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Study on effect of semi-angle between non-parallel walls on magneto hydro dynamic Jeffery Hamel flow using semi-analytical approach

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In this paper, magneto hydro dynamic (MHD) Jeffery-Hamel flow with nanoparticle has been investigated and its nonlinear ordinary differential equation has been solved through homotopy perturbation method (HPM). The governing boundary layer equations are written into a dimensionless form by similarity transformations. The effect of semi-angle has also been investigated in both convergent and divergent channel. A closed agreement between the obtained results and forth order Runge Kutta solution has been established. The proposed procedure can be applied to investigate the effect of other parameters on current problem.

Key words: Magneto hydro dynamic flow, Jeffery Hamel problem, homotopy perturbation method, analytical solution.

INTRODUCTION

The incompressible viscous fluid flow through convergent divergent channels is one of the most applicable cases in fluid mechanics, civil, environmental, mechanical and biomechanical engineering. After introducing the problem of the fluid flow through a divergent channel by Jeffery and Hamel in 1915 and 1916, respectively, it is called Jeffery-Hamel flow. The effect of magnetic field on the velocity profile is an interesting topic for research (Anwari et al., 2005; Cha et al., 2002; Homsy et al., 2005; Ganji and Azimi, 2013; Makinde, 2003; Makinde and Motsa, 2002). Magneto hydro dynamics (MHD) is important in the magnetic confinement of plasmas in experiments of controlled thermonuclear fusion. MHD principles are used for MHD power generator, light-ion-beam powered inertial confinement, and ion thrusters for spacecraft propulsion. Most of the phenomena in engineering such as present problem are essentially nonlinear. Up to now, it is very difficult to obtain analytical approximations of nonlinear ordinary and partial differential equations even though there are high performance computers and computation software. Because of the difficulties of solving nonlinear equations, using helpful and simple approaches are very important. Recently a great deal of interest has been focused on the application of homotopy perturbation method to solve a wide variety of problems (Shakeri et al., 2012; Si et al., 2010).

In this work homotopy perturbation method is used to investigate the hydro magnetic dynamics Jeffry Hamel flow. In present study, we have a study on the effect of semi-angle between non-parallel walls on magneto hydro dynamic Jeffery Hamel flow using HPM. In special case, the obtained results is compared with forth order Runge Kutta method which is different. Present approach can be

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used for the other cases.

PROBLEM DESCRIPTION

In this part of the work, we will consider the physic of problem. The geometry of problem can be seen in Figure 1. At first, we assume that the velocity is only along the radial direction and depends on *r* and *θ* and it is assumed that there are no changes with respect to z so that v = *(u(r, θ),* 0*)*, then the governing equations can be introduced as follows:

$$\frac{\rho}{r}\frac{\partial}{\partial r}\left(ru(r,\theta)\right)=0 \quad (1)$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r}= -\frac{1}{\rho}\frac{\partial P}{\partial r}+\nu\left[\nabla^2 u(r,\theta)-\frac{u(r,\theta)}{r^2}\right]-\frac{\sigma B_0^2}{\rho r^2}u(r,\theta) \quad (2)$$

$$-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+\frac{2\nu}{r^2}\frac{\partial u(r,\theta)}{\partial \theta}=0 \quad (3)$$

Here B_0 the electromagnetic induction, P is the fluid pressure, σ the conductivity of the fluid, ν is the coefficient of kinematic viscosity and ρ the fluid density.
From Equation 1:

$$u(r,\theta)=\frac{f(\theta)}{r} \quad (4)$$

Using dimensionless parameters:

$$F(\eta)=\frac{f(\theta)}{U_{\max}},\quad \eta=\frac{\theta}{\alpha} \quad (5)$$

Where α is the semi-angle between the two inclined walls. The resulting non-linear differential system is of the following form:

$$F'''(\eta)+2\alpha\,\mathrm{Re}\,F(\eta)F'(\eta)+(4-H)\alpha^2F'(\eta)=0 \quad (6)$$

$$F(0)=1,\quad F'(0)=0,\quad F(1)=0 \quad (7)$$

We introduce the Reynolds number and The Hartman number based on the electromagnetic parameters as follows, respectively:

$$\mathrm{Re}=\frac{f_{\max}\alpha}{\nu}=\frac{r\alpha U_{\max}}{\nu} \quad (8)$$
$$\left(\begin{array}{l} Divergen-Channel:\alpha>0,\ f_{\max}>0\\ Convergen-Channel:\alpha<0,\ f_{\max}<0\end{array}\right)$$

