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Application of Scheffe's model in optimization of compressive strength of lateritic concrete

Elvis M. Mbadike[1]* and N. N. Osadebe[2]

[1]Department of Civil Engineering, Michael Okpara University of Agriculture, Umudike, Umuahia, Nigeria.
[2]Department of Civil Engineering, University of Nigeria, Nsukka, Enugu State, Nigeria.

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In this research work, the use of Scheffe's simplex theory for the optimization of the compressive strength of lateritic concrete was investigated. The objective of the study is to develop a model that can predict the mix ratio when the desired compressive strength is known or vice-versa. A total of sixty (60) concrete cubes were cast. For each of the twenty mix ratios, three cubes were cast and the average determined. The first thirty cubes were used to determine the coefficients of the model while the other thirty cubes were used to validate the model (control test). The optimum compressive strength of concrete at 28 days curing was found to be 25.04 N/mm^2 and the corresponding mix ratio was 0.6:1:1.75:1.75 (water, cement, laterite, granite). The model was found to be adequate for prescribing concrete mix ratios, when the desired compressive strength is known and vice-versa.

Key words: Compressive strength, Scheffe's model, lateritic concrete, curing.

INTRODUCTION

Concrete mix design could be carried out using either the empirical or statistical experimental method (Slmon et al., 1997). For instance, optimization of mix proportions of mineral aggregates for use in polymer concrete was attempted using statistical techniques (Mohan et al., 2002). There have been some advances in statistical experimental design for performing tests on concrete but these do not explicitly take into consideration the chemistry involved (Simon, 2003). The supplementary cementitious materials optimization system has been developed (Malhorta and Mehta, 2002). The method is a decision making system that enables the reduction of portland cement in concrete by determining the optimum level of replacement by supplementary cementitious materials. New mix designs for fresh and hardened concrete were developed in order to create construction materials with high performance (Bloom and Bentur, 1995). Some of the statistical experimental methods include simplex design (Scheffe, 1958, 1963) and (Obam, 1998), axial design, mixture experiments involving process variables, mixture models with inverse terms (Draper and St John, 1997) and K-model (Draper and Pukelsheim, 1997). Empirical methods are prone to trial and error which results in material wastage whenever they are used (Ezeh and Ibearugbulem, 2009). Sequel to this, statistical experimental method could be adopted using simplex design. The materials used in such experiments include water, cement, laterite and granite. There is the need to formulate mathematical models that will prescribe concrete mix ratios, when the desired compressive strength is known and vice-versa. Similarly, the need to determine the combination of the materials that would give the highest compressive strength should be met.

In this paper, the Scheffe's mathematical model was adopted in the optimization of compressive strength

*Corresponding author. E-mail: elvis_mbadike@yahoo.co.uk.

of lateritic concrete.

LITERATURE REVIEW

Concrete is a mixture of several component such as cement, fine aggregate, coarse aggregate and water. According to (Onyenuga, 2001), concrete is known to be a composite inert material comprising binder course (cement) and mineral filler (body) or aggregate and water. Admixture could be added but for given set of materials the proportion of the components influences the properties of the concrete mixture, hence, the need to optimize concrete properties such as strength. Mathematical modeling is the process of creating a mathematical representation of some phenomenon in order to gain a better understanding of that phenomenon (Osunade, 1994). Lasis and Ogunjimi (1984) described a model as an abstract that uses mathematical language to control the behaviour of a giving system. According to Osadebe (2003), modeling is mathematical equation of dependent variable (Response) and independent variable (Predictor). Manasce et al., 1994) from their studies refers to it as a representation of a system. Simon et al. (1997) stated that the area of application of mathematical modeling includes engineering and natural sciences.

Simon et al. (1997) studies on high performance concrete, which contains many constituents and which are often subjected to several performance constraints can be a difficult and time consuming task. Different works by Ezeh and Ibearugbulem (2009) and Osadebe (2003) demonstrated the application of mathematical modeling in civil engineering.

MATERIALS AND METHODS

Simplex design formulation

The relation between the actual components and pseudo components is according to Scheffe (1958)

$$Z = AX \qquad (1)$$

Z and X are four element vectors, where A is a four by four matrix. The value of matrix A will be obtained from the first four mix ratios. The mix ratios are Z_1 [0.5:1:1:1], Z_2 [0.55:1:1.5:2], Z_3 [0.65:1:2:1.5], Z_4 [0.6:1:1.5:1.5].

