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Investigation of pounding based on finite element analyses of two adjacent buildings, considering new equation of motion to measure impact

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Building pounding philosophy is the collision of adjacent buildings when earthquake take place and buildings show lateral movement. Providing suitable separation distance, impact force precludes and reduces the failure of construction elements. In this paper, building pounding between two reinforced concrete buildings with insufficient gap size is numerically investigated in order to study the impact force during seismic excitations. For this challenge, two to three story concrete buildings are modeled by finite element software. On the other hand, two mathematic models of RC buildings with lumped masses are considered to get the results of pounding and effect of impact on structural behavior. A mathematic program is used based on single degree-of-freedom (SDOF) equations to analyze the failure of models. Since calculation of impact forces depends significantly on link element assigned between two bodies, special link element is utilized. Used link element between two buildings is included in a spring with high stiffness and a viscous impact dashpot, which were located parallel in order to absorb the energy loss during impact. Finally, a new equation of motion is suggested to calculate the impact force. The accuracy of suggested equation is confirmed by energy equation.

Key words: Building pounding, impact, dissipated energy, finite element.

INTRODUCTION

Commonly, during seismic excitation buildings make lateral displacement, which can cause the collision of adjacent buildings. This behavior of buildings that have been found to impact each other during earthquakes was reported by engineering and researchers, and it can inflict significant damage to structures and even collapse in some cases. Since all the structures can exhibit dramatic movements when they vibrate under earthquakes, building pounding is a special event for engineers to investigate and to assess the numerical study of collision. If adjacent buildings do not have sufficient separation distance from each other, earthquake can be provided large lateral displacement and buildings can be damaged considering building pounding, even if they are well designed and well-constructed. The numerical formulation of building pounding is solved by a matrix equation of building motion. The matrices include buildings mass, their stiffness and their damping. The

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Figure 1. Building pounding (New Zealand, 2009).

instantaneous loading can be of two different types: random and periodic. Many researchers have used numerical modeling of building pounding to investigate the power of impact. It is of interest to predict impacts, and to provide some solutions to avoid the resulted damages. Anagnostopoulos (1998) was among the first researchers, who showed the effect of impact and displacement in building model with distributed mass. He also suggested a formulation for damped linear contact element. He further described that damping constant can be related to the coefficient of restitution (Anagnostopouls, 2004).

Watanaba and Kawashima (2005) have investigated pounding of distributed masses to model colliding bridge decks. They showed that five elements were used per deck; the collision element stiffness would be five times of diagram stiffness. Cole and Dhakal (2009) have indicated that building pounding and its impact depends on the structural properties and on collision velocity of both buildings. They suggested a plan to control the impact. A theoretical maximum collision force has been determined by them, for a system with two distributed masses. Velocity, mass and stiffness at time of impact have a relation, and the number and magnitude of the impacts depend on these three options mentioned. Mortezaei and Zahrai (2009) have investigated a concrete building with respect to pounding, and have decreased the displacements by using a viscous-elastic damper. Since a separation distance or gap between two buildings could decline the risk of building pounding, it must be confirmed if code regulatory provisions are adequate under all combinatorial design situations and circumstances. Barros and Khatami (2012), considering Iranian earthquake code, have modeled two adjacent concrete buildings and have shown that suggested gap size is not sufficient and buildings collide several times under different earthquake (Figure 1).

Many researchers have focused on the link element and have suggested different formula to get the best available estimate of impact force. Komodromosans and Polycarpou (2010) have suggested a new formula of damping ratio to calculate impact force using hysteretic damping. Ye and Li (2009) have also simulated a new relation to get ξ, which is caused to change impact force in mathematic equations. Barros and Khatami (2012) estimated the effect of damping ratio on the numerical study of impact forces between two adjacent concrete buildings subjected to pounding. In yet another study, Barros and Khatami (2012) compared results of two single degree-of-freedom (SDOF) frames with different link elements based on mathematic relations. In some of their analyses, structures were modeled as single-degree-freedom systems and collision was simulated using linear visco-elastic models of impact force. Barros and Khatami (2012) suggested a new equation by using harmonic process to get the best estimation of impact damping ratio. Naderpour et al. (2013) modified a new link element based on Kelvin-Voit model and could confirm their suggested equation by calibrating the results and coefficient and restitution. In this paper, two

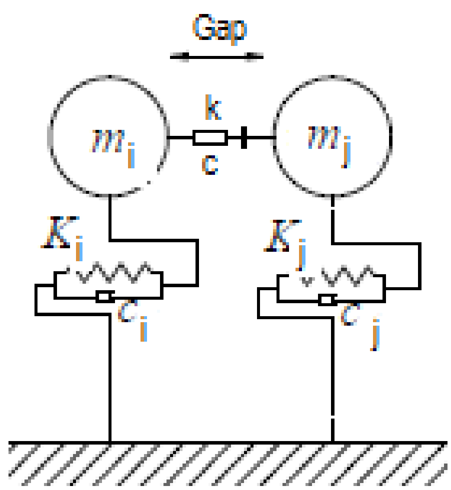

Figure 2. Building pounding with schematic dynamic analysis.

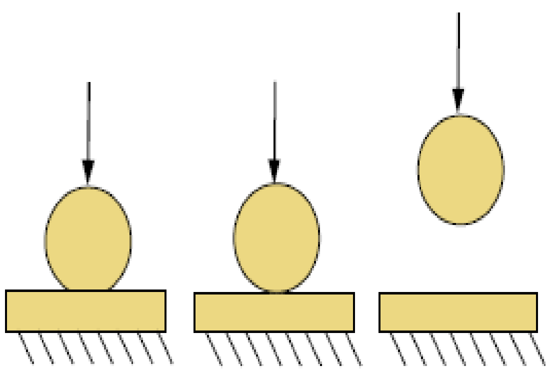

Figure 3. The schematic of impact between body and rigid surface.

adjacent buildings by having three stories are considered and a new equation of motion is suggested to calculate the impact damping ratio, subsequently, calculation of damping factor and finally, impact force.

The concept of building pounding

Lateral loads cause large lateral displacements which eventually collide adjacent buildings. Structural pounding is a complex phenomenon involving plastic deformation at contact points, local cracking or crashing, fracturing due to impact and friction. Forces crashed by collisions are applied and removed during a short interval of time initiating stress waves, which travel away from the region of contact. The process of energy transfer during impact is highly complicated, which makes the mathematical analysis of this type of problem very difficult.