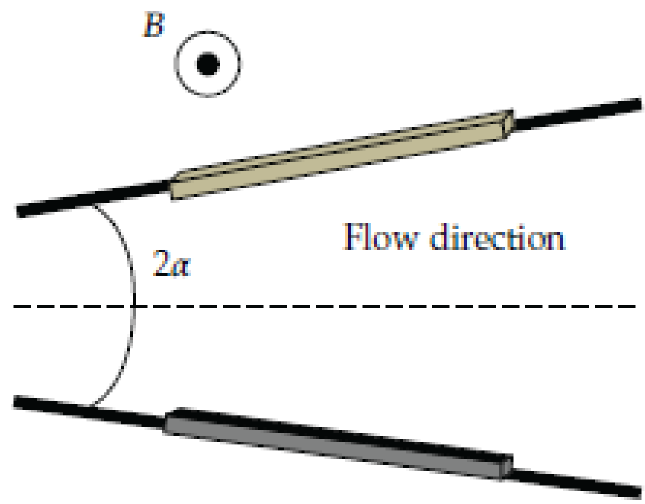

Figure 1. Sgematic of problem.

$$H=\sqrt{\frac{\sigma B_0^2}{\rho\nu}} \quad (9)$$

SOLUTION PROCEDURE

To explain the basic idea of this method, we consider the following nonlinear differential equation:

$$A(u)-f(r)=0,\quad r\in\Omega \quad (10)$$

Subject to boundary condition:

$$B(u,\partial u/\partial n)=0,\quad r\in\Gamma \quad (11)$$

Where A, B, $f(r)$ and Γ are a general differential operator, a boundary operator, a known analytical function, and the boundary of domain Ω.

Generally speaking the operator A can divided into a linear part L and a nonlinear part $N(u)$. Equation 10 can so, be rewritten as:

$$L(u)+N(u)-f(r)=0 \quad (12)$$

We construct a homotopy of Equation 6 $\nu(r,p):\Omega\times[0,1]\to R$ which satisfied:

$$H(\nu,p)=L(\nu)-L(u_0)+pL(u_0)+p[N(\nu)-f(r)]=0 \quad (13)$$

Where p is embedding parameter and u_0 is an initial guess approximation of Equation 6 which satisfies the boundary condition. According to Modified homotopy perturbation method, the solution is expanded into series of p in the form:

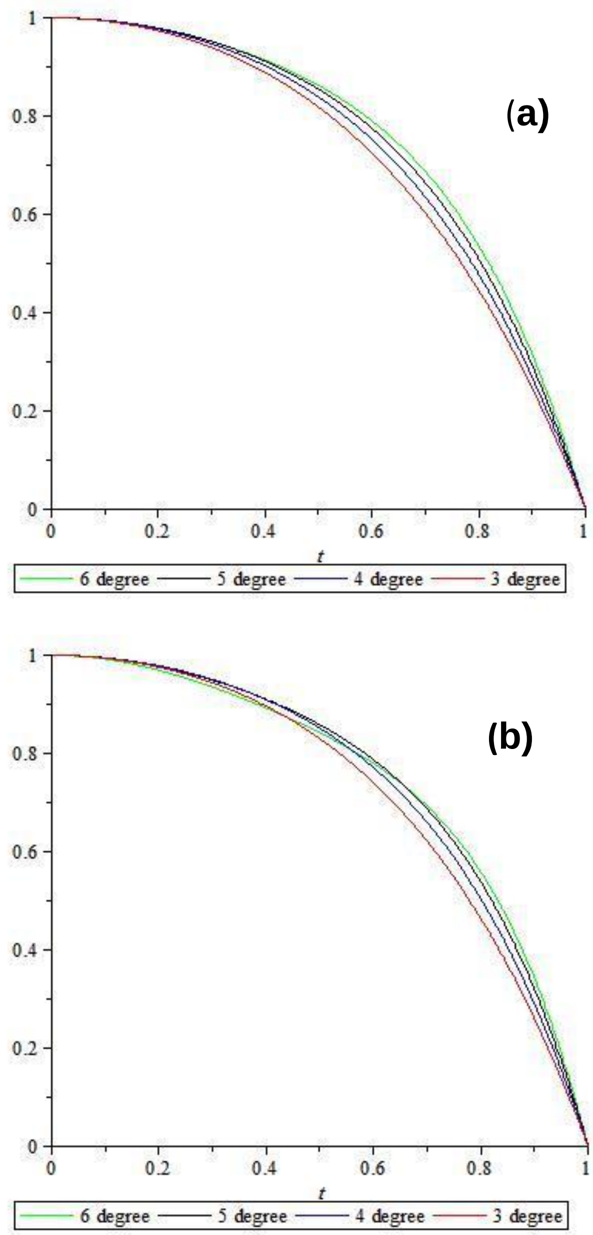

Figure 2. Effect of the semi-angle between the two walls on velocity profile in convergent channel a) Re=50, H=500, b) Re=50, H=1000.

$$u=\sum_{i=0}^{n} p^i v_i \tag{14}$$

Setting $p=1$ results in the approximate solution of Equation 12:

$$u=v_0+pv_1+p^2v_2+\cdots \tag{14}$$

Now, we will use the homotopy perturbation method in order to obtain the solution to Equation 6.

The governing ordinary differential equations are:

$$P^0: F_0'''=0 \tag{15}$$

$$F_0(0)=1,\ \ F_0'(0)=0,\ \ F_0(1)=0$$

$$P^1: F_1'''+(4-H)\alpha^2F_0'+2\alpha\,\mathrm{Re}\,F_0F_0'=0 \tag{16}$$

$$F_1(0)=0,\ \ F_1'(0)=0,\ \ F_1(1)=0$$

$$P^2: F_2'''+(4-H)\alpha^2F_1'+ 2\alpha\,\mathrm{Re}(F_1F_0'+F_0F_1')=0 \tag{17}$$

$$F_2(0)=0,\ \ F_2'(0)=0,\ \ F_2(1)=0$$

Solving above equations, gives following results:

$$F_0(\eta)=1-\eta^2 \tag{18}$$

$$F_1(\eta)=-\frac{\alpha\,\mathrm{Re}}{30}\eta^6+\left(\frac{2\alpha^2}{6}+\frac{\mathrm{Re}\,\alpha}{6}-\frac{a^2H}{12}\right)\eta^4-\left(\frac{2}{15}\mathrm{Re}\,\alpha+\frac{1}{3}\alpha^2-\frac{1}{12}Ha^2\right)\eta^2 \tag{19}$$

$$\begin{aligned}F_2(\eta)=&-\frac{\mathrm{Re}^2\alpha^2}{1350}\eta^{10}+\left(-\frac{H\,\mathrm{Re}\,\alpha^3}{280}+\frac{\mathrm{Re}\,\alpha^3}{70}+\frac{\mathrm{Re}^2\alpha^2}{140}\right)\eta^8\\&-\left(\frac{H^2\alpha^4}{360}-\frac{H\,\mathrm{Re}\,\alpha^3}{60}+\frac{\mathrm{Re}\,\alpha^3}{15}-\frac{2\alpha^4H}{45}+\frac{\mathrm{Re}^2\alpha^2}{50}+\frac{2\alpha^4}{45}\right)\eta^6\\&+\left(\frac{H^2\alpha^2}{144}-\frac{\mathrm{Re}\,\alpha^3H}{40}-\frac{H\alpha^4}{18}+\frac{\alpha^2\mathrm{Re}^2}{45}+\frac{\mathrm{Re}\,\alpha^3}{10}+\frac{\alpha^4}{9}\right)\eta^4\\&-\left(\frac{163\,\mathrm{Re}^2\alpha^2}{18900}+\frac{\alpha^2\,\mathrm{Re}}{21}+\frac{\alpha^4}{15}+\frac{4\alpha\,\mathrm{Re}}{15}+\frac{\alpha}{3}-\frac{H\alpha^3\,\mathrm{Re}}{84}+\frac{H^2\alpha^4}{240}+\frac{\mathrm{Re}\,\alpha^3}{21}-\frac{H\alpha^4}{30}\right)\eta^2\end{aligned} \tag{20}$$

