

Full Length Research Paper

Reliability assessment of progressive failure of a low rise framed building on weak soil-foundation interaction

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A 4-storey reinforced concrete framed buildings was modelled in 3-dimensional analysis with the same sections and loadings for both rigid and weak foundations respectively using ETABS and FORM5 Softwares in accordance with Euro code provisions. The weak foundation was initially analysed and designed as a fixed column-foundation joint and later re-analysed and redesigned as a hinged column-foundation joint. The ETABS software was used to obtain the most critical component member forces and bending moments while FORM5 software was used to obtain the reliability indexes. The results revealed that due to the effect of weak soil safe bearing capacities, allowable maximum displacement was exceeded resulting in lower predicted reliability indexes and higher probability of failures that enhanced progressive failure. The reliability index recommended by Euro code was not achieved due to the effect of weak soil-structure interactions which showed that it will be very disastrous if rigid soil-structure interactions were assumed for a weak soil safe bearing capacities. The use of a hinge column-foundation joint for structural analysis will produce increased sections and reinforcement areas in reinforced-concrete frames. These would consequently improve the reliability indices of structures built on weak soils and reduces its probability to fail. Therefore, it was concluded that, a hinged joint should be adopted as column-foundation connection when the soil is generally weak. The findings of this study would be a useful guide and reference materials for structural safety and reliability analysis with regards to variation of soil type.

Key words: Soil structure interaction, soil bearing capacity, reliability index, probability of failure.

INTRODUCTION

Schulze (1943), a prominent historical figure in soil mechanics and foundation engineering in Germany, stated in 1943 that "For the determination of allowable

bearing pressure, the geophysical methods, utilizing seismic wave velocity measuring techniques with absolutely no disturbance of natural site conditions, may

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yield relatively more realistic results than those of the geotechnical methods, which are based primarily on borehole data and laboratory testing of so-called undisturbed soil samples. Therefore, Tezcan and Ozdemi (2011) based on a variety of case histories of site investigations, including extensive borehole data, laboratory testing and geophysical prospecting at more than 550 construction sites, an empirical formulation is proposed for the rapid determination of allowable bearing pressure of shallow foundations in soils and rocks. The proposed expression collaborates consistently with the results of the classical theory and is proven to be rapid, and reliable. Plate load tests have been also carried out at three different sites, in order to further confirm the validity of the proposed method. It consists of only two soil parameters, namely, the in situ measured shear wave velocity and the unit weight. The unit weight may be also determined with sufficient accuracy, by means of other empirical expressions proposed (Tezcan and Ozdemi, 2011).

According to Halabian et al. (2003), statistics and probabilistic analyses and risk assessments can be very useful decision-making tools when dealing with structural-geotechnical problems. Wind loads, dynamic properties of soil underneath the structure and material characteristics of the structure are important factors that affect the wind action on the structure and consequently the structural wind-induced response. The main associated uncertainties that are very useful in the estimation of these factors are human error or inherent variability criteria which are at the forefront for the use of reliability approaches to evaluate the risk of failure during the service period of the structure under consideration. They performed the probabilistic base force analyses for the tall structure; the substructure approach in which the soil supporting the foundation is modelled by the foundation compliances as functions of soil shear wave velocity is used to account for the soil–structure interaction efficiently. A three main variable probabilistic approach is used to account for the uncertainties in shear wave velocity of the soil underneath the foundation, the concrete strength and the design wind speed on the calculated response and the base forces (Halabian et al., 2003).

The result of the investigation show that the dynamic response of the tower increases as soil shear wave velocity decreases. For the range of soil shear wave velocity encountered in practice, the base forces of the structure may increased by up to 20% as a result of the foundation flexibility. For the limit state considered in this study, it was found that the reliability index decreases by up to 15% and the probability of failure increases by up to one order of magnitude as a result of the soil–structure interaction effect (Halabian et al., 2003).

Jihong and Liqiang (2018) defined the failure or damage of a structure as consisting of a series of failure incidents, including the transition from rigid joints to

pinned ones and component fractures. They used the Improved Structural Vulnerability Theory (SVT) in which failure processes of essential components is defined according to damage characteristics of their ductile and brittle members. The improved method accurately identified possible collapse mode of steel moment frame because of the transformation processes of the failure of a 4-storey steel framed building.

The collapse mode caused by failure scenarios near the joints in the bottom story had the maximum vulnerability index. Therefore, such a failure mode should be avoided during structural design because the first story is likely to be the weakness or the weakest link for the point of unzipping of the structure which could trigger collapse mechanism. On the other hand, the collapse mode with a “beam plastic hinge” failure as the expected failure mode had the minimum vulnerability index. Thus, the improved method theoretically verified the rationality of the seismic design concepts of “strong-joint weak-member” and “strong-column weak-girder”, as found in the codes of different countries (Jihong and Liqiang, 2018).

Jiang et al. (2019) similarly used the Improved Structural Vulnerability Theory (ISVT) to analyse a pedestrian bridge on the campus of Florida International University (FIU Bridge) that collapsed during construction, and 6 victims were identified in this disaster event. By comparing the collapse mode identified by ISVT and the actual collapse scenario of the FIU Bridge, it was found that ISVT could effectively identify the weakness and predict the possible collapse modes of the FIU Bridge by a quantitative vulnerability index. Then, parametric analyses were conducted for the bridge to different unforeseen damage events, and the collapse mechanism of the failure characteristics of the FIU Bridge is also revealed by the ISVT. The result show that, if a component of the FIU Bridge is damaged in an unforeseen damage event, the maximum vulnerability index would be increased 10~419%. The increments are more obviously for the cases that the damaged component is located at mid-span. Once one of these components is damaged, the maximum vulnerability index of the bridge is increased dramatically. Thus, these key components should be properly designed by the researchers and engineers, both in construction stage and servicing stage. It is also recommended that to avoid the relative slender components, rigid frame bridges should be designed because the relative slender components would be easily buckled if an unforeseen damage event occurred. Once these components damaged, the collapse risk of the bridge would be increased uncontrollably (Jiang et al., 2019).

Wang et al. (2010) describes reliability as the ability of a system or component to function under stated conditions for a specified period of time. The term reliability, in an engineering sense, refers to the probability that a structure will not reach one or several specified

limit states during its service life. For a long time, the concept of “reliability” has been used to evaluate the quality of engineering structures. However, due to the variations and uncertainty of material properties and load, alongside various types of possible errors during construction and usage; from the engineering point of view, a structural problem can be considered as “uncertain” when some lack of knowledge exists about the theoretical model which describes the structural system and its behavior, either with respect to the model itself, or to the value of its significant parameters (Wang et al., 2010).

Due to the uncertainty in materials and soil parameters, geometric dimension, loads and other parameters in a real structure, fragility analysis should be performed to evaluate the collapse probability. The structural performance and response curves are firstly obtained through progressive collapse analyses. Thereafter the fragility functions are created according to the key points in structural performance curves. Lastly, the collapse probabilities of the structures are quantified by substituting the most disadvantageous responses into the fragility functions. Such progressive collapse analyses include push-down analysis (PDA) and nonlinear time-history analysis (NTHA). Push-down analysis (PDA, correspondence to the structural performance) results represent the collapse resistance of the steel frame structure, and the nonlinear time-history analysis (NTHA, correspondence to the structural response) can obtain the dynamic responses of the structure (Grierson et al., 2005; Liu et al., 2010; Xu and Ellingwood, 2011).

In the past decades, several guidelines have been developed for designs against progressive collapse based on a number of experimental and numerical studies that have been conducted. However, majority of the previous experimental and numerical investigations focused on the response of buildings under the scenarios of loss of interior or exterior columns and robustness of the framed building (Lu et al., 2012). The loss of a corner column in the event of a terrorist attack or accident can also trigger progressive collapse given the paucity of surrounding elements that could help to redistribute the axial forces and bending moments initially resisted by the lost corner column (Menzies, 2005; Agarwal and England, 2008).

According to Adesanya and Olanrewaju (2014), quacks in the building industry most time execute building construction without necessarily investigating the geologic and geotechnical nature of the soil. However, weak soil is more likely to trigger progressive collapse, if a rigid joint of foundation is erroneously assumed on a weak insitu soil; this can lead to progressive failure through the foundation abrupt or excessive settlement, translation and rotation that can trigger the collapse of the whole building through the dislocation from the weakest links located at critical positions within the structural system. Most of these weak soils are usually within the active zones of the soil profiles and are not strong

enough to effectively carry the weight of the building super-imposed on them. If this fact is not considered during the structural analysis and design phases of the project, they might lead to differential settlement and other limit state conditions that might eventually lead to progressive structural failure. In some areas, due to weak nature of the soil, building might even sink.

In general, the foundations of low rise framed buildings are designed as rigid foundation-soil interaction. This assumes by analysis that the whole footing will not have infinitesimal settlements, translations or rotation because it is a rigid element; whereas, if the soil is weak, the foundation-soil interaction will have some degree of flexibility. This may lead to differential settlements, translations and or rotation. The study focused on the response of buildings under the scenarios of loss of interior or exterior columns at the ground or first story level. If a rigid joint of foundation soil interaction is erroneously assumed on a weak insitu soil, it is likely to trigger progressive failure that could lead to eventual collapse of the structural system. Such progressive failures arise from uncertainties that affect the structural performance of buildings when a rigid soil–structure interaction was assumed for a weak soil instead of a hinge. The structural loadings were provided according to Euro code 1(2002) and members were analyzed, designed and detailed in accordance with Euro code 2 (2004).

