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A multi-purpose water resources optimization programme: A case study at Oji-River

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This paper presents a household centered water resource approach jointly proposed by Federal Ministries of Water Resources and Power. Moreover, the steady increasing demand to achieve optimum operating policy on water resources and power forces engineers and planners of both ministries to contemplate and propose even more comprehensive and ambitious plans for mitigated water supply, power and flood. This paper provides a mathematical optimization model that can be used to determine optimal policy for constructing multi-purpose dams using deterministic method and dynamic programming in system analysis which involves maximizing revenue from power production, using utility and disincentive functions recursively.

Key words: Water resources, power, optimal policy, multi-purpose, dynamic-programming.

INTRODUCTION

Water resource system is a system engineering that provides utilities to people in either its natural form or created in piecemeal/whole by hydraulic engineering structures such as dams (Hall and Dracup, 1979). The concept of the proposed project was developed by the Federal Ministries of Water Resources and Power under the auspices of united Nation Development Programme (UNDP). It is a multi-purpose project designed to increase benefits, and thus embrace socio-economic justification of the project. (WHO/UNICEF, 2000).

The people of southeastern Nigeria have suffered from erratic power and inadequate water supply. Therefore, the Federal Government decided to give them a face lift by constructing a dam at Oji River. The dam will increase power supply by 200 Mw per day and sustain ultra-modern water treatment plants to supply safe water to both Anambra and Enugu states (Sanga and Fregene, 1977).

Federal Ministry of Water Resources (2000) in her national water supply and sanitation policy, supported the programme packaged principally to provide support to institutional and policy reform, demonstrate innovative approach to service delivery including implementation of water supply and sanitation projects to be co-financed by Federal Government and UNDP in accordance with cost-sharing provision in the National Water and Sanitation Policy 2000. The policy involves demand responsive approach and management of water supply and sanitation facilities at the lowest level possible. According to Federal Ministry of Water Resources (2005) in her rural water supply and sanitation initiative, programme implementation manual, which describes the various intake structure already commonly used in Nigeria, particular emphasis to water resources protection, considering in design, construction and maintenance methods, which contributed to poverty eradication, sustainable development and to achieve the Millennium Development Goals. This is to be achieved through increase in access to safe, adequate and sustainable water and sanitation service in the two States (Anambra and Enugu), (Sanga and Fregene 1977).

Objectives of the multi-purpose project

(i) Maximize output from national/state investments in water resources,
(ii) Generate income and ensure equitable redistribution,
(iii) Promote and sustain economic growth via increased commercial and industrial activities,
(iv) Create useful employment for the populace, and
(v) Improve healthcare delivery systems in the region.

Analysis of water resource systems involves techniques that ensure dynamic planning and operation policies of complex reservoir systems, provide optimum hydroelectric power and water, and determine required flood mitigation contributions. All of these can be optimized for maximum returns at firm levels (Umolu, 1977).

The cost characteristics of water system elements are equally important to water resources engineering and the nature of cost functions associated with each component. Thus cost functions in water resources generally do not show marginal cost increase, on which current economic theories are mostly based. Compared with operation and maintenance costs, most components of river systems involve huge investment costs. Such cost aspects of completed projects are therefore critical and make corresponding decisions economically rigid and merely irreversible.

The research task of this paper was to develop a realistic method for optimal water resources policies in hydropower, water supply and flood control engineering.

METHODOLOGY

System engineering focuses on quantitative analysis of building models, which optimizes the overall performance of a system. It is about optimal use of resources such as manpower, finance, machinery and other materials (Yeh and Becker, 1974).

This system is implicitly dynamic as such is characterized by

(i) A rule which determines if the object is to be considered a part of the system or of the environment (system boundary).
(ii) A statement of input and output interactions with the environment.
(iii) A statement of interrelationship between the elements of the system, the inputs and the outs, including external interaction between input and output.

This system engineering has input (fund) and outputs (power, water supply and irrigation).

The inputs are classified as controllable while outputs are classified as desirable, the controllable is called decision variables and when each decision variable is assigned particular value, while the resulting set of decision is called policy. The decision variable at any given stage is allowed to take on only a finite set of discrete values of dynamic programming problem. Therefore, the solution is always maximum or minimum regardless of magnitude of problem (Yakowitz, 1982).

Terminology

(i) Decision variables are variables that are freely controlled by decision-makers.
(ii) Constraints are algebraic equations or inequalities that variables must satisfy.

(iii) Objective function is the measure of effectiveness or value associated with particular variable combinations.
(iv) Optimization technique is a problem that seeks for maximum or minimum value for the proper functioning of variables while at the same time satisfying imposed requirements.
(v) Dynamic programming is the optimization of multi-stage decision processes. Multi-stage decision processes are separable into sequential steps that could be completed in one or several ways.

