

Review

# The analysis of shallow shells with random variable curvature by asymptotic method

Ahmad Akbarlou

Islamic Azad University, Tabriz Branch, Tabriz, Iran. E-mail: Ahmad.akbarlou@gmail.com. Tel: +989143119781.

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The paper intends to survey and assess the random change of the form in the shallow shells, which is caused by the usual faults in the technology construction, the influence of which is ignored by the use of safety factors. The surface of the shallow shells with the random function is modulated by the asymptotic method, surveying the stress and strain in the shell. The results of the research can be used in the structural plans of the important projects.

**Key words:** Shallow shells, asymptotic, stress, strain.

## INTRODUCTION

The shallow shells, having very light weight, enjoy a high loading capability (Losif and Lebedev, 1999; Peter and Andrei, 2001; Bucco et al., 1982), random curving may occur in the construction of the planes and ships, and in the thin-walled structures and shells because of some executive conditions, construction, form and the shape of the structure, as contrasted to the ideal model (Mohamad, 1992; Xingwu et al., 2011; Olson and Lindberg, 1988; Pin and Maoguang, 1992). This probability may cause an incident and it may even destroy the structure. The random deforming factor may happen in the automobile.

The problem is solved by the asymptotic method, suggested by Kasumov and Sobolev (1991), Kasumov (1996), Bathe and Ho (1980) and Delpark (1980). According to the mentioned analysis, the mode of stress and strains of shallow shells is investigated on the basis of the following differential equations:

$$\begin{cases} DL W(x, y) - L_k \Phi(x, y) = q(x, y) \\ D_2 L \Phi(x, y) + L_k W(x, y) = 0 \end{cases} \quad (1)$$

Solution of differential Equation 1 should satisfy the following boundary condition on the boundary  $G_s$  of the shell: (Figure 1)

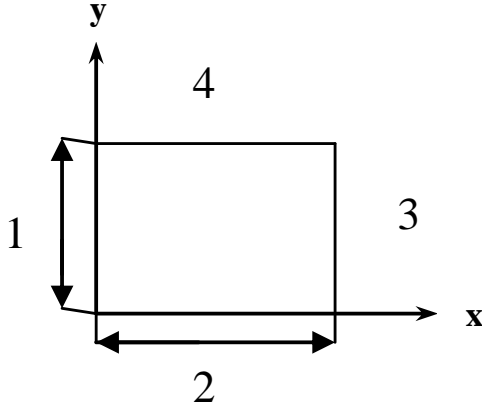
$$y = 0, b \quad x = 0, a \quad \text{for}$$

$$\begin{cases} l_{1x}, l_{*1x} W = 0, & \begin{cases} l_{1y}, l_{*1y} W = 0, \nu = 1, 2 \\ l_{3y}, l_{*3y} \Phi = 0 \\ l_{4y}, l_{*4y} \Phi + l_{3x}, l_{5y} W = 0 \end{cases} \\ l_{3x}, l_{*3x} \Phi = 0 \\ l_{4x}, l_{*4x} \Phi + l_{5x} W = 0 \end{cases} \quad (2)$$

Here

$$\begin{aligned} L &= \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2, \\ l_{1x}, l_{*1x} &= D \left( \frac{\partial^2}{\partial x^2} + \nu \frac{\partial^2}{\partial y^2} \right) + C_{1,3y} \frac{\partial^3}{\partial x \partial y^2}; \\ l_{2x}, l_{*2x} &= D \left( \frac{\partial^3}{\partial x^3} + (2 - \nu) \frac{\partial^3}{\partial x \partial y^2} \right) + B_{1,3y} \frac{\partial^4}{\partial y^2}; \\ l_{3x}, l_{*3x} &= \frac{\partial^2}{\partial x^2} - \nu \frac{\partial^2}{\partial y^2} + \frac{h}{F_{1,3y}} \frac{\partial}{\partial x}, \\ l_{4x}, l_{*4x} &= D \left( \frac{\partial^3}{\partial x^3} + (2 + \nu) \frac{\partial^3}{\partial x \partial y^2} \right) - \frac{1}{B_{1,3h}}; \\ D &= Eh^3 / 12(1 - \nu^2), \quad D_2 = 1 / (Eh), \\ D &= Eh^3 / 12(1 - \nu^2), \quad D_2 = 1 / (Eh), \end{aligned} \quad (3)$$

-flectural rigidity and tension stiffness respectively.