Foundation failure

The foundations transfer and spread the loads from a structure’s column and walls into either wider areas or deeper profiles of the ground. The safe bearing capacity of the soil must not be exceeded, otherwise excessive uneven settlements and rotations might occur, resulting in damage to the building and its service facilities, such as the water and gas mains could in turn aggravate the fluidity and lowering of the soil safe bearing capacity. Foundation failure can also affect the overall stability of a structure so that it is liable to slide, lift vertically, or even overturn. Applying a bearing pressure which is safe with respect to failure does not ensure that settlement of the foundation will be within acceptable limits. Therefore, settlement analysis should generally be performed since most structures are sensitive to uneven excessive settlement. The safe bearing capacity and the allowable differential settlement must together be paramount parameters for consideration at all times. Table 1 was obtained from Mosley et al. (2007) and it shows the allowable bearing capacities of rocks and various soils; all soils less than 100 kN/m² are generally considered as weak.

Probability of failure and reliability index

The most important term used in the theory of structural

Table 1. Typical allowable bearing capacity of rocks and soil.

Rock or Soil	Typical bearing capacities (KN/m ²)
Massive Igneous rock	10,000
Sand Stone	2000 to 4000
Shales and mudstone	600 to 2000
Gravel, sand and gravel, compact	600
Very stiff clay	300 to 600
Medium dense sand	100 to 300
Stiff clay	150 to 300
Loose fine sand	Less than 100
Firm clay	75 to 150
Soft clay	Less than 75

Source: Mosley et al. (2007).

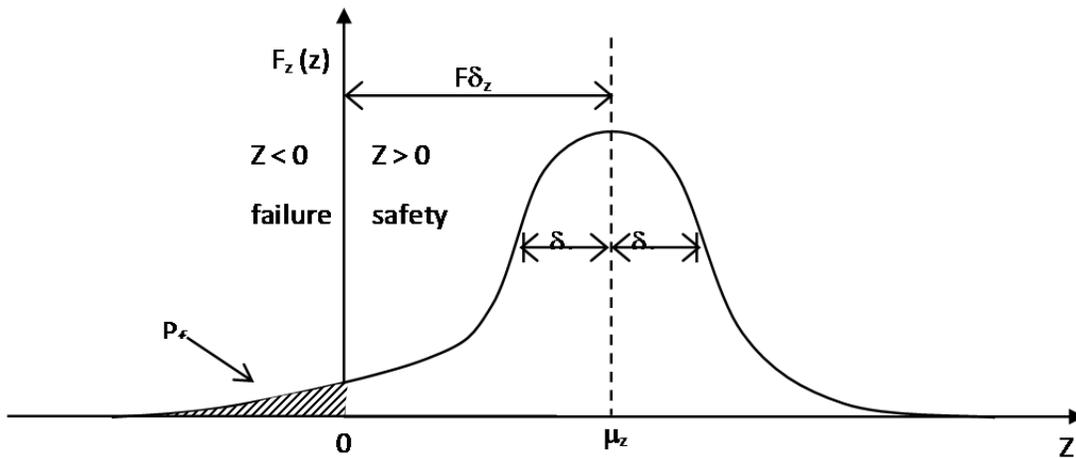


Figure 1. Distribution of safety margin.

reliability is evidently the probability of failure P_f . In order to define P_f properly, it is assumed that structural behavior may be described by a set of basic variables $X = [X_1, X_2, \dots, X_n]$ characterizing actions, mechanical properties, geometrical data and other model uncertainties. Furthermore, it is assumed that the limit state (ultimate and serviceability) of a structure is defined by the limit state function (or the performance function), usually written in an implicit form as Equation 1 (Figure 1) (Euro code 1, 2002).

$$Z(X) = 0 \tag{1}$$

The limit state function $Z(X)$ should be defined in such a way that for a favorable (safe) state, the function is positive ($Z(X) \geq 0$) and for an unfavorable (failure) state of the structure the limit state function is negative

$$Z(X) < 0 \tag{2}$$

For most limit states (ultimate, serviceability), the

probability of failure can be expressed as

$$P_f = P\{Z(X) < 0\}. \tag{3}$$

The probability of failure P_f can be assessed, if basic variables $X = [X_1, X_2, \dots, X_n]$ are described by appropriate probabilistic models. Assuming the basic variables $X = [X_1, X_2, \dots, X_n]$ are described by independent joint probability density function $\phi_x(x)$, then the probability P_f can be determined using the integral

$$P_f = \int \phi_x(x) dx; \quad Z(X) < 0 \tag{4}$$

An equivalent to the failure probability is the reliability index β , formally defined as a negative value of a standardized normal variable corresponding to the probability of failure P_f . Thus, the following relationship may be considered as a definition

$$\beta = -\phi_u^{-1}(P_f) \tag{5}$$

Here, $\phi_u^{-1}(P_f)$ denotes the inverse standardized normal

Table 2. Reliability classification for different reference periods.

Reliability classes	Failure consequences	Reliability indexes		Examples
		1 year	50 years	
RC3	High	5.2	4.3	Bridges, public buildings
RC2	Medium	4.7	3.8	Residences, offices
RC1	Low	4.2	3.3	Agricultural buildings

RC= Reliability class.
Source: Eurocode 2 (2004).

distribution function. At present, the reliability index β defined by Equation 5 is commonly used as a measure of structural reliability. The basic recommendation concerning a required reliability level is often formulated in terms of the reliability index β related to a certain design working life; T_d (Milan and Ton, 2015; Euro code 2, 2004) recommends the target reliability index for two reference periods (1 and 50 years), as shown in Table 2. For a structure of Reliability Class 2 (RC2-residences and offices), the minimum reliability index $\beta_{min} = 3.8$ used should be provided such that probabilistic models of basic variables are related to the return period of 50 years. The same reliability level should be reached when $\beta = 4.7$ are applied using the theoretical models for one year recurrent interval. Note that the couples of β -values correspond to the same reliability level only when failure probabilities in individual time intervals (basic reference periods for variable loads) are independent. Considering a reference period equal to the remaining working life, it might be understood from Euro code 1 (2002) that the reliability level corresponding to an arbitrary remaining working life can be calculated using the following expression:

$$\beta_{t_{ref}} = \Phi^{-1} \{ [\Phi(\beta_1)]^{t_{ref}} \} \tag{6}$$

Where β_1 = target reliability index taken from Table 2 (Eurocode 2, 2004) for a relevant reliability class and the reference period $t_{ref} = 1$ year. For the model structure, it follows that $\beta \approx 4.1$ should be considered for $t_{ref} = 15$ yr (Sýkora et al., 2011).

The First-order Reliability Method (FORM)

FORM is an abbreviation for the first-order reliability method. It approximates the limit-state function somewhere on the limit-state surface, that is, at a point where $g=0$ instead of at the mean. The limit-state surface, which separates the failure domain from the safe domain, is shared by all equivalent limit-state functions (Figures 2 and 3). Given the prologue, two questions appear: Which point on the limit-state surface to select, and thereafter, how to obtain the failure probability. The answer to both questions is found in the “standard normal space.” This is

a space of uncorrelated standard normal random variables, in which realizations are denoted (y) and the joint Probability Density Function (PDF) is given by the following equation (Baecher and Christian, 2003):

$$\varphi(y) = \frac{1}{\sqrt{(2\pi)^n}} \exp \left(-\frac{1}{2} y^T y \right) \tag{7}$$

The transformation from the original x -space to the standard normal y -space is adopted for two reasons (Der Kiureghian, 2005):

1. The joint PDF in the standard normal space is rotationally symmetric and decays in the radial and tangential directions. Consequently, the point on the limit-state surface that is closest to the origin is the point in the failure domain with highest probability density. As a result, the point closest to the origin is an appealing point for approximating the limit-state function, because that is where a significant portion of the failure probability density is located.

2. In the standard normal space, it is possible to develop a formula for the probability content outside a hyper-plane, which is used in FORM, and outside a hyper-parabolic, which is used in Second Order Reliability Method (SORM). The probability content outside a hyper-plane is

$$P_f = \Phi(-\beta) \tag{8}$$

Where, β is the distance from the origin to the closest point on the hyper-plane. The limit-state function is denoted $g(x)$ in the original space, and it is denoted by capital letter $G(y)$ in the standard normal space. Figure 1 graphically estimates the limit state probability.