The corresponding pseudo mix ratios are X_1(1:0:0:0], X_2[0:1:0:0], X_3[0:0:1:0], X_4[0:0:0:1]. Substitution of X_i and Z_i into Equation 1 gives the values of A as

$$A = \begin{pmatrix} 0.5 & 0.55 & 0.65 & 0.6 \\ 1 & 1 & 1 & 1 \\ 1 & 1.5 & 2 & 1.5 \\ 1 & 2 & 1.5 & 1.5 \end{pmatrix} \qquad (2)$$

The first four mix ratios are located at the vertices of the tetrahedron simplex. Six other pseudo mix ratios located at mid points of the lines joining the vertices of the simplex are X_{12} [$^1/_2$: ½:0:0], X_{13} [$^1/_2$:0: $^1/_2$:0], X_{14} [$^1/_2$:0:0: $^1/_2$], X_{23}[0: $^1/_2$: $^1/_2$:0] X_{24}[0: $^1/_2$:0: $^1/_2$],X_{34}[0:0: $^1/_2$: $^1/_2$].

Substituting these values into Equation 1 will give the corresponding actual mix ratios, Z as

Z_{12} [0.525:1:1.25:1.5] Z_{13} [0.575:1:1.5:1.25]
Z_{14} [0.55:1:1.25:1.25] Z_{23} [0.6:1:1.75:1.75]
Z_{24} [0.575:1:1.5:1.75], Z_{34} [0.625:1:1.75:1.5]

No pseudo component according to Scheffe (1958) should be more than one or less than zero. The summation of all the pseudo components in a mix ratio must be equal to one (Scheffe, 1958; Obam, 1998).

That is

$$0 \le X_i \le 1 \qquad (3)$$

$$\sum X_i = 1 \qquad (4)$$

The general equation for regression is given as

$Y = b_o + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ij} k x_i x_j$ XK+--------+ ∑bi1, i2---- inxi, xi2---, xin+e (5)

Where $1 \le i \le q$, $1 \le i \le j \le q$, $1 \le i \le j \le k \le q$ and 1≤i1≤i1≤-----≤in≤q respectively (Simon et al., 1997).

Expanding Equation 5 up to second order polynomial for four component mixture, we obtain:

$$\begin{aligned} Y = b_o &+ b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_{11}x_1^2 + b_{12}x_1x_2 \\ &+ b_{13}x_1x_3 + b_{14}x_1x_4 + b_{22}x_2^2 + b_{23}x_2x_3 + b_{24}x_2x_4 \\ &+ b_{33}x_3^2 + b_{34}x_3x_4 + b_{44}x^2_4 + e \end{aligned} \qquad (6)$$

Multiplying Equation 4 by b_o, we obtain

$$b_o = x_1b_o + x_2b_o + x_3b_o + x_4b_o \qquad (7)$$

Multiplying Equation 4 again by x_i and re-arranging we obtain

$$x_1^2 = x_i - x_1x_1 - x_2\,x_i \qquad (8)$$

Substituting Equation 7 and 8 into Equation 6 and collecting like terms together, we obtain

$$\begin{aligned} Y = \theta_1x_1 &+ \theta_2x_2 + \theta_3x_3 + \theta_4x_4 + \theta_{12}x_1x_2 + \theta_{13}x_1x_3 \\ &+ \theta_{14}x_1x_4 + \theta_{23}x_2x_3 + \theta_{24}x_2x_4 \\ &+ \theta_{34}x_3x_4 + e \end{aligned} \qquad (9)$$

Where $\theta_i = b_o + b_i + b_{ii}$ and $\theta_{ij} = b_{ij} - b_i; - b_{ij}$ without loss of generality; e is the estimated error and could be dropped from Equation 9.

Hence

$$\begin{aligned} Y = \theta_1x_1 &+ \theta_2x_2 + \theta_3x_3 + \theta_4x_4 + \theta_{12}x_1x_2 + \theta_{13}x_1x_3 \\ &+ \theta_{14}x_1x_4\ \theta_{23}x_2x_3 + \theta_{24}x_2x_4 + \theta_{23}x_3x_4 \end{aligned} \qquad (10)$$

