

Full Length Research Paper

## Computer simulation for structural analysis

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**A computer simulation is a computer program that attempts to simulate an abstract model of a particular system. Computer simulations have become a useful part of mathematical modeling of many natural systems in structural analysis. The main aim of this paper is to provide a new efficient method to all aspects of analysis of structures by computer simulation. The use of a new method is necessary to accuracy and stability to improve performance of these structures. The approach of proposed method is based upon the principle of conservation of energy.**

**Key words:** Fletcher-Reeves method, optimization of energy, perturbation technique, analyzing modal, nonlinear dynamic response.

### INTRODUCTION

The new sets of eigenvectors and eigenvalues must be calculated at each time step and the stiffness matrix must be reevaluated at end of each time step. The non-linear systems have no fixed sets of eigenvectors and eigenvalues. This makes the use of conventional methods extensively time consuming and costly. The dynamic response analysis of non-linear system is based on the evaluation of response for a series of short time intervals using different types of time integration techniques (Aschheim et al., 2007).

The dynamic problems do not have a single solution like static counterparts. The analyst must establish a succession of solution corresponding to all times of interest in the response period. In the dynamic problems, the differential equations arising from the equilibrium of the dynamic forces acting on the mass is solved by implicit or explicit methods. The implicit or explicit method provides numerical solutions to the equations of motion set up for one interval of time. They assume the structural properties to remain constant during the interval, but reevaluate them at the end of time step. This may not be sufficient for highly non-linear structures. It is important to reevaluate both the stiffness and damping during the time step. The reevaluation process makes the methods more

expensive to use. The implicit method offers unconditional stability at the expense of operating with relatively dense decomposed matrices when applied to linear structures, but lose the advantage of unconditional stability when applied to non-linear systems (Buchholdt and Moossavinejad, 1982). The explicit methods, on the other hand, use relatively less computer storage, but are hampered by instability which limits the size of the time steps. The implicit methods when applied to non-linear structures require the solution of a set of non-linear equations whilst most explicit methods require the inversion of a non-diagonal matrix. Hence, it is impossible to choose any of these methods as the best, unless the type of structure to be analyzed is specified. As latter, reduction of time consuming, costly, and high accurate result justify the used of indirect methods such as optimization theory (Ha, 2005).

The present research is based on application of optimization theory. The optimization theory and techniques is used a real-valued objective function and decrease time consuming and costly. A new algorithm is proposed which converges more rapidly to the neighborhood of solution. The developed method is found to be suitable technique for minimization of total potential

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energy function especially in cases where the number of variables is large and the structure is highly nonlinear. The present method decreases computational time and number of iterations required per time step.

The tension structures have many advantages such as prefabrication, ease of transportation and erection, relatively low cost and provision for coverage of large clear spans and high strength, large flexibility and elasticity. The design process is a relatively complex problem. In the present study, we consider the effect of dynamic loads in tension structures and describe a Fletcher-Reeves method for the determination of free and forced vibration analysis of structures. The Fletcher-Reeves method belongs to a group of methods which attempt to locate a local minimum function. Fletcher-Reeves algorithm is applied to calculate the set of displacements to minimize the energy of structural system (Hashamdar al., 2011a; Kukreti, 1989). The proposed theory for nonlinear analysis of 3D space structure is based on minimization of the total potential dynamic work. The minimization of the total potential dynamic work is indirect method which based on principle of convergence of energy in structures. Conventional methods such as superposition methods are direct method. They are usually employed for the solution of equilibrium equations of structures.

However, the conventional methods use for structural analysis of nonlinear structures over estimates the displacements when the structures is stiffening and under estimate when it is softening (Bradford and Yazdi, 1999). For the conventional method, the number of iteration increases with increase in degree of freedom and these methods need large computer storage for solution of equation of motion. The cable structure belonged to tension structure.

### Equation of motion for a system

The cable structure is multi degree system and the equation of motion for a multi degree system can be written as:

$$M\ddot{X} + C(t)\dot{X} + K(t)X = P(T) \quad (1)$$

Where  $M$  = mass matrix,  $C(t)$  = Damping matrix,  $K(t)$  = stiffness matrix,  $X$  = Displacement vector  $\dot{X}$  = Velocity vector,  $\ddot{X}$  = Acceleration vector,  $P(t)$  = Load vector.