Pounding is one of the primary reasons of many building damages in earthquake. This phenomenon has been reported in many earthquakes between two adjacent buildings. Seismic pounding occurs during earthquake, in building with different dynamic characteristics, adjacent buildings' vibrating out of phase and in building in at-rest separation is insufficient.

Minimum separation distance is given by two main mathematical formulations, which could be written as:

$$S = u_i + u_j \tag{1}$$

$$S = \sqrt{u_i^2 + u_j^2} \tag{2}$$

Where S is separation distance and u_i, u_j denotes peak lateral displacement in buildings. The first equation called ABS (ABSolute sum) and second equation is termed as SRSS (Square Root of Sum of Squares).

The schematic analysis of building by mathematical method is shown in Figure 3. In order to investigate collision between two elements, a conditional spring was assigned between two bodies and to absorb energy a dashpot was used as shown in Figure 2. Governing differential equation of motion of the two adjacent building can be written similarly:

$$m_i\ddot{u}_i + (c_i + c)\dot{u}_i + (k_i + k)u_i = -m_i\ddot{u}_g \tag{3}$$

$$m_j\ddot{u}_j + (c_j + c)\dot{u}_j + (k_j + k)u_j = -m_j\ddot{u}_g \tag{4}$$

Equation is found to be in matrix form as:

$$\begin{bmatrix} m_i & 0 \\ 0 & m_j \end{bmatrix}\begin{bmatrix} \ddot{u}_i \\ \ddot{u}_j \end{bmatrix} + \begin{bmatrix} c_i + c & -c \\ -c & c_j \end{bmatrix}\begin{bmatrix} \dot{u}_i \\ \dot{u}_j \end{bmatrix} + \begin{bmatrix} k_i + k & -k \\ -k & k_j \end{bmatrix}\begin{bmatrix} u_i \\ u_j \end{bmatrix} = \begin{bmatrix} f_i \\ f_j \end{bmatrix} \tag{5}$$

where m denotes masses of the structures, k and c are the linear stiffness and damping contact, respectively. As researchers analyze structures by dynamic methods, they have tried to use single degree of freedom (SDOF) models. That is, a tall building included many stories could be modeled with many lumped-mass-spring systems.

Impact force

In order to calculate the impact force of two buildings during seismic excitation, a special link element is devised in conjunction level of two models and its mathematic relation is written by:

$$F_c(t) = k_h.\delta(t)^n + c\dot{\delta}(t) \quad (6)$$

Where k_h describes stiffness of nonlinear spring, n is recommended to be 1.5 and c is damping coefficient of damper, which could be shown as:

$$E = f_D(t) \rightarrow E = c\dot{\delta}(t) \rightarrow c = \zeta\delta(t)^{1.5} \quad (7)$$

In fact, used lateral displacement and velocity are determined by given equations:

$$\delta(t) = \delta_i(t) - \delta_j(t) - gap \quad (8)$$

$$\dot{\delta}(t) = \dot{\delta}_i(t) - \dot{\delta}_j(t) \quad (9)$$

In Equation (8), gap is a separation distance between two buildings, δ_i and δ_j are also lateral displacements of building *i* and *j*, respectively.

Energy dissipation

During earthquake, when two buildings collide with each other, dynamic model is assumed to be two lumped masses. In this situation, dissipated energy during impact between two bodies will be written by:

$$\Delta E = \frac{m.(1-e^2)}{2}.(\frac{d\delta}{dt})^2 \quad (10)$$

Where m is equivalent masses, e is a vector, which is described in the following paragraph and $(\frac{d\delta}{dt})$ is the relative velocity of the two velocities of the bodies just before impact.

Damping coefficient

Damping coefficient is calculated by two parameters, lateral displacement and an unknown parameter, which is shown by ζ. Moreover, it needs to be determined by mathematics equation. A Rayleigh damping matrix is calculated as a linear combination of the stiffness matrix scaled by a user specific coefficient, and the mass matrix scaled by a second user specific coefficient. The two coefficients are computed by specifying equivalent fractions of critical model damping at two different frequencies (Warrnote, 2008). Stiffness and mass proportional damping is linearly to frequency and period, respectively. In fact, it could be written by:

$$c = a_0 m + a_1 k \quad (11)$$

The factors a_0 and a_1 are calculated from:

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{2\zeta}{w_n + w_m} \begin{bmatrix} w_n w_m \\ 1 \end{bmatrix} \quad (12)$$

By having above equation, damping ratio would be determined by:

$$\begin{bmatrix} \zeta_n \\ \zeta_m \end{bmatrix} = \begin{bmatrix} \frac{1}{2w_n} & w_n \\ \frac{1}{2w_m} & w_m \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad (13)$$

Nerveless, a summary of previous studies about impact damping ratio of pounding have shown that formulas are based on coefficient of restitution (*e*) and calculate the impact damping ratio by mathematic equations. Coefficient of restitution is a factor, which simulate a relation between velocities before and after collision. This equation could be written as:

$$0 < e = \frac{v_{before}}{v_{after}} < 1 \quad (14)$$

As it is shown, coefficient of restitution is calculated to be in the range of 0 and 1. If *e* becomes equal to 0, collision is perfectly plastic and if *e* becomes equal to 1, collision shows an elastic behavior.

Suggested equations

Many researchers have studied to get the best estimation of ζ to simulate impact and calculate impact damping coefficient. In order to predict the value of impact and also dissipated energy, suggested ζ is used to base on different equation of motions. Jankowski (2006) presented an idea, which a nonlinear viscose damper parallel locates to the spring in order to absorb an energy dissipation mechanism.

$$c = 2\zeta\sqrt{k_k\sqrt{\delta(t)}\frac{m_1 m_2}{m_1 + m_2}} \quad (15)$$

$$\zeta = \frac{9\sqrt{5}}{2}\frac{1-e^2}{e(e(9\pi-16)+16)} \quad (16)$$

Mentioned equation depends on *e* factor. Undoubtedly, selecting different *e*, the results of impact forces would be various and it could not be satisfied to get the optimize results. On the other hand, a new pounding analytical model is described in this part based on Equation (7). Hertz damped model with nonlinear damping was suggested by Muthukumur (2006) to calculated impact force between two dynamic models. In this equation, the nonlinear damping ratio is explained as follows:

$$\zeta = \frac{3k_h(1-e^2)}{4v_{imp}} \tag{17}$$

In order to improve Equation (17), Ye and Li (2009) have suggested a new ξ, which can be written as:

$$\zeta = \frac{3k_k(1-e)}{2.e.v_{imp}} \tag{18}$$

PROPOSED MODEL

Impact between two buildings is assumed to be a dynamic collision between two bodies with lumped masses, which is devised at the top of a column. In fact, column shows stiffness and damping of buildings as it was shown in Figure 2. To measure the impact force and simplify the contact between two bodies, it is assumed that two masses joint with each other to make an equivalent mass $(m = \frac{m_i m_j}{m_i + m_j})$ and a rigid surface.

