### Full Length Research Paper

# Importance of spatial variability of seismic ground motion effects on long beams response

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Rigorous seismic analysis of long structures requires a complete description of the free field ground motion. At a given site, the recorded motions at distinct points are different; this may induced by loss of seismic wave's coherency, wave passage effect, site response effect and attenuation of seismic waves. In this paper, the effects spatial variability of ground motion (SVGM) on the stochastic response of structures subjected to this phenomenon (wave propagation and incoherence), using response spectrum method are analysed. The responses are evaluated along a two-span beam. After detecting the critical sections, the responses at these sections are studied in more details according to the fundamental period of the beam. SVGM have beneficial effects on displacement of structure founded on stiff soil and unbeneficial effects in presence of soft soil. The interplay between dynamic and pseudo-static responses may either control how the spatial variability ground motion excitation effects are favourable or not depending on different sections along the two spans beam and the rigidity of the soil.

Key words: Spatial variability, mean maximum response, two-span beam, soil, fundamental period.

#### INTRODUCTION

A rigorous seismic analysis of extended structures such as bridges, dams and large industrial buildings require a rational knowledge of the free surface seismic motion. Recent observations during earthquakes showed that at a given site, the recorded movements at distinct points are different, in amplitude, duration and frequency content (Loh et al., 1982; Harichandran and Vanmarke, 1986; Abrahamson et al., 1991; Pitilakis et al., 1994). Important advances in computing power in the early 1990s allowed more involved studies to be performed. The spatial variability models of excitation allowed phenomena which are responsible for the variability of seismic movements to be incorporated into studies using a stochastic field based on random vibration theory based on the Luco and Wong coherency function (Luco and Wong, 1986).

Several other coherence functions have been proposed (Oliveira et al., 1991; Kiureghian et al., 1992). Kiureghian

suggests that several phenomena are responsible for the spatial variability of seismic movements (Kiureghian, 1996). The first one is the difference in arrival times of seismic waves to different recording stations. The second is the loss of coherency of the seismic movement due to the multiple refractions and reflections of waves in the heterogeneous media. The third phenomenon is the difference in soil mechanical properties under different points of recording, which induce seismic movements characterised by different spectral amplitudes and frequency contents. Finally, the last phenomenon that causes spatial variability is attenuation of seismic waves due to soil damping as well as the dissipation of energy affecting the seismic wave amplitude.

The effects of the spatial variability of seismic excitation on structure response subjected to such excitations are important and cannot be neglected (Abdelghaffar and Rubin, 1982; Zerva et al., 1988; Hao, 1989; Zerva, 1990; Harichandran and Wang, 1990; Berrah and Kausel, 1992; Kiureghian and Neuenhofer 1992; Monti et al., 1994; Haricahndran et al., 1996; Kiureghian et al., 1997; Kahan et al., 1998; Sextos, 2001; Sextos et al., 2003; Zanardo et al., 2002; Chouw and Hao, 2005; Lupoi et al., 2005). One

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of the studies based on the random vibration theory approach done by Zerva (1990) evaluates the structural response of beams with varied lengths for seismic excitation with cases of incoherence. The results showed that the effects of spatial variability of ground motion on response are very complex. Either increasing or decreasing the structural response depends on many parameters.

Results tend to conclude that the more the ground excitation differs from support to another the more the structure is subjected to lower inertial forces and therefore its dynamic response is favourable. The reduction of inertial forces is due to pseudo-static forces increase. The combination of the inertial forces reduction and pseudo-static forces increase may be conservative or unconservative, depending on the structural configuration and properties of the structure (structural system, dynamic characteristics,...) as well as on the ground motion characterization (apparent wave velocity, soil conditions,...) (Zerva et al., 1988; Hao 1989; Kiureghian et al., 1997; Monti et al., 1994; Zanardo et al., 2002)

Also, multiple support excitations may trigger higher modes of structure vibrations and in some cases the dynamic response is dominated by such modes (Calvi and Pinto, 1996; Kahan et al., 1998; Price and Eberahard, 1998; Sextos, 2001).

Although significant aspects of the effects of spatial variability have been already clarified, there is still a need for more research, which would contribute to a more refined and reliable seismic design of long structures. The present paper describes a parametric study on the effects of ground motion spatial variability on the response of long span structures (continuous two-span beam).

The attention is focused on wave propagation and incoherence effects on stochastic response of extended structures. The approach takes into account the cross-correlation terms between both the participant modes and the support excitations in the response calculation. A general formulation of maximum response including the dynamic and pseudo-static components as well as the cross-correlation between them is used. Finally, parametric study is performed to show the response at the previously determined critical sections.

#### **EQUATIONS OF MOTION**

The following hypotheses are considered in this work:

- 1. The seismic input is considered probabilistic, and propagates in homogeneous random field. The excitation and the response are stationary, Gaussian with zero mean value;
- 2. The spatial variation of the seismic movement is characterised by the power cross-spectral density function of soil accelerations. This function is related to the power auto-spectral density function (PSD) by the coherency function;

3. The PSD is supposed to be the same in all supports as soils conditions are identical.

The dynamic part of the structure response subjected to multiple support excitations may be obtained from the differential equations of the motion (Clough and Penzien, 1993):

$$M\ddot{X} + C\dot{X} + KX = \left(MR + M_c\right) \ddot{u} \left(CR + C_c\right) \dot{u} \tag{1}$$

with: 
$$X = X^t$$
  $X^s$ ,  $X^s = Ru$  and  $R = K^1 K_c$ 

Where  $x^t$  is the total displacement vector of the structure degrees of freedom (free degrees); x is the dynamic component of the displacement;  $x^s$  is the pseudo-static component, expressed according to the influence matrix R and the displacement of degrees of freedom u attached to the soil; M, C and K are respectively mass, damping and stiffness matrices associated to structure degrees of freedom;  $M_c$ ,  $C_c$  and  $K_c$  are respectively mass, damping and stiffness coupling matrices associated to the imposed degrees of freedom.