Second moment concept

With resistance R and load effect Q , each second moment random variable (that is, having normal distribution) the limit state equation is the safety margin. $Z = R - Q$ and the probability of failure P_f is;

$$P_f = \Phi(-\beta) \tag{8a}$$

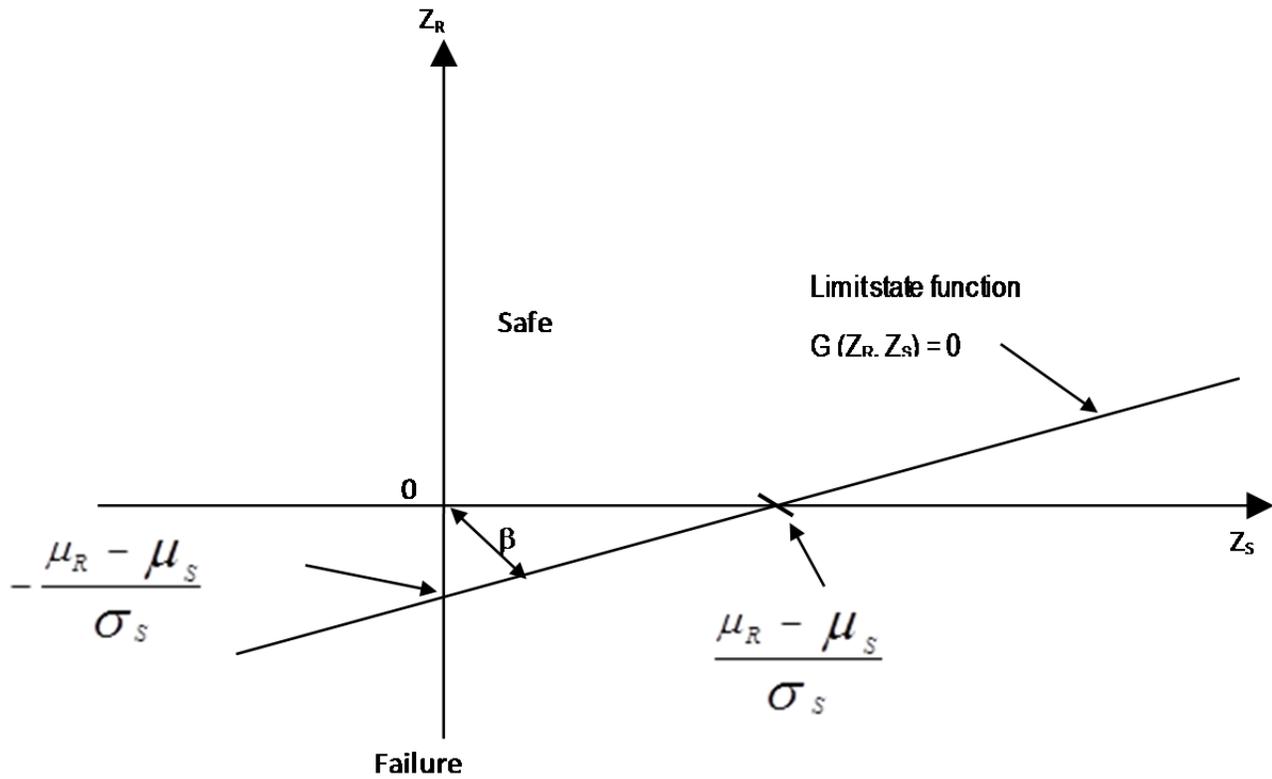


Figure 2. Reliability index defined as the shortest distances in the space of the reduced variables.

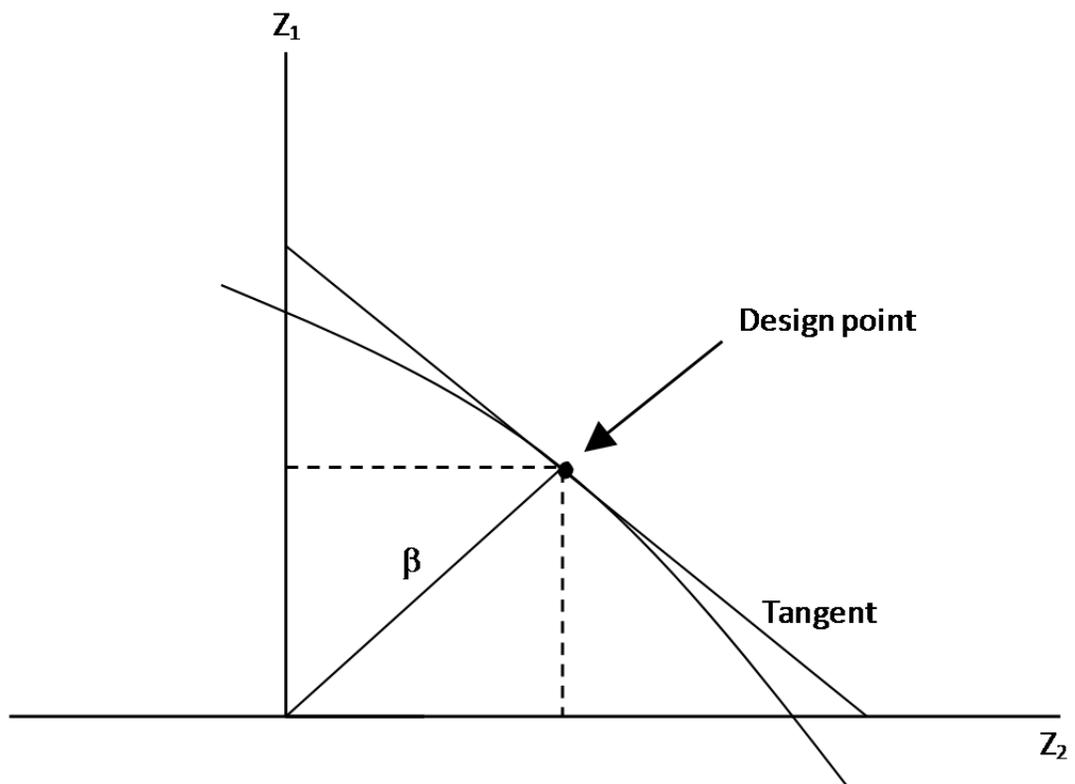


Figure 3. Hasofer-Lind reliability index.

$$\beta = \frac{\mu_z}{\sigma_z} \tag{8b}$$

Where, β is the safety index (reliability index);
 ϕ is the standard normal distribution function;
 μ_z is the mean of the safety margin (z);
 σ_z is the standard deviation of (z).

The above equation yields the exact probability of failure when both R & Q are normally distributed. However, P_f defined in this way is only a nominal failure probability for other distributions R & Q. Conceptually, it is probably better in this case not to refer to the probabilities at all but simply to β , the safety index (Melchers, 1987).

However, serious difficulties with the second moment format were discovered in the development of practical examples. First, it was not obvious how to define a reliability index in the cases of multiple random variables e.g. when more than two loads were involved. More disturbingly, Ditlevsen (1973) and Lind (1995) independently discovered problems of invariance. Cornell's index was not constant when certain simple problems were reformulated in a mechanically equivalent way, yet no other safety index would remain constant under other mechanical admissible transformation (Madsen et al., 1986).

Reliability Index

As shown in Figures 1 and 2, β is simply a measure (in standard deviation σ_z units) of the distance that mean μ_z is away from the origin $Z = 0$. This point marks the boundary to the failure region. Hence β is direct measure of the safety of the structural element and greater β represents greater safety or lower normal probability of failure.

Hasofar and Lind (1974) defined reliability index as the shortest distance from the origin of reduced variables to the line $g(Z_R, Z_S) = 0$ as will be shown in Figure 2 (Nowak and Collins, 2000).

Using geometry, we can calculate the reliability index (shortest distance) from Equation 8c:

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{8c}$$

where β is the inverse of coefficient of variation of the function $g(R, S) = R - S$ when R and S are uncorrelated. The definition for a two-variable case can be generalized for n variables as follows:

Consider a limit state function $g(x_1, x_2, \dots x_n)$ where the

x_1 variables are all uncorrected. The Hasofer and Lind (1974) reliability index is defined as follows:

i) Define the set of reduced variables ($z_1, z_2 \dots Z_n$) using

$$Z_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \tag{8d}$$

ii) Redefine the limit state function by expressing it in terms of the reduced variable ($z_1, z_2, \dots Z_n$).
 iii) The reliability index is the shortest distance from the origin in the n-dimensional space of the reduced variables to the curve described by $g(z_1, z_2, \dots Z_n)$.

First-Order Second-Moment Reliability Index

Linear limit state function

Consider a function $g(z_1, z_2, \dots z_n)$

$$= a_0 + a_1 x_1 + a_2 x_2 + \dots + a_n x_n$$

$$= a_0 + \sum_{i=1}^n a_i x_i \tag{8e}$$

Where the a_i terms ($i = 0, 1, 2, \dots n$) are constants and the x_i terms are uncorrelated random variables. If we apply the three steps procedure outlined above for determining the Hasofer and Lind (1974) reliability index, we would obtain the following expressions.

$$\beta = \frac{a_0 + \sum_{i=1}^n a_i \mu_{x_i}}{\sqrt{\sum_{i=1}^n (a_i \sigma_{x_i})^2}} \tag{8f}$$

Where,

β is called a second moment measure of structural safety, because only the first two moments (mean and variance) are required to calculate β . For a non-linear limit state function, we can obtain an approximate answer by linearizing the non-linear function using tailors series expansion. The result is:

$$\beta = \frac{g(\mu_{x_1}, \mu_{x_2}, \dots \mu_{x_n})}{\sqrt{\sum_{i=1}^n (a_i \sigma_{x_i})^2}} \tag{8g}$$

where $a_i = \left. \frac{dg}{dx_i} \right|_{\text{evaluated at mean values}}$

The first order second moment mean values have some disadvantages.