Since  $m$  is a non-zero constant value, both sides of Equation 1 can be divided by  $m$ , and for:

$$P = \frac{C(t)}{M} \quad Q = \frac{K(t)}{M} \quad F = \frac{P(t)}{M}$$

Equation 1 can be written as:

$$\ddot{X} + P\dot{X} + QX = F \quad (2)$$

The mathematical solution of Equation 2 depends on the values of  $P$ ,  $Q$  and  $F$ . Equation 2 is a linear differential equation if  $P$  and  $Q$  are independent of  $x$  and remains so even if  $P$  and  $Q$  are functions of  $t$ .

### THE FLETCHER-REEVES METHOD

This method avoids explicit construction and inversion of the Hessian matrix  $k$ , by using the iterative formula:

$$X_{k+1} = X_k - H_k g_k \quad (3)$$

$$H_k = I + \sum_i^{k-1} A_i \quad (4)$$

$$A_i = \frac{V_i V_i^T}{V_i^T x_i} - \frac{H_i \gamma_i \gamma_i^T H_i}{\gamma_i^T H_i \gamma_i} \quad (5)$$

$$\gamma_i = g_{i+1} - g_i \quad (6)$$

In the first iteration  $H_1 = I$ , the identity matrix. Thus, the first step is in the direction of steepest descent. The slow convergency of the steepest descent method is then overcome by choosing the sequence of  $H$  such that as  $i$  approach  $k$ ,  $H_k$  becomes approximately equal to  $k^{-1}$ . For linear problem the method converges in  $n+1$  steps in which case  $H_{n+1} = k^{-1}$ . It finds the solution to the second equation that is closest to the current estimate and satisfies the curvature condition (Daston, 1979; Farshi and Alinia-Ziazi, 2010). This update maintains the symmetry and positive definiteness of the Hessian matrix. The essential feature of the method is a recursion formula for updating an initial approximation to the Hessian matrix of second partial derivatives of the function to be minimized. The iterative method applied ensures that each step in the procedure leads to a function decrease until a stationary point is reached. The function to be minimized is  $f(x)$  where  $x$  denotes the argument vector of the decision variables  $x_1, x_2, \dots, x_n$ .

### The expression for the total potential energy

The total potential energy is written as:

$$W = U + V \quad (7)$$

Where,  $W$  = the total potential energy;  $U$  = the strain energy of the system, and  $V$  = the potential energy of the loading.

Taking the unloaded position of the assembly as datum:

$$W = \sum_{n=1}^m U_n + \sum_{j=1}^j \sum_{i=1}^3 F_{ji} X_{ji} \quad (8)$$

Where,  $M$  = total number of members,  $J$  = total number of cable joints,  $F_{ji}$  = external applied load on joint  $j$  in direction  $i$ , and  $X_{ji}$  = displacement of joint  $j$  in direction  $i$ .

The condition for structural equilibrium is that the total potential energy of the system is a minimum, and is written as:

$$\partial W / \partial X_{ji} = 0 \quad (9)$$

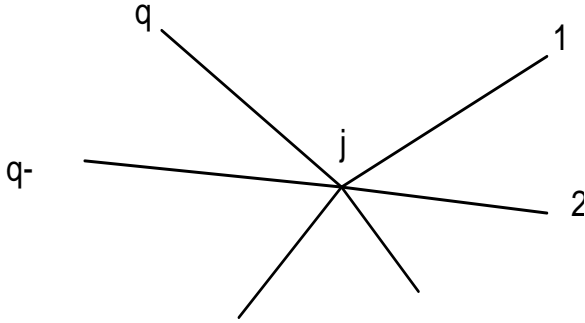


Figure 1. General view member's connection.

Thus, the solution is when the gradient vector of the total potential energy function is zero.

**The gradient of the total potential energy**

Differentiating Equation (9) with respect to  $X_{ji}$  gives the  $g_{ji}$  element of the gradient vector  $g$  as:

$$g_{ji} = \partial W / \partial X_{ji} = \sum_{n=1}^q \partial U_n / \partial X_{ji} - F_{ji} \tag{10}$$

Let,  $T^{°jn}$  the initial tension in member  $jn$ ,  $T^{jn}$  the instantaneous tension in member  $jn$ ,  $e^{jn}$  elastic elongation of member  $jn$ ,  $E$  = young Modulus of Elasticity,  $A$  = cross-sectional area of cable,  $L^{jn}$  length of member  $jn$ , and  $Q$  = number of member meeting at joint  $j$  as shown in Figure 1.