An approximation of the right-hand member of equation (1) is introduced: the quantity  $(CR + C_c)\dot{u}$  can be neglected compared to inertial forces.

The response spectrum method requires the modal approach. Using the transformation  $x = \varphi$  y in equation (1), where  $\varphi$  is the modal matrix and y is the generalised displacement vector, the uncoupled equations of the motion are obtained:

$$\ddot{y}_{i} + 2\varsigma_{i}\omega_{i}\dot{y}_{i} + \omega_{i}^{2}y_{i} = \sum_{k=1}^{m}\beta_{ki}\ddot{u}_{k}$$
 (2)

 $\omega_i$ ,  $\varsigma_i$  i=1, n are natural frequencies and modal damping ratios of the structure, and m is the number of supports of the structure,

 $\beta_{ij}$  is the modal participation factor given

by 
$$\beta_{ki} = \frac{\varphi_i^T (Mr_k + M_c i_k)}{\varphi_i^T M \varphi_i}$$
,

 $r_{k}$  is the  $k^{th}$  column of the influence matrix R, and  $i_{k}$  is the  $k^{th}$  column of the identity matrix.

Let us define the normalised modal response  $s_{ki}(t)$  as  $y_{i}(t) = \sum_{k=1}^{m} \beta_{ki} s_{ki}(t)$ , thus rewriting (2):

$$\ddot{S}_{ki} + 2\zeta_i \omega_i \dot{S}_{ki} + \omega_i^2 S_{ki} = \ddot{U}_k \tag{3}$$

The responses of the structure z(t) (nodal displacement, strain component, internal efforts ...) are linearly expressed

according to the total displacement:

$$Z(t) = q^{\mathsf{T}} X^{t}(t) = q^{\mathsf{T}} (X^{s} + X)$$
(4)

where q is a vector dependent of the geometry and mechanical properties of the structure.

Substituting the pseudo-static and dynamic component by their values, equation (4) becomes:

$$Z(t) = \sum_{k=1}^{m} a_k u_k(t) + \sum_{k=1}^{m} \sum_{i=1}^{n} b_{ki} s_{ki}(t)$$
 (5)

where  $a_k$  is the effective influence coefficient, and  $b_{ki}$  is the effective modal participation factor, given respectively by:

$$a_{k} = q^{\mathsf{T}} r_{k}$$

$$b_{ki} = q^{\mathsf{T}} \varphi_{i} \beta_{ki}$$
(6)

Based on random vibration theory, the response spectra method incorporating the spatial variability of ground excitation developed by Der Kiureghian and Neuenhofer (1992), provides a combination rules include the contributions of cross-correlation between: (1) modes, (2) movements of supports and (3) the pseudo-static and dynamic response component. It is assumed that the excitations are stationary stochastic processes with zero mean. The peak response mean value is obtained by multiplying mean square response by the corresponding peak factor. In general, peak factors depend on the characteristics of each process and their ratios are around unity (Der Kiureghian and Neuenhofer, 1992), thus the mean maximal response value is given by:

$$E[\max|z(t)|] = \sum_{k=1}^{m} \sum_{l=1}^{m} \sum_{i=1}^{m} a_{k} a_{l} \rho_{u_{k}u_{l}} u_{k,\max} u_{l,\max} + 2 \sum_{k=1}^{m} \sum_{l=1}^{m} \sum_{j=1}^{m} \sum_{i=1}^{m} a_{k} b_{lj} \rho_{u_{k}s_{lj}} u_{k,\max} D_{l}(\omega_{j}, \varsigma_{j}) + \sum_{k=1}^{m} \sum_{l=1}^{m} \sum_{i=1}^{m} \sum_{j=1}^{m} b_{ki} b_{lj} \rho_{s_{ki}s_{lj}} D_{k}(\omega_{i}, \varsigma_{i}) D_{l}(\omega_{j}, \varsigma_{j}) \right]^{1/2}$$

$$(7)$$

In which:

 $u_{k,\max}$  is the mean value of the maximum ground displacement at station k,  $D_k(\omega_i, \varsigma_i)$  is the ordinate of the displacement response spectrum associated to the support k of the mode i.  $\rho_{u_k u_i}$  is the cross-correlation coefficient for the ground displacements at two stations k and l. It depends on the cross-power spectral density of displacements  $u_k$  and  $u_i$  It is equal to unity in the case of uniform excitation and insignificant for stiff soil conditions without site effects (Der Kiureghian and Neuenhofer, 1992), and defined by:

$$\rho_{u_k u_l} = \frac{1}{\sigma_{u_k} \sigma_{u_l}} \int_{-\infty}^{+\infty} S_{u_k u_l}(\omega) d\omega$$
(8)

 $\rho_{u_k s_{ij}}$  is the cross-correlation coefficient between the displacement at support k and the response of oscillator  $(\omega_j, \zeta_j)$  at support l. It depends on the dynamic properties of both the soil and the oscillator, and is given by:

$$\rho_{u_k s_{ij}} = \frac{1}{\sigma_{u_k} \sigma_{s_{ij}}} \int_{-\infty}^{+\infty} H_j(-\omega) S_{u_k u_i}(\omega) d\omega$$
(9)

 $\rho_{s_u s_y}$  is the cross-correlation coefficient between the two oscillators  $(\omega_i, \zeta_i)$  and  $(\omega_i, \zeta_i)$ . It depends on the power spectral density (PSD) of individual support accelerations  $\ddot{u}_k$  and  $\ddot{u}_i$ , and is given by:

$$\rho_{s_{ki}s_{lj}} = \frac{1}{\sigma_{s_{ki}}\sigma_{s_{lj}}} \int_{-\infty}^{+\infty} H_i(\omega)H_j(\omega)S_{u_ku_l}(\omega)d\omega$$
(10)