- i) Results are inaccurate, if the tails of the distribution functions cannot be approximated by a normal distribution.
- ii) There is invariance problem: the value of the reliability index depends on the specific form of the limit state function.

Hasofer and Lind (1974) proposed a modified reliability index that did not exhibit the invariance problem. The correction is to evaluate the limit state function at a point known as the design point instead of the mean values. The design point is a point on the failure surface $g = 0$ (Figure 3). Since this point is generally not known a priori, an iteration technique must be used to solve for the reliability index.

Structural Reliability Assessment

Reliability analysis evaluates the probability of structural failure by determining whether the limit state functions are exceeded. Reliability analysis is not limited to calculation of the probability of failure (Jihong and Liqiang, 2018). Evaluation of various statistical properties, such as probability distribution functions and confidence intervals of structural responses, plays an important role in reliability analysis. When a structure exceeds a specific limit, the structure is unable to perform as required and then the specific limit is called a *limit-state*.

The structure will be considered unreliable, if the failure probability of the structure limit-state exceeds the required value. For most structures, the limit-state can be divided into two categories:

i) Ultimate limit-states: are related to a structural collapse of component part or all of the structure. Examples of the most common ultimate limit-states are corrosion, fatigue, deterioration, fire, plastic mechanism, progressive collapse, fracture, etc. Such a limit-state should have a very low probability of occurrence, since it may risk the loss of life and major financial losses.

ii) Serviceability limit-states: are related to disruption of the normal use of the structures. Examples of serviceability limit-states are excessive deflection, excessive vibration, drainage, leakage, local damage, etc. Since there is less danger than in the case of ultimate limit-states, a higher probability of occurrence may be tolerated in such limit-states. However, people may not use structures that yield, excessive deflections,

vibrations, etc.

Generally, the limit-state indicates the margin of safety between the resistance and the load of structures. The limit-state function, $g(\cdot)$, and probability of failure, P_f , can be defined as

$$g(x) = R(X) - S(X) \quad (9)$$

$$P_f = [g(\cdot) < 0] \quad (10)$$

Where R is the resistance and S is the loading of the system. Both $R(\cdot)$ and $S(\cdot)$ are functions of random variables X . The notation $g(\cdot) < 0$ denotes the failure region. Likewise, $g(\cdot) = 0$ and $g(\cdot) > 0$ indicate the failure surface and safe region, respectively.

Another well-known definition of reliability analysis is the *safety factor*, F :

$$F = \frac{R}{S} \quad (11)$$

Failure occurs when $F = 1$, and if the safety factors are assumed for normally distributed, the safety index defined from load and resistance parameters with means μ_S and μ_R , and standard deviations σ_S and σ_R , respectively, the reliability index β is given by Equation 12

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad (12)$$

Axioms, Theoretical Assumptions and Analysis

If the soil is weak and the first story is the weakness or the weakest link due largely to the point of unzipping of the structure, it could trigger collapse mechanism. When unzipping Jihong and Liqiang (2018) or a plastic hinge formation of the structure occurs at the first storey, the column element which was originally designed as rigidly fixed at both ends with effective length equal to $0.5L$. If the unzipping or a plastic hinge is to happen at the two end of a column resting on an unbraced isolated or pad footing, the effective length would be gradually transformed into L and this transformation will have serious implication on the Euler critical load of the element subjected to compression and structure robustness integrity deterioration and after such long term progressive deterioration, could probably be sliding into collapse mechanism. Therefore, two important conditions are hereby postulated:

i) If the reinforced concrete column resisting an isolated or pad footing is effectively braced along both axes (x and y) by supporting reinforced concrete ground and underside of the floor of the first storey, or in addition, the pad footing is supported on pier or pile and beam skeleton to transfer the load from the isolated column to soil with higher safe bearing pressure, assuming a rigidly

fixed support reaction during the analysis is considered best and adequate; Whereas,

ii) If the reinforced concrete column resisting an isolated or pad footing is unbraced along both axes (x and y) by supporting reinforced concrete ground and underside of the floor of the first storey, or no additional support to the pad footing with pier or pile and beam skeleton to transfer the load from the isolated column to soil with higher safe bearing pressure, assuming a hinged support reaction during the analysis is considered as though not as safer as case (i) but better and would be safer than being analysed as a rigidly fixed support reaction during the analysis.

iii) The plastic or progressive failure of a framed building can be analyzed either by kinematic or statical methods but the statical method which is based on the force method of analysis was used. In this approach, Nisimov (1984) created the following relationships between the load magnitudes for a multi-bay structure.

Let us consider a multi-bay frame with varied span (l_i), column height (h_i) and unequal magnitude of evenly distributed loadings (q_i) along the vertical axis and a point load either from the effect of wind or seismic loadings (V_{bf}) along the horizontal axis. The varied multi-bay frame is subjected to a progressive failure mechanism due to unzipping and formation of plastic hinges (l_i) along the cross section caused by the following conditions (Figure 4a and b) respectively. In summary

$$i. \quad \eta = \frac{V_{bf}}{q_{Tp}} ; \quad \xi = \frac{q_{ip}}{q_{1p}} \quad (13a)$$

Where:

- q_{Tp} = uniformly distribution load (UDL) at various spans;
- η = ratio of the concentrated wind load at floor levels and the UDL at various spans;
- ξ = ratio of q_{Tp} which is variation of uniformly distributed loads;
- V_{bf} = the concentrated load from wind at floor level.

$$ii. \quad w = \sum \frac{M_{TAi} + M_{TBi} + M_{TC(i+1)}}{M_{TB1} + M_{TC1}} - \sum \frac{\xi_i l_i}{h_i} \quad (13b)$$

For the first bay M_{TBi} , M_{TB1} and first column M_{TCi} and M_{TC1} ; and

$$iii. \quad \beta_i = \frac{y_{ci}}{y_{Cl}} = \sqrt{\frac{M_{TBi} + M_{TCi}}{\xi(M_{TB1} + M_{TC1})}} \quad (13c)$$

$$iv. \quad y_{Ci} = \sum \frac{\xi_i \beta_i l_i}{w} \left[-1 + \sqrt{1 - \frac{w(2\eta + \sum \frac{\xi_i l_i^2}{h_i})}{(\sum \frac{\xi_i \beta_i l_i}{h_i})^2}} \right] \quad (13d)$$

v. Ultimate UDL for the first Bay

$$q_{ib} = \frac{2(M_{TB1} + M_{TC1})}{y^2 C_i} \quad (13e)$$

vi. Ultimate UDL q_{ib} and V_b

$$q_{ib} = \xi_i q_{1b} \quad (13f)$$

and

$$V_b = \eta q_{1b} \quad (13g)$$

Structural Modelling and Reliability-Based Analysis of RC Framed Structure

A four-storey reinforced concrete-framed building designed for office use is studied to demonstrate application of the proposed progressive collapse assessment method. The structural layout and modeling of the building are produced as Figures 5a and b. The building is designed in accordance with the requirements of current Euro code 2 (2004). All floors were designed to carry equal gravity but variable live loads along with the effects of wind pulsation. The methodology adopted is in three stages:

- a) structural modelling
- b) structural analysis and
- c) reliability analysis.

The materials used at these stages were wo-computer software:

- i) Extended 3D Analysis of Building Systems (ETABS) for the structural modeling, analysis and design, while
- ii) First Order Reliability Method (FORM) was used for the reliability analysis. This computer-based software is commercially available and can be easily procured.

Figure 5b is the structural model of the 30 m RC four storeys framed building as obtained from the ETABS structural analysis (ETABS Version, 2015).

Design specifications

The structural configuration is a reinforced concrete three bays in the Y-direction and five bays in the X-direction. It has a storey height of 3 m but a building overall height of 12 m. The building is subjected to both dead, live and wind load respectively. Other information to be obtained includes the design loads as recommended by Euro code 1 (2002); these include the imposed live, dead and wind loads respectively. The wind gust for Maiduguri was obtained from published literature (Onundi et al., 2009). The study is considering a model consisting of six frames spaced at 8 m centers for a building length 30,000 mm (6000 m x 5) and width of 9600 mm (2 x 3600 mm + 2400 mm) as shown in Figures 5a and b.