Let n_i be the experimental compressive cube strength of any of the first four mix ratios, and n_{ij} be the experimental compressive strength of the remaining six mix ratios that were used in this model formulation. Substituting for n_i and the corresponding pseudo mix ratio into Equation 10 gives

$$n_i = \theta_i \qquad (11)$$

In the same way, substituting n_{ij} and the correspond pseudo mix ratio into Equation 10 gives

$$n_{ij} = 0.50_i + 0.05\theta_j + 0.25\theta_{ij} \quad (12)$$

Rearranging Equations 11 and 12, we obtain

$$\theta_i = n_i \quad (13)$$

$$\theta_{ij} = 4n_{ij} - 2n_i - 2n_j \quad (14)$$

Substituting Equation 13 into 14 gives

$$\theta_{ij} = 4n_{ij} - 2n_i - 2n_j \quad (15)$$

Substituting Equations 13 and 15 into Equation 10 and collecting like terms will give

$$\begin{aligned} F(x) = {} & n_1x_1 (1 - 2x_2 - 2x_3 - 2x_4) + n_2x_2 (1\text{-}2x_1 - 2x_3\text{-}2x_4) \\ & + n_3x_3 (1\text{-}2x_1 - 2x_2 - 2x_4) + n_4x_4 (1\text{-}2x_1 - 2x_2 - 2x_3) \\ & + 4n_{12}x_1x_2 + 4n_{13}x_1x_3 + 4n_{14}x_1x_4 + 4n_{23}x_2x_3 \\ & + 4n_{24}x_2x_4 + 4n_{34}x_3x_4 \end{aligned} \quad (16)$$

Now, multiplying Equation 4 by 2 and subtracting 1 from both sides, we obtain

$$2x_1 + 2x_2 + 2x_3 + 2x_4 - 1 = 1 \quad (17)$$

Rearranging Equation 17, we obtain

$$2x_2 - 1 = 1\text{-}\ 2x_1 - 2x_2 - 2x_4 \quad (18)$$

Similarly

$$2x_2 \text{ -}1 = 1 \text{ -}2x_1 - 2x_3 \text{ -}2x_4 \quad (19)$$

$$2x_3 - 1 = 1\text{-}2x_1 - 2x_2 - 2x_4 \quad (20)$$

$$2x_4 - 1 = 1\text{-}2x_1 - 2x_2 - 2x_3 \quad (21)$$

Substituting Equations 18, 19, 20 and 21 into Equation 16, we obtain

$$\begin{aligned} F(x) = {} & n_1x_1 (2x_1\text{-}1) + n_2x_2 (2x_2\text{-}1) + n_3x_3 (2x_3\text{-}1) + n_4x_4 (2x_4\text{-}1) \\ & + 4n_{12}x_1x_2 + 4n_{13}x_1x_3 + n_{14}x_1x_4 + 4n_{23}x_2x_3 + 4n_{24}x_2x_4 \\ & +4n_{34}x_3x_4 \end{aligned} \quad (22)$$

Equation 22 is the mathematical model equation.