The expression for  $g^{ji}$  can then be written as:

$$g_{ji} = \sum_{n=1}^q \frac{\partial U_n}{\partial e_{jn}} \cdot \frac{\partial e_{jn}}{\partial X_{ji}} - F_{ji} \tag{11}$$

The strain energy of member  $jn$  is given as :

$$U_{jn} = T_{°jn} e_{jn} + \frac{EA}{2L_{jn}} e_{jn}^2 \tag{12}$$

Differentiating  $U_{jn}$  with respect to  $e_{jn}$  yields:

$$\partial U_{jn} / \partial e_{jn} = T_{°jn} + \frac{EA}{L_{jn}} e_{jn} = T_{jn} \tag{13}$$

The initial and elongated length of member  $jn$  may be expressed as:

$$L_{jn}^2 = \sum_{i=1}^3 (X_{ni} - X_{ji})^2 \tag{14}$$

$$(L_{jn} + e_{jn})^2 = \sum_{i=1}^3 (X_{ni} - X_{ji} + x_{ni} - x_{ji})^2 \tag{15}$$

Where  $X_{ji}$  is the coordinate of joint  $j$  in direction  $i$ . Simplifying Equation 15 and substituting for  $L^{jn}$  from Equation 14 yields the following expression for  $e^{jn}$  :

$$e_{jn} = \frac{1}{2L_{jn} + e_{jn}} \sum_{i=1}^3 \{ (X_{ni} - X_{ji}) (2X_{ni} - 2X_{ji} + X_{ni} - X_{ji}) \} \tag{16}$$

Differentiating Equation 10 with respect to  $X_{ji}$  yields:

$$\partial e_{jn} / \partial X_{ji} = \frac{-1}{L_{jn} + e_{jn}} (X_{ni} - X_{ji} + X_{ni} - X_{ji}) \tag{17}$$

Substituting Equations 10 and 17 into Equation 18 yields the expression for the gradient as:

$$g_{ji} = - \sum_{n=1}^q t_{jn} (X_{ni} - X_{ji} + X_{ni} - X_{ji}) - F_{ji} \tag{18}$$

Where  $t_{jn} = T_{jn} / (L_{jn} + e_{jn})$  is the tension coefficient of member  $jn$ .

**Total potential energy in the direction of descent**

The correct value of  $X$  for which  $W$  is a minimum can be found by the iterative process:

$$X_{ji(k+1)} = X_{ji(k)} + S_{(k)} V_{ji(k)} \tag{19}$$

Where the suffices  $(k)$  and  $(k+1)$  denote the  $(k)$ th and  $(k+1)$ th iterate respectively and where  $V_{ji}$  is the element of the direction vector,

and  $S^{(k)}$  = the step length which defines the position along  $V_{ji(k)}$  where the total potential energy is a minimum. The expression for  $V_{ji}$  is, if the Fletcher-Reeves formulation method is used (Fletcher, 2007) given by:

$$V_{ji(k)} = -g_{ji(k)} + \frac{\sum_{j=1}^J \sum_{i=1}^3 g_{ji(k)} g_{ji(k)}}{\sum_{j=1}^J \sum_{i=1}^3 g_{ji(k-1)} g_{ji(k-1)}} V_{ji(k-1)} \tag{20}$$

The stationary point in the direction of descent can be found by expressing the total potential energy as a function of the step length along  $V_{ji}$ . Thus the required value of  $S^{(k)}$  can be determined by the condition and is given (Gloeckner et al., 1976):

$$\partial W_{(k)} / \partial S_{(k)} = 0 \tag{21}$$

**Calculation of the step length**

The required polynomial for step length is found by substituting the expression for  $X_{ji(k+1)}$  given by Equation 19 into a suitable expression for the total potential energy  $w$ . Writing the strain energy term in Equation 16 as a function of the elongation, Equation 21, and at the same time substituting for  $X_{ji}$  using Equation 19 lead to the first expression for the elongation as a function of  $S$  as given by Kirsch and Bogomolni (2007):

