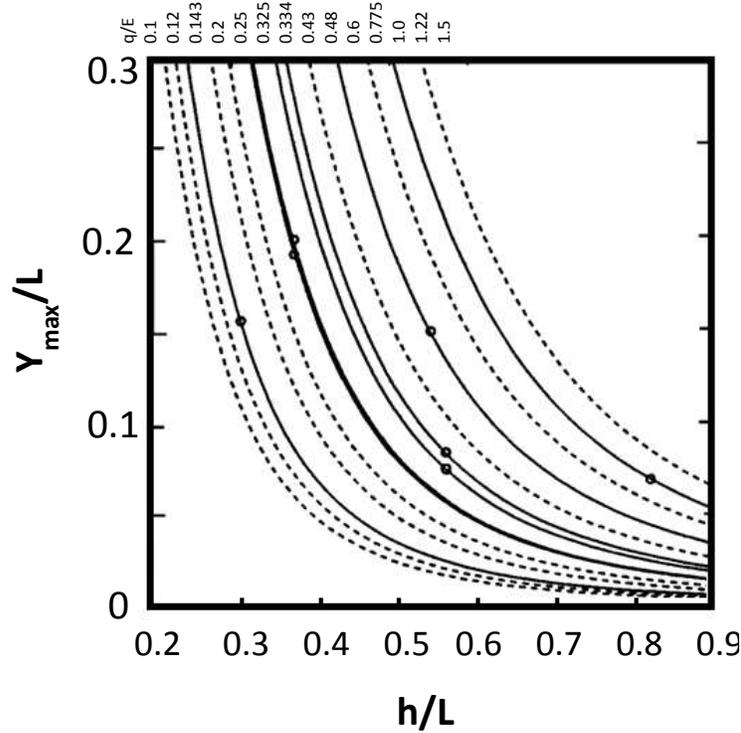


Figure 5. The normalized relationship between the beam height and displacement.

Where q , is the weight of the beam per unit span length and expressed as:

$$q = \rho_t \cdot h = 1.365 \cdot h \text{ (gf/cm}^2\text{)} \tag{2}$$

The cross-sectional secondary moment (I) of the beam is:

$$I = \frac{h^3}{12} \tag{3}$$

Equations 2 and 3 are used to calculate the parameters in Equation 1. The beam-shaped structure prepared in the laboratory, consists of the sand and the reinforcement material. For this reason, it is referred to as a compound material in this paper. The compound material reinforced by geosynthetics, can never be accepted as a linear elastic body (Ohta and Iizuka, 1991). It is rather non-linear accompanied by irreversible deformation. The final deformation of each beam was measured, as shown in Table 2 and the macroscopic elastic coefficients equivalent to such deformations were calculated.

Analysis of the test results

The Young's Moduli shown in Table 2 can be accepted as secant deformation moduli of compound material.

A set of curves in Figure 5 shows the relationship between Y_{max}/L and h/L , calculated by using Equations 1 and 3. The plots in Figure 5 are the monitored displacements throughout a series of experiments. The broken lines show the theoretical curves calculated by Equation 4, and the solid lines are the calculated theoretical curves that overlaps with the experiment results.

$$\frac{Y_{max}}{L} = \frac{q}{32 \cdot E} \times \frac{1}{(h/L)^3} \tag{4}$$

In the model, if both-sides fixed beam (Figure 4) is admissible, the displacement of the beam becomes larger, as the value h/L becomes smaller.

In Figure 6, the equivalent Young's modulus (E) estimated from the global moment stiffness is plotted against the number of reinforcement materials, n , laid in the sand per unit height of the structure. The results in Figure 6 indicate that, reinforcement contributes an increase in the equivalent Young's modulus but, there seems to be a limiting value of Young's modulus.

In order to explain the above experimental results, we introduce the following general expression:

$$E = f(E_g, E_s, n/h) \tag{5}$$

E : the global Young's modulus; E_g : Young's modulus of

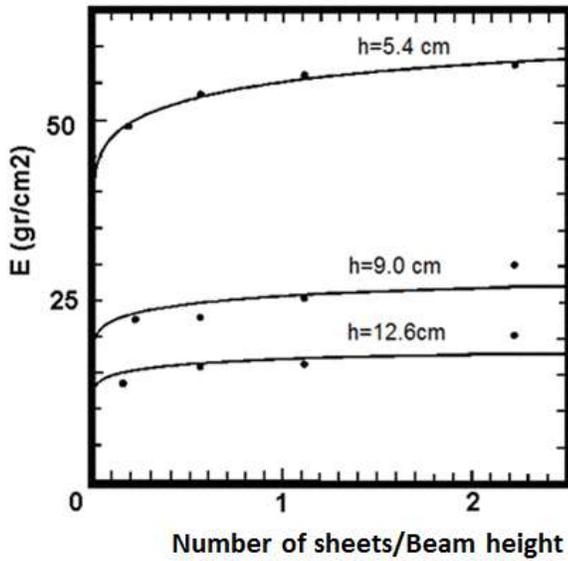


Figure 6. The effect of reinforcement on the stiffness of the beam.