PROGRAM FOR COMPRESSVE STRENGTH

```
Private Sub ENDMNU_Click()
End
End Sub

Private Sub STARTMNU_Click()
 Rem ONE COMPONENT
     Cls
     '  SCHEFFE'S SIMPLEX MODEL
   Print "    THE PROGRAM WAS WRITTEN BY"
   Print: Print
   Print "    MBADIKE ELVIS"
   Print:
     WWWWW = InputBox("CLICK OK. TO CONTINUE"): Cls
   Print: Print "     THIS PROJECT IS  A RESEARCH PROJECT"
   Print "  CIVIL ENGINEERING"
   WWWWW = InputBox("CLICK OK. TO CONTINUE"): Cls
   Print "    I ACKNOWLEDGE REV. PROF. NKEMAKOLAM NWAOLISA OSADEBE"
   Print "    FOR INITIATING AND SUPERVISING THIS WORK"
   WWWWW = InputBox("CLICK OK. TO CONTINUE"): Cls

'  CIVIL ENGINEERING DEPARTMENT, FUTO

   CT = 0: OPSTRENGTH = 0
   ReDim X(10), A(4, 4), Z(4), N(10), B(4, 4), ZZ(4): QQQ = 1

   Cls
   N1 = 21.48: N2 = 13.63: N3 = 14.96: N4 = 18.07: N5 = 19.85
   N6 = 21.18: N7 = 16.30: N8 = 25.04: N9 = 20.74: N10 = 20.59


4  QQ = InputBox("WHAT DO YOU WANT TO DO? TO CALCULATE MIX RATIOS GIVEN DESIRED COMPRESSIVE
STRENGTH OR CALCULATING COMPRESSIVE STRENGTH GIVEN MIX RATIO?", "IF THE STRENGTH IS KNOWN
TYPE 1 ELSE TYPE 0", "TYPE 1 OR 0 and CLICK OK")
   If QQ <> 1 And QQ <> 0 Then EE = InputBox("No Way! You must ENTER 1 or 0", , "CLICK OK and do so"):
GoTo 5
   If QQ = 0 Then GoTo 900
```

```
    Rem   ***   CONVERSION MATRIX   ***
     A(1, 1) = 0.5: A(1, 2) = 1: A(1, 3) = 1: A(1, 4) = 1
     A(2, 1) = 0.55: A(2, 2) = 1: A(2, 3) = 1.5: A(2, 4) = 2
     A(3, 1) = 0.65: A(3, 2) = 1: A(3, 3) = 2: A(3, 4) = 1.5
     A(4, 1) = 0.6: A(4, 2) = 1: A(4, 3) = 1.5: A(4, 4) = 1.5


    YY = InputBox("WHAT IS THE DESIRED STRENGTH?"): YY = YY * 1

      Rem ONE COMPONENT
      Q = -5: R = 1: E = 1
50  For I = 1 To 4: X(I) = 0: Next I
    X(E) = 1
    If Q = 0 Then GoTo 60
    GoTo 2000
55 E = E + 1: Q = Q + 1: GoTo 50
60  Rem TWO COMPONENTS
    R = R + 1: F = 1: E = 2: T = 1: J = 1: W = 1: K1 = 0.9: K2 = 0.1: V = 6
65  For I = 1 To 4: X(I) = 0: Next I
    X(F) = K1: X(E) = K2
    If T = 6 Then GoTo 70
    If J = V Then GoTo 80
    If W = 5 Then GoTo 90
    GoTo 2000
67  T = T + 1: K1 = K1 - 0.1: K2 = K2 + 0.1: GoTo 65
70  J = J + 1: E = E + 1: K1 = 0.9: K2 = 0.1: T = 1: GoTo 65
80  J = 1: V = V - 1: F = F + 1: E = F + 1: T = 1: W = W + 1: K1 = 0.9: K2 = 0.1: GoTo 65

90  Rem THREE COMPONENTS
    R = R + 1: E = 2: T = 1: J = 1: W = 1
    K1 = 0.89: K2 = 0.01: K3 = 0.1
100 For I = 1 To 5: X(I) = 0: Next I
    If E = 5 Then X(2) = K3: X(1) = K1: X(E) = 0.01: GoTo 110
    X(1) = K1: X(E) = 0.01: X(E + 1) = K3
110 If T = 99 Then GoTo 120
    If J = 5 Then GoTo 130
    If W = 10 Then GoTo 140
    GoTo 2000
115 T = T + 1: X(1) = X(1) - 0.01: X(E) = X(E) + 0.01: GoTo 110
120 T = 1: J = J + 1: E = E + 1: GoTo 100
130 J = 1: T = 1: W = W + 1: E = 2: K1 = K1 - 0.1: K3 = K3 + 0.1: GoTo 100
140 Rem THREE COMPONENTS CONTINUED
    R = R + 1: E = 2: T = 1: J = 1: W = 1
    K1 = 0.69: K2 = 0.11: K3 = 0.2
150 For I = 1 To 5: X(I) = 0: Next I
    If E = 4 Then X(2) = K3: X(1) = K1: X(E) = K2: GoTo 160
    X(1) = K1: X(E) = K2: X(E + 1) = K3
160 If T = 99 Then GoTo 170
    If J = 4 Then GoTo 180
    If W = 8 Then GoTo 190
    GoTo 2000
165 T = T + 1: X(1) = X(1) - 0.01: X(E) = X(E) + 0.01: GoTo 160
170 T = 1: J = J + 1: E = E + 1: GoTo 150
180 J = 1: T = 1: W = W + 1: E = 2: K1 = K1 - 0.1: K3 = K3 + 0.1: GoTo 150