Therefore, for the analysis, the following were also used ETABS software version (2015):

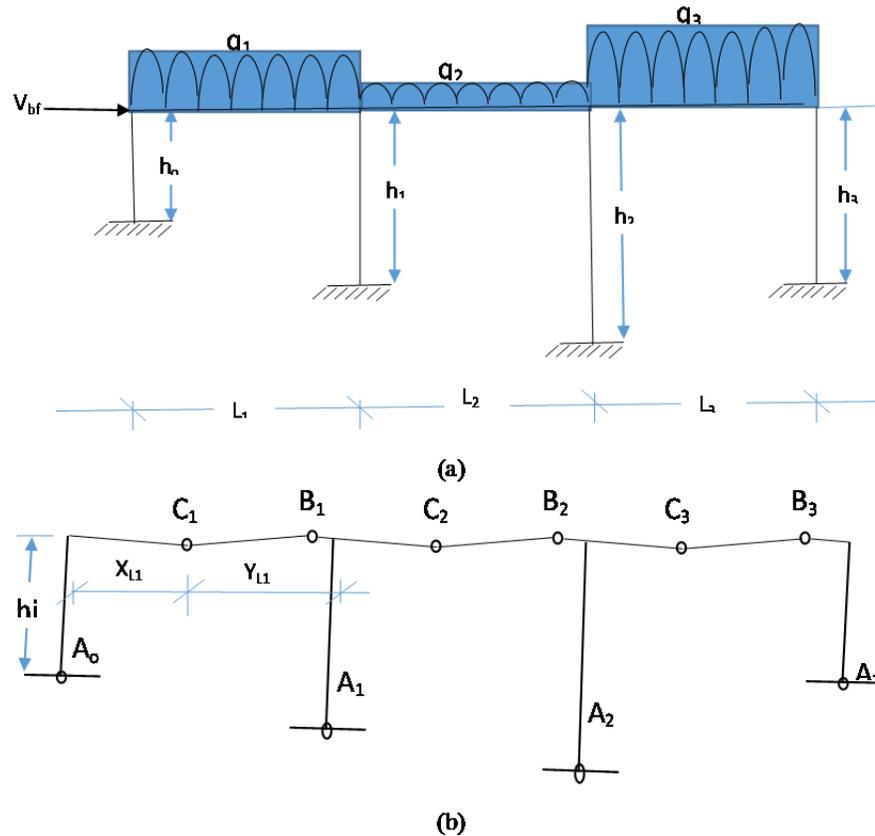


Figure 4. Varied multi-bay framed structure (a) of the actual structure and (b) with collapse mechanism (that is, plastic equilibrium).

- i) The base joints or degree of freedom were assumed to be hinged for the weak and modified weak foundation and rigidly fixed for the foundation with high safe bearing capacity.
- ii) The live load on the slab was assumed to be 4 kN/m^2 (Mosley et al., 2007).
- iii) Self-weight (dead) of the reinforced concrete slab (assuming 150 mm thick and density of 24 kN/m^3) is 3.75 kN/m^2 .
- iv) Super dead load which comprises of finishes and partitions and services = 3.0 kN/m^2 .
- v) The beams were assumed as $300 \times 550 \text{ mm}$ for the hinged and $230 \times 500 \text{ mm}$ for the fixed. The concrete columns were assumed to be 300×450 for the hinged and the fixed columns were assumed to be $230 \times 400 \text{ mm}$.

Developed model and assumptions using Euro code

The applied forces are shown in Figure 6. The load combination factors are applied to the forces and moments obtained from the associated load cases and are then summed to obtain the factored design forces and moments for the load combination. The following ten

loading conditions as shown in Table 3 were used in the ETABS software during the structural analysis in order to obtain the most critical component member forces and bending moments.

The design load combinations are used for determining the various combinations of the load cases for which the structure needs to be designed and checked. The load combination factors used vary with the selected design code (Eurocode 0, 2002).

Case studies and development of their probability objective functions and reliability indexes

Three case studies labelled A, B and C were structurally modelled, analyzed and designed using ETABS Software (2015) in accordance with Euro code provisions. Reliability analysis was carried out based on the First Order Reliability Analysis Method FORM 5 (Gollwitzer et al., 1988).

- i) Case study "A" represent building frames with rigid foundation (that is, representing a soil with high safe bearing capacity and designed as a rigidly fixed column-foundation joint);

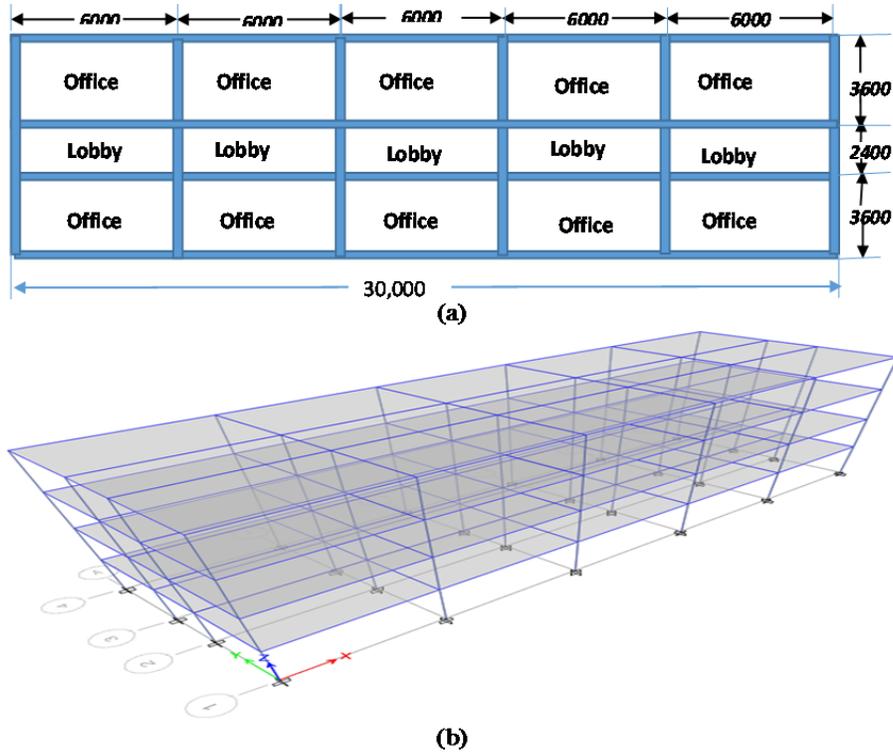


Figure 5. (a) Structural layout and (b) Three-dimensional models of the 30 m four storeys RC framed building.

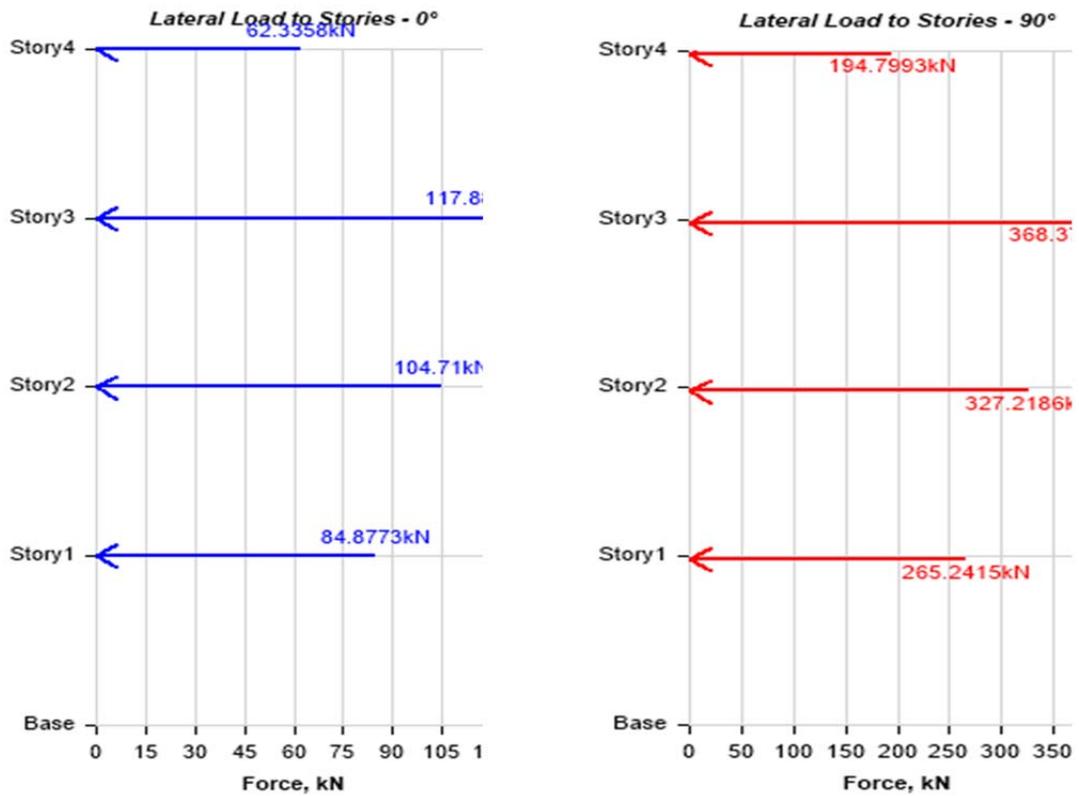


Figure 6. Total applied story forces to be distributed to the axes frames.

Table 3. Load combinations.