So, the approximate solution can be yield as:

$$F(\eta)\approx F_0(\eta)+F_1(\eta)+F_2(\eta) \tag{21}$$

RESULTS AND DISSCUSSION

The aim of the present study is to investigate the magneto hydro dynamics flow between two non-parallel walls by using analytical method. In this section, we want to study the effect of semi-angle between two walls on velocity profile. We also want to check the accuracy and validity of obtained results in comparison with numerical solution in numerical case.

Figures 2 and 3 show the semi-angle effect on the velocity profiles for divergent and convergent channels. The velocity curves show that the rate of transport is

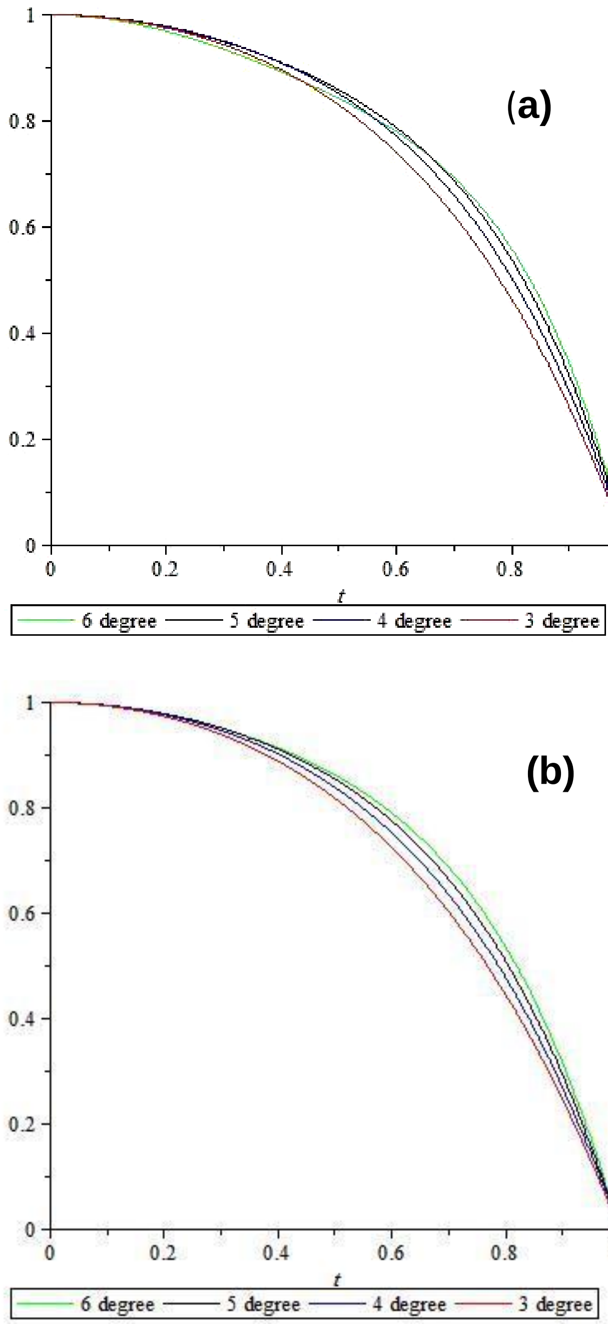

Figure 3. Effect of the semi-angle between the two walls on velocity profile in divergent channel a) Re=50, H= 500, b) Re=50, H=1000.

considerably increased with increase of the semi-angle between the two walls.

In Figure 4 the velocity profiles for $\mathrm{Re}=110$, $H=1000$, $\alpha=5$ is illustrated and The Analytical Results are compared with numerical ones.

