

*Full Length Research Paper*

# Performance analysis of constricted tube ultrafiltration unit for production of food-grade protein concentrates from food wastes

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**Production of protein concentrate from food wastes such as cheese whey, using improved design of tubular ultrafiltration (UF) modules that employ constricted tubes (diverging-converging tubes) has been analysed. The performance characteristics of such UF modules are first simulated mathematically using a numerical algorithm that involves an iterative procedure (a segment to segment computation). The values of system or operating parameters predicted by the developed simulation model agree closely with those obtained from laboratory experiments, the maximum deviation being  $\pm 12\%$ . It is observed that UF modules of proposed configuration provide significantly higher permeate flux and solute rejection. Problems due to concentration polarization and those due to membrane fouling are also at a minimum. The protein concentrate produced is of superior quality with little protein denaturation and containing little salt, lactic acid and lactose, making it suitable for recommendation for human consumption.**

**Key words:** Ultrafiltration, constricted UF module, cheese whey concentration, performance augmentation.

## INTRODUCTION

Production of food-grade protein concentrate from food wastes such as cheese whey is gaining momentum. Cheese whey is the mother liquor left behind after the coagulation and separation of casein from milk. Though more than 70% of the nutritive value of milk (mostly lactose, 20% protein and most of the vitamins and minerals) remains behind in the whey and its yield is quite high (typically, 17 kg of milk yields 1 kg of whey solids), it is often disposed off as waste. In addition to the loss of potentially valuable food products, this also causes serious environmental damage. Thermal concentration of whey by evaporation and spray drying is expensive and in addition, the high salt and lactic acid content of dried whey makes it unsuitable for human consumption. Ultrafiltration (UF) is, by far, the most efficient and economical method for the concentration of cheese whey since it is not only concentrated on the whey, but also prevents denaturation of proteins.

Conventional UF modules are spiral-wound, tubular or hollow fibre systems. The permeate flux often decreases due to solute build-up at the membrane surface and consequent concentration polarization and membrane

clogging. Use of turbulent promoters (Shen and Probst, 1979) and static mixers (Pitera and Middleman, 1973) has been recommended, particularly in tubular modules. However, this increases the fluid shear and concomitantly, the pressure drop and the operating cost also tends to cause denaturation of proteins. In the present paper, production of protein concentrate using tubular UF modules that employ constricted (diverging-converging) tubes is presented, in an attempt to analyse their superior performance characteristics in providing enhanced permeate flux, significantly larger solute rejection and improved product quality. Tubes of such diverging-converging geometry have been successfully employed for the improved design of heat exchangers, condensers, thin film evaporators, falling film absorbers, solar collectors and tower fermenters (Narayanan, 1987, 1988, 1998).