**Figure 1.** Shows the boundary Gs of the shell surface in vertical plan with Cartesian coordinate of the shell.

$$\ell_{5x}, \ell_{*5x} = k_y \frac{\partial}{\partial x}$$

$w, \Phi$

$B_{1,3V}, B_{1,3h}$  –

$$L = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2,$$

$$\ell_{1x}, \ell_{*1x} = D \left( \frac{\partial^2}{\partial x^2} + \nu \frac{\partial^2}{\partial y^2} \right) + C_{1,3y} \frac{\partial^3}{\partial x \partial y^2};$$

$$\ell_{2x}, \ell_{*2x} = D \left( \frac{\partial^3}{\partial x^3} + (2-\nu) \frac{\partial^3}{\partial x \partial y^2} \right) + B_{1,3y} \frac{\partial^4}{\partial y^2};$$

$$\ell_{3x}, \ell_{*3x} = \frac{\partial^2}{\partial x^2} - \nu \frac{\partial^2}{\partial y^2} + \frac{h}{F_{1,3y}} \frac{\partial}{\partial x},$$

$$\ell_{4x}, \ell_{*4x} = D \left( \frac{\partial^3}{\partial x^3} + (2+\nu) \frac{\partial^3}{\partial x \partial y^2} \right) - \frac{1}{B_{1,3h}};$$

In above cited expressions, torsional stiffness of shell ribs on the edges parallels to axis oy sectional area of these

$$L = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2,$$

$$\ell_{1x}, \ell_{*1x} = D \left( \frac{\partial^2}{\partial x^2} + \nu \frac{\partial^2}{\partial y^2} \right) + C_{1,3y} \frac{\partial^3}{\partial x \partial y^2};$$

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$$\ell_{4x}, \ell_{*4x} = D \left( \frac{\partial^3}{\partial x^3} + (2+\nu) \frac{\partial^3}{\partial x \partial y^2} \right) - \frac{1}{B_{1,3h}};$$

shells, and correspondingly, flextural stiffness of the vertical and horizontal sections.  $\nu$  E, h poisson ratio, modulus of elasticity and thickness of shell.  $K_y$  being the curvature of the shell in the direction of the axis y and q being the intensity of the load, and the displacement in the drection of axis z and stress function correspondingly. Assume that the neutral plane of the shell with stochastic function.

$$F(x, y) = \bar{F}(x, y) + \tilde{F}(x, y); \quad \bar{F}(x, y) = \langle \tilde{F}(x, y) \rangle \quad (4)$$

$F(x, y)$  is described as follows:

$$K_F = \sigma_F^2 \exp \left\{ -\alpha \left[ (x-x')^2 + (y-y')^2 \right]^{1/2} \right\} \quad \langle \tilde{F} \rangle = 0 \quad (5)$$

We assume that the ordinates of stationary function F according to Gauss law development and they are set by correlation function.

$$\tilde{F}(x, y)$$

We will present the stochastic function in the form of orthogonal series and it is displayed as follows:

$$\tilde{F}(x, y) = \sigma_F \sum_{k=0}^{K_s} \lambda_k \left( \tilde{a}_k \cos \omega_k r + \tilde{b}_k \sin \omega_k r \right), \quad (6)$$

$$r = (x^2 + y^2)^{1/2}, \quad \omega_k = k\pi / d$$

Where:

Here  $\lambda_k, \Phi_k$  standard deviation of section of function  $F(x, y)$ , and  $\sigma_F$

$$\Phi_k(x, \omega_k) = \lambda_k \int_{-d}^d K_F(x-x', y-y') \Phi(x', y', \omega_k) dx' dy'$$

are defined as follows:

$$\lambda_0 = 1/(\alpha d), \quad \lambda_k = 2\alpha / (d \omega_k^2 + \alpha^2)$$

$$\Phi(r, \omega_k) = \begin{cases} \cos \omega_k r \\ \sin \omega_k r \end{cases}$$

$$K_F = \exp \left\{ -\alpha \left[ (x-x')^2 + (y-y')^2 \right]^{1/2} \right\} \quad (7)$$

$\tilde{a}_k, \tilde{b}_k$  – Random quantity of Gauss and they are determined as follows:

$$\langle \tilde{a}_i, \tilde{b}_j \rangle = \langle \tilde{b}_i, b_j \rangle = \langle E \rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

<E>-unitary matrix

$$\langle \tilde{a}_k \rangle = \langle \tilde{b}_k \rangle = \langle \tilde{a}_k, \tilde{b}_k \rangle = 0, \quad k, i, j = \overline{1, k}$$

$$L_k, \quad l_{5x}, \quad l_{5y}$$

On the basis of Equation 4 for stochastic operators, we have:

$$L_k = \overline{L}_k + \tilde{L}_k,$$

$$\overline{L}_k = \langle L_k \rangle = K_y \frac{\partial^2}{\partial x^2} + K_x \frac{\partial^2}{\partial y^2}$$

$$\overline{l}_{5x} = \langle l_{5x} \rangle = K_y \frac{\partial}{\partial x}, \quad \overline{K}_y = \langle K_y \rangle = \frac{\partial^2 F}{\partial y^2} = const,$$

$$l_{5x} = \overline{l}_{5x} + \tilde{l}_{5x}$$

$$l_{5y} = \overline{l}_{5y} + \tilde{l}_{5y},$$

$$\overline{l}_{5y} = \langle l_{5y} \rangle = K_x \frac{\partial}{\partial x}, \quad \overline{K}_x = \langle K_x \rangle = \frac{\partial^2 F}{\partial y^2} = const,$$

From the condition of stationary of function F there follows stationary of on element of curvature of the shell, we get.

$$\langle \tilde{K}_x, \tilde{K}_y \rangle = 0, \quad K_{kxky}(x, y, x', y') = K_{kxky}(x - x', y - y')$$

The following definitions are offered for the function of shell curvature:

$$\tilde{K}_x = \sigma_F \sum \lambda_k \left[ \tilde{a}_k \alpha_{xk}(x, y) + \tilde{b}_k \beta_{xk}(x, y) \right]$$

$$K_y = \sigma_F \sum \lambda_k \left[ \tilde{a}_k \alpha_{yk}(x, y) + \tilde{b}_k \beta_{yk}(x, y) \right]$$

$$\alpha_{xk} = \omega_k \left[ \omega_k \varepsilon_1(x, y) \cos \omega_k r + \varepsilon_2(x, y) \sin \omega_k r \right],$$

$$\beta_{xk} = \omega_k \left[ -\omega_k \varepsilon_1(x, y) \sin \omega_k r + \varepsilon_2(x, y) \cos \omega_k r \right] \tag{8}$$

$$\alpha_{yk} = \omega_k \left[ \omega_k \varepsilon_3(x, y) \cos \omega_k r + \varepsilon_4(x, y) \sin \omega_k r \right]$$

$$\beta_{yk} = \omega_k \left[ -\omega_k \varepsilon_3(x, y) \sin \omega_k r + \varepsilon_4(x, y) \cos \omega_k r \right]$$

$$\varepsilon_1(x, y) = x^2 / (x^2 + y^2); \quad \varepsilon_2(x, y) = \frac{[1 - \varepsilon_1(x, y)]}{\sqrt{x^2 + y^2}}$$

$$\varepsilon_3(x, y) = y^2 / (x^2 + y^2); \quad \varepsilon_4(x, y) = \frac{[1 - \varepsilon_3(x, y)]}{\sqrt{x^2 + y^2}}$$