$$e_{jn} = \frac{1}{2L_{jn} + e_{jn}} (a_1 + a_2 S + a_3 S^2)$$

$$a_1 = \sum_{i=1}^3 (2(x_{ni} - x_{ji})(x_{ni} - x_{ji}) + (x_{ni} - x_{ji})(x_{ni} - x_{ji})) \tag{22}$$

$$a_2 = \sum_{i=1}^3 2((x_{ni} - x_{ji} + x_{ni} - x_{ji})(v_{ni} - v_{ji}))$$

$$a_3 = \sum_{i=1}^3 (v_{ni} - v_{ji})^2$$

And secondly to the expression for  $W$  in terms of the step length  $S$  and its derivative with respect to  $S$  as given below:

$$W = C_1 S^4 + C_2 S^3 + C_3 S^2 + C_4 S + C_5 \tag{23}$$

$$\partial W / \partial S = 4C_1 S^3 + 3C_2 S^2 + 2C_3 S + C_4 \tag{24}$$

Where,

$$C_1 = \sum_{n=1}^m \left( \frac{EA}{2L(2L+e)^2} a_3^2 \right)_n$$

$$C_2 = \sum_{n=1}^m \left( \frac{EA}{L(2L+e)^2} a_2 a_3 \right)_n$$

$$C_3 = \sum_{n=1}^m \left( \frac{T_o}{2L+e} a_3 + \frac{EA}{2L(2L+e)^2} (a_2^2 + 2a_1 a_3) \right)_n$$

$$C_4 = \sum_{n=1}^m \left( \frac{T_o}{2L+e} a_2 + \frac{EA}{L(2L+e)^2} a_1 a_2 \right)_n - \sum_{j=1}^J \sum_{i=1}^3 F_{ji} V_{ji}$$

$$C_5 = \sum_{n=1}^m \left( \frac{T_o}{2L+e} a_1 + \frac{EA}{2L(2L+e)^2} a_1^2 \right)_n - \sum_{j=1}^J \sum_{i=1}^3 F_{ji} X_{ji}$$

**NUMERICAL AND EXPERIMENTAL TESTING**

The analytical method is used to experiment with mathematical model and experimental work.

**Theoretical**

**Theoretical analysis (Mathematical modelling)**

The theoretical result based on proposed theory is calculated by the structural property matrices below for a pin jointed member with three degrees of freedom at each end as follows;

**The lumped mass matrices for a pin jointed member**

$$\frac{\bar{m} L}{3} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

Where  $\bar{m}$  is the mass and  $L$  is the length of member.

**The orthogonal damping matrices**

This damping matrix in which as many modes can be given by:

$$C = M \left( \sum_{n=1}^N \frac{2\varepsilon_n \omega_n}{\phi_n^T M \phi_n} \phi_n \phi_n^T \right) M \tag{26}$$

Where,  $n$  = the mode number;  $\phi_n$  = the  $n$ th mode shape vector;  $M$  = diagonal mass matrix.

**The stiffness matrix for a pin jointed member**

$$= \frac{EA}{L} \begin{bmatrix} \lambda_1^2 & \lambda_1 \lambda_2 & \lambda_1 \lambda_3 \\ \lambda_2 \lambda_1 & \lambda_2^2 & \lambda_2 \lambda_3 \\ \lambda_3 \lambda_1 & \lambda_3 \lambda_2 & \lambda_3^2 \end{bmatrix} + \frac{T}{L} \begin{bmatrix} 1 - \lambda_1^2 & -\lambda_1 \lambda_2 & -\lambda_1 \lambda_3 \\ -\lambda_2 \lambda_1 & 1 - \lambda_2^2 & -\lambda_2 \lambda_3 \\ -\lambda_3 \lambda_1 & -\lambda_3 \lambda_2 & 1 - \lambda_3^2 \end{bmatrix} \tag{27}$$

Where,  $T$  is the axial force in the axial force and  $\lambda_1, \lambda_2$  and  $\lambda_3$  are the corresponding direction cosines.

**Experimental work**

The mathematical model chosen is a 7\*5 flat net with 105° of freedom. The 7\*5 net was built as an experimental model and tested in order to verify the static and dynamic nonlinear Fletcher-Reeves theory. The construction of the experimental model is shown in Figure 2.