## MATHEMATICAL SIMULATION

The proposed design of the UF module is sketched in

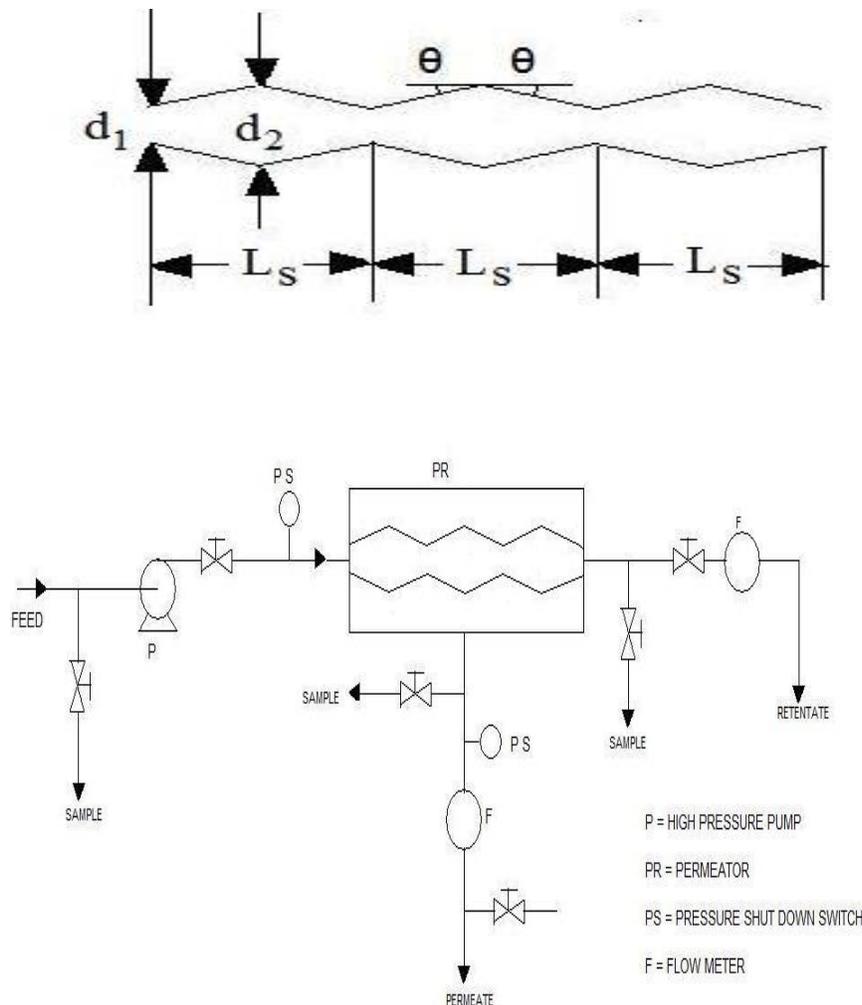


Figure 1. Schematic of constricted geometry and experimental setup.

Figure 1. A large number of tubes of this geometry are enclosed in a shell to constitute a module. The number of tubes or tubular membranes ( $n$ ) could range from 20 to as high as 150. Each tubular membrane has a maximum diameter  $D_2$  and a minimum diameter  $D_1$ , such that:

$$\tan(\theta) = (D_2 - D_1)/L_s = 1/12 \quad (1)$$

or

$$\theta \cong 5^\circ \quad (2)$$

Where  $L_s$  is the segment length and  $\theta$  is the angle of convergence or divergence. Note that the proposed geometry differs from straight, cylindrical geometry by not more than  $5^\circ$ .

Let us consider a differential segment of thickness  $\Delta z$  at a distance  $z$  from the entrance to the tubular module.

This segment is designated as  $i$  -  $th$  segment. The diameter of the tubular membrane at  $z$  shall be

$$D(z) = D_1 + 2z \tan(\theta), \quad \text{for } z \leq L_s/2 \quad (3)$$

$$= D_2 - 2(z - L_s) \tan(\theta), \quad \text{for } L_s/2 \leq z \leq L_s \quad (4)$$

Though the cross-sectional area does change from  $z$  to  $(z + \Delta z)$ , since  $\Delta z$  is extremely small, this change is neglected. The velocity of the flow of solution through the tubular module ( $U$ ) changes continuously with  $z$  due to two reasons:

- (a) The area of cross section of the tube changes with  $z$ .
- (b) The protein-free solution permeates continuously through the membrane and this causes decrease in volumetric flow rate of feed solution in the axial direction.

We have to therefore follow a segment to segment computational procedure. To start with, the following parameters are defined:

$$\eta = [K_m \Delta P / c] \quad (5)$$

$$\phi = B/(c \Delta P) \quad (6)$$

$$\theta^* = \xi / \eta \quad (7)$$

$$\lambda(i) = k_{con}(i)/(\eta \theta^*) \quad (8)$$

$$\alpha(i) = 2 \eta \Delta z / [D(z)U(i-1)] \quad (9)$$

The molar density ( $c$ ) of the solution is assumed to remain more or less constant. Further, the osmotic pressure of the solution ( $\pi$ ) is assumed proportional to the molar concentration of proteins ( $C_A$ ) such that:

$$\begin{aligned} \pi &= C_A R T \\ &= R T (C_A / c) = B(C_A / c) \end{aligned} \quad (10)$$