Discussed link element in previous paragraph is located between body and surface as it is seen in Figure 3. According to Equations (17) and (18), impact damping ratio depends significantly on *e*, impact velocity and stiffness of spring. It seems that these equations are not correctly able to show impact force and determine dissipated energy as one of the most important parameter in terms of impact is not used. Undoubtedly, masses of investigated bodies are effectiveness which is neglected in suggested equations. It is assumed that the impact damping ratio depends on the same parameters that calculate dissipated energy as in the following formula:

$$\zeta = f(e, m, k, \delta_{\max}, v_{imp}) \tag{19}$$

Based on Equations (7) and (10), dissipated energy can be written by follow the equation:

$$\Delta E = \int_0^{\delta_{\max}} c\dot{\delta}d\delta \tag{20}$$

where *c* is damping coefficient and $\dot{\delta}$ denotes velocity during impact. On the other hand, the damping ratio is a parameter, usually denoted by ζ that characterizes the frequency response of a second order ordinary differential equation. The damping ratio provides a mathematical means of expressing the level of damping in a system relative to critical damping. For a damped harmonic oscillator with mass *m*, damping coefficient *c*, and spring constant *k*, it can be defined as the ratio of the damping coefficient in the system's differential equation to the critical damping coefficient:

$$c = 2\zeta.m.w \tag{21}$$

Using the natural frequency of the simple harmonic oscillator, *w* can be followed as:

$$w = \sqrt{\frac{k}{m}} \tag{22}$$

Subsequently, according to Equation (20) into Equation (21) we have:

$$\Delta E = \int_0^{\delta_{\max}} 2\zeta m w\dot{\delta}d\delta \tag{23}$$

After integral transformation, dissipated energy would be:
After modifying Equation (10) to Equation (24), it would be as:

$$2\zeta\dot{\delta}\delta_{\max}\sqrt{k.m} = \frac{m.(1-e^2)}{2}(\frac{d\delta}{dt})^2 \tag{25}$$

In fact, by using Equation (25), damping ratio is appeared to be:

$$\zeta = \frac{(1-e^2).v_{imp}}{4.\delta_{\max}}.\sqrt{\frac{m}{k}} \tag{26}$$

As it is seen, impact damping ratio depends on five different parameters, which are coefficient of restitution, impact velocity, lateral displacement, masses and stiffness of spring (Figure 4).

As it is shown, there is a negative part of enclosed energy in hysteresis loop. It is obviously specified that impact cannot provide negative energy. Consequently, it seems that the rendered equation needs to improve based on velocity.

According to impact model, it is assumed that velocity is positive. Based on mentioned assumption, the equation will be changed:

$$2\zeta\delta_{\max}\sqrt{k.m} = \frac{m.(1-e^2)}{2.e}v_{imp} \tag{27}$$

$$\begin{cases} \zeta = \sqrt{\frac{m}{k}}\frac{(1-e^2).v_{imp}}{4.\delta_{\max}.e} \rightarrow v > 0 \\ \zeta = \sqrt{\frac{m}{k}}\frac{(1-e^2).v_{imp}}{4.\delta_{\max}} \rightarrow v < 0 \end{cases} \tag{28}$$

$$2\zeta\delta_{\max}\sqrt{k.m} = \frac{m.(1-e^2)}{2.e}v_{imp} \tag{27}$$

$$\begin{cases} \zeta = \sqrt{\frac{m}{k}}\frac{(1-e^2).v_{imp}}{4.\delta_{\max}.e} \rightarrow v > 0 \\ \zeta = \sqrt{\frac{m}{k}}\frac{(1-e^2).v_{imp}}{4.\delta_{\max}} \rightarrow v < 0 \end{cases} \tag{28}$$

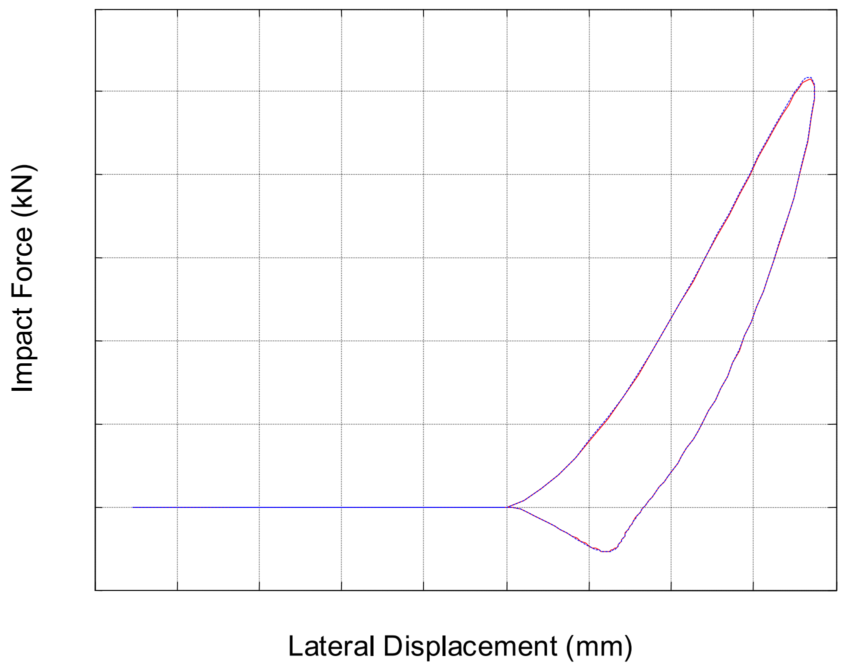

Figure 4. The results of analyses used Equation (26).