 $\sigma_{u_k}$  is the variance of soil displacement  $u_k(t)$  given by:

$$\sigma_{u_k}^2 = \int_{-\infty}^{+\infty} S_{u_k u_k}(\omega) d\omega \tag{11}$$

 $\sigma_{s_{ki}}$  the variance of normalised modal response  $s_{ki}(t)$  given by:

$$\sigma_{s_{ki}}^{2} = \int_{-\infty}^{+\infty} |H_{i}(\omega)|^{2} S_{u_{k}u_{k}}(\omega) d\omega$$
(12)

 $H_i(\omega)$  is the model frequency response function:  $H_i(\omega) = (\omega^2 + 2i\zeta_i\omega_i\omega)^{-1}$ 

 $S_{xy}(\omega)$  is the cross-power spectral density of processes x and y. The auto power spectral densities are the same in case of identical site conditions.  $S_{xy}(\omega)$  is related to auto PSD by the relation:

$$S_{\ddot{u}_{k}\ddot{u}_{l}}(\omega) = \gamma_{kl}(d_{kl}, \omega)S_{\ddot{u}_{q}}(\omega)$$
(13)

we note that:

$$S_{u_k u_l}(\omega) = \omega^4 S_{\ddot{u}_k \ddot{u}_l}(\omega) \tag{14}$$

$$S_{u_k \bar{u}_l}(\omega) = \omega^2 S_{\bar{u}_k \bar{u}_l}(\omega)$$
 (15)

 $\gamma_{_{kl}}(d_{_{kl}},\omega)$  is the coherency function characterizing the spatial variability of the ground motion between the stations k and l. Generally, it is complex value, which modulus characterises the incoherence effect, and the phase angle characterises the wave passage effect. It depends on soil frequency and distance between stations

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The first term on the right-hand side of equation 7 with double summations over the support degrees of freedom k and l is the pseudo static component of the response, the term with summations over the support degrees of freedom and modes of structure is the dynamic component and the second term is the dynamic component arising from the covariance between the pseudo static and dynamic components. This response spectrum method employs a simple combination rule in terms of mean value of peak ground displacement ( $u_{k,\max}$ ,  $u_{l,\max}$ ) and the displacement response spectra ( $D_k(\omega_i,\varsigma_i)$ ,  $D_l(\omega_i,\varsigma_i)$  at supports k and l

#### **APPLICATION**

The studied structure is a continuous two-span beam, which has uniform mass and stiffness properties.

The Natural frequencies of the system are (Clough and Penzien, 1993):

$$\omega_{j} = \left[\frac{2\pi}{L}\right]^{2} \sqrt{\frac{EI}{ml}} \left(\frac{1}{4} \left[2j + 1 + \frac{1 - (-1)j}{2}\right]\right)^{2}_{j=1,n}$$

and the modal shape functions are:

$$\varphi_j = \sin \frac{2\lambda_j x}{L} \frac{\sin \lambda_j}{\sinh \lambda_i} \sinh \frac{2\lambda_j x}{L}$$
 j=1,n.

Where *EI* denotes the flexural rigidity of the beam, *mI* the mass per unit length, *L* is the length of the span, and

$$\lambda_j = \frac{L}{2} \sqrt{\omega_j} \left( \frac{ml}{EI} \right)^{\frac{1}{4}}$$

The pseudo-static effective influence factors are third degree polynomials.

The model of the PSD function of the support acceleration is the Kanai-Tajimi modified model (Clough and Penzien, 1993):

$$S_{u_s}(\omega) = S_0 \frac{1 + 4\zeta_s^2 \left(\frac{\omega}{\omega_s}\right)^2}{\left(1 - \left(\frac{\omega}{\omega_s}\right)^2\right)^2 + 4\zeta_s^2 \left(\frac{\omega}{\omega_s}\right)^2} \frac{\left(\frac{\omega}{\omega_f}\right)^4}{\left(1 - \left(\frac{\omega}{\omega_f}\right)^2\right)^2 + 4\zeta_f^2 \left(\frac{\omega}{\omega_f}\right)^2}$$
(16)

Where  $S_0$  is the white process bedrock excitation amplitude, which value is taken so that the maximal acceleration does not exceed 0,35g,  $\omega_g$  and  $\zeta_g$  are the soil dynamic characteristics;  $\zeta_f$  and  $\omega_f$  are parameters of the corrective filter, given by (Der Kiureghian et al., 1997).

For the stiff soil,  $\omega_g=15$  rad/s,  $\zeta_g=0.6$ ,  $\omega_f=1.5$  rad/s and  $\zeta_f=0.6$  and for the soft soil,  $\omega_g=5$  rad/s,  $\zeta_g=0.2$ ,  $\omega_f=0.5$  rad/sand  $\zeta_f=0.6$  The model of coherency function is assumed to have the following form (Luco and Wong, 1986):

$$\gamma(\omega, d_{kl}) = \exp\left[-(\alpha \frac{\omega d_{kl}}{v_s})^2\right] \exp\left(i\omega \frac{d_{kl}^L}{v_{app}}\right)$$
(17)

Where  $\alpha$  is the incoherence factor,  $V_s$  the shear wave velocity,  $V_{app}$  the apparent velocity of the predominant wave,  $d_{kl}$  the horizontal distance between stations k and l, and  $d_{kl}^{L}$  is its projection along the longitudinal direction of wave propagation.

In the case of analyzing the importance of wave passage effect (case 2), it is assumed that the wave propagates from the support k to the support I and that  $d_{ij}^{L} = d_{ij}$ . In the case 3, it is assumed

vertically propagating waves ( $^{V_{app}}=\infty$ ). The following values are used in the study: wave velocities are 100 m/s in soft soil, and 500 m/s in stiff soil, structure fundamental period is 1s with a span length of 100 m and modal damping ratio equal to 5%.

