

Full Length Research Paper

Analysis by homogenisation method of structures in reinforced soil and behaviour interfaces of soil/reinforcement

T. Karech

Université de Batna Dep de Génie-Civil Algérie. E-mail: karech@hotmail.com. Tel: 00213 (0) 33 86 93 12.

Accepted 16 September, 2011

The reinforced soil is a composite material formed by the addition to a non cohesive soil of steel strip reinforcement which is able to withstand important tension forces. Through friction between the soil and the reinforcement, the ground transmits the tension forces, which develop into the mass that cannot be supported, to the steel reinforcement. The stretched reinforcements thus confer to the ground some cohesion along their direction. Therefore providing reinforcements, improves the global mechanical properties of the ground. To oppose lateral expansion of the soil (reinforced ground of artificial filling materials, nailed ground of excavation and slopes) or its movement (blasted columns or micro-piers), the friction “soil-reinforcement” confers to the reinforced ground material an anisotropic cohesion. In a similar manner as with the reinforced and prestressed concrete, the bond between the soil and the reinforcement is an important phenomenon. An interaction analysis involves, separately, the behaviour of two present materials. From such analysis, and contrary to the previous one, appears the real composite behaviour of reinforced soils.

Key words: Homogenisation, interfaces, shearing, finite elements, tension, cohesion, friction.

INTRODUCTION

Many authors were interested in the implementation of a module of homogenisation in a computer code. The application to cases of walls in reinforced ground was undertaken by Cardoso and Carreto (1989), Sawicki (1990). The aforementioned studies made the assumption of perfect adherence between the matrix of the reinforced medium and the elements of reinforcement. Certain authors introduced the possibility of slip between these two materials, this new assumption allowing not over-estimating the resistance of the medium reinforced (De Buhan and Talierco, 1991). Hermann and Al Yassin (1978) on the basis of a computer code based on the finite elements took into account a displacement relating to the interface in the stiffness matrix. They then carried out a comparison with a model of inclusions that are discretized which lead to identical results. The method of homogenisation allows a considerable saving of time in the resolution. Sudret and De Buhan (1999) presented a multiphase model which gives a polar micro description of the reinforced material. Their module not only makes it possible to take into account the relative slip (of elastoplastic type) between the ground and

inclusions, but also the effect of the shearing forces and bending moments. Parametric studies were undertaken on networks of intersected piles and inclusions. The principal interest of the implementation of a module of homogenisation lies in the fact that one can take into account, in an axisymmetric configuration, the longitudinal and radial reinforcement (bolting in the tunnels) which makes it possible to avoid carrying a three-dimensional calculation. This makes parametric studies possible considering a short time of resolution of such an approach. In addition to the sophistication of these modules, the study of a displacement relating to the interface soil/reinforcement and even the inflection in inclusions is also possible.

Approach taking into accounts the complete modeling of the ground, inclusion and their interaction

In this technique, the two components (solid mass and reinforcement) are discretized then assembled by means

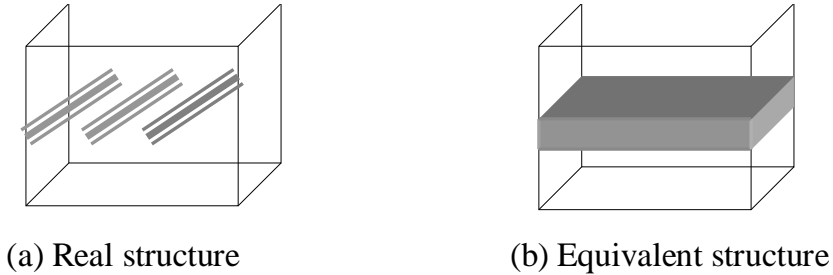


Figure 1. Modeling in 2d with an equivalent plate (Al Hallak, 1999).

of 2d or 3d elements (Chaoui, 1992; Ho and Smith, 1993) or by bar elements. The contributions of these approaches are multiple; they can allow, in particular taking into account the relative displacement of the soil/reinforcement by the introduction of interface elements and the calculation of the efforts mobilized in the reinforcement. The use of these methods contributes to a better estimate of the contribution of the reinforcement to the limitation of the deformations.