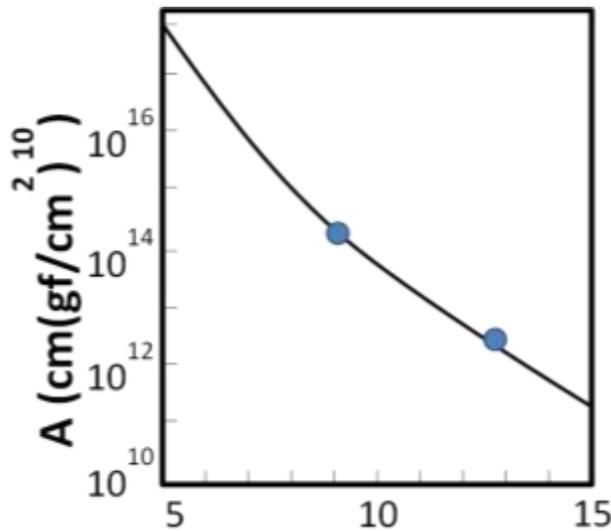


Figure 7. Parameter (A) plotted against beam height (h).

geosynthetics; E_s : Young's modulus of sand; n : Number of geosynthetic layers; h : B Beam height.

If the effects of E_g and E_s can be combined into a parameter A , Equation (5) can be written as:

$$E = f(A, n/h) \tag{6}$$

In the case of the present experiment applying Equation (6) to the curves in Figure 6, Equation (7) is obtained:

$$E = \{A \cdot (n/h)\}^{0.1} \tag{7}$$

in which parameter A , has a relation with the height of the structure as indicated by the plots in Figure 6. This relation can be described by the equation:

$\log(A) = a_0 + a_1 \times \log(h)$ as shown by the solid line in the Figure 7, Equation (8) can be derived from Equation (7):

$$E = \alpha \cdot h^\beta (n/h)^m \tag{8}$$

in which α , β and m are constants. By arranging the results obtained from the experiment based on Equation (8), the effect of the height of the structure and the number of geosynthetics laid in the sand on the global Young's modulus, can be quantitatively shown as in Figure 7, where in the present experiment $\alpha = 2.33 \times 10^6$ (gf/cm² cmm^{-*}) and $\beta = -1.4$, $m = 1.0$.

However, we should note here that Equation (8) is an empirical equation that can only be applied to some special cases. In the finite element program named "DACSAR", an elasto-viscoplastic constitutive model proposed by Iizuka and Ohta (1987) is employed to model the mechanical behavior of clay materials.

In this study the materials used were sand and reinforcement materials. The sand was modeled by a non-linear elastic model using hyperbolic stress-strain relations (Duncan and Chang, 1970). Since reinforcement materials can be regarded as linearly elastic materials based on the uni-extension test result, then the geosynthetics can be modeled by a linearly elastic bar element under two dimensional spaces. Accordingly the hyperbolic constitutive model for the sand is newly implemented in DACSAR.

The simulation program was modified to incorporate non-linear elastic constitutive relations as shown in Figures 7 and 8, to model the sand and the reinforcement materials, respectively. The model for the sand (Figure 8) is described as:

$$\frac{q}{p_0} = \frac{\varepsilon_a}{a + b\varepsilon_a} \tag{9}$$

in which q is the stress difference between principal stresses, p_0 is the initial mean stress, ε_a is the axial strain and a and b are the material constants.

The parameters a and b are determined from the triaxial CU test stress and strain data. In the two dimensional finite element programming, Equation (9) is rewritten in terms of the generalized stress deviator q (the deviatoric stress component) and the generalized strain deviator γ (the deviatoric strain component) and then the incremental form of it is employed in the step by step calculation scheme.

$$q = \sqrt{\frac{3}{2} s_{ij} s_{ij}}, \quad s_{ij} \text{ deviatoric stress component}$$

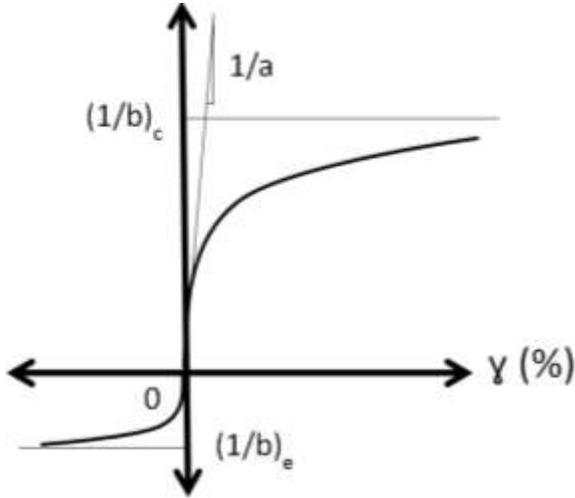


Figure 8. Stress strain model for sand.