```

```
190 Rem FOUR COMPONENTS
   R = R + 1: E = 2: T = 1: J = 1: W = 1
   K1 = 0.79: K2 = 0.01: K3 = 0.1: K4 = 0.1
200 For I = 1 To 5: X(I) = 0: Next I
   If E = 4 Then X(1) = K1: X(2) = K4: X(E) = K2: X(E + 1) = K3: GoTo 210
   If E = 5 Then X(1) = K1: X(2) = K3: X(3) = K4: X(E) = K2: GoTo 210
   X(1) = K1: X(E) = K2: X(E + 1) = K3: X(E + 2) = K4
210 If T = 99 Then GoTo 220
   If J = 5 Then GoTo 230
   If W = 9 Then GoTo 240
   GoTo 2000
215 T = T + 1: X(1) = X(1) - 0.01: X(E) = X(E) + 0.01: GoTo 210
220 T = 1: J = J + 1: E = E + 1: GoTo 200
230 J = 1: T = 1: W = W + 1: E = 2: K1 = K1 - 0.1: K4 = K4 + 0.1: GoTo 200
240 Rem FOUR COMPONENTS CONTINUED
   R = R + 1: E = 2: T = 1: J = 1: W = 1
   K1 = 0.59: K2 = 0.01: K3 = 0.2: K4 = 0.2
250 For I = 1 To 4: X(I) = 0: Next I
   If E = 4 Then X(1) = K1: X(2) = K4: X(E) = K2: X(E + 1) = K3: GoTo 260

   X(1) = K1: X(E) = K2: X(E + 1) = K3: X(E + 2) = K4
260 If T = 99 Then GoTo 270
   If J = 5 Then GoTo 280
   If W = 7 Then GoTo 290
   GoTo 2000
265 T = T + 1: X(1) = X(1) - 0.01: X(E) = X(E) + 0.01: GoTo 260
270 T = 1: J = J + 1: E = E + 1: GoTo 250
280 J = 1: T = 1: W = W + 1: E = 2: K1 = K1 - 0.1: K4 = K4 + 0.1: GoTo 250

290 Rem FOUR COMPONENTS CONTINUED AGAIN
   R = R + 1: E = 2: T = 1: J = 1: W = 1
   K1 = 0.29: K2 = 0.01: K3 = 0.4: K4 = 0.3
300 For I = 1 To 4: X(I) = 0: Next I
   If E = 4 Then X(1) = K1: X(2) = K4: X(E) = K2: X(E + 1) = K3: GoTo 310

   X(1) = K1: X(E) = K2: X(E + 1) = K3: X(E + 2) = K4
310 If T = 99 Then GoTo 320
   If J = 5 Then GoTo 330
   If W = 4 Then GoTo 340
   GoTo 2000
315 T = T + 1: X(1) = X(1) - 0.01: X(E) = X(E) + 0.01: GoTo 310
320 T = 1: J = J + 1: E = E + 1: GoTo 300
330 J = 1: T = 1: W = W + 1: E = 2: K1 = K1 - 0.1: K4 = K4 + 0.1: GoTo 300
340 Rem FOUR COMPONENTS CONTINUED AGAIN AGAIN
   R = R + 1: E = 2: T = 1
350 For I = 1 To 4: X(I) = 0.25: Next I
   X(E) = 0
360 If T = 6 Then GoTo 370
   GoTo 2000
365 T = T + 1: E = E + 1: GoTo 350
         Rem PRINTING OF RESULTS

      For I = 1 To 4: Z(I) = 0: Next I
      For I = 1 To 4: For JJ = 1 To 4: Z(I) = Z(I) + A(I, JJ) * X(JJ): Next JJ: Next I
```

```
    If Z(1) < 0 Or Z(2) < 0 Or Z(3) < 0 Or Z(4)  < 0 Then GoTo 830
    If X(1) < 0 Or X(2) < 0 Or X(3) < 0 Or X(4) < 0 Then GoTo 830
    If X(1) > 1 Or X(2) > 1 Or X(3) > 1 Or X(4)  > 1 Then GoTo 830
    'If Z(2) + Z(3) < 1 Then GoTo 830
    If Z(2) > 1 Then GoTo 830
    Y = X(1) * (1 - 2 * X(2) - 2 * X(3) - 2 * X(4) -  * N1 + X(2) * (1 - 2 * X(1) - 2 * X(3) - 2 * X(4) -  * N2 + X(3) * (1 -
       2 * X(1) - 2 * X(2) - 2 * X(4) - 2 * X(5)) * N3
    Y = Y + X(4) * (1 - 2 * X(1) - 2 * X(2) - 2 * X(3) -  * N4 + ) * (1 - 2 * X(1) - 2 * X(2) - 2 * X(3) - 2 * X(4)) * + 4 *
       N6 * X(1) * X(2) + 4 * N7 * X(1) * X(3)
    Y = Y + 4 * N8 * X(1) * X(4) + 4 * N9 * X(1) * X(5) + 4 * N10 * X(2) * X(3)
    If Y > OPSTRENGTH Then For I = 1 To 4: ZZ(I) = 0: Next I
    If Y > OPSTRENGTH Then OPSTRENGTH = Y: For I = 1 To 4: For JJ = 1 To 4: ZZ(I) = ZZ(I) + A(I, JJ) * X(JJ): Next
JJ: Next I

    If Y > YY - 0.05 And Y < YY + 0.05 Then GoTo 810 Else GoTo 830

810   CT = CT + 1
    For I = 1 To 4: Z(I) = 0: Next I
    For I = 1 To 4
    For JJ = 1 To 4
    Z(I) = Z(I) + A(I, JJ) * X(JJ)
    Next JJ
    Next I
    'If Z(2) > 1.01 Or Z(2) < 0.9998 Then GoTo 830