Two-dimensional models

A calculation in plane deformation is a priori acceptable only for the two-dimensional elements of reinforcement (geotextile sheet, mesh wire) which are continuous in their plan on the scale of the work. Two major methods in plane deformations exist for modeling the reinforced solid masses. The first consists of replacing a discontinuous steel sheet by a continuous sheet, whose macroscopic properties are equivalent to those of the real sheet by formulating some assumptions recalled by Chaoui (1992), Unterreiner (1994) for a reinforced solid mass. The composite material "ground + steel" is replaced by a homogeneous plate having properties different from those of the ground and steel (Figure 1).

The second approach consists in studying the section S-S where the ground is not broken when modeling the influence of steels in the section of the ground. Two methods are proposed. The first method "slipping strip analysis" presented by Naylor (1978) is based on the study of a vertical section halfway between two vertical lines of reinforcement. The interaction between the ground and the vertical line of steel is modeled by a vertical zone of interface. With this method the reinforcements are placed out of the section of the studied ground and using a kind of load transfer function to model the interaction between the ground and steels. This approach preserves the vertical continuity of the ground

The second method is proposed by Unterreiner (1994) which considers that it is not necessary to introduce a continuous vertical zone of interface but it is sufficient to model the interaction between the section of ground S-S and each steel by the load transfer function. This one must be calculated in a suitable way or measured starting

from wrenching tests on a solid mass.

Method of homogenisation

In the field of the reinforcement of the grounds, the technique of the homogenisation was developed in particular by Buhan and Al (1989). Greuell (1993), Bernaud and Al (1995) and Wong (1997) presented specific approaches for the reinforcement of the grounds. Their models, developed in cases of very simple configurations and boundary conditions, authorize analytical or semi-analytical solutions. From these studies, a model of homogenized behavior of the soil/reinforcement in our computer code is proposed. The possibility of a slip between the reinforcement and the ground is also considered.

Field of validation of the method of homogenisation by the numerical modeling of the reinforced grounds

The homogenisation of a solid mass of reinforced ground consists in replacing the two materials by an equivalent homogeneous material, representing the ground, the reinforcements and their interactions (Figure 2). This approach, however, assumes that various conditions are observed, relating in particular to the periodicity and the density of inclusions.

Representatives of the basic cell

First of all, we will define the concept of the basic cell (Romstad, 1976). This term represents the elementary structure of the composite soil/reinforcement. It is the smallest volume containing the two constitutive materials of the reinforced ground. Figure 2 illustrates in an explicit way a case of reinforcement. The basic cell is composed of two materials

The representatives of this basic cell define the ability of this one to represent the reality the whole reinforced solid mass. While knowing by advance that this condition cannot be strictly met, it is essential nevertheless that inclusion is distributed more or less in a regular way so

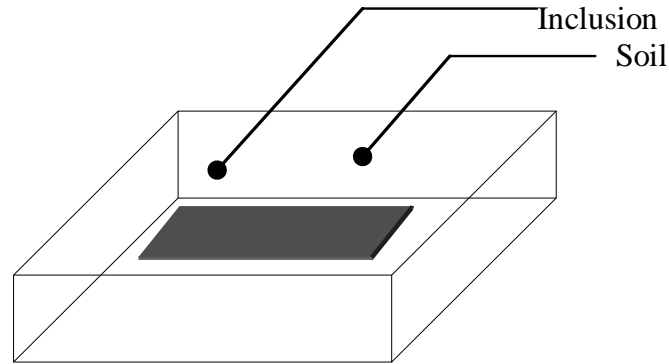


Figure 2. Basic cell representative of the reinforced ground.

that we can model the reinforced ground like a material with periodic structure. It is one of the necessary conditions to the existence of a basic cell representative of the reinforced solid mass.

Nevertheless the use of techniques of discretization in discontinuous elements (finite elements or finite differences) makes it possible to vary, to a certain extent, the density of reinforcement and its orientation in each element, which is impossible in the simplified analytical approaches based on the homogenisation.