Five cases are considered in the present study:

Case 1: uniform (fully coherent) excitation at all the supports  $(\gamma(\omega, d_{kl}) = 1)$ ;

Case 2: only wave passage effects ( $\alpha$ =0);

Case 3: only incoherence effects ( $V_{app} = \infty$ );

Case 4: both effects wave passage and incoherence ( $\alpha$ =0,25);

Case 5: support motions are mutually independent  $(\gamma(\omega, d_{\omega}) = 0)$ 

The results are shown in term of normalised mean values of the maximum (peak) responses (displacement, bending moment and shear forces) to different excitation cases in relation to uniform response (case 1). From hereafter the following abbreviations are used: MMXR where MM is mean of maxima, X is a variable describing the component (D - dynamic, DS- dynamo-static, PS - pseudo-static, and T - total), and R is the response (D - displacement, M - bending moment, and T - shear forces).

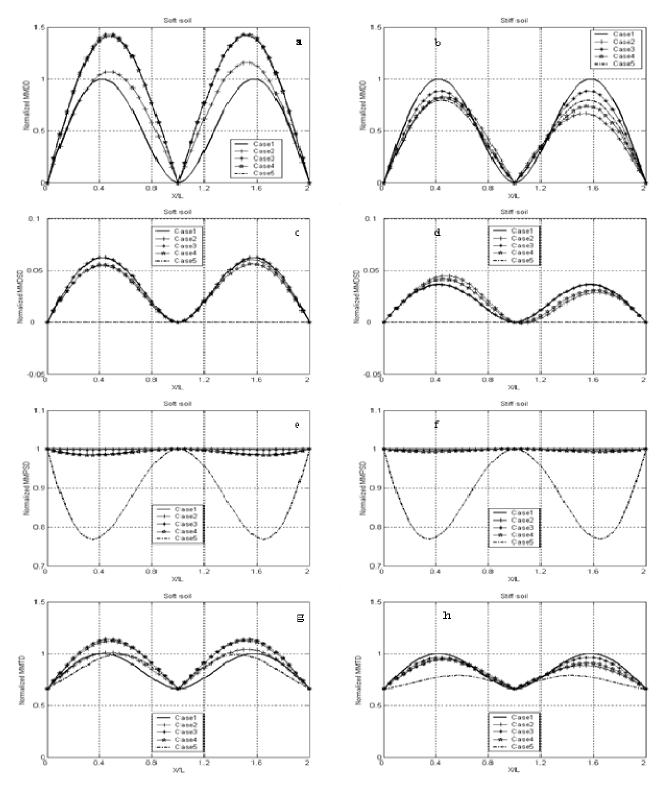
#### Spatial variability effects along the beam

#### Effects on normalised mean maximum displacement

The effects of the spatial variability of ground motion tend to increase the dynamic component in case of soft soil and reduce it in the case of stiff soil (Figures 1a and b). The decrease of dynamic component was related by some authors (Hao, 1989; Monti et al., 1994; Der Kiureghian et al., 1997) and it is due to the presence of stiff soil (Der Kiureghian et al. (1997) consider a stiff soil "type 2" recommended by SEAOC). Due to the symmetry of the beam and lack of the wave passage effect, responses in the cases 1, 3 and 5 are symmetric. In all cases, the maximum values are found at mid-spans.

The dynamo-static component (Figures 1c and 1d) is nil in the case of mutually independent excitation (case 5) while it contributes to the total displacement in all other cases with 6 - 7% (of dynamic component) for soft soil and less than 5% for stiff soil. This component is more sensitive to incoherence effect in case of stiff soil in term of loosing symmetry.

The pseudo-static shown in Figures 1e and 1f is constant in case 1 because of rigid body motion and reduced in the other cases (differential displacement). The normalisation here is done in relation to this case. The increase in differential displacement is maximal in case 5



**Figure 1.** Normalised mean maximum displacement along the two-span beam (a - b: dynamic component, c - d: dynamo-static component, e - f: pseudo-static component and g - h: total displacement).

induces more sensitivity of this component to spatial variability of ground motion.

The total maximum displacement on Figures 1g and h

is qualitatively the same as the dynamic component. In the case of soft soil, the uniform excitation (case 1) reduces the total displacement, while the incoherence effect of excitation (cases 3 and 4) increases it. In case of stiff soil, the contrary is observed. Note that the total displacement is dominated by the pseudo static component. Displacement induced by uniform excitation can be unconservative in case of continuous long beam models founded on soft soil. Analogous results have been obtained by Saxena et al. (2000). The determination of the pseudo static component is essential because of its domination in total response as recommended by Abbas and Manohar (2002) and it causes differential displacements along the supports which may cause failure of structures (Nuti and Vanzi, 2005).

## Effects on normalised mean maximum bending moment

For the dynamic bending moment component, uniform excitation overestimates the response around the middle support whatever the soil, it undere stimates it at midspan for soft soil and overestimates it for stiff soil. The dvnamo-static component (Figures 2c and contributes to the total response with negative peaks around the middle support and positive values elsewhere. Dynamic and pseudo-static bending moment components are negatively correlated. The maximum negative contribu-tion is due to uniform excitation, around 10% of dynamic component for soft soil and 7% for stiff soil. Totally, incoherent excitation (case 5) produces nil dynamo-static components and maximum pseudo static component; therefore, in this case the total mean maximum bending moment is conservative along all the beam for soft soil and only around the middle support for stiff soil (Figures 2g and h). Also, as the pseudo-static bending moment component is almost nil in case 1, the normalization is done according to the dynamic component. The pseudo-static response under case 5 is very important (around 30%) compared to other cases (around 5%). Generally, bending moment induced by uniform excitation can be unconservative along the beam founded on soft soil in case of mutually independent ground motion. In other cases (2 - 4), the increased contribution of pseudo static component is significant only around mid spans. The differences between case 1 and others cases are reduced around the middle support compared to the dynamic component because of the negative contribution of the dynamo static component. Analogous results have been obtained in many studies; the case of totally inde-pendent excitations increases the total bending moment compared to uniform excitation especially for beams founded on soft structures (Dumanoglu and Soyluk, 2003).