    'If Z(1) < 0 Or Z(2) < 0 Or Z(3) < 0 Or Z(4) < 0 Or Z(4) < 0 Then GoTo 830
    If QQQ = 25 Then QQQQ = InputBox("PRESS OK TO CONTINUE", , , 5500, 8500): QQQ = 1: Cls
    QQQ = QQQ + 1

820   Print "  Y = "; Format(Y, "0.00#"),
    Print "        WATER    "; Format(Z(1), "0.00#"), ;
    Print "    CEMENT     ", Format(Z(2), "0.00#");
    Print "    LT    ", Format(Z(3), "0.00#");
    Print "    CA      "; Format(Z(4), "0.00#");


830
    If R = 1 Then GoTo 55
    If R = 2 Then GoTo 67
    If R = 3 Then GoTo 115
    If R = 4 Then GoTo 165
    If R = 5 Then GoTo 215
    If R = 6 Then GoTo 265
    If R = 7 Then GoTo 315
    If R = 8 Then GoTo 365
    If R = 9 Then GoTo 395
    If R = 10 Then GoTo 455
    If R = 11 Then GoTo 515
    If R = 12 Then GoTo 525
2100
    Print: Print
    If CT = 0 Then Print "   ***   SORRY THE HARDNESS IS OUTSIDE THE FACTOR SPACE   ***"
    Print: Print
    Print " OPTIMUM STRENGTH PREDICTABLE BY THIS MODEL IS "
    Print OPSTRENGTH: Print
    Print " THE CORRESPONDING MIXTURE RATIO IS AS FOLLOWS:"
```

```
      Print "     WATER ="; ZZ(1); "    CEMENT ="; ZZ(2); "    LT ="; ZZ(3);
      Print "   CA ="; ZZ(4); "
      GoTo 22222

900
      Cls
   Y = 0
   For I = 1 To 5: X(I) = 0: Next I
   Rem   ***  RESPONSE AT THE CHOSEN 10 POINTS ON THE FACTOR SPACE FOR THE MODEL  ***

   N1 = 21.48: N2 = 13.63: N3 = 14.96: N4 = 18.07: N5 = 19.85
   N6 = 21.18: N7 = 16.30: N8 = 25.04: N9 = 20.74: N10 = 20.59
   GoTo 3010
3010    Rem   ***  CONVERSION MATRIX   ****

B(1, 1) = 4.680851064: B(1, 2) = 3.358297872: B(1, 3) = -0.323404255: B(1, 4) = -1.787234043: B(1, 5) = -
2.740425532
B(2, 1) = -6.382978723: B(2, 2) = -12.83404255: B(2, 3) = 3.531914894: B(2, 4) = 4.255319149: B(2, 5) =
5.191489362
B(3, 1) = -1.276595745: B(3, 2) = 16.19319149: B(3, 3) = -4.093617021: B(3, 4) = -3.14893617: B(3, 5) = -
4.161702128
B(4, 1) = 2.127659574: B(4, 2) = -4.855319149: B(4, 3) = -0.510638298: B(4, 4) = 1.914893617


      Rem   ***   ACTUAL MIXTURE COMPONENTS   ****
      Z(1) = InputBox("ENTER THE VALUE OF Water")
      Z(2) = InputBox("ENTER THE VALUE OF Cement")
      Z(3) = InputBox("ENTER THE VALUE OF LT")
      Z(4) = InputBox("ENTER THE VALUE OF CA")

      Rem   ***   PSEUDO MIXTURE COMPONENTS   ***
      For I = 1 To 4
      For JJ = 1 To 4
      X(I) = X(I) + B(I, JJ) * Z(JJ)
      Next JJ
      Next I

      Rem   ***  CALCULATING THE STRENGTH (RESPONSE)  ****
      Y = X(1) * (1 - 2 * X(2) - 2 * X(3) - 2 * X(4) - 2 * X(5)) * N1 + X(2) * (1 - 2 * X(1) - 2 * X(3) - 2 * X(4) - 2 * X(5))
         * N2 + X(3) * (1 - 2 * X(1) - 2 * X(2) - 2 * X(4) - 2 * X(5)) * N3
      Y = Y + X(4) * (1 - 2 * X(1) - 2 * X(2) - 2 * X(3) - 2 * X(5)) * N4 + X(5) * (1 - 2 * X(1) - 2 * X(2) - 2 * X(3) - 2 *
         X(4)) * N5 + 4 * N6 * X(1) * X(2) + 4 * N7 * X(1) * X(3)
      Y = Y + 4 * N8 * X(1) * X(4) + 4 * N9 * X(1) * X(5) + 4 * N10 * X(2) * X(3)

      Print "  Y = "; Format(Y, "0.00#"),
      Print "         WATER     "; Format(Z(1), "0.00#"), ;
      Print "    CEMENT     ", Format(Z(2), "0.00#");
      Print "    LT     ", Format(Z(3), "0.00#");
      Print "    CA      "; Format(Z(4), "0.00#");

      For I = 1 To 4: Print X(I),: Next I

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End
```