Total character of the representation

Contrary to what was presented in numerical calculation taking account of inclusions modeled individually making it possible to locally evaluate the contribution of steels inside an element of ground, the technique of homogenisation allows us to be interested only in the total values inside the cell. In other words, it allows obtaining inside an element of ground only the average force taken by the steel located inside the element since they are also regarded as distributed in the volume of ground. This method thus has an interest only if one is interested in the total sizes (or averages) in the work.

Scale effect

The scale is directly connected to the density of steel D_b in other words the number of inclusions per square meter of wall. This density of reinforcement must be rather high so that the method of homogenisation can be employed (the surface fraction of reinforcement

$$d = \frac{\text{Section reinforcement}}{\text{Section cellule}} \quad (\text{must be sufficiently weak } d \ll 1)$$

The observation is based on a comparative study between the experimental results obtained by Siad (1987) on the reinforced earth and theoretical approach

by homogenisation carried out by Buhan (1989), which has established a good agreement between their results. Let us specify nevertheless that the scale effect is also related to the size of the studied field, it is thus appropriate not to consider the absolute value of L_b (length of reinforcement) but the relative one compared to the volume of studied ground, i.e. the relative homogeneity of the studied solid mass. Thus, as specified by Jassonnesse (1998), it is appropriate "to consider more objective concepts" than the scale effect as those which we approach under the following conditions.

Mesh smoothness of the numerical model

The finite element or the finite differences method forces to divide the studied continuous medium into a more or less great number of elements representing the mesh. This quantity of elements chosen by the user according to the desired precision defines the smoothness of the mesh. This concept especially dedicated to numerical calculation brings the idea of minimal length on which the digital model provides information. This size must also "be relativized" compared to dimensions of the studied work; while netting very finely. Clearly, it does not seem very useful to go down below the size of the cell but a too loose mesh can lead to a loss of information. Bernaud and Al (1995) propose in the case of a circular tunnel reinforced by radial bolting to keep the same smoothness of mesh as in the case of a non reinforced tunnel.

Period of the basic cell

The last of the conditions to satisfy to homogenize the reinforced solid mass is that the period of reinforcement (the dimension of the basic cell) is small compared to the scale of the work. The dimensions of the basic cell increase with the reduction of the number of the inclusions imbedded in the solid mass until a certain limit that validates the homogenisation method.

If these conditions are satisfied, we can then use the

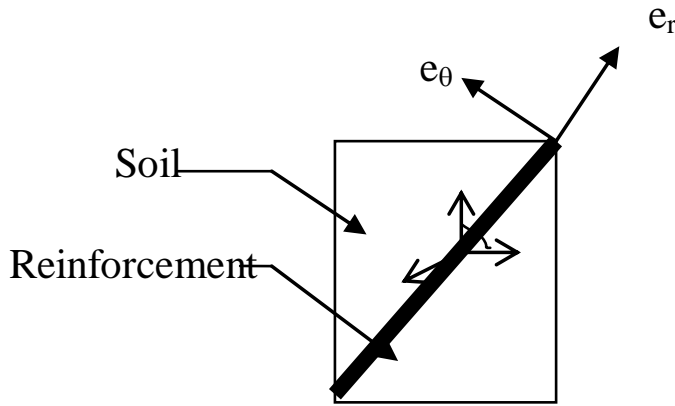


Figure 3. Anisotropic homogenized element.
 θ slope of the reinforcement with the horizontal line.

homogenisation of the periodic mediums in order to analyze the behavior of an anisotropy homogeneous composite material having the same geometry, boundary and loading conditions as the reinforced soil.

METHODOLOGY

We now will be interested in the study of the case of a medium to be homogenized in an axisymmetric configuration. Because of the axisymmetric conditions, we are interested, in the same way as Bernaud and Al (1995), only to the radial and axial directions of the reinforcement (Figure 3).

All the configurations of reinforcement are nevertheless possible and, in particular, the association of radial and axial inclusions (case of tunnels) on distinct volumes of ground.