#### Effects on normalised mean maximum shear forces

The dynamic component (Figures 3a and 3b) under case 1 overestimates the other cases for both soft and stiff soil.

The maximum values are around the middle support in all cases. Dynamic and pseudo-static shear forces are negatively correlated around the middle support (Figures 3c and 3d); the correlation is maximal in case 1, minimal in case 3 and nil in case 5. Maximum pseudo-static component (Figures 3e and 3f) is around 8% of dynamic component in case 5 and nil in case 1. The total mean maximum shear forces (Figures 3g and 3h) is slightly greater under case 5 in presence of soft soil, especially around the middle support and stays lower than the response under uniform excitation in case of stiff soil.

#### Spatial variability effects at critical sections

After analyzing the effects of the spatial variability (various cases of excitation) on the mean maximal responses along the beam and detecting the critical sections, it is interesting to study in more details the responses at these sections when the fundamental period of the beam varies.

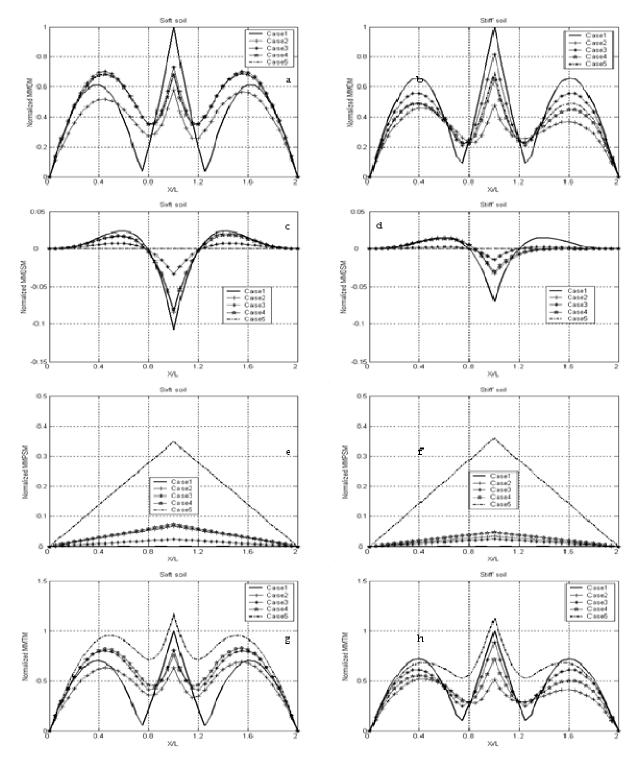
#### Effects on normalised mean maximum displacement

Figure 4 shows displacement components due to different spatial variability effects for both soft and stiff soils at middle of first span (X/L = 1/2). In all cases, for structures with fundamental period less than 1 s, the dynamic response is the same (Figures 4a and b). It becomes different only for soft soil and higher periods. The effect of wave passage is more important in the 1-2 s bands, where the response is overestimated. Beyond this period, the response under incoherence effect dominates especially in case 3 for which the incoherence is taken alone.

The curves present slight inflexion points in the vicinity of the ground period (  $^{T_g}=2\pi/\omega_g$  ), around 0.42 s for stiff soil and 1.26 s. for soft soil). This inflexion is accentuated in the case of soft soil, which means that the response of large structures (bridges) founded on soft soil exhibits weak variations for fundamental periods higher than that of the soil, to the contrary of the stiff soil case where the difference between cases remains constant. Dynamo-static displacements are more important in case 3 (Figures 4c and d). They are weak compared to the dynamic component. They appear more insignificant for rigid structures and more when the structure is founded on stiff soil.

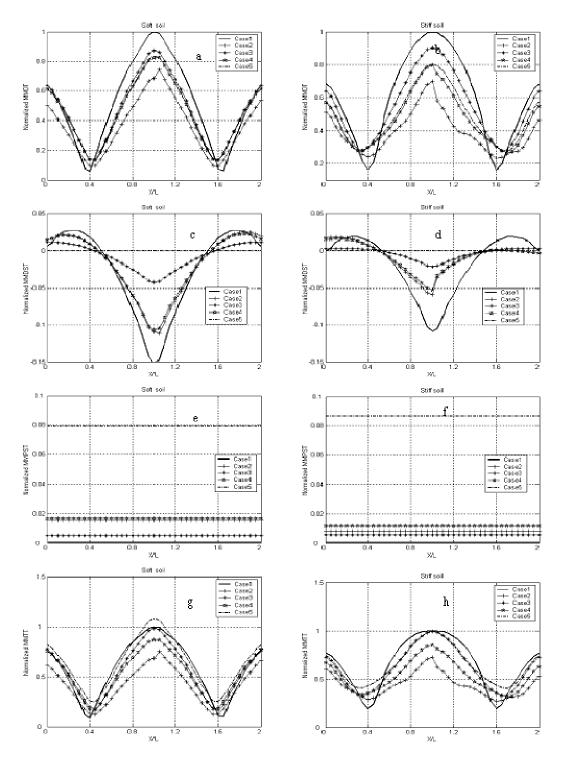
The pseudo-static components are constant in all cases (Figures 4e and 4f); they are overestimated compared to the case 1 on soft soil.