Name	Load Case/Combo	Scale factor	Type	Auto
DCon1	Dead	1.35	Linear Add	Yes
DCon2	Dead	1.35	Linear Add	Yes
DCon2	Live	1.5		No
DCon3	Dead	1.35	Linear Add	Yes
DCon3	Live	1.5		No
DCon3	Wind	0.9		No
DCon4	Dead	1.35	Linear Add	Yes
DCon4	Live	1.5		No
DCon4	Wind	-0.9		No
DCon5	Dead	1.35	Linear Add	Yes
DCon5	Live	1.05		No
DCon5	Wind	1.5		No
DCon6	Dead	1.35	Linear Add	Yes
DCon6	Live	1.05		No
DCon6	Wind	-1.5		No
DCon7	Dead	1.35	Linear Add	Yes
DCon7	Wind	1.5		No
DCon8	Dead	1.35	Linear Add	Yes
DCon8	Wind	-1.5		No
DCon9	Dead	1	Linear Add	Yes
DCon9	Wind	1.5		No
DCon10	Dead	1	Linear Add	Yes
DCon10	Wind	-1.5		No

ii) Case study “B” represent frames with weak foundation (that is, representing a soil with low safe bearing capacity but the design has erroneously assumed this as a soil with high safe bearing capacity by initially representing it with a rigidly fixed column-foundation joint but re-analyzed and tested for reliability as hinged column-foundation joint with the same quantity of reinforcement required for the rigidly fixed column-foundation joint) whereas;

iii) Case study “C” represent frames with modified weak foundation (that is, representing a soil with low safe bearing capacity whose design correctly assumed the soil as low safe bearing capacity and represented it as a hinged column-foundation joint to facilitate possible rotations of the joints due to the nature of the soil and also provided it with the required appropriate quantity of reinforcement to absorb the increase in forces and moments for stability and safety).

Limit State Equation for the Most Critical Beam at the Rigid Foundation Frame (R_{BEAM} Case Study A)

$$1.7301 \frac{f_{ck}b}{Br^2} - gk(1.35 + 1.05\alpha) = 0 \quad (14)$$

Where, b is the beam width, B_r is basic ratio = $\frac{\text{Span}}{\text{Effective length}}$,

α is load ratio = $\frac{qk}{gk}$ and f_{ck} is characteristic compressive strength of concrete. gk and qk are characteristic dead and live loads respectively.

The reliability analysis parameters used in FORM5 software for Equation 10 (represented by Equation 14) are presented in Table 4.

Limit State Equation for the Most Critical Beam at the Weak Foundation Frame (W_{BEAM} , Case study B)

$$1.547 \frac{f_{ck}b}{Br^2} - gk(1.35 + 1.05\alpha) = 0 \quad (15)$$

The reliability analysis parameters used in FORM5 software for Equation 15 are presented in Table 5.

Limit State Equation for the Most Critical Beam at the Modified Weak Foundation Frame (MW_{BEAM} Case Study C):

$$1.623 \frac{f_{ck}b}{Br^2} - gk(1.35 + 1.05\alpha) = 0 \quad (16)$$

The reliability analysis parameters used in FORM5 software for Equation 16 are presented in Table 6.

Table 4. Reliability analysis parameters for the most critical beam in case study A.

Physical meaning	Type IV (i)	Mean EX (i)	Standard deviation (Sx)	COV	Variable type	Reference
Basic Ratio (Br)	Normal =2	8.18 mm	0.164	0.02	X1	Mirza and MacGregor (1997)
Width of Beam (b)	Normal =2	230.0 mm	4.600	0.02	X2	Sanjoyan (2004)
Characteristics strength of concrete (f_{ck})	Normal =2	25.00 N/mm ²	4.500	0.18	X3	Mirza and MacGregor (1999)
Dead Load (g_k)	Log Normal =3	23.85 kN	2.385	0.1	X4	Mirza and MacGregor (2000)

Reliability Index β obtained from the analysis is 3.722 and its equivalent probability of failure P_f is 0.988E-04.

Table 5. Reliability analysis parameters for the most critical beam in case study B.

Physical meaning	Type IV (i)	Mean EX (i)	Standard deviation Sx (i)	COV	Variable type	Reference
Basic Ratio (Br)	Normal =2	8.18 mm	0.164	0.02	X1	Mirza and MacGregor (1997)
Width of Beam (b)	Normal =2	230.0 mm	4.600	0.02	X2	Sanjoyan (2004.)
Characteristics strength of concrete (f_{ck})	Normal =2	25.00 N/mm ²	4.500	0.18	X3	Mirza and MacGregor (1999)
Dead Load (g_k)	Log Normal =3	23.85 kN	2.385	0.1	X4	Mirza and MacGregor (2000)

Reliability Index β obtained from the analysis =3.336 and its equivalent probability of failure P_f =0.425E-03.

Table 6. Reliability analysis parameters for the modified weak beam in case study C.

Physical meaning	Type IV (i)	Mean EX (i)	Standard deviation Sx (i)	COV	Variable type	Reference
Basic Ratio (Br)	Normal =2	7.4 mm	0.15	0.02	X1	Mirza and MacGregor (1997)
Width of Beam (b)	Normal =2	300 mm	6.00	0.02	X2	Sanjoyan (2004)
Characteristics strength of concrete (f_{ck})	Normal =2	25 N/mm ²	4.500	0.18	X3	Mirza and MacGregor (1999)
Dead Load (g_k)	Log Normal =3	28.35 kN	2.835	0.1	X4	Mirza and MacGregor (2000)

Reliability Index β obtained from the analysis in case study C = 4.181 and its equivalent probability of failure P_f =0.145E-04.

Limit State Functions and Reliability analysis parameters for analysis and design of columns

From Figure 7, members were designed as a doubly reinforced section to resist M_a acting by itself using Equation 17.

$$M_a = 0.167f_{ck}bd^2 + 0.87f_{yk}A's (d-d') \tag{17}$$

Where, b is width, d is the effective depth and d' is the depth of compression reinforcement.

$A's$ is the area of compression reinforcement and f_{yk} is characteristic compressive strength of steel.

Limit State Equation for the Most Critical Column for the Rigid Foundation Frame (Case Study A)

$$M_a = 0.167f_{ck}bd^2 + 0.7165f_{yk}A'sd - 0.0591(1.0N_k + 1.5W_k)L^2 \tag{18}$$

Where, N_k is uniformly distributed axial force, w_k is characteristic wind load and L is the length of the column.

The reliability analysis parameters used in FORM5 software (Gollwitzer, et al., 1988) for Equation 18 are presented in Table 7.

Limit State Equation for the Most Critical Column for the Weak Foundation Frame (Case Study B)

$$M_a = 0.167f_{ck}bd^2 + 0.7165f_{yk}A'sd - 0.0913(1.0N_k + 1.5W_k)L^2 \tag{19}$$

The reliability analysis parameters used in FORM5 software (Gollwitzer, et al., 1988) for Equation 19 are presented in Table 8. Reliability Index β obtained from the Column reliability analysis if a reinforcement value of 3700 mm² is used in case study "B"= 4.27. However, if 2100 mm² reinforcement as used in case study "A" (for rigid foundation base) was used in case study "B", the β will be 2.45.

Limit State Equation for the Most Critical Column for the Modified Weak Foundation Frame (Case Study C)

$$M_a = 0.167f_{ck}bd^2 + 0.7362 f_{yk}A'sd - 0.0893(1.0N_k + 1.5W_k)$$

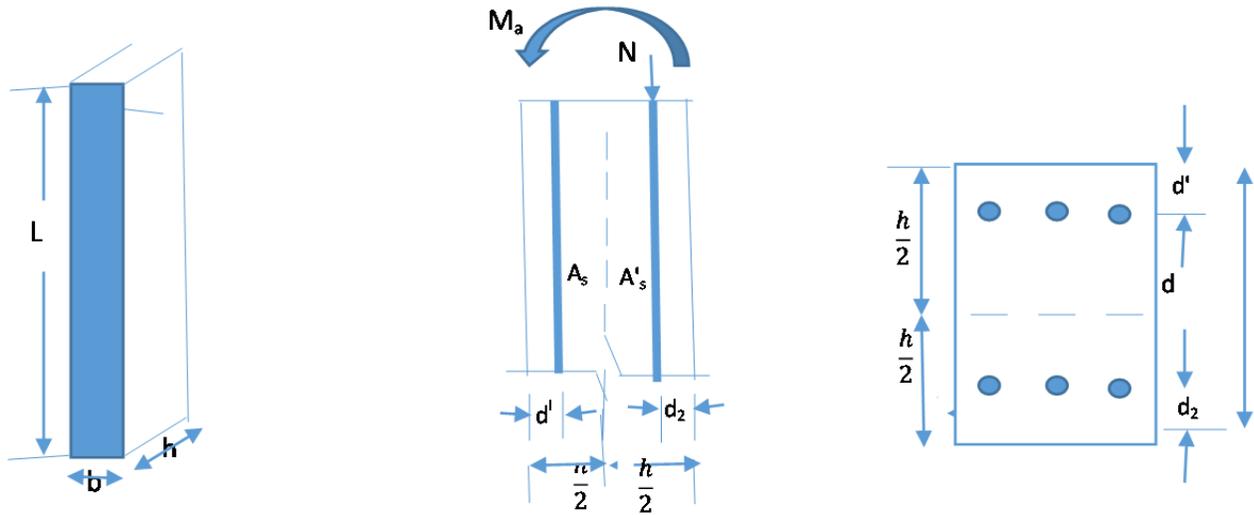


Figure 7. Simplified column design method.

Table 7. Reliability parameters for rigid foundation - column (Case Study A).