As seen, during the setting of the surface of the shell Z = F(x, y) by stochastic function of curvature there will be also stochastic functions. In special cases for the shells with transferred surfaces and constant curvature with the constant curvature Equation 1 will be with constant coefficients. Now we will proceed to the solution of stochastic boundary problem for shallow shells by asymptotic method. Solution of the problem adds up to the solution of the system.

$$\begin{cases} D\overline{L}W - \overline{L}_k \phi - \varepsilon \tilde{L}_k = q \\ D_2 L \phi + \overline{L}_k W - \varepsilon \tilde{L}_k W = 0 \end{cases} \tag{9}$$

The solution of this problem considered in the form of  $\phi$

$$W = \sum_{j=0}^g \varepsilon^j W^{(j)}; \quad \phi = \sum_{j=0}^g \varepsilon^j \phi^{(j)} \tag{10}$$

Taking it into account, this solution adds up to the analysis of shallow shells with constant curvature on the random load,

$$\begin{cases} DLW^{(0)} - \overline{L}_k \phi^{(0)} = q \\ D_2 L \phi^{(0)} - \overline{L}_k W^{(0)} = 0 \end{cases} \tag{11}$$

For null approximation, and random load, we have:

$$\begin{cases} DLW^{(i)} - \overline{L}_k \phi^{(i)} = \tilde{L}_k \phi^{(i-1)} \\ D_2 L \phi^{(i)} - \overline{L}_k W^{(i)} = -\tilde{L}_k W^{(i-1)} \end{cases} \tag{12}$$

To estimate  $i$ , the boundary conditions change correspondingly. The solution of the system (12) is considered in the form of

$$\begin{aligned}
 W_0^{(i)} &= \sum \sum B_{mn} \sin \varphi_m x \sin \psi_n y; \\
 W^{(i)} &= W_0^{(i)} + W_*^{(i)}; \\
 W_*^{(i)} &= \sum [X_{1m} a_{1m}(\eta) + X_{5m} a_{5m}(\eta) + X_{4m} a_4(\eta) + X_{8m} a_8(\eta)] \sin \pi m \zeta + \\
 &+ \sum [X_{9n} a_{9n}(\zeta) + X_{13n} a_{13n}(\zeta) + X_{12n} a_{12}(\zeta) + X_{16n} a_{16}(\zeta)] \sin n \pi \eta; \\
 \Phi_0^{(i)} &= \sum \sum A_{mn} \sin(m \pi \zeta) \sin(n \pi \eta); \\
 \Phi^{(i)} &= \Phi_0^{(i)} + \Phi_*^{(i)}; \\
 \Phi_*^{(i)} &= \sum [X_{3m} a_{3m}(\eta) + X_{7m} a_{7m}(\eta) + X_{2m} a_2(\eta) + X_{6m} a_6(\eta)] \sin m \pi \zeta + \\
 &\sum [X_{11n} a_{11n}(\zeta) + X_{15n} a_{15n}(\zeta) + X_{10n} a_{10}(\zeta) + X_{14n} a_{14}(\zeta)] \sin n \pi \eta; \\
 a_{1m} &= (1-\eta) \left[ \frac{v}{6} \left( \frac{m \pi}{\gamma} \right)^2 H_1(\eta) \right]; & a_{16} &= -\frac{a^2}{6D} H_2(\zeta);
 \end{aligned}$$

Where,

$$\begin{aligned}
 a_{5m} &= \eta + \frac{v}{6} \left( \frac{m \pi}{\gamma} \right)^2 H_2(\eta) & a_{12} &= \frac{a^2}{6D} H_1(\zeta);
 \end{aligned} \tag{13}$$