Inclusions are laid out radially around the spherical cavity $0^\circ < \theta < 90^\circ$ of section S_{bs} and density in D_{bs} variable with the length of the reinforcement:

*Radial inclusion: disposed $\alpha\tau \theta = 90^\circ$ of section S_{bs} and density in bet D_{bs} variable with the radius:

$$D_{bs}(r) = D_{bs}(R/r)^2 \tag{1}$$

*Axial inclusion: laid out at $\theta = 0^\circ$, of section S_{bs} and of wall constant density D_{bs} for any distance to the wall.

Law of homogenized behavior

We define the behavior of a cell of a homogenized medium from validate relations for each of its basic components, namely the ground and inclusion. We limited our study to the one-way reinforcement by plane inclusions.

Determination of stress fields and deformations in the homogenized medium

In the macroscopic scale of the structure, the reinforced ground can be considered in general as an anisotropic homogenized continuous medium, this in spite the fact that the ground and

inclusions are isotropic materials. We can thus substitute the initial heterogeneous by a homogenized material inside which the stress and strain states are defined respectively by the symmetrical tensors $\underline{\Sigma}_{\eta\sigma\mu} \quad \alpha\nu\delta, \underline{\varepsilon}_{\eta\sigma\mu}$

The components of these tensors being as follows:

$$[\Sigma_{rr} \quad \Sigma_{\theta\theta} \quad \Sigma_{zz} \quad \sqrt{2} \Sigma_{r\theta} \quad \sqrt{2} \Sigma_{rz} \quad \sqrt{2} \Sigma_{\theta\rho}] \text{ for stress} \tag{2}$$

$$[\varepsilon_{\rho\rho} \quad \varepsilon_{\theta\theta} \quad \varepsilon_{z\zeta} \quad \sqrt{2} \varepsilon_{\rho\theta} \quad \sqrt{2} \varepsilon_{\rho\zeta} \quad \sqrt{2} \varepsilon_{\theta\zeta}] \text{ for strain} \tag{3}$$

In order to be able to simplify the writing of the tensor of the stresses, it is necessary that simultaneously the surface fraction of reinforcement (with S_b section of the reinforcement and, S_{CB} section of base) be very weak $d \ll 1$, and that the stiffness of steels is much larger than that of the ground ($E_{\text{steel}} \gg E_{\text{ground}}$). Gruell showed that if these two conditions are joined together, the reinforced material behaves in a macroscopic scale like a transverse isotropic elastic medium around the axis. This demonstration established by using a variational approach makes it possible to establish a relation between the tensors $\underline{\Sigma}_{\text{hom}}$ and, $\underline{E}_{\text{hom}}$. The tensor of the stresses in homogenized material comes from the sum of the contribution of each of two materials:

$$\underline{\Sigma}_{\eta\sigma\mu} = \underline{\Delta} \underline{\varepsilon}_{\eta\sigma\mu} + K(\rho) \underline{\varepsilon}_{\eta\sigma\mu} \tag{4}$$

$$K(\rho) = \Delta_{\beta\sigma}(\rho) \Sigma_{\beta} E_{\beta} \tag{5}$$

Where: $D_{bs}(R)$ = density of reinforcement (constant when it is axial); S_b = section of the reinforcement and E_b = Young's Modulus of the steel.

Anisotropic field of elasticity

Let us define initially, the behavior of two materials constituting the homogenized material:

1. The isotropic linear elastic ground defined by the Young's modulus E_s and by the Poisson's ratio ν_σ
2. Inclusions: linear elastic bands one-way (direction \vec{e}_r) defined by the Young's modulus E_b

Elastoplastic behavior of the homogenized medium

As a criterion of plasticity for the ground, we have adopted that of Mohr-Coulomb. We know that this elastic perfectly plastic criterion is well adapted to the study of the grounds or tender rocks having a coherent / friction behavior.

$$f(\underline{\underline{\sigma}}_s) = (\sigma_1)_s - \frac{1 + \sin \varphi}{1 - \sin \varphi} (\sigma_3)_s - \frac{2C \cos \varphi}{1 - \sin \varphi} \tag{6}$$

Where: $(\sigma_1)_s$ = Major principal stress in the ground and $(\sigma_3)_s$ = Minor principal stress.