Physical meaning	Type IV (i)	Mean EX (i)	Standard deviation Sx (i)	COV	Variable type	Reference
f_{yk}	Log Normal= 3	500 N/mm ²	25	0.05	x ₁	Mirza and MacGregor (1999)
Width of column (b)	Log Normal= 3	230 mm	4.6	0.02	x ₂	Sanjoyan (2004)
Depth of column (d)	Log Normal= 3	340 mm	6.8	0.02	x ₃	Sanjoyan (2004.)
Characteristics strength of concrete (f_{ck})	Normal = 2	25 N/mm ²	4.50	0.18	x ₄	Mirza and MacGregor (1999)
UD Axial Force (N_k)	Log Normal= 3	103.13 kN	5.16	0.05	x ₅	JCSS (2000)
UD Wind Load (W_k)	Gamma = 5	96.40 kN	35.67	0.37	x ₆	Munich (1988)
Length of Column	Log Normal= 3	3000 mm	150	0.05	x ₇	Sanjoyan (2004)

Reliability Index β obtained from the column reliability analysis in case study A = 4.25.

Table 8. Reliability parameters for weak foundation frame - column (Case Study B).

Physical meaning	Type IV (i)	Mean EX (i)	Standard deviation Sx (i)	COV	Variable type	Reference
f_{yk}	Log Normal=3	500 N/mm ²	25	0.05	x ₁	Mirza and MacGregor (1999)
Width of column (b)	Log Normal=3	230 mm	4.6	0.02	x ₂	Sanjoyan (2004)
Depth of column (d)	Log Normal=3	340 mm	6.8	0.02	x ₃	Sanjoyan (2004)
Characteristics strength of concrete (f_{ck})	Normal =2	25 N/mm ²	4.50	0.18	x ₄	Mirza and MacGregor (1999)
UD Axial Load (N_k)	Log Normal=3	104.48 N/mm	5.22	0.05	x ₅	JCSS (2000)
UD Wind Load (W_k)	Gamma	96.40 N/mm	35.67	0.37	x ₆	Munich (1988)
Length of Column (l)	Log Normal=3	3000 mm	150	0.05	x ₇	Sanjoyan (2004)

$$L^2 \quad (20)$$

The reliability analysis parameters used in FORM5 software for Equation 20 are presented in Table 9. Reliability Index β obtained from the Column reliability analysis in case study "C"= 4.14.

INTERPRETATION OF THE RESULTS

The research studied a model consisting of six frames spaced at 6 m centers, for a building length 30,000 mm (6000 mm x 5) and width of 9600 mm (2 x 3600 mm + 2400 mm) as shown in Figure 5a and b and Table 10

Table 9. Reliability parameters for modified weak foundation column (Case Study C).

Physical meaning	Type IV (i)	Mean EX (i)	Standard deviation Sx (i)	COV	Variable type	Reference
f_{yk}	Log Normal =3	500 N/mm ²	25	0.05	x ₁	Mirza and MacGregor (1999)
Width of column (b)	Log Normal =3	300 mm	6.00	0.02	x ₂	Sanjoyan (2004)
Depth of column (d)	Log Normal =3	390 mm	7.8	0.02	x ₃	Sanjoyan (2004)
Characteristics strength of concrete (f_{ck})	Normal = 2	25 N/mm ²	4.500	0.18	x ₄	Mirza and MacGregor (1999)
UD Axial Force (N _k)	Log Normal =3	108.23 N/mm	5.41	0.05	x ₅	JCSS (2000)
UD Wind Load (W _k)	Gamma = 5	96.40 N/mm	35.67	0.37	x ₆	Munich (1988)
Length of Column (l)	Log Normal = 3	3000 mm	150	0.05	x ₇	Sanjoyan (2004)

respectively.

Three case studies labelled A, B and C were structurally modelled, analyzed and designed using ETABS Version (2015) in accordance with Euro code provision for structural analysis and assessed for reliability with FORM5 software (Gollwitzer, et al., 1988).

i) Case study A represent building frames with rigid foundation (that is, representing a soil with high safe bearing capacity and designed as a rigidly fixed column-foundation joint);

ii) Case study B represent frames with weak foundation (that is, representing a soil with low safe bearing capacity but the design has erroneously assumed this as a soil with high safe bearing capacity by initially representing it with a rigidly fixed column-foundation joint but re-analyzed and tested for reliability as hinged column-foundation joint with the same quantity of reinforcement required for the rigidly fixed column-foundation joint); whereas

iii) Case study C represent frames with modified weak foundation (that is, representing a soil with low safe bearing capacity whose design correctly assumed the soil as low safe bearing capacity and represented it as a hinged column-foundation joint to facilitate possible rotations of the joints due to the nature of the soil and also provided it with the required appropriate increased section and quantity of reinforcement to absorb the increase in forces and moments for stability and safety).

Table 10a shows the building length of 30,000 x 9,600 mm was assessed in case studies A, B and C with columns and beams sections of 400 x 230 mm and 500 x 230 mm respectively for Case studies A and B; while column and beams sections of 450 x 300 mm and 550 x 300 mm respectively, were assumed in case study C. Ultimate load combination that gave the highest maximum displacement for all cases for analysis was 1.35 gk + 1.05 qk + 1.5 wk. The allowable maximum displacement for the analyzed model was 24 mm. Therefore, from the cases deflection analyses in Table 10b, it can be observed that case study B exceeded the allowable maximum displacement by 42% due to the effect of weak foundation but case studies A and C are

less than the allowable maximum displacement by between -20 to -26% respectively. Many engineers normally use maximum displacement or structures maximum horizontal sway to verify, if a low, medium or high rise building is safe or unsafe as under pulsating wind loading, the vibration likely to be generated might exceed the allowable threshold. In extreme situation, this phenomenon might of course contribute significantly to building collapse or fear and Norcia to occupants living or working at the upper floors of such building. Therefore, judging from Table 10b, the afore-mentioned assertion inferred that, it will be very disastrous to assume a very rigid foundation on a weak soil.

Table 11a presents the corresponding values of the quantities of reinforcements used for the design of the columns, computed reliability indexes (β) and probability of failures (P_f) on the implications of the three cases [that is, Rigid Foundation (Case A₁), Weak Foundation (Weak Foundation Case B₁) and Modified Weak Foundation (Case C₁)] considered for investigation in the research. In Table 11b, the relationship between the probability of failure P_f and reliability index β recommended by Baecher and Christian (2003) is also presented.

Actual reliability values obtained from the FORM5 (Gollwitzer and Rackwitz, 1988) analysis and the corresponding A_s values are highlighted in colours in order to compare the values with the recommended values from EURO CODE. The quantity of reinforcement actually required is $A_s = 2000 \text{ mm}^2$ and the β value for Case study A is 4.1 while the β value of Case study B is 2.3, using A_s value of 2000 mm^2 . Therefore, reliability index for case study B is exactly 44% lower than case study A and 40% lower than recommended value 3.8 by (Euro code 2 EN 1992-1-1, 2004).

However, in order to correct the erroneous assumption in Case study B and to take care of all the uncertainties due to the development of likely plastic equilibrium that could lead to progressive collapse of structure, Case study C was proposed with higher section and quantity of reinforcement A_s value of 2400 mm^2 to produce a higher target reliability index of 4.0 as shown in Table 11a. Case study B have lower index of reliability β and higher probability of failures P_f values due to the effect of weak foundation erroneously assumed as rigid in the analysis

Table 10a. Comparison of design parameters and deflection results.

Foundation type	Case study	Frame spacing	Building (mm)		Column (mm)		Beam (mm)		Displacement (mm)	
			Length	Width	Depth	Width	Depth	Width	Designed maximum	Allowable maximum
Rigid	A	6.000	30.000	9.600	400	230	500	230	17.1	24
Weak	B	6.000	30.000	9.600	400	230	500	230	34.0	24
Modified Weak	C	6.000	30.000	9.600	450	300	550	300	19.1	24

Table 10b. Variation of allowable maximum displacements and percentage differences.

Case study	Maximum displacement (mm)	Allowable displacement (mm)	Difference	Percentage difference
A	17.1	24	-6.9	-29
B	34.0	24	10	42
C	19.1	24	-4.9	-20

Table 11a. β and P_f values of the columns for case studies A, B and C.

A_s (mm ²)	Rigid Foundation (A_1)		Weak Foundation (B_1)		Modified Weak Foundation (C_1)	
	β	P_f	β	P_f	B	P_f
1000	2.302	1.070E-02	0.611	0.271	1.994	2.310E-02
1200	2.717	3.290E-03	1.005	0.157	2.336	9.740E-03
1400	3.101	9.640E-04	1.368	8.57E-02	2.658	3.930E-03
1600	3.458	2.720E-04	1.704	4.42E-02	2.961	1.540E-03
1800	3.791	7.500E-05	2.017	2.19E-02	3.247	5.840E-04
2000	4.104	2.030E-05	2.310	1.04E-02	3.518	2.180E-04
2200	4.399	5.650E-06	2.586	4.85E-03	3.775	7.990E-05
2400	4.678	1.450E-06	2.847	2.21E-03	4.021	2.900E-05
2600	4.942	3.860E-07	3.094	9.87E-04	4.255	1.050E-05
2800	5.194	1.030E-07	3.329	4.35E-04	4.479	3.750E-06
3000	5.435	2.750E-08	3.554	1.90E-04	4.694	1.340E-06
3200	5.665	7.370E-09	3.768	8.22E-05	4.901	4.780E-07
3400	5.886	1.990E-09	3.974	3.54E-05	5.100	1.700E-07
3600	6.098	5.400E-10	4.172	1.51E-05	5.291	6.090E-08

Table 11b. Recommended relationship between the probability of failure P_f and reliability index β .

p_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
B	1.3	2.3	3.1	3.7	4.2	4.7	5.2

Source: Baecher and Christian (2003).

and design of the model.