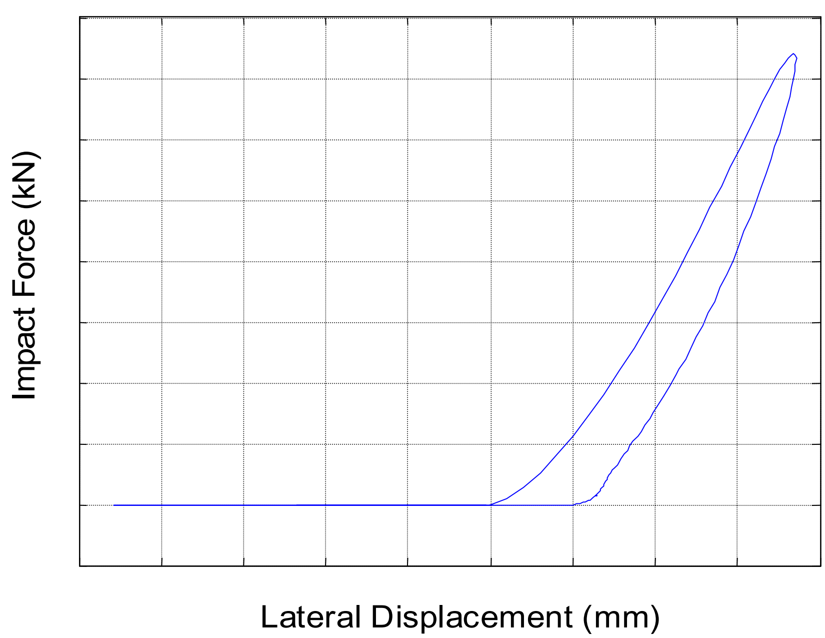

Figure 5. The results of analyses used Equation (28).

Researchers assume coefficient of restitution among 0 to 1 based on their numerical analyses. By using suggested equation of motion to calculate impact force and selecting different *e*, the results of analyses have various responses and it could not be accepted. Undoubtedly, in order to measure impact force and evaluate the accuracy of their responses, it seems that it is a need to provide a reference curve based on coefficient of restitution and the relation between impact velocity and maximum lateral displacement. Although, the mentioned parameters seem quite effective and promising for calculation of impact between two bodies, they are not suitable to determine pounding (Figure 5).

Subsequently, a new nonlinear curve is created to get the best

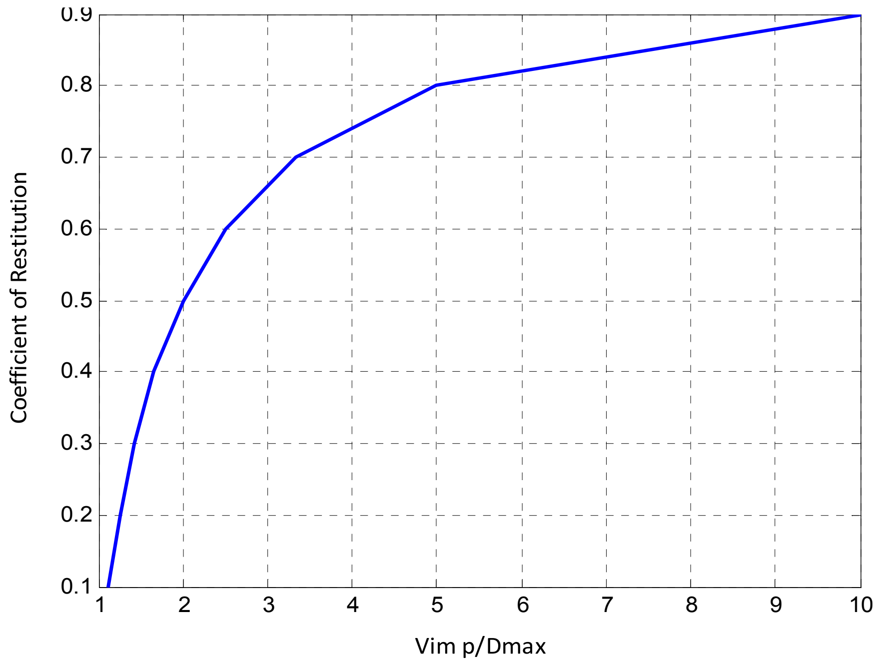

Figure 6. Suggested curve for e and $\frac{v_{imp}}{\delta_{\max}}$.

estimation of impact velocity and maximum lateral displacement which could confirm the selected coefficient of restitutions. In fact, by selecting different coefficient of restitutions, it could be assumed that an approximately relation between impact velocity and maximum lateral displacement is appeared.

A numerical algorithm is written to get the relation between impact velocity and maximum lateral displacement. After getting discussed relation, coefficient of restitution is selected and new *e* is modified (Figure 6). New analyses are carried out and new results are observed. The results show that by using depicted curve, the following formula has been obtained regarding the coefficient of restitution. The estimated formula can be written as:

$$e = 0.0069\left(\frac{v_{imp}}{\delta_{\max}}\right)^3 - 0.127\left(\frac{v_{imp}}{\delta_{\max}}\right)^2 + 0.724\left(\frac{v_{imp}}{\delta_{\max}}\right) - 0.5156 \quad (29)$$

The written equation show that selected coefficient of restitution depends on $\frac{v_{imp}}{\delta_{\max}}$. For instance, in order to have same responses by having different bodies, properties of material, lateral load and damping, stiffness of spring, the lateral displacement is determined and the peak lateral displacement is seen. By having the time of collision, impact velocity is obviously calculated as derivation of displacement.

It is assumed that the impact force is exponentially increased with the indentation. By having damping ratio equation of motion, hysteresis curve of impact force is depicted. The enclosed area is the area of the hysteresis loop and expresses the dissipated energy during impact.

In order to investigate the accuracy of the suggested equation of motion, the value of dissipated energy is determined by hysteresis loop of impact force-lateral displacement curve. This value of energy is compared with calculated value of energy from Equation (10).

For this challenge, a value of *e* is considered and impact force is calculated. Hysteresis curve is also depicted. Enclosed area of curve is determined and dissipated energy is found. On the other hand, considering calculated energy and using Equation (10), new *e* is determined. Assumed *e* and calculated *e* are compared with each other and are derived in Figure 7.

BUILDING MODEL

The reference model is represented by two reinforced concrete frame having three floors, which are called 3-3 M, with a gap of 10 cm, that are seismic moment resistant frames of medium level of ductility. Both three floor frames has one 4-meter spans in the X direction. Height of each floor is considered to be 3 m and the use of the frames is assumed to be residential. Complementary to this assumption, concrete compressive strength is 25 MPa and yielding strength of steel is 400 MPa. The columns are modeled to be 30*30 and 40*40 for right and left buildings, respectively. All of beams are assumed to be 30*40 cm.