Mathematical program

Optimal:  \[ Z = f_1(x_1) + f_2(x_2) + f_3(x_3) + \cdots + f_n(x_n) \]  \hspace{8cm} (1)

Conditional:  \[ x_1 + x_2 + x_3 + \cdots + x_n \leq b \]  \hspace{8cm} (2) with all variables being non-negative and integer such that \[ f_1(x_1) + f_2(x_2) + f_3(x_3) + \cdots + f_n(x_n) \]  are known (non-linear) functions of a given variable, and b is an input non-negative constraint.

The term \( x_i \) is the allocation amount to the project \( i \); \( i = 1 \) for hydropower, \( i = 2 \) for water supply, and \( i = 3 \) for flood control; \( U(x) \) is net utility spending in \( \text{billions} \) on project \( i \); and then \( m_i > 0 \) minimal amount spent on project \( i \). Hence models are important elements of multi-stage decision processes.

Regardless of given decisions, optimum policies enter a state at a specific stage. Then all supporting decisions must constitute to the optimum policies of exit. Therefore, policies must begin with states with least \( n \)-stage processes and determine for the states the best exit policies. Assuming that all proceeding stages are optimally completed and results applied in succeeding stages, then the process can be deemed successfully complete. Then the optimal allocation \( x_i \) to a project \( i \) could maximize net utility (Hall et al., 1968).

Dynamic programming

Maximal:  \[ Z = \sum_{i=1}^{3} u_i(x_i) \]  \hspace{8cm} (2)

Conditional:  \[ x_1 + x_2 + x_3 \leq \alpha \]  \hspace{8cm} \text{total amount}

\[ x_i \geq m_i \quad (i = 1, 2 \text{ or } 3) \]  \hspace{8cm} (3)

Gross utility function:  \[ U_{1i}(x) = a_i(1 - e^{-b_i x_i}) \]  \hspace{8cm} (4)

Disincentive function:  \[ U_{2i}(x) = C_i x_i^d \]  \hspace{8cm} (5)

Net utility function:

\[ U_i(x) = U_{1i}(x) - U_{2i}(x) = a_i(1 - e^{-b_i x_i}) - C_i x_i^d \]  \hspace{8cm} (6)

Maximal:  \[ Z = \sum_i [a_i(1 - e^{-b_i x_i}) - C_i x_i^d] \]  \hspace{8cm} (7)

Conditional:  \[ x_1 + x_2 + x_3 \leq \alpha; \text{ where } x_1 \geq m_i \]
The proposed allocation among the several uses fixes the minimal price for power and water supply, and contributes to required flood mitigation via the benefit method. Therefore the government minimal allocation of multi-purpose water resources development comprises of three decision variables. These are: hydropower = ₦4 billion, water supply = ₦2 billion and flood control = ₦1 billion (all totaling ₦7 billion for minimal allocation).

Delays in implementation add inflation cost, which must be considered in integrated interest rates of re-allocated uses as:

\[ F = P(1 + i)^n \]

where \( F \) is future value; \( P \) is present/existing value which is ₦7 billion; \( i \) is CBN inflation rate which is 15%; \( n \) is the period/year which is 6 years. The new total then is given as: \( F = 7(1 + 0.15)^6 \), which is equal to ₦16 billion. ₦16 billion is the construction and maintenance cost for 10 years.

**Net utility function**

\[ N = a_i (1 - e^{-b_i x_i}) - C_i x_i^d_i \]

where \( N \) is net utility function; \( a_i, b_i, c_i \) and \( d_i \) are known position constants where \( d_i \leq 1 \);

\( t_i \) is state variable of money available for the remaining \( i \) projects.

\[ x_i = y_i + m_i, \text{ hence } f_i(y_i) = f_i(y) \]

\[ f_i(y_i) = \max \{ U_i(y_i, m_i) + f_2(y_2) \} \quad 0 \leq y_i \leq t_i \]

\[ \min \{ U_i(y_i, m_i) + f_2(t_i - y_i) \} \]

Let \( t_i = a - (m_i + m_2 + m_3) \), then \( 16 - (4 + 2 + 1) = 9 \)

**Recursively**

\[ f_3(y_3) = \max \{ U_3(y_3, 1) \} \quad 0 \leq y_3 \leq t_3; \text{ is the maximum utility derived from project 3 given available } t_3 \]

\[ f_2(y_2) = \max \{ U_2(y_2, 2) + f_3(f_2 - y_2) \} \quad 0 \leq y_2 \leq t_2; \text{ is the maximum utility derived from project given available allocation } t_2 \]

\[ f_1(y_1) = \max \{ U_1(y_1, 4) + f_2(t_1 - y_1) \} \quad 0 \leq y_1 \leq t_1; \text{ is the maximum utility from projects 3, 2 and 1 from their allocation given available total allocation } t_1 \]

Let \( A = 16, m_1 = 4, m_2 = 2, m_3 = 1 \)