The equation $\underline{\underline{\sigma}}_b = E_b \underline{\underline{\varepsilon}}_{xx} \geq R_b$ defines the acceptable field in the inclusion.

$$f(\underline{\underline{\Sigma}}_{\text{hom}} - K(x) \underline{\underline{\varepsilon}}_{xx}) \leq 0 \text{ defines the elastic range } G_s$$

$$\text{and } f(\underline{\underline{\Sigma}}_{\text{hom}} - K(x) \underline{\underline{\varepsilon}}_{xx}) = 0 \text{ working limit of stresses in the ground}$$

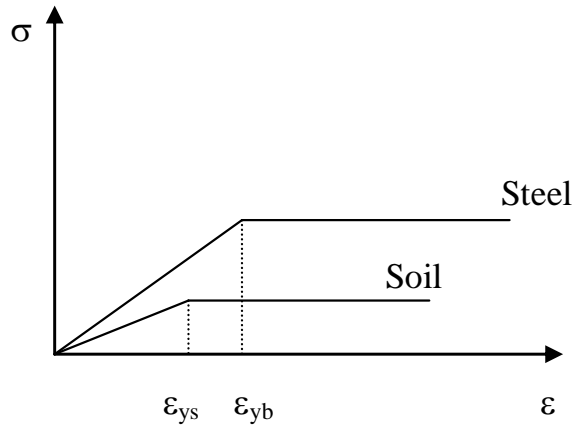


Figure 4. Uni-dimensional behavior of the ground and steels (Wong, 1995).

When a principal direction of the stresses coincides with the direction of reinforcement, the border of the acceptable field for a homogenized material is thus reached when simultaneously the criterion of rupture of the ground is obtained $f(\underline{\sigma}_s)=0$ and, tensile stress in inclusions reaches the maximum value R_b (Figure 4).

The basis of the limit of load transfer

The concept of limit of transfer of load developed by Jassonnesse (1998) introduces a limitation to σ_β representing in fact a possible slip between inclusion and the ground and thus an imperfect transfer of load from steel to the ground, which limits the resumption of the effort by steels.

The limit of the transfer of load corresponds to the introduction of a rigid-plastic friction/slip law between the ground and the inclusion and, results from the equilibrium of inclusion (Figure 5).

By putting τ friction with the interface soil/inclusion, P_b the perimeter of inclusion, σ_b the stress in the reinforcement and, S_b its section, we obtain the following relations:

$$F(x)=p_b\tau(x)et, T(\xi)=\sum_{\beta} \sigma_{\beta}(\xi) \quad (7)$$

The inclusion is put into tension by friction τ on the interface inclusion/soil, the equilibrium thus leads to:

$$T(x+dx)-T(x)=F_s(x).dx \Leftrightarrow \frac{d\sigma_b}{dx} = -\frac{p}{S_b} \tau(x) \quad (8)$$

Modeling of the wall in a reinforced soil

Here, we will compare our theoretical results with those obtained by SHAFIEE (1985). These results show the general aspects of the behavior of the wall in a reinforced soil. We treat successively:

1. The evolution of tension T_s in the reinforcements (of the work),
2. The location of the maximum tensions T_{max} in the armed wall,
3. The distribution of displacements of the wall (for various heights of the armed wall).

The work considered is a wall 5 m high, reinforced with 5 beds of

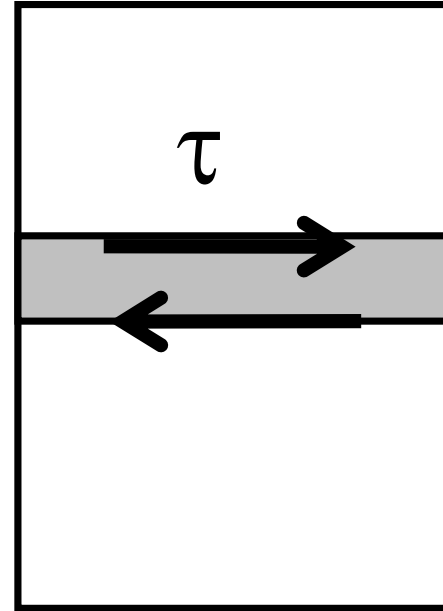


Figure 5. Equilibrium of inclusion.

reinforcement of 5.1 m in length spaced vertically and laterally (ΔH and e) at 1 m. The facing is in concrete scales of 0.1 m in thickness (Table 1).