The contact element used between the frames is assumed to be a link element called ´Kelvin-Voigt´, represented schematically in Figure 8. The nonlinear viscose elastic link model with gap uses a dashpot damper and spring with high stiffness, which is devised for energy dissipation and for restraint of lateral displacements.

In order to define contact element between two surfaces, roof to roof, or two nodes of surface, it used a special element, which can be shown by Figure 9. As it was noted, two different frames are selected for investigating the influence of impact between two buildings. The frames are modeled by finite element software, using the 3D element. This element is capable of cracking in tension and crushing in compression. In concrete applications, for instance,

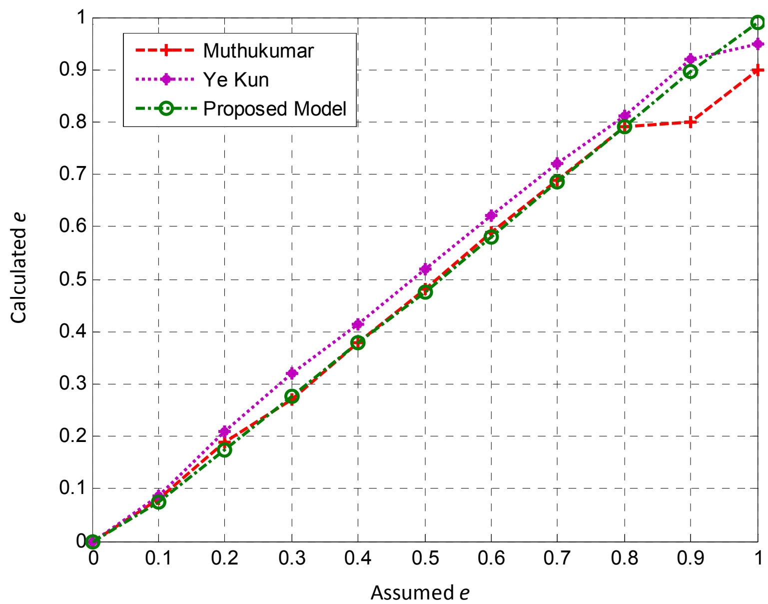

Figure 7. Comparison of assumed and calculated coefficient of restitution.

the solid capability of the element may be used to model the concrete while the rebar capability is available for modeling reinforcement behavior. Other cases for which the element is also applicable would be reinforced composites and geological materials (such as rock). The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y and z directions. Up to three different rebar specifications may be defined. Nonlinear behavior, initial deflections and imperfections, and creep are other capabilities available for this element.

Considering dynamic structural properties present in the matrix equation of motion of the seismically excited buildings, the three dynamic matrices of mass, damping and stiffness can be written as:

$$m = \begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{32} \end{bmatrix}$$

$$c = \begin{bmatrix} c_{11}+c_{21}+c & -c_{21} & 0 & c & 0 & 0 \\ -c_{21} & c_{21}+c_{31}+c & -c_{31} & 0 & c & 0 \\ 0 & -c_{31} & c_{31}+c & -c_{12} & 0 & c \\ c & 0 & -c_{12} & c_{12}+c_{22}+c & -c_{22} & 0 \\ 0 & c & 0 & -c_{22} & c_{22}+c_{32}+c & -c_{32} \\ 0 & 0 & c & 0 & -c_{32} & c_{32}+c \end{bmatrix}$$

$$k = \begin{bmatrix} k_{11}+k_{21}+k & -k_{21} & 0 & k & 0 & 0 \\ -k_{21} & k_{21}+k_{31}+k & -k_{31} & 0 & k & 0 \\ 0 & -k_{31} & k_{31}+k & -k_{12} & 0 & k \\ k & 0 & -k_{12} & k_{12}+k_{22}+k & -k_{22} & 0 \\ 0 & k & 0 & -k_{22} & k_{22}+k_{32}+k & -k_{32} \\ 0 & 0 & k & 0 & -k_{32} & k_{32}+k \end{bmatrix} \quad (30)$$

Where m_i is lumped mass of each story of the systems used as sample structures; c_i represent building damping coefficient and finally, k_i denotes stiffness in both buildings. Also *c* and *k* denote damping and stiffness of used linked element, respectively.

Used model is considered to be a multi-degree-of-freedom (MDOF) system. For this challenge, the building is model by lumped masses at the floor level. A link element is located between masses, which is devised by a spring and damper parallel with each other. Poundings are assumed to happen if the available seismic gap is exceeded. The equation of motion of MDOF system is followed to be:

$$F_I(t) + F_D(t) + F_E(t) = 0 \quad (31)$$

here $F_I(t), F_D(t), F_E(t)$ are inertia, damping and elastic forces of building, respectively. In this equation, it could be written as:

$$F_I(t) = \bar{M}\ddot{\bar{U}}(t) + \bar{M}.\bar{a}.\ddot{\bar{U}}_g(t) \quad (32\text{-a})$$

$$F_D(t) = \bar{C}.\dot{\bar{U}}(t) + \bar{b}.F_{Dimp}(t) \quad (32\text{-b})$$

$$F_E(t) = \bar{K}.\bar{U}(t) + \bar{b}.F_{Eimp}(t) \quad (32\text{-c})$$

Where

$$\bar{a} = [1 \quad 1 \quad 1 \quad .... \quad 1]^T$$

$$\bar{b} = [1 \quad 0 \quad 0 \quad .... \quad 0]^T$$

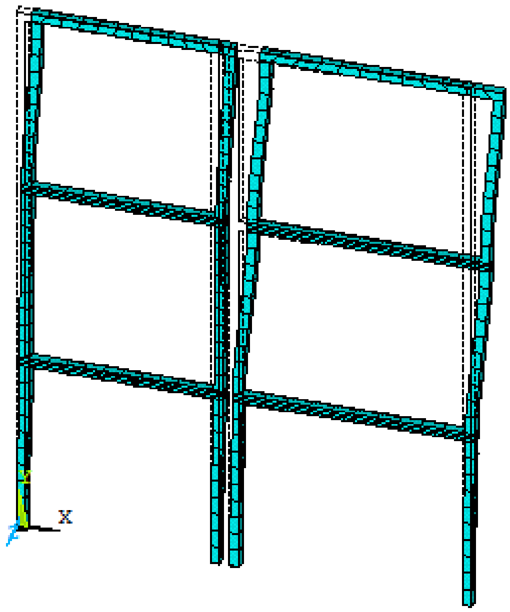

Figure 8. The finite element model.