The specifications of erected rectangular net and cables are given in Table 1. Each steel cable was initially tensioned to about 1 KN and then left for 2 weeks to permit the individual wires in the strands to bed in. Then, the tensions on the cables were readjusted to 11.5 KN. This tension was maintained throughout the test

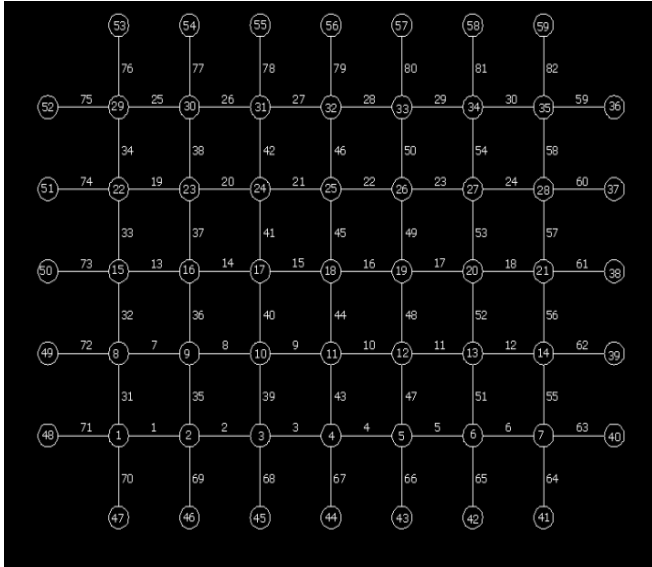


Figure 2. Grid lines of the flat net.

Table 1. The specifications of flat net and cables.

Description	Details
Overall dimensions	3000*4000
Spacing of the cables	mm
Spacing of the cables	500 mm
Number of free joints	35
Number of fixed joints	24
Number of links	82
Diameter(mm)	15.34
Section Area (mm <sup>2</sup> )	142.90
Y/Strength 1% (kN)	244.40
Young's Modulus	192.60 KN/mm <sup>2</sup>
Y/Strength	244.40 KN
Pretension	11500 N/link



Figure 3. Construction of frame steel.

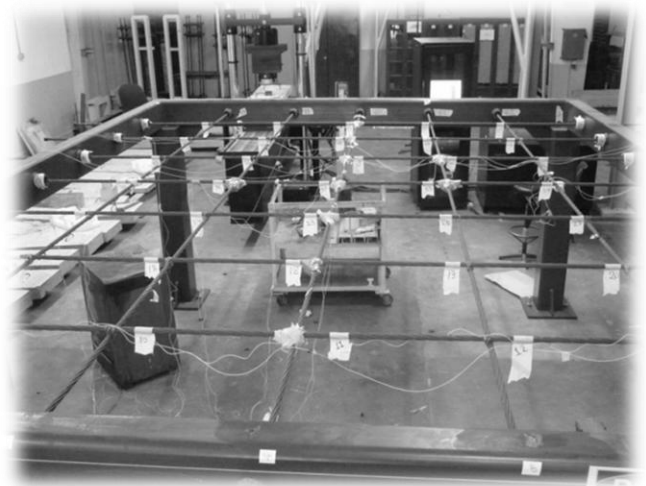


Figure 4. General view of steel frame.

Table 2. Features of steel frame made.

Frame supported specification	
Column	1400 mm (box) height
Beam	300 mm x 400 mm (box) length
Beam size	100 x 200 x 9 mm (hollow section)
Column size	200 x 200 x 9 mm (hollow section)

programme by checking at interval times. The wedge and barrel used on hollow cylindrical steel to provide endcaster degree of freedom for boundary condition of cables. Endcaster joints are used to fix boundary condition. General view of steel frame is shown in Figures 3 and 4. Specifications of steel frame made are given in Table 2.

The material is homogeneous and isotropic. The stress strain relationship of all material remains within the linear elastic range during the whole nonlinear response. The external loads are displacement independent.

## RESULTS AND DISCUSSION

### Static test

Any deficiency in the model could influence the dynamic behavior and make subsequent comparison of experimental and theoretical values difficult. Hence, a Static test is carried out to investigate the degree of symmetric behavior on the frame. The investigation consisted of checking the degree of symmetric behavior about the major and minor axes. The degree of symmetric behavior about the minor axis is investigated by first placing an increasing load on joint 11 and then compares the resultant displacement with those obtained by placing similar loads on joint 25. The degree of symmetric behavior about the major axis is similarly studied by loading first joint 16 and then joint 20. Figure 5



Figure 5. Linear variable differential transformer used on steel frame.