As it can be seen in Figures 2 and 3 in high Hartman numbers the variation of the semi-angle between two walls is more effective on velocity profile for both convergent $\alpha<0$ and divergent $\alpha>0$ cases.

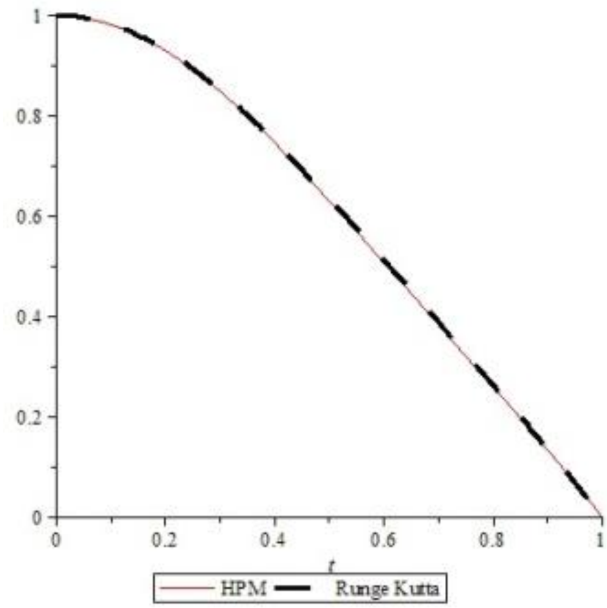

Figure 4. Comparison between Homotopy Perturbation method and Numerical solution for $\mathrm{Re}=110, H=1000, \alpha=5$.

As it can be seen in Figure 4 there is good agreements between the numerical solution obtained by the fourth-order Runge-Kutta method and HPM.

Conclusions

In this paper, we used an analytical approach to investigate the effect of some parameters on the velocity profile for magneto hydro dynamic Jeffery Hamel flow in both convergent and divergent channel and to yield approximate solution for it. In order to obtain the approximate solution Homotopy Perturbation Method is used. Homotopy Perturbation method is a powerful approach for solving MHD Jeffery- Hamel flow in different numerical cases, and it can be observed that there is a good agreement between the present and numerical results.

REFERENCES

Anwari M, Harada N, Takahashi S (2005). Performance of a magnetohydrodynamic accelerator using air-plasma as working gas. Energy Convers. Manag. 4:2605–2613.

Cha JE, Ahn YC, Kim MH (2002). Flow measurement with an electromagnetic flow meter in two-phase bubbly and slug flow regimes. Flow Meas. Instrum. 12(5-6):329–339.

Ganji DD, Azimi M (2013). Application of DTM on MHD Jeffery Hamel Flow with Nanoparticle, UPB Sci. Bull. Ser. A.75:223-230.

Hamel G (1916). Spiralf¨ormige Bewgungen z¨aher Fl¨ussigkeiten, Jahresber. Deutsch. Math. Verein. 25:34–60.

Homsy A, Koster S, Eijkel JCT, Ven der Berg A, Lucklum F, Verpoorte E, de Rooij NF (2005). A high current density DC magnetohydrodynamic (MHD) micropump. Royal Soc. Chem. Lab Chip 5:466–471.

Jeffery GB (1915). The two-dimensional steady motion of a viscous fluid. Phil. Mag. 6:455–465.

Makinde OD (2003). Magneto-hydrodynamic stability of plane-Poiseuille flow using multi-deck asymptotic technique. Math. Comput. Model. 37(3-4):251–259.

Makinde OD, Motsa SS (2002). Hydromagnetic stability of plane Poiseuille flow using Chebyshev spectral collocation method. J. Instrum. Math. Comput. Sci. 12(2):175–183.

Shakeri F, Ganji DD, Azimi M (2012). Application of HPM-pade to Jeffery Hamel Flow Problem. Int. Rev. Mech. Eng. 6(3):537-540.

Si XH, Zheng LC, Zhang XX, Chao Y (2010). Perturbation solution to unsteady flow in a porous channel with expanding or contracting walls in the presence of a transverse magnetic field. Appl. Math. Mech. Eng. Ed. 31(2):151–158. DOI 10.1007/s10483-010-0203-z.