F_{Dimp} and F_{Eimp} are the elastic and damping contact force during impact, respectively. In order to make same terms in equations, second terms of equations are changed to be matrix.

ANALYZES AND RESULTS

The models were analyzed to simulate the impact during collision. Nonlinear viscose elastic link element was installed to calculate the lateral displacement and impact force between two buildings. In order to determine impact force and value of energy absorption, seismic response of models is represented in this part. The vector of stress is shown in Figure 10. As it is seen, the top story of buildings has maximum lateral displacement and the maximum stress has caused in three story building, where they collide with each other during seismic loading. For this challenge, a mathematic program is used to simulate impact between two considered bodies and calculate impact force during collisions based on nonlinear lateral displacement and time steps. The impact force at each time step is computed by the following expression:

$$F(t+\Delta t) = k.\delta(t)^{1.5} + c_{imp}\dot{\delta}(t) \tag{33}$$

By using Equations (29) and (7) into Equation (30), impact force appeared to be:

$$F_{imp} = k\delta^{1.5} + \zeta\delta^{1.5}.\dot{\delta} \tag{34}$$

Firstly, lateral displacement curves show nonlinear behavior in buildings. Models have inherently shown an irregular fluctuation, which is calibrated by Fourier series (Figure 11).

By having lateral displacement curves, the maximum part of depicted curve is selected and calibrated by Fourier series. These equations could be written as:

$$y_1 = 1.4\cos(3.39t) - 31\sin(3.39t) - 30\cos(6.8t) + 3.7\sin(6.8t) + 20.8$$
$$y_2 = 1.66\cos(2.25t) - 24\sin(2.25t) - 28\cos(4.5t) + 3.1\sin(4.5t) + 14.23 \tag{35}$$

Two calculated lateral displacements based on Fourier series are jointed with each other to get the best estimation of link element lateral displacement. By having $\delta(t)$, velocity is determined by derivation of displacement. Both terms of Equation (35) depend on time, which appeared in $\delta(t)$ and $\dot{\delta}(t)$. First term is divided to two parts, stiffness of spring and lateral displacement. Consequently, the result of impact has a linear behavior. Second term has a different situation as it has various parameters such as coefficient of restitution, impact velocity, and maximum displacement, stiffness of spring and body masses. Collision describes an energy dissipation, which is absorbed by damper. Energy dissipation is shown by hysteresis curve and is considered by enclosed area of curve. In fact, both terms are added with each other to show impact force and dissipated energy. Based on both mentioned terms, all parameters used are investigated. The effectiveness of these parameters is evaluated and the results of different situation are compared with each other. Firstly, in order to predict an estimated result of impact force and dissipated energy between two bodies during collision, an equivalent mass is selected to be 1000 kN. It is approximately assumed that stiffness of spring is 1000. Impact velocity and coefficient of restitution are considered to be 10 m/s^2 and 0.5, respectively.
The collision is simulated and the maximum impact force is calculated. This value of impact is $3*10^7$ kN, which has occurred at the time of 4.6 ^{sec} (Figure 12).

Effect of stiffness of spring

In order to investigate the effectiveness of stiffness of spring, three different type of stiffness are selected, which are 1000, 2000 and 3000. Other parameters are assumed to be same with previous analysis. The results of analyses show that by increasing stiffness of spring, impact force is increasing and subsequently, dissipated energy is declining. Impact forces are $3*10^7$, $4.1*10^7$ and $5.45*10^7$ for stiffness of springs 1000, 2000 and 3000, respectively. Consequently, increase of the stiffness of spring has not shown normal result (Figure 13).

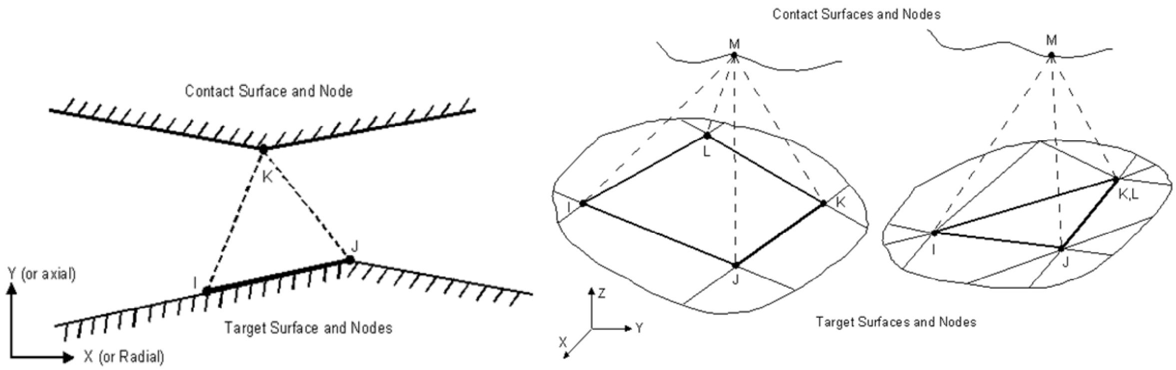

Figure 9. Contact element in finite element software [13].

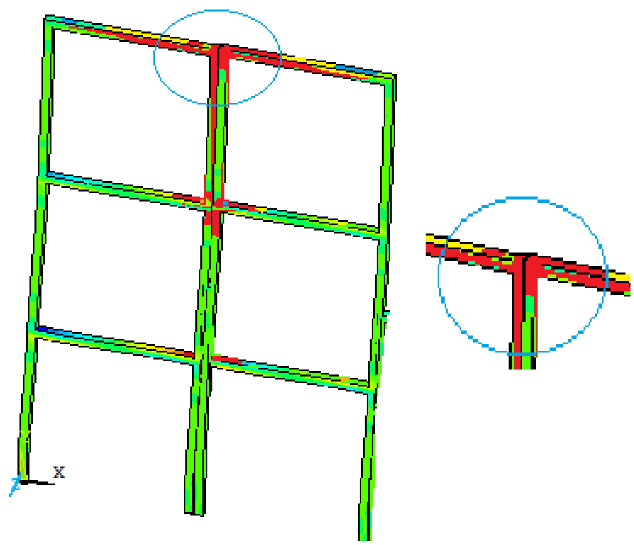

Figure 10. Stress vector of impact.