\( a^i = (200, 150, 100), b^i = (25, 20, 15) \)

\( b^f = (03, 04, 02), C_i = (0.4, 07, 05) \)

\( U_1(t_1) = 200 (1 + e^{-0.3 t_1}) - 25 t_1 \)

\( U_2(t_2) = 150 (1 + e^{-0.4 t_2}) - 20 t_2 \)

\( U_3(t_3) = 100 (1 + e^{-0.2 t_3}) - 15 t_3 \)

**DISCUSSION**

Dynamic Programming is optimization tool in determination of the optimal operating policy. The computational time in this programming or increase geometrically as dimensionality of the problem will be increased. The dimensionality problem also decreases by reducing some decision variables to parameters (Yeh and Becker, 1974).

In this programme analysis, the reservoir operation satisfies the following needs as, power production, water supply and flood control which include irrigation as in Figure 1. These three purposes will be kept constant, to avoid dimensionality problem, while the level of water supply treated as a parameter. The programme problem will be reduced to finding the optimal monthly water release from the reservoir in order to maximize the revenue from power production while satisfying the constant. Therefore, with the knowledge of the unit price for water and energy, the annual revenue can be calculated for different operational policies, and the optimum one can be selected, moreover the constraints of system include storage constraint, constraint on release of water and energy generation (Gundiri, 2001).

The release constraints require that the allowance releases for each month must neither be less than the minimum release that will bring the reservoir to its maximum storage level for the next month or will bring the reservoir to a minimum storage level. Also, evaporation losses from the surface of the reservoir will have a significant effect on the total inflow, considering tropical climate of Nigeria (Mohammed and Sera, 2001; Abam, 1999).

Moreover, Kayagu and Sansom (2003) in their public/private partnership in small towns water supply stated that private sector participation in the delivery of public service like water supply can:

(i) Inject technical/managerial expertise into the water sector and the transfer of technology innovations.

(ii) Improve the economic efficiency of the sector, in terms of both operating performance and the use of capital investment.

(iii) Inject large-scale investment capital into the sector, or to create access to private capital markets.
Table 1. Optimization for decision variable 3.

<table>
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$U_3(t_3) = \{100[1-e^{0.2(y_3+1)}]-15(y_3+1)^{2.5}\}.$

Table 2. Optimization for decision variable 2.

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$U_2(t_2) = \{150[1-e^{-0.8(y_2+2)}]-20(y_2+2)+f_3(t_3)\}.$

Table 3. Optimization for decision variable 1.

<table>
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$U_1(y_1) = \{200[1-e^{-0.8(y_1+4)}]-25(y_1+4)+f_2(t_1-y)\}.$

3rd stage for decision variable 1

From Table 3, optimal allocation $y_1^* = 4$ (with max utility 273.8)

$x_1^* = y_1^* + m_1 = 4+4 = 8$

2nd stage for decision variable 2

$t_2 = t_1 - y_1^* = 9 - 4 = 5$

$t_2 = 5 \Rightarrow$ from Table 2, $y_2^* = 2$ (with maximum utility 92.01)

$x_2^* = y_2^* + m_2 = 2 + 2 = 4$

1st stage for decision variable 3

$t_3 = t_2 - y_2^* = 5 - 2 = 3$ (with maximum utility 39.03)

$t_3 = 3 \Rightarrow$ from Table 1, $y_3^* = 3$

$x_3^* = y_3^* + m_1 = 3 + 1 = 4$

Hence optimal or maximum allocation

$X_1^* = 8 \Rightarrow$ N8b for Hydropower

$X_2^* = 4 \Rightarrow$ N4b for water supply

$X_3^* = 4 \Rightarrow$ N4b for flood control

Benefit

(i) Maximizes water and electricity supply.
(ii) Maximizes profits from developed water resources and generated services.

(iii) Minimizes unit delivery cost or retail cost.

(iv) Minimizes unit cost of production, meaning uninterrupted mainte

(v) Determines optimum required contribution for flood mitigation.

Conclusion

This operation research program will increase benefit with a non-proportional increase in cost. It therefore enhances economic justification of the project, power and water supply, and flood mitigation. Moreover, the research broadens the project scope and suggests the best/optimal solution to contemplated problems by the Federal Ministries of Power and Water Resources. Also, it opens up employment opportunities for young graduates, especially fresh and energetic engineers (Neefjes, 2000).

The private sector participation in the water sector will reduce the level of public subsidies to the sector, and the redirection of these subsidies from the group currently served to the poor and those not currently served. This will distant the sector from short term political intervention in the operation of utilities for intervention of powerful vested interest (White and Fane, 2001).

Consequently, the quest for increase in Mega Watt electricity production, to improve productivity of local industries and encourage the growth of small scale industries will be on the increase. More so, the problems of water and sanitation will be reduced to the bearest minimum in Anambra and Enugu states.

REFERENCES