RESULTS AND DISCUSSION

The evolution of tension in the 5 reinforcements is presented on (Figure 7). This evolution, in conformity with the experimental observations, separates the active zone from the passive one by a line of maximum tension. This vertical line is located at a distance D of the facing. Our results are compared to those of SHAFIEE. Our model slightly underestimates the calculated tensions. This (light) difference is due, in our opinion, to the nature of the calculation (homogenisation, elasto-perfectly plastic with a criterion of Mohr-Coulomb taking into account the effect of the interface). Distribution (non-dimensional) of maximum tension T_{max}^* (we standardize all T_{max} by the maximum value of T_{max} calculated) related to the depth Z/H (Figures 6 and 8). The values determined by our program are very close to those obtained by the "CLOUTERRE" program used by SHAFIEE. We, however, note that when we check the usual mode of displacements of a reinforced earth wall (Figure 9), the values determined by our computer code remain slightly lower than the computed values by "CLOUTERRE" (the difference is about 15% in the case of taking into account the effect of the interface and 10% for a perfect adherence).

Conclusions

The behavior of the soils reinforced with linear inclusion

Table 1. Modeling of the wall in a reinforced soil.

| Soil (non-cohesive) | Reinforcements in (steel) | Facing |
|--|--|--|
| $E=10 \text{ MPa}$ $\gamma= 16 \text{ KN/m}^3$ $\nu=0.33$ $\phi=30^\circ$ | $E_b=2.10^5 \text{ Mpa}$ $\nu_b=0.25$ $(S_b \phi 50\text{mm})$ | $E_p=2.10^4 \text{ Mpa}$ $\nu_p=0.25$ |

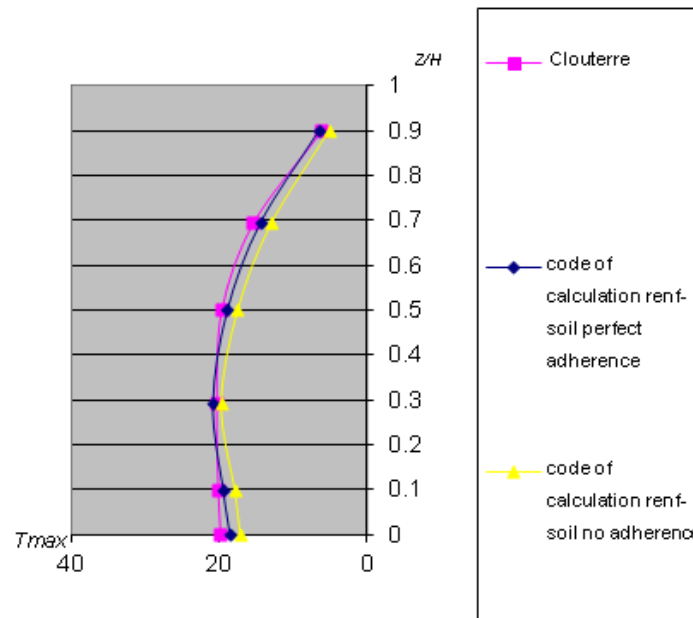


Figure 6. Maximum tension T_{max} .