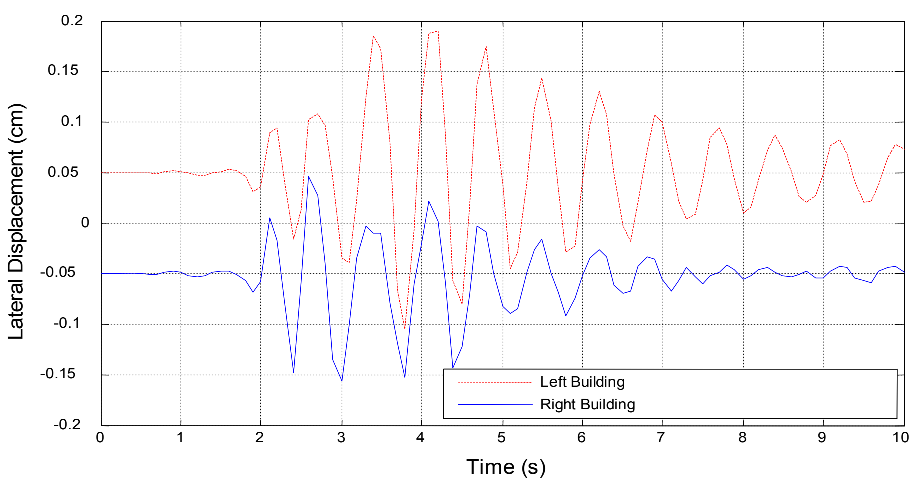

Figure 11. Lateral displacement of analyses.

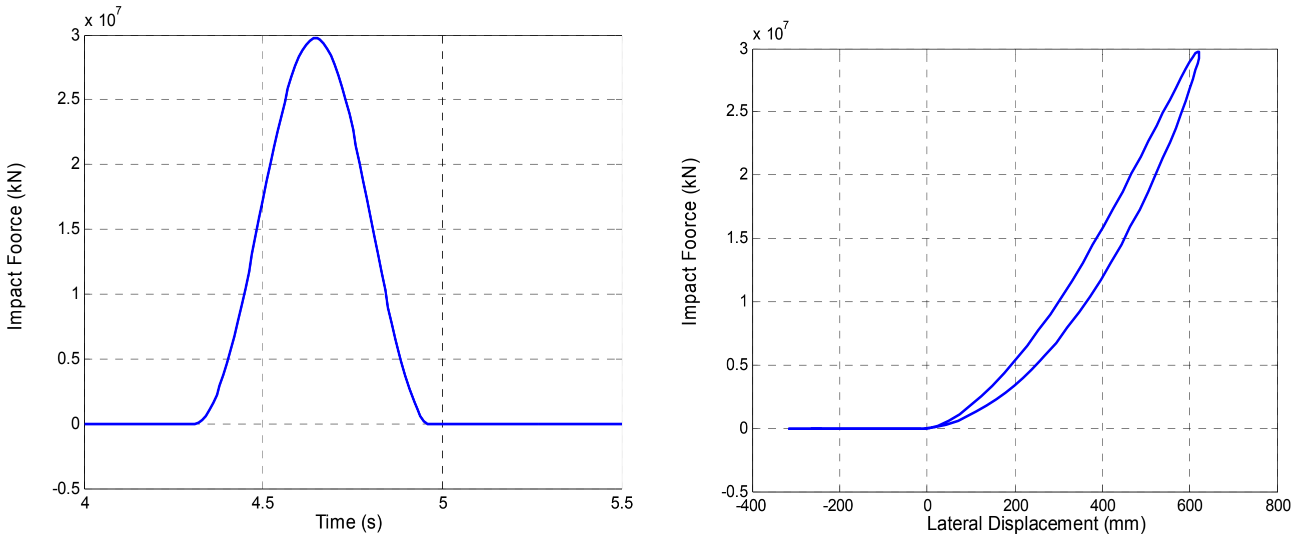

Figure 12. The results of analyses.

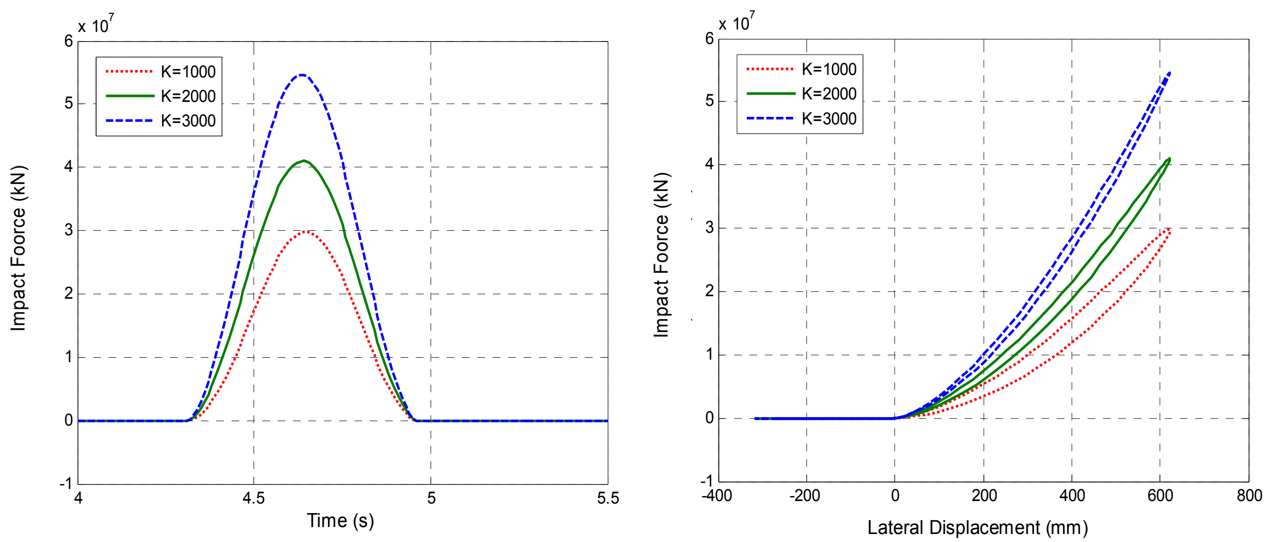

Figure 13. Comparison of the results of stiffness.