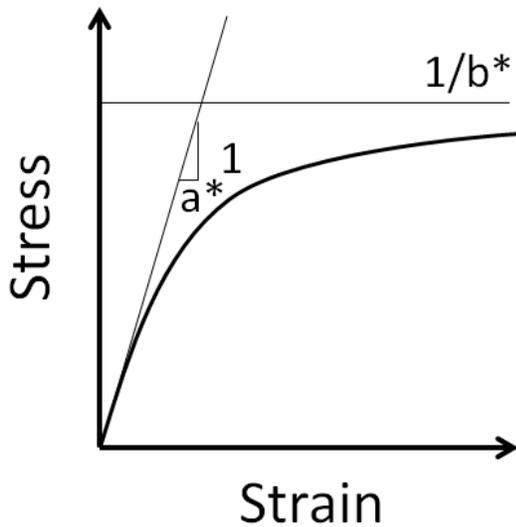


Figure 9. Stress strain model for tissue paper.

$$\gamma = \sqrt{\frac{3}{2}} e_{ij} e_{ij}, e_{ij} \quad \text{generalized strain deviator}$$

Therefore, the constant modulus (elastic shear modulus) between dq and $d\gamma$ can be expressed as:

$$G = \frac{3ap_0}{(3a + 2b\gamma)^2} \quad (10)$$

In the present simulation, the input parameters in equation (10) are determined from the triaxial CU test results for Toyoura sand reported by Fukushima and Tatsuoka (1984), and are as follows: $a = 2.72 \times 10^{-3}$ and

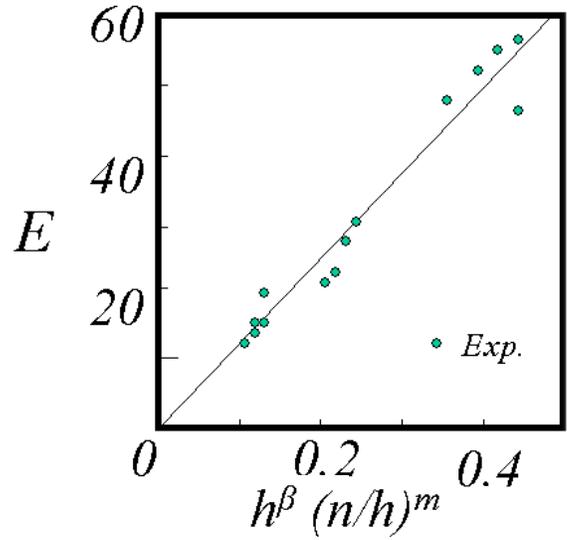


Figure 10. Dependency of n and h ($E = 125.6[\exp(-0.164h)(n/h)^{0.1}]$).

$b = 3.78 \times 10^{-1}$. The non-linear elastic constitutive relation for the geosynthetic material is represented by:

$$\sigma = \frac{\varepsilon}{a^* + b^* \varepsilon} \quad (11)$$

ε in Equation (11) is the axial strain and a and b are calculated using Figures 8 and 9. Therefore, Young's modulus in a certain step is expressed as:

$$E = d\sigma/d\varepsilon = E_i(1 - \sigma/\sigma_{max})^2 \quad \text{and} \quad E_i = 1/a^*, \sigma \quad (12)$$

$$\sigma_{max} = 1/b^*$$

The input parameters needed in the computation in Equation (12) are determined from the experimental results in Figure 10 as $E_i = 5130$ (tf/m²), $\sigma_{max} = 151$ (tf/cm²).

In the simulation, the whole composite structure was modeled by 340 4-node quadrilateral constant strain elements and the geosynthetic reinforcement was modeled by bar elements. Figure 11 shows the experimental and finite element results of the effect of reinforcement (n : number of layers), on the maximum deformation at the center (Y_{max}) of the structure, and Figure 12 illustrates the predicted contribution of reinforcement to the elastic stiffness of the structure.

The Young's moduli in Figure 12 are not directly obtained from the simulation or experiment but estimated from the maximum deformation at the center of the structure, based on linearly elastic beam theory. The numerical simulation using non-linear elastic models can successfully explain observed results in the laboratory experiment so far.