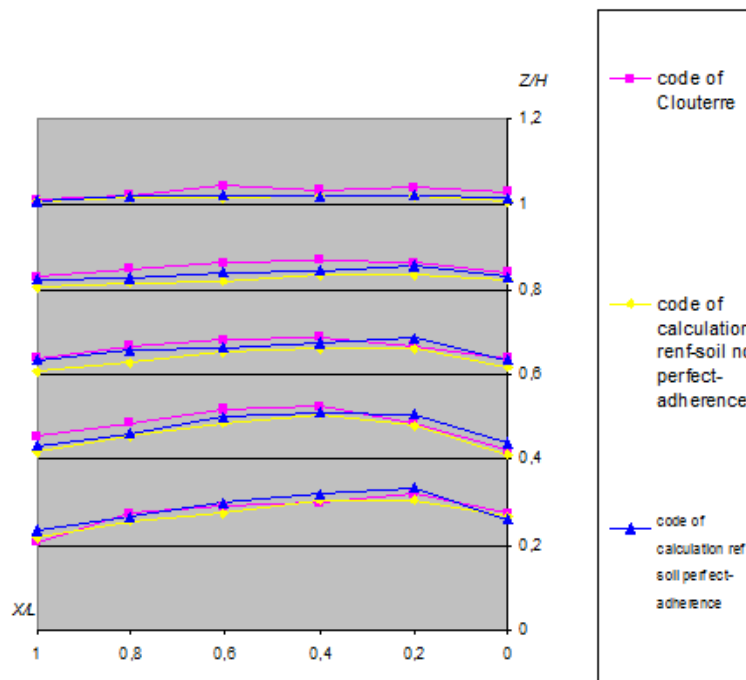


Figure 7. Evolution of tension T_s in the reinforcements.

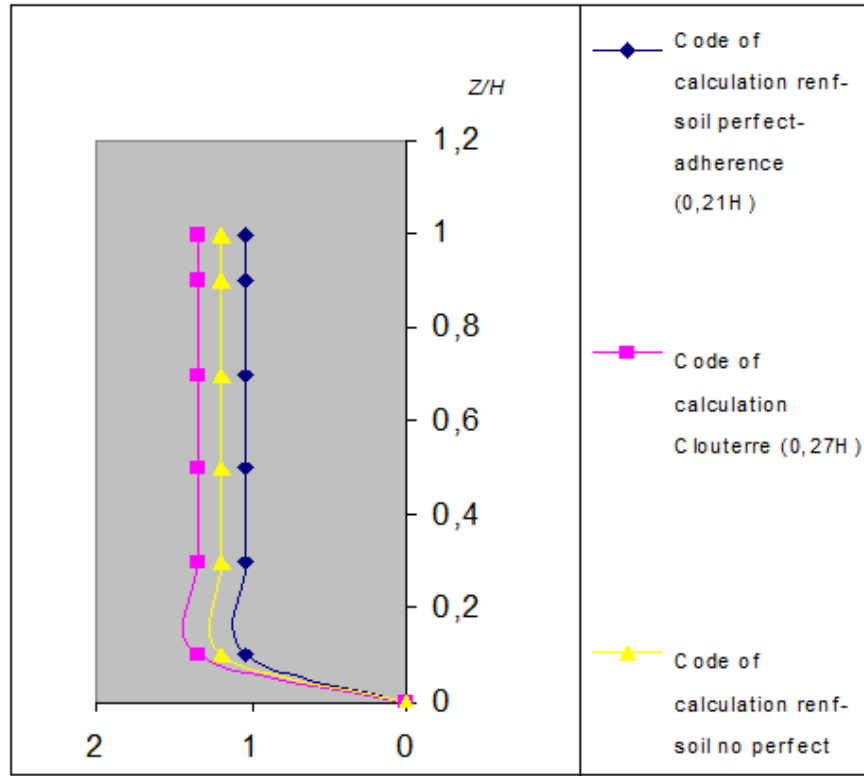


Figure 8. Location of maximum tension.