For soft soil, the effects of spatial variability are important, the dynamic mean maximum total displacement increases when those effects (cases 2, 3 and 4) are taken into consideration (Figures 4g and 4h) for different structures fundamental periods. For structures on stiff



**Figure 2.** Normalised mean maximum bending moment along the two-span beam (a - b: dynamic component, c - d: dynamo-static component, e - f: pseudo-static component and g - h: total bending moment).

soil, the effects appear important for flexible structures and can be neglected for rigid structures. In soft conditions of soil, the total displacement at mid first span is overestimated independently of the structure rigidity under cases 2 - 4 and totally incoherent ground motion reduces the total displacement. In presence of stiff soil, the difference between different cases of excitation appears only for flexible structures.

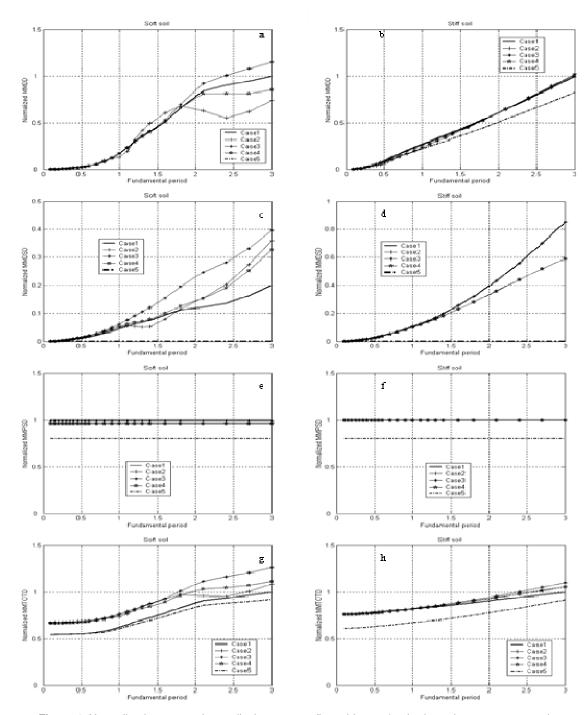


**Figure 3.** Normalised mean maximum shear forces along the two-span beam (a - b: dynamic component, c - d: dynamo-static component, e - f: pseudo-static component and g - h: total shear forces).

## Effects on normalised mean maximum bending moment

For soft soil and up to its fundamental period (1.26 s), the dynamic response at middle support is practically the

same under all cases of excitation (Figure 5a). However, a clear difference appears in case 2 starting at the soil fundamental period, from which the case 2 response becomes higher than all others. For flexible structures, the case 2 responses drops and is lower than that of all



**Figure 4**. Normalised mean maximum displacement at first mid-span (a - b: dynamic component, c - d: dynamo-static component, e - f: pseudo-static component and g - h: total displacement).

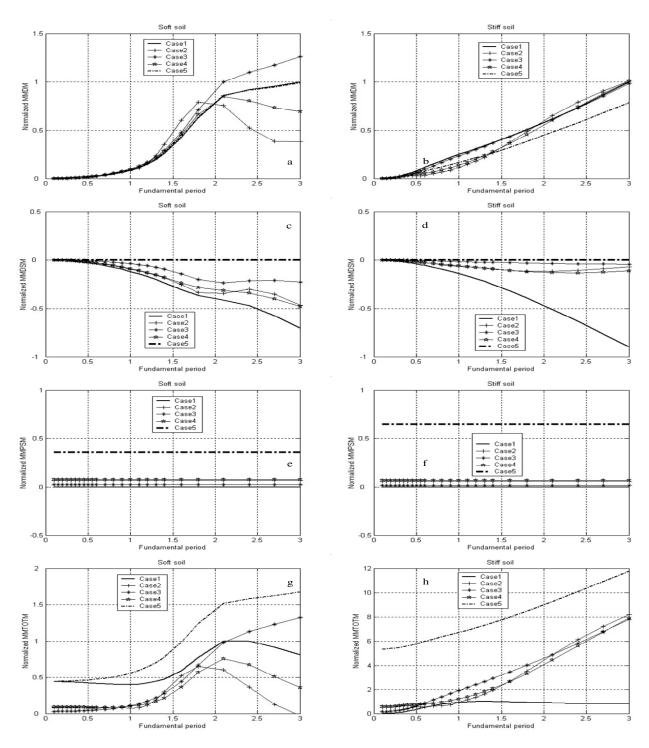
other cases, while the case 3 response overestimates the others.

For stiff soil (Figure 5b), a slight inflection is observed around the fundamental soil period (0.42 s) and the response under uniform excitation overestimates all the other cases. The maximum effects of spatial variability are observed for structures with periods ranging from 0.42 to 1.8 s.

The dynamic and pseudo-static bending moment components are negatively correlated (Figures 5c and 5d). Spatial variability tends to minimise the dynamostatic components for both soft and stiff soil.

The pseudo-static components (Figures 5e and 5f) are constant, nil in the case of uniform excitation and maximal in the case of totally independent excitation.

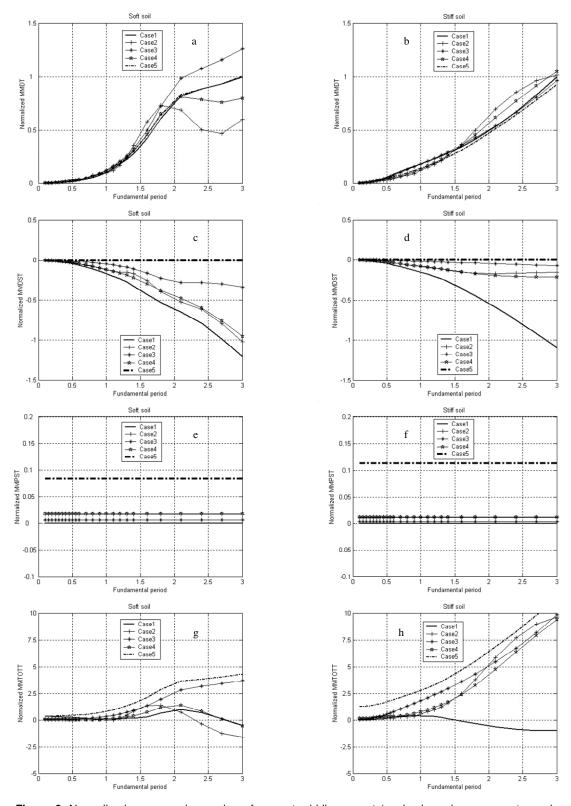
Total mean maximum bending moment (Figures 5g and



**Figure 5**. Normalised mean maximum bending moment at middle support (a - b: dynamic component, c - d: dynamo-static component, e - f: pseudo-static component and g - h: total bending moment).