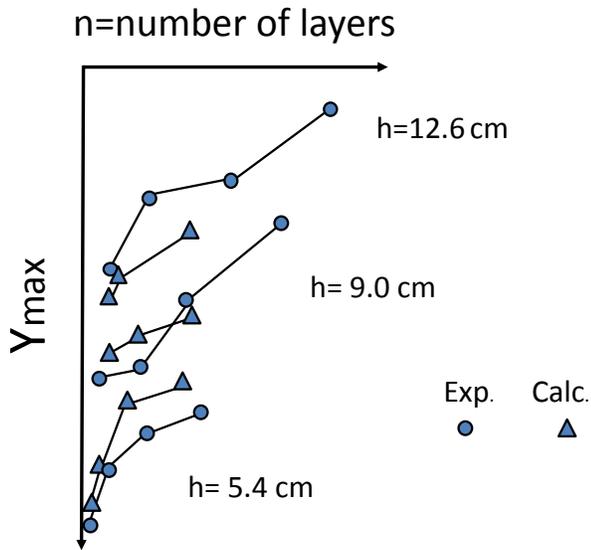


Figure 11. The effect of reinforcement on the deformation.

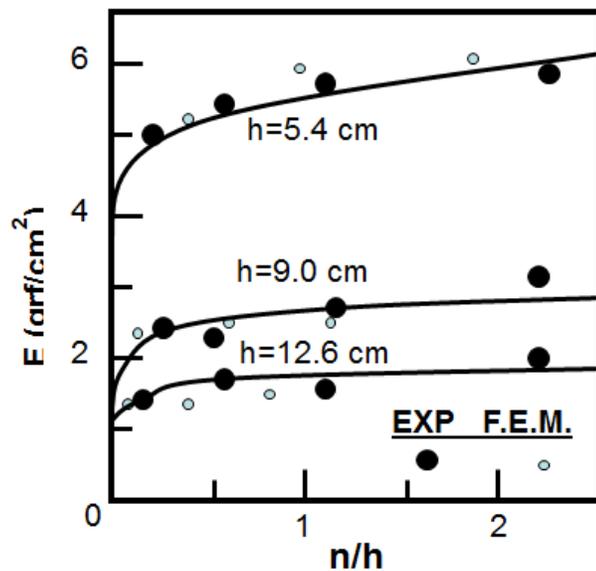


Figure 12. The comparison of computed and monitored effect of reinforcement on the stiffness.

DISCUSSION

This study represents the laboratory model tests which are carried out to examine the global stiffness of the soils, reinforced by geosynthetics against the bending moment and a series of two dimensional finite element simulations. The laboratory test can be used to provide basic data on the momentum stiffness of soils, reinforced by geosynthetics to design the full scale in-situ tests, to be used for further safe and economic designs of soil structures.

The sand and the geosynthetic cannot resist independently against bending moments. However, their composite material shows a fairly strong bending resistance. As a conclusion from the tests, it was understood that, sand reinforced with tissue papers gain stiffness. But how strong the bending resistance seems, depend on how well the compacted sand was wrapped by the geosynthetics.

The relation between the number of the reinforcement materials used and the equivalent Young's modulus of the beam-shaped structure was calculated. The technique of numerical simulation, considering the effect of confining pressure on the stress and strain for soil behavior is developed successfully. The numerical simulation indicates that, the effect of soil confinement by geosynthetic material is particularly significant.

Conclusion

In this study, a series of finite element simulations was carried out, where non-linear constitutive models considering the effect of confining pressure were employed. The computed results are in good agreement with the measured results in the laboratory model test.

In the experiment, the number of layers of reinforcement materials (tissue paper) laid in the sand was 1 to 28 layers, while the span of the structure was 16 to 24 cm and the height of the structure was changed from 4.9 to 13.1 cm.

It was observed that the displacement changes according to the height of the beam, not according to the number of the reinforcement materials (tissue paper). From the results of many experiments, it was observed that the effect of the number of sheets was less. The displacement of the beam with many sheets does not mean less displacement and even with much less reinforcement it can be possible to retain the stiffness of the structure. Generally these results are obtained from this research:

1. Sand reinforced with tissue papers gain stiffness.
2. If it is reinforced with tissue paper, it is possible to build a beam-shaped structure.
3. Moment stiffness of the structure can be improved considerably by using more reinforcement materials.
4. However, there is a limit for the moment stiffness increased by reinforcement materials.
5. Improvement of elasticity coefficient decreases rapidly after a certain point, even if the number of reinforcement materials increased.
6. The relation between the number of the reinforcement materials used and equivalent Young's modulus of the beam-shaped structure was calculated.
7. According to this, the stiffness of the beam-shaped structure can be increased by adjusting the height and the number of the reinforcement materials, of the beam-shaped structure.

8. From the physical experiments of the materials, the sand and the tissue paper are described by a hyperbolic model. By the help of this, the deformation of the beam-shaped sand structure can be calculated by finite element methods.

9. It is possible to express the effect of reinforcement, by using the sand's confining pressure.

CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

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