Table 1. Pseudo and actual mix ratios for the control test.

Points	Ratio of materials							
	Water	Cement	Laterite	Granite	Water	Cement	Laterite	Granite
	X_1	X_2	X_3	X_4	Z_1	Z_2	Z_3	Z_4
C_1	0	0.5	0	0.5	0.575	1	1.5	1.75
C_2	0.15	0.15	0.35	0.35	0.595	1	1.6	1.5
C_3	0.4	0.1	0.2	0.3	0.565	1	1.4	1.35
C_4	0.1	0.1	0.4	0.4	0.34	1	1.65	1.5
C_5	0.25	0.3	0.25	0.2	0.5725	1	1.5	1.525
C_6	0.16	0.34	0.22	0.28	0.578	1	1.53	1.59
C_7	0.17	0.31	0.25	0.27	0.58	1	1.54	1.57
C_8	0.32	0.18	0.33	0.17	0.5755	1	1.505	1.43
C_9	0.26	0.34	0.30	0.10	0.572	1	1.52	1.54
C_{10}	0.5	0.25	0.25	0	0.55	1	1.375	1.375

To validate the model, extra ten mix ratios (control) were determined and used in the ANOVA test. The aim of the test was to ascertain whether the difference between the results of compressive strength from experiment and model was significant or not. If the difference between the two results is significant, alternative hypothesis will be adopted. If the difference between the two results is not significant, null hypothesis will be adopted. The mix ratios are shown in Table 1.

Compressive strength test

The materials used for this test includes:

i) Granite, which is free from deleterious substance with a maximum size of 20 mm.
ii) The cement used is Dangote cement which is a brand of Ordinary Portland Cement and conforms to (BS 12, 1978).
iii) The laterite used was obtained from Nekede in Owerri North L.G.A of Imo State, Nigeria.
iv) The water used is clean water from bore-hole.

The materials were batched by weight. Mixing was done manually using spade and hand trowel. 150 × 150 × 150 mm concrete moulds were used for casting the concrete cubes. The concrete cubes were cured in water for 28 days. At the end of the hydration period the cubes were crushed and their compressive strength were determined according to the requirement of (BS 1881, 1986).

RESULTS AND DISCUSSION

The compressive strength test results are shown in Table 2. Higher compressive strength was recorded at point 23 (25.04 N/mm^2). Although the mix ratio of 0.6:1:1.75:1.75 had low cement content, the high water/cement ratio of 0.6 seemed to be the major reason for this. Point 1 should have given higher compressive strength going by its high cement content but it had low water cement ratio of 0.5, and it could be observed that water/cement ratio and cement content were not the only factors responsible for the behaviour of the compressive strength. This is so because the strength at point 23 is higher than that at point 1 irrespective of the fact that point 1 has lower water/cement ratio and higher cement content than point 23. According to Ezeh and Ibearugbulem (2009), the highest compressive strength predicted by the application of Scheffe's model in optimization of compressive strength of River Stone aggregate concrete is 37.62 N/mm^2 when the mix ratio is 0.5:1:2.4:3.6 (water-cement ratio, cement, river sand, river stone). Also the result obtained when Osadebe (2003) model was used to predict the compressive strength of concrete containing water-cement ratio, cement, fine aggregate, coarse aggregate in a mix ratio of 0.6:1:0.7:2.5 is 25.39 N/mm^2. These results show that both Scheffe and Osadebe's model can be used in the optimization of compressive strength of concrete containing four or five mixture ingredients.