5h) is maximal in the case of totally independent excitation because of the strong pseudo-static contribution. Compared to other cases, the uniform excitation is conservative for structures having fundamental periods higher than 2 s for soft soil. For stiff soil, the important negative correlation between the dynamic and

pseudo-static components resulting from uniform excitation makes total mean maximum bending moment underestimated for structures with fundamental periods higher than 1.0 s. Totally independent excitation is more conservative on total bending moment at the middle support. Regarding the other cases, the maximum bending



**Figure 6**. Normalised mean maximum shear forces at middle support (a - b: dynamic component, c - d: dynamo-static component, e - f: pseudo-static component and g - h: total shear forces).

moment around the middle support of flexible structures on soft soil can be amplified due to incoherence effect. In

case of stiff soil, the effect of spatial variability of ground motion is more significant even for rigid structures that

are underlined by Abbas and Manohar (2002).

#### Effects on normalised mean maximum shear forces

For fundamental periods up to 1.5 s, the differences in dynamic response under all cases are not significant. For higher periods, they appear more drastically especially for soft soil (Figures 6a and 6b). Generally, the incoherence effects (case 3) tend to overestimate the response while the wave passage effects (case 2) tend to underestimate the dynamic shear forces (Figures 6a and 6b). The dynamo-static shear forces component (Figures 6c and 6d) exhibits negative values due to the nature of the correlation between the dynamic and pseudo-static components. The correlation is maximal in case of uniform excitation and reaches 100% of dynamic component for very flexible structures (T > 2.7 s). This correlation makes the total mean maximum shear forces (Figures 6a) and h) underestimated. The latter reaches negative values for periods higher than 1.6 s. The pseudo-static component (Figures 6e and 6f) overtakes 10% of the dynamic component for soft soil and 8% for stiff soil. As a conclusion, totally independent excitation is more conservative case on total shear forces at the middle support. The effect of spatial variability of ground motion on shear forces around the middle support is more significant in presence of stiff soil especially for flexible structures.

#### Conclusion

The mean maximum responses of the structure have studied using response spectrum method incorporating the spatial variability of ground excitation. Variations of ground motion due to wave passage effect and incoherence effect are considered for both soft and stiff soil. The relative influence of each effect on the three components of the total response (dynamic component, pseudo-static component and the cross-term between the dynamic and pseudo-static) are examined. The response analysis of a structure with a fundamental period of 1.0 s excited with five different cases has been carried out in the first part of this work. It found in general that maximal displacement induced by uniform excitation can be unconservative in case of continuous long two spans beam founded on soft soil and conservative in case of stiff soil. Analogous results have been obtained in some previous studies. The determination of the pseudo static component is essential because of its domination in total maximal displacement and due to spatial variability of around motion differential displacements along the supports may cause failures. Asynchronous support excitation overestimates the total dynamic bending moment component at mid-span and underestimates it around the middle support. It generally has beneficial effects for structures founded on stiff soil. The pseudo-static bending moment component, which does not participate

to the total response in the case of uniform excitation, appears in other cases with important contribution, especially for structures founded on soft soil.

Uniform excitation produces highest dynamic shear forces around the middle support for both soft and stiff soil. With asynchronous excitation, pseudo-static shear forces appear and that reduces the differences between total shear forces under uniform excitation and other cases.

The interplay between dynamic and pseudo-static responses may either control how the spatial variability ground motion excitation effects are beneficial or not depending on different sections along the two spans beam and the rigidity of the soil.

In the second part, the effects of non uniform excitation on structures with different fundamental period have been studied. The mean maximum displacements were analysed at first mid-span and both mean maximum bending moment and shear forces at middle support.

For structures with fundamental periods less than the soil period  $(T_g),$  dynamic response components (displacement, shear forces and bending moment) are practically the same for all studied cases. Using uniform excitation (case1) in seismic analysis of such sections will thus give accurate responses. The wave passage effect dominates for structures with fundamental periods in  $T_g$  - 1.8 s range. For more flexible structures, dynamic displacement is dominated by the incoherence effects may be because of the high value of the incoherence coefficient  $(\alpha=0,25)$ 

As for the total response, the uniform excitation (case 1) underestimates the displacement compared to cases 2, 3 and 4 and overestimates it compared to case 5. Total displacement of structures founded on stiff soil is less sensitive to spatial variability of ground motion effects.

The pseudo-static displacement in case of uniform excitation does not produce shear forces nor bending moment, in all other cases, a differential displacement is produced, which stimulates additional shear forces and bending moment. In the case of totally independent excitation (case 5), these additional internal forces control both the total bending moment and shear forces.

On soft soil, for structures with fundamental periods less than 2.1 s, total bending moment under case 1 overestimates all the other cases except case 5. Beyond this fundamental period, incoherence effects dominate. For stiff soil the effects of spatial variability of ground motion on total bending moment are dominating. The reduction of the total response under uniform excitation is related to the negative contribution of the dynamo-static component. This reduction is more important for stiff soil compared to soft soil.

As for shear forces, important additional pseudo-static forces and negative correlation components indicate that the ground motion spatial variability has to be accounted for. Totally independent excitation is more conservative case on total shear forces at the middle support. Non

uniform excitation effect on shear forces around the middle support is more significant in presence of stiff soil especially for flexible structures.