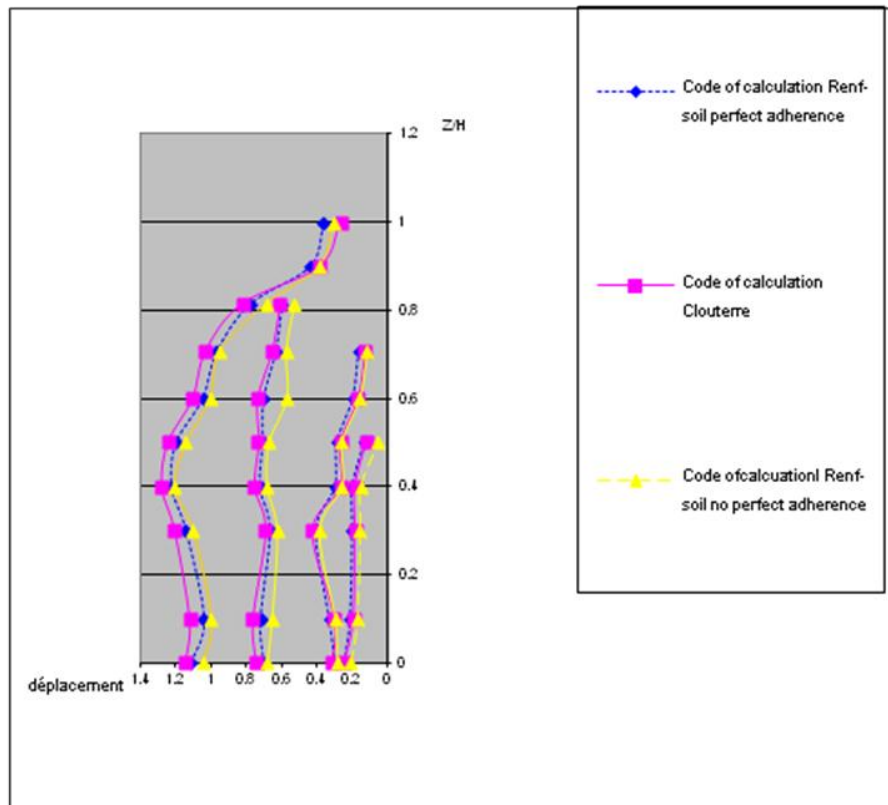


Figure 9. Evolution of displacements of the wall.

is complex and requires taking into account the transfers of the efforts to the interface soil/inclusions. The approaches of the type calculation to rupture aim to determine the equilibrium of the solid mass, but do not allow to evaluate the state of its deformations. The modeling in deformations takes into account the various elements: soil, inclusions and their connection, and leads to two types of approaches; analytical and numerical ones. The homogenisation of the periodic mediums is another approach which makes it possible to consider the composite soil and reinforcement at the macroscopic level as an equivalent material whose global behavior gives an account of that of the soil and the inclusions.

REFERENCES

- Bernaude and Al (1995). Numerical simulation of the convergence of a bolt-supported tunnel through a homogenization method, *Tnt. J. Num. Anal. Methods Geomech.*, 19: 267-288.
- Chaoui F (1992). Etude tridimensionnelle du comportement des pieux dans les pentes instables, Thèse de doctorat, Ecole Nationale des Ponts et Chaussées, pp. 355.
- Greuell NE (1993). Etude du soutènement des tunnels par boulons passifs dans les sols et les roches tendres par une méthode d'homogénéisation, Thèse de doctorat de l'Ecole Polytechnique, Palaiseau. pp. 199
- Herman LR, Al Yasin Z (1978). Numerical analyses of reinforced soil systems, *Proc. Symp. Earth Reinforced*, Pittsburgh, pp. 428-570
- Jassonnesse C (1998). Contrôle de la déformation du massif renforcé par boulonnage au front de taille d'un tunnel, Thèse de doctorat, INSA de Lyon, pp. 234
- Shafiee S (1986). Simulation numérique du comportement des sols cloués interaction sol-renforcement et comportement de l'ouvrage , Thèse DDI, ENPC. pp. 278
- Sawicki A (1990). Development of failure in reinforced soil structures, *Performance of reinforced soil structures*, Glasgow, pp. 31-40.
- Sudret B, de Buhan P (1999). Modélisation multiphasique de matériaux renforcés par inclusions linéaires, *C. R. Acad. Sci.*, Paris, t. 327, Série Hb, pp. 7-12.
- Unterreiner Ph (1994). Contribution à l'étude et à la modélisation numérique des sols clones : application au calcul en déformation des ouvrages de soutènement, thèse de doctorat, ENPC., 2 : 499.