Where,

$$\begin{aligned}
 a_{5m} &= \eta + \frac{v}{6} \left( \frac{m \pi}{\gamma} \right)^2 H_2(\eta) & a_{12} &= \frac{a^2}{6D} H_1(\zeta); \\
 a_4 &= \frac{b^2}{6D} H_1(\eta); & a_{13n} &= \zeta + \frac{v}{6} \left( \frac{n \pi}{\gamma} \right)^2 H_2(\zeta); \\
 a_8 &= -\frac{b^2}{6} H_2(\eta); & a_{9n} &= 1 - \zeta - \frac{v}{6} \left( \frac{n \pi}{\gamma} \right)^2 H_2(\zeta); \\
 a_{3m} &= -a^2 \left[ (1-\eta) / (m \pi)^2 + \frac{v}{6 \gamma^2} H_1(\eta) \right]; & a_{14} &= \frac{Eha^2}{6} H_2(\zeta); \\
 a_{7m} &= -a^2 \left[ \frac{\eta^2}{(m \pi)^2} - \frac{v}{6 \gamma^2} H_2(\eta) \right]; & a_{10} &= -\frac{Eha^2}{6} H_1(\zeta); \\
 a_2 &= -\frac{Eha^2}{6} H_1(\eta); & a_{15n} &= -a^2 \left[ \frac{\zeta}{(m \pi)^2} - \frac{v}{6} H_2(\zeta) \right]; \\
 a_6 &= \frac{Eha^2}{6 \gamma^2} H_2(\eta); & a_{11n} &= -a^2 \left[ \frac{1-\zeta}{(m \pi)^2} + \frac{v}{6} H_2(\zeta) \right]; \\
 \zeta &= \frac{x}{a}; \quad \eta = \frac{y}{b}; \quad H_1(z) = z^3 - 3z^2 + 2z; \\
 H_2(z) &= z^3 - z
 \end{aligned}$$

The Following equations exists between  $X_{mn}, B_{mn}, A_{mn}$  :

The following equations exists between  $X_{mn}, B_{mn}, A_{mn}$  :

$$\begin{aligned}
 B_{mn} &= (\Delta_{mn} C_{1mn} / Eh - \Delta_{kmn} C_{2mn}) / X_{mn}; \\
 A_{mn} &= (D \Delta_{mn} C_{21mn} - \Delta_{kmn} C_{1mn}) / X_{mn}; \\
 X_{mn} &= \frac{D}{Eh} \Delta_{mn} - \Delta_{kmn}; \\
 \Delta_{mn} &= m^2 + \psi_n^2; \quad \Delta_{mn} = K_x \psi_n^2 + K_y m^2; \\
 C_{1mn}, C_{2mn} &= \iint F_1, F_2(x, y) \sin m x \sin \psi_n y \, dx \, dy; \\
 F_1 &= \tilde{F}_k \varphi^{(0)}; \quad F_2 = -\tilde{L}_k W^{(0)}
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 \tilde{L}_k &= \frac{\partial}{\partial x} \left( K_y \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_x \frac{\partial}{\partial y} \right) = \sum \lambda_k \left\{ a_k \left[ A_{kx} \frac{\partial}{\partial x} + A_k \frac{\partial^2}{\partial x^2} + C_{ky} \frac{\partial}{\partial y} + C_k \frac{\partial^2}{\partial y^2} \right] + \right. \\
 &\left. + b_k \left[ B_{kx} \frac{\partial}{\partial x} + B_k \frac{\partial^2}{\partial x^2} + D_{ky} \frac{\partial}{\partial y} + D_k \frac{\partial^2}{\partial y^2} \right] \right\};
 \end{aligned}$$

$$A_k = \frac{1}{a^2} \left\{ -\omega_{0k} \left[ \left( \frac{\eta}{\gamma} \right)^2 r_0^{-2} \cos \omega_{0k} r_0 + \left( \zeta^2 r_0^{-3} / \omega_{0k} \right) \sin \omega_{0k} r_0 \right] \right\}$$

$$B_k = \frac{1}{a^2} \left\{ \omega_{0k} \left[ -\left( \frac{\eta}{\gamma} \right)^2 r_0^{-2} \sin \omega_{0k} r_0 + \left( \zeta^2 r_0^{-3} / \omega_{0k} \right) \cos \omega_{0k} r_0 \right] \right\}$$

$$C_k = \frac{1}{a^2} \left\{ -\omega_{0k} \left[ \zeta^2 r_0^{-2} \cos \omega_{0k} r_0 + \left( \frac{\eta}{\gamma} \right)^2 r_0^{-3} / \omega_{0k} \sin \omega_{0k} r_0 \right] \right\};$$