Spatial variability of ground motion affects structure response in a very significant way and must definitively be taken into account for the design of long structures.

#### **REFERENCES**

- Abbas AM, Manohar CS (2002). Investigation into critical earthquake load models within deterministic and probabilistic framework, Earthquake Eng. Struct. Dyn. 31: 813-832.
- Abdel-Ghaffar AM, Rubin LI (1982). Suspension bridge response to multiple support excitation. J. Eng. Mechanics Division (ASCE), 108: 419–435.
- Abrahamson NA, Schneider J-F, Stepp JC (1991). Empirical spatial coherency functions for soil-structure interaction analyses. Earthquake Spectra, 7(1): 1–27.
- Berrah MK, Kausel E (1992). Response Spectrum Analysis of Structures Subjected to Spatially Varying Motions. Earthquake Eng. Struct. Dyn. 21(6): 461-470.
- Calvi M, Pinto P (1996). Experimental and numerical investigations on the seismic response of bridges and recommendations for code provisions. Prenormative Research in Support of Eurocode 8, Report 4, European Consortium of Earthquake Shaking Tables.
- Chouw N, Hao H (2005). Estimation of required setting length of bridge girders under non-uniform ground excitation and different ground conditions. JSCE J. Appl. Mech. 8: 709-718
- Clough RW, Penzien J (1993). Dynamics of Structures, McGraw-Hill. New York.
- Der Kiureghian A, Neuenhofer A (1992). Response Spectrum Method for Multi-Support Seismic Excitations. Earthquake Eng. Struct. Dyn. 21: 715-740.
- Der Kiureghian A (1996). A Coherency Model for Spatially Varying Ground Motions. Earthquake Eng. Struct. Dyn. 25: 99-111.
- Der Kiureghian A, Keshishian P, Hakobian A (1997). Multiple support response spectrum analysis of bridges including the site response effect & MSRS code. Report N° UCB/EERC-97/02, University of California, Berkeley.
- Saxena V, Deodatis G, Shinozuka M (2000). Effect of spatial variation of earthquake ground motion on the nonlinear dynamic response of highway bridges. Proceeding of 12th World Conference on Earthquake Engineering, Auckland, New Zealand.
- Dumanoglu AA, Soyluk K (2003). A stochastic analysis of long span structures subjected to spatially varying ground motions including the site-response effect. Eng. Struct. 25: 1301–1310.
- Hao H (1989). Effects of spatial variation of ground motions on large multiply-supported structures. Report N° UBC/EERC-89/06, University of California, Berkeley.

- Harichandran RS, Vanmarke E (1986). Stochastic variation of earthquake ground motion in space and time. J. Eng. Mechanics (ASCE); 112:154–174.
- Harichandran RS, Wang W (1990). Response of indeterminate twospan beam to spatially varying seismic excitation. Earthquake Eng. Struct. Dyn. 19: 173–187.
- Harichandran RS, Hawwari A, Sweidan BN (1996). Response of longspan bridges to spatially varying ground motion. J. Struct. Eng. (ASCE); 122(5): 476–484.
- Kahan M, Gibert R, Bard PY (1998). Influence of spatial wave variability on bridges: a sensitivity analysis. Earthquake Eng. Struct. Dyn. 25(8): 795–814.
- Loh CH, Penzien J, Tsai YB (1982). Engineering analysis of SMART-1 array accelerograms. Earthquake Eng. Struct. Dyn. 10: 575–591.
- Luco JE, Wong HL (1986). Response of a rigid foundation to a spatially random ground motion. Earthquake Eng. Struct. Dyn. 14:891–908.
- Lupoi A, Franchin P, Pinto PE, Monti G (2005). Seismic Design of Bridges Accounting for Spatial Variability of Ground Motion, Earthquake Eng. Struct. Dyn. 34: 327–348.
- Monti G, Nuti C, Pinto P, Vanzi I (1994). Effects of non-synchronous seismic input on the inelastic response of bridges. Proceedings of the 2nd International Workshop on Seismic Design of Bridges, vol. 1, Queenstown, New Zealand, 90–107.
- Nuti C, Vanzi IE (2005). Influence of earthquake spatial variability on differential soil displacements and SDF system response, Earthquake Eng. Struct. Dyn. 34: 1353–1374
- Pitilakis K, Hatzidimitriou D, Bard PY, Manos G, Jongmans D (1994). EUROSEISTEST Volvi-Thessaloniki—A European test site for engineering seismology, earthquake engineering and seismology. Earthquake Resistant Construction and Design, Savidis (ed). Balkema: Rotterdam.
- Price TE, Eberahard MO (1998). Effect of varying ground motions on short bridges. J. Struct. Eng. (ASCE); 124(8): 948–955.
- Sextos A (2001). Effect of spatial variability, local soil conditions and soil-structure interaction on the inelastic dynamic analysis of RC bridges. Ph.D. Thesis, Aristotle University of Thessaloniki, Greece.
- Sextos AG, Pitilakis KD, Kappos AJ (2003). Inelastic dynamic analysis of RC bridges accounting for spatial variability of ground motion, site effects and soil–structure interaction phenomena. Part 1: Methodology and analytical tools. Earthquake Eng. Struct. Dyn. 32: 607–628.
- Zanardo G, Hao H, Modena C (2002). Seismic response of multi-span simply supported bridges to a spatially varying earthquake ground motion. Earthquake Eng. Struct. Dyn. 31: 1325–1345.
- Zerva A, Ang AHS, Wen YK (1988). Lifeline response to spatially variable ground motions. Earthquake Eng. Struct. Dyn. 16: 361–379.
- Zerva A (1990). Response of multi-span beams to spatially incoherent seismic ground motions. Earthquake Eng. Struct. Dyn. 19(6): 819–832