Therefore, there is a noticeable difference in reliability indexes between case studies B and C which were built on same foundation condition (that is, a weak soil safe bearing capacity) but with improved analyses, member sections and increased quantities of reinforcement to ensure better safety and reliability of the model (structural system). The arrows show minimum area of

reinforcements required for safety of the column.

Frame forces under the most critical load combinations obtained from structural analyses using ETABS software were used to develop limit state functions, which were submitted to FORM5 reliability software to obtain the P_f and β values. As shown in Figures 8 and 9, the areas of reinforcement (A_s) between 1000 and 3600 mm where progressively varied to determine the equivalent β and P_f

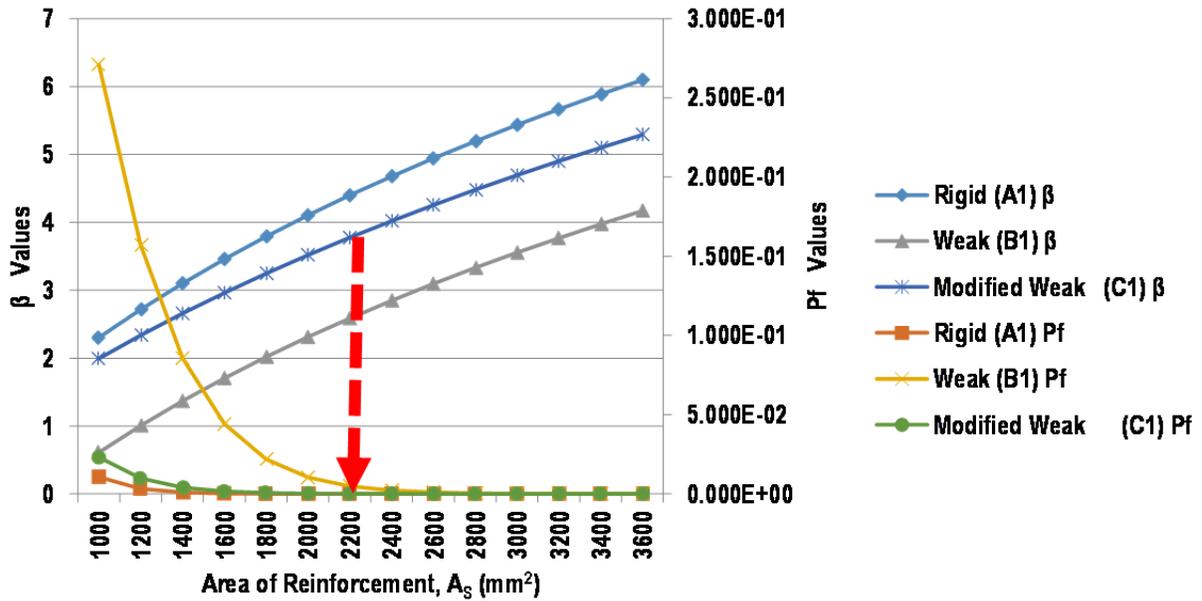


Figure 8. Reliability Index β and P_f values of the columns for case studies A_1 , B_1 and C_1 .

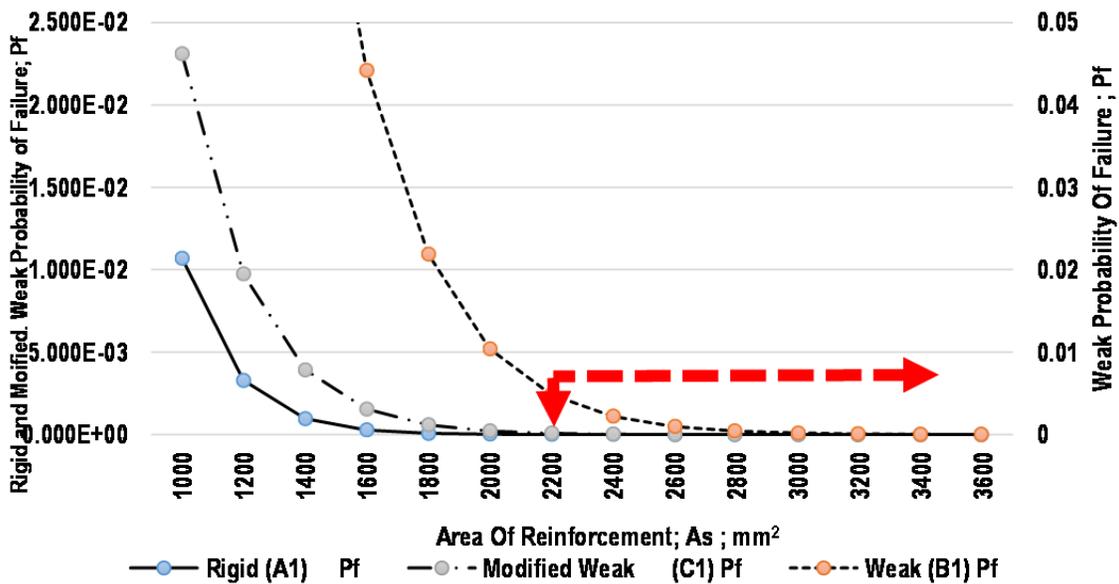


Figure 9. Column probability of failure for case studies A, B and C.

values for all the case studies. Euro code 2 (2004) recommends a minimum value of 3.8 reliability index for residential and office buildings for a reference period of 50 years as shown in Table 2. Therefore, a minimum reliability index $\beta \geq 3.8$ is the value recommended for the present study.

Therefore, it can be inferred that the use of a hinge column-foundation joint for structural analysis will produce increased sections and reinforcements in reinforced-

concrete composite frames. These would consequently improve the reliability indexes of structures built on a weak soil safe bearing capacity and reduces its probability to fail. Similarly, a slight difference was also observed between the P_f and β values of cases studies A and C. This implies that the rigid foundation gives higher reliability index but equivalent reliability index can be obtained from a weak soil using appropriate design model, sections and reinforcement.

Euro code 2 (2004) recommended a minimum value of 3.8 reliability index β for residential and office buildings for a reference period of 50 years. From Table 2, it is very clear that, any building constructed using undersized reinforcement will definitely collapse before its design life span. Therefore, the following minimum reinforcement is recommended for this model: Case studies - A = 2000 mm², B=3400 mm² and C=2400 mm²; however, 2200 mm² has approximately satisfied cases A and C as shown in Figures 8 and 9 respectively.

Concluding remarks

i) Typical 4-storey RC framed buildings modelled in 3-dimensional analysis with the same sections and loadings for both rigid and weak foundation respectively were conducted on the case studies using ETABS 2015 and FORM5 Softwares in accordance with Euro codes provisions. Limit state equations developed were used to perform reliability analysis to obtain the variable reliability indexes and probability of failures which proved to be significantly affected by the soil-structure interactions.

ii) The results revealed that due to the effect of weak soil safe bearing capacities, allowable maximum displacement was exceeded. In case study B, the allowable maximum displacement was exceeded by 42% due to the effect of weak foundation but case studies A and C are less than the allowable maximum displacement by between -20 to -26% respectively. Many engineers normally use maximum displacement or structures maximum horizontal sway to verify, if a low, medium or high rise building is safe or unsafe as under pulsating wind loading, the vibration likely to be generated might exceed the allowable threshold. In extreme situation, this phenomenon might of course contribute significantly to building collapse or fear and Norcia to occupants living or working at the upper floors of such building.

iii) Apart from effect on the deflection, the results led to variable quantity of reinforcements required as $A_s = 2000$ mm² and the β value for Case study-A is 4.1 while the β value of case study - B is 2.3, using A_s value of 2000 mm². Therefore, reliability index for case study B is exactly 44% lower than case study A and 40% lower than recommended value 3.8 by Euro Code.

iv) Therefore, the reliability index of $\beta = 3.8$ recommended by Euro code was not achieved due to the effect of weak soil-structure interactions which showed that, it will be very disastrous, if rigid soil-structure interactions were assumed for a weak soil.

v) A hinge should be generally adopted as column-foundation joint for Maiduguri since the soil is generally weak; similarly, increasing the sections and reinforcement in reinforced concrete frames improved the reliability indexes and reduced the probabilities to fail.

vi) It is clear from Figures 7 and 8 and Table 11a and b that Case Study B have low reliability index due to the effect of weak foundation or weak soil safe bearing

capacity which caused a high probability of failure. Considering the target safety index of 3.8 specified, it can be concluded that, based on the results obtained from this study, design criteria for case studies C are adequate for construction of a four storey framed building on a weak soil safe bearing capacity.

RECOMMENDATION

Further research should consider different methods of reliability-based assessment of framed buildings in order to ensure quality assurance of structural design and execution of construction works to achieve an acceptable quality of buildings in the construction industry.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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