$$D_k = \frac{1}{a^2} \left\{ \omega_{0k} \left[ -\zeta^2 r_0^{-2} \sin \omega_{0k} r_0 + \left( \frac{\eta}{\gamma} \right)^2 r_0^{-3} / \omega_{0k} \cos \omega_{0k} r_0 \right] \right\};$$

$$\begin{aligned}
 A'_{kx} &= \frac{1}{a^3} \left\{ \omega_{0k} \zeta r_0^{-3} \left[ \omega_{0k} \left( \zeta^2 - 2(\eta/\gamma)^2 \right) r_0^{-1} \cdot \cos \omega_{0k} r_0 + \right. \right. \\
 &\left. \left. + \left( 2 - \omega_{0k}^2 \left( \frac{\eta}{\gamma} \right)^2 - 3\zeta^2 r_0^{-2} \right) \sin \omega_{0k} r_0 \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 B'_{kx} &= \frac{1}{a^3} \left\{ \omega_{0k} \zeta r_0^{-3} \left[ \omega_{0k} \left( \zeta^2 - 2(\eta/\gamma)^2 - \zeta^2 \right) r_0^{-1} \cdot \sin \omega_{0k} r_0 + \right. \right. \\
 &\left. \left. + \left( 2 - \omega_{0k}^2 \left( \frac{\eta}{\gamma} \right)^2 - 3\zeta^2 r_0^{-2} \right) \cos \omega_{0k} r_0 \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 C'_{ky} &= -\frac{1}{a^3} \left\{ -\omega_{0k} \left( \frac{\eta}{\gamma} \right)^2 r_0^{-3} \left[ \omega_{0k} \left( \left( \frac{\eta}{\gamma} \right)^2 - 2\xi^2 \right) r_0^{-1} \cos \omega_{0k} r_0 + \right. \right. \\
 &+ \left. \left. \left( 2 - \omega_{0k}^2 \xi^2 - 3 \left( \frac{\eta}{\gamma} \right)^2 r_0^{-2} \right) \sin \omega_{0k} r_0 \right] \right\} \\
 D'_{ky} &= \frac{1}{a^3} \left\{ \omega_{0k} \left( \frac{\eta}{\gamma} \right)^2 r_0^{-3} \left[ \omega_{0k} \left( 2\xi^2 - \left( \frac{\eta}{\gamma} \right)^2 \right) r_0^{-1} \sin \omega_{0k} r_0 + \right. \right. \\
 &+ \left. \left. \left( 2 - \omega_{0k}^2 \xi^2 - 3 \left( \frac{\eta}{\gamma} \right)^2 r_0^{-2} \right) \cos \omega_{0k} r_0 \right] \right\} \\
 \gamma &= a/b; \quad \omega_{0k} = k\pi/\rho_0; \quad \rho_0 = d/a; \\
 r_0 &= \left( \xi^2 + \left( \frac{\eta}{\gamma} \right)^2 \right)^{1/2}; \quad \xi = x/a; \quad \eta = y/b.
 \end{aligned}$$

The probable characteristics of the stochastic function  $X_i^{(1)}; (i=1,6)$  are determined from the  $x_i^{(1)}; (\gamma=1,6)$  probabilistic characteristics of the following equations:

$$\begin{aligned}
 &\int_a^b l_{1x}, l_{*1x} w^{(1)}(x, y) \sin \psi_n y dy \Big|_{x=0, a} = 0. \quad \nu = 1, 2 \\
 &\int_a^b l_{3x}, l_{*3x} \Phi^{(1)}(x, y) \sin \psi_n y dy \Big|_{x=0, a} = 0. \\
 &\int_a^b [l_{4x}, l_{*4x} \Phi^{(1)}(x, y) + \tilde{l}_{5x} w^{(1)}(x, y)] \sin \psi_n y dy \Big|_{x=0, a} = \\
 &= -\int_0^b [\tilde{l}_{5x} w^{(0)}(x, y)] \sin \psi_n y dy \Big|_{x=0, a} \\
 &\int_0^a l_{1y}, l_{*1y} w^{(1)}(x, y) \sin \psi_m x dx \Big|_{y=0, b} = 0. \quad \nu = 1, 2 \\
 &\int_a^b l_{3y}, l_{*3y} \Phi^{(1)}(x, y) \sin \psi_m x dx \Big|_{y=0, b} = 0. \\
 &\int_0^a [l_{4y}, l_{*4y} \Phi^{(1)}(x, y) + \tilde{l}_{5y} w^{(1)}(x, y)] \sin \psi_m x dx \Big|_{y=0, b} \\
 &= -\int_0^b [\tilde{l}_{5y} w^{(0)}(x, y)] \sin \psi_m x dx \Big|_{x=0, b}
 \end{aligned} \tag{16}$$

Or in more abbreviated form <sup>TM</sup>

$$\delta_{03}(n) = \frac{1}{Eh} \sum \left[ \frac{X^{(1)}(m, n) \Delta_{mm} - Eh K_2 \phi_m Z^{(1)}(m, n)}{u(m, n) - \frac{a}{E} \int \tilde{l}_{5y} w^{(0)} \sin \phi_n y dy \Big|_{x=0; a}} \right] \tag{17}$$

$$[A_k] \{X_k^{(1)}\} = \{\delta_k^{(1)}\}$$

$$\{\delta_k^{(1)}\} = \{\delta_{01} \dots \delta_{04}; \delta_{a1}, \dots, \delta_{a4}, \delta_{\nabla 1}, \dots, \delta_{\nabla 4}, \delta_{b1}, \dots, \delta_{b4}\};$$

$$\delta_{01}(n) = a \sum \phi_m Z^{(1)}(m, n) / u(m, n)$$

$$\delta_{02}(n) = a^3 \sum Z^{(1)}(m, n) [\phi_m^3 + (2-\nu)\phi_m \psi_n^2]$$

$$\delta_{ai} = \delta_{0i}; \quad i=1,3$$

$$\delta_{01}(m) = a \sum \psi_m Z^{(1)}(m, n) / u(m, n);$$

$$\delta_{02}^{(m)} = a \sum Z^{(1)}(m, n) [\psi_n^3 + (2-\nu)\phi_m^2 \psi_n] / u(m, n);$$

$$\delta_{03}^{(m)} = \frac{1}{Eh} \sum \psi_n Z^{(1)}(m, n) / u(m, n);$$

$$\delta_{03}^{(m)} = \frac{\frac{a}{E} \sum X^{(1)}(m, n) \Delta_{mm} - Eh K_1 \psi_n Z^{(1)}(m, n)}{u(m, n)} - \frac{a}{E} \int \tilde{l}_{5y} w^{(0)} \sin \phi_m x dx \Big|_{y=0, b}$$

$$\delta_{bi} = \delta_{0i}; \quad i=1,3$$

$$Z^{(1)}(m, n) = \frac{1}{Eh} (\Delta_{mm}^2 C_{1mm} - \Delta_{km} C_{2mm})$$

$$X^{(1)}(m, n) = D \Delta_{mm}^2 C_{1mm} + \Delta_{km} C_{2mm}$$

### CONCLUSION

The elements of matrix [A] are defined exactly. Other approximations are built analogically. Every approximation is realized by method of statistical experiments of Monte Carlo. According to the results of the investigation the following conclusions are derived. During the investigation of MD of shallow shellts with random variable curvature with asymptotic method the decision is brought to the calculation of the shellt on random load. As unknowns of resolving system of algebraic equations varies, during their determination,

one should use method of statistical testing of Monte Carlo. It is determined that during the investigation of MD by asymptotic method after null and first approximation, building of algebraic equations for successive approximation is practically impossible. It is natural, when the neutral surface is non stationary function, two mentioned approximations will differ markedly from true. Transformation of neutral surface for each realization makes the decision more difficult and makes it practically unsolvable by asymptotic method.

In spite of the indicated disadvantages of using asymptotic method, it allows tracing all the peculiarities of the investigated problem.

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