The ANOVA tests of the generated data are shown in Table 3.

$$S_E^2 = \frac{\sum(YT - YAT)^2}{N-1} = \frac{157.5374}{9} = 17.504$$

$$S_T^2 = \frac{\sum(YE - YAE)^2}{N-1} = \frac{364.4661}{9} = 40.496$$

Hence $S_T^2 = S_j^2 = 40.496$
$S_E^2 = S_2^2 = 17.504$

$$\therefore \frac{S_1^2}{S_2^2} = \frac{40.496}{17.504} = 2.31$$

F-value from the table is F_α (V_1, V_2)
$= F_{0.05}$ (9,9) = 3.18

$$\frac{1}{F} = \frac{1}{3.18} = 0.3145$$

Table 2. Compressive strength results.

Points	Replicate 1 (N/mm^2)	Replicate 2 (N/mm^2)	Replicate 3 (N/mm^2)	Average compressive strength (N/mm^2)
1	19.69	23.40	21.36	21.48
2	13.58	11.82	15.50	13.63
3	17.10	14.00	13.78	14.96
4	17.00	19.30	17.91	18.07
12	18.90	20.90	19.76	19.85
13	25.40	17.15	21.00	21.18
14	12.60	20.20	16.09	16.30
23	24.76	22.05	28.31	25.04
24	20.45	15.78	26.00	20.74
34	17.72	24.15	19.90	20.59
C_1	17.52	26.10	20.81	21.48
C_2	29.10	22.85	28.92	26.96
C_3	21.90	28.10	23.33	24.44
C_4	5.60	3.70	4.03	4.44
C_5	20.45	29.10	26.90	25.48
C_6	25.60	18.21	25.98	23.26
C_7	26.41	24.00	19.81	23.41
C_8	20.75	25.15	27.00	24.30
C_9	19.85	27.74	23.96	23.85
C_{10}	23.90	19.08	25.90	22.96

Table 3. ANOVA test.

Points	YE	YT	YE-YAE	YT-YAT	(YE-YAE)^2	(YT-YAT)^2
C_1	21.48	21.48	-0.578	1.696	0.3341	2.6764
C_2	26.96	13.63	4.902	-6.154	24.030	37.8717
C_3	24.44	14.96	2.382	-4.824	5.6739	23.2718
C_4	4.44	18.07	-17.618	-1.714	310.394	2.9378
C_5	25.48	19.85	3.422	0.066	11.710	0.00436
C_6	23.26	21.18	1.202	1.406	1.4448	1.9768
C_7	23.41	16.30	1.352	-3.484	1.8279	12.1383
C_8	24.30	25.04	2.242	5.256	5.0265	27.6255
C_9	23.85	26.74	1.792	6.956	3.2113	48.3859
C_{10}	22.96	20.59	0.902	0.806	0.8136	0.6496
Total	**220.58**	**197.84**			**364.4661**	**157.5374**
Average	**22.058**	**19.784**				

YE = Experimental strength; YAE = average of the experimental strength; YT = the model strength; YAT = average of the model strength; N = number of points of observation, V = degree of freedom, α = significant level.

Consequently,

$$\frac{1}{F} \leq \frac{S_1^2}{S_2^2} \leq F$$

$\therefore$ $0.3145 \leq 2.31 \leq 3.18$

Therefore, the difference between the experiment result and the model result was not significant. Hence, the model is adequate for use in predicting the probable compressive strength when the mix ratio is known and vice-versa.

Conclusion

The study revealed that the highest compressive strength predicted by this model is 25.04 N/mm^2. The corresponding mix ratio is 0.6:1:1.75:1.75 [water, cement, laterite granite]. Again, the Scheffe's model formulated was adequate and reliable at 5% risk for predicting the compressive strength of concrete made with the above mentioned materials.

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