Effect of coefficient of restitution

Evaluation of simulated impact force by having different coefficient of restitutions has inflicted that by increasing discussed parameter, the impact force and dissipated energy are increasing. Impact forces are 2.4, 3 and $3.55*10^7$ for *e* equal to be 0.3, 0.5 and 0.7, respectively. It is obviously seen that selected coefficient of restitutions is very important as different researchers use various coefficient of restitutions among zero to one. It seems that it is a vital need to justify the value of *e* to have coordinate results (Figure 14).

Effect of body masses

Last investigated parameter is mass. Three different masses are considered to analyze and evaluate the effectiveness of mass in terms of impact and dissipated energy. Masses are 1000, 2000 and 3000 and also other different parameters are assumed to be same by reference analysis. In this investigation, collision between two bodies has shown that by increasing the value of mass, impact force and dissipated energy are increased (Figure 15).

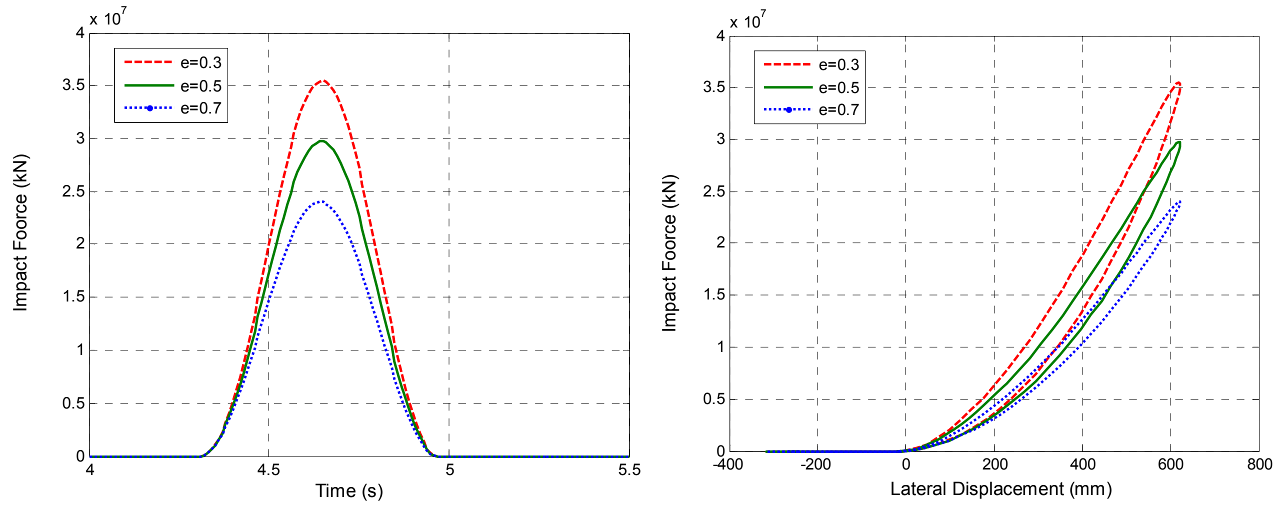

Figure 14. Comparison the results of coefficient of restitution.

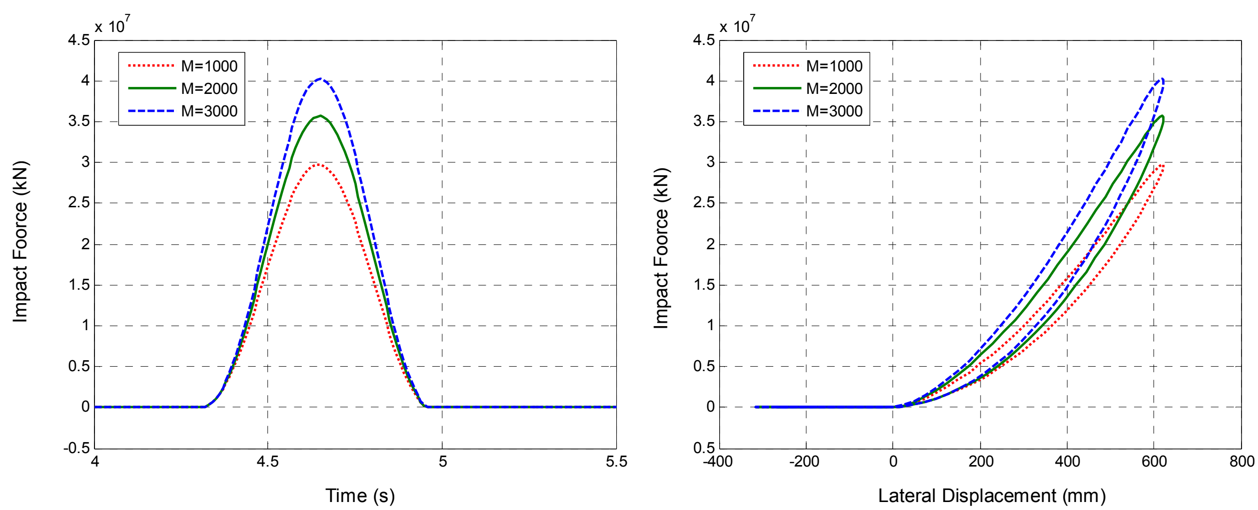

Figure 15. Comparison the results of masses.

Conclusion

In this paper, a nonlinear hysteretic impact model between two MDOF systems is considered for the simulation of impact between adjacent buildings. Two to three story buildings are modeled and a link element is located. The link element is assumed to have a spring and a damper, which are located parallel with each other. In order to measure impact force and energy dissipation, a new equation of motion is calculated based on masses, lateral displacement, impact velocity, coefficient of restitution and stiffness of spring. All of matrixes are written and by using finite element software, buildings collide with each other during given seismic excitation. In order to investigate the accuracy of rendered equation, impact during collision is depicted by hysteresis loop and dissipated energy is calculated. Determined energy is compared by calculated energy value based on physics roll. As the equation depends on coefficient of restitution, assumed and calculated coefficient of restitution are compared. The results show that suggested equation could be satisfied.

In order to simulate impact force and dissipated energy, lateral displacements are determined and calibrated based on Fourier series. All of parameters, used in suggested equation are investigated and the effectiveness of them is evaluated. A new equation of motion is justified between coefficient of restitution and two other parameters, maximum displacement and impact velocity. By using mentioned equation and selecting coefficient of restitution, it is possible to measure a relation between them. The equation could be helped to have same results by using different coefficient of restitution.

Conflict of Interest

The authors have not declared any conflict of interests.

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