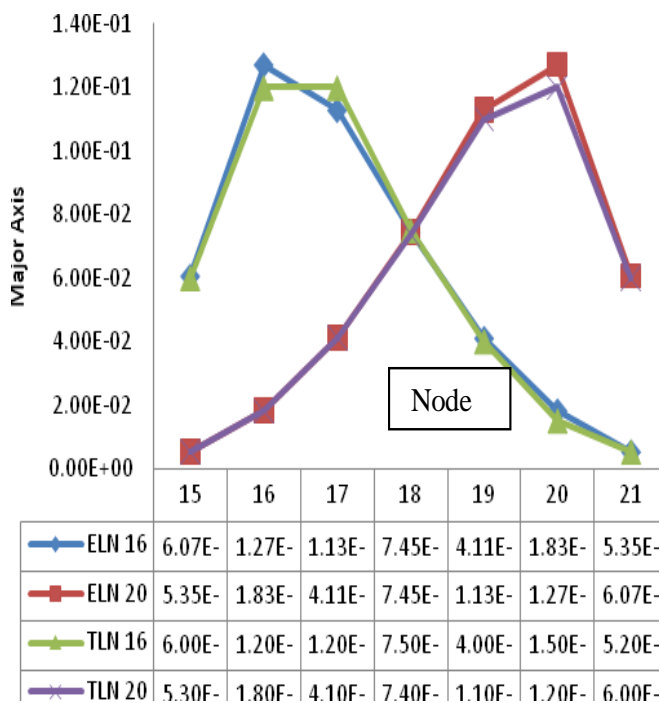


Figure 6. Degree of symmetric about major axis when the load is placed on node 16 and 20. ELN 16, 20: Experimental result of load on nodes 16, 20; TLN 16, 20: Theoretical result of load on nodes 16, 20.

shows the relationship between loads and deflection in major axis.

When the concentrated load is placed on node 20, the deflection gradually increased from 0.535 mm on node 15 it reached a peak of 12.7 mm on node 20. From this point

onwards, it is projected to drop sharply until it reached 0.607 mm on the node 15. When concentrated load is placed on node 16, the deflection from about 0.607 mm on node 15 rapidly rose to reach a peak of 12.7 mm on node 16. From this point onwards, it is project to fell slightly until it reached 0.535 mm units on node 21. Degree of symmetry about the major, minor axes Joint 11 (Figure 6), deflections due to concentrated load on joint 11 is given in Table 3.

The values between the calculated and measured static deflections are in the same value to each other. A static test checked the stiffness of the boundary and then shows the degree of error for any elastic deformation of the frame is zero. The result verifies the frame is symmetric. Test with different pattern and intensities of static loading in order to compare the experimental and theoretical values of the static deformation showed that the deflection calculated by the proposed nonlinear method gives reasonably accurate results.

### Conclusion

The values between the calculated and measured static deflections are in good agreement. The comparison of experimental and theoretically predicted values of dynamic response shows that the response calculated by the proposed nonlinear method gives reasonably accurate results. The proposed method was found to be stable for time steps equal to or less than half the smallest time period of the system. The experimental work carried out by static and dynamic testing of the flat net showed good agreement between the experimental result and theoretically predicted values. The percentage

**Table 3.** Degree of symmetry about the major, minor axes Joint 11, deflections due to concentrated load on joint 11.

Load (n) = 2400	Theoretical (t) z axis (m)	Experimental (e) z axis (m)	( t – e ) / t*100
Deflections node 18	127.9e-03	125.2e-03	2.11
Deflections node 11	142.3e-03	141.5e-03	0.56
Deflections node 4	67.78e-03	65.28e-03	3.69
Deflections node 25	78.68e-03	77.28e-03	1.78
Deflections node 32	29.54e-0	29.24e-0	1.02
Deflections node 15	20.70e-03	20.32e-03	1.84
Deflections node 16	54.46e-03	53.23e-03	2.26
Deflections node 17	104.2e-03	101.5e-03	2.59
Deflections node 19	104.2e-03	102.1e-03	2.02

differences between the theoretical and experimental results did not in any case exceed 10%. This is thought to be acceptable.

Finally, it be concluded that, the Fletcher-Reeves algorithm is the more efficient in terms of computing time and storage practically in high nonlinear structures.

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