If  $P$  is the total volume of permeate collected per unit time (in  $m^3/s$ ), then we may define the permeate recovery (PR) as:

$$PR = P / Q_0 \quad (11)$$

Where  $Q_0$  is the volume flow rate of feed solution at module inlet. As stated earlier, the volume flow rate ( $Q$ ) and velocity ( $U$ ) of the solution varies in the axial direction, such that:

$$Q(i) = Q(i-1)[1 - PR(i)] \quad (12)$$

$$U(i) = U(i-1)[1 - PR(i)] \quad (13)$$

Accordingly,

$$\alpha(i+1) = \alpha(i)[1 - PR(i)] \quad (14)$$

The permeate recovery for the  $i$ -th segment, PR ( $i$ ), may be expressed in terms of the protein concentration in permeate [ $C_P(i)$ ] and the previously defined parameters as:

$$PR(i) = 2 \theta^* \alpha(i) / [\phi C_P(i) + \theta^*] \quad (15)$$

The protein concentration in solution leaving the  $i$ -th segment is obtained from a simple mass balance as:

$$[C_F(i) - C_P(i)] = [C_P(i)/\phi_C] \exp\left[-\frac{1}{\lambda(i)\phi_C}\right] \quad (16)$$

Where:

$$\phi_C = [\phi C_P(i) + \theta^*]$$

Also,

$$C_F(i-1) = PR(i) C_P(i) + [1 - PR(i)]C_F(i) \quad (17)$$

Since the equations are mutually coupled with each other, it is necessary to resort to an iterative, trial and error procedure for the computation of the permeate recovery for each segment and protein concentration in permeate from each segment, as shown below:

Step- 1: A value of  $C_P(i)$  is first assumed.

Step- 2: Now, PR ( $i$ ) is computed from equation (15).

Step- 3: Assuming a performance ratio (SR) = 3.75, the mass transfer coefficient is computed as

$$k_{con}(i) = (SR) k_{st}(i) \quad (18)$$

The value of  $k_{st}(i)$  may be computed from one of the experimental correlations reported in literature such as that proposed by Kimura and Sourirajan (1968) or that proposed by Porter (1972). The former is reproduced below:

$$\alpha_0(Sh) = 1.3 / (\beta)^{1/3} \text{ For laminar flow} \quad (19)$$

$$= 0.184 (Re)^{1/4} (\beta)^{-1/3} \text{ For turbulent flow} \quad (20)$$

Where Sh = Sherwood number

$$= D_S k_{st}(i) / D_{AS} \quad (21)$$

$$\beta = 2 \eta \Delta z / [4D_S U(i-1)(\alpha_0^2)] \quad (22)$$

$$\alpha_0 = 2 D_{AS} / (\eta D_S) \quad (23)$$

$$Re = D_S U(i-1) \rho_f / \mu_f \quad (24)$$

$D_S$  = surface diameter of constricted tubular module

By surface diameter, we mean the diameter of a straight, cylindrical tube having the same surface area per unit length as the constricted tube. From simple geometry, it can be deduced that:

$$D_S = [D_2^2 - D_1^2] / [2L_S \sin \theta] \quad (25)$$

The values of SR have been estimated experimentally using a batch permeator (composed of two segments of constricted geometry) and are illustrated in Figure 2. The experimental procedure is elaborated subsequently in this paper. It can be seen that the values of SR range from 3.75 to 4.0, within the Reynolds number ( $Re_m$ ) range

$$Re_m = 4 Q_0 \rho_f / [\pi D_S \mu_f] \quad (26)$$

Accordingly, a typical value of SR = 3.75 is used in the computations.

Step- 4: Now, the protein concentration in solution leaving the segment, namely  $C_F(i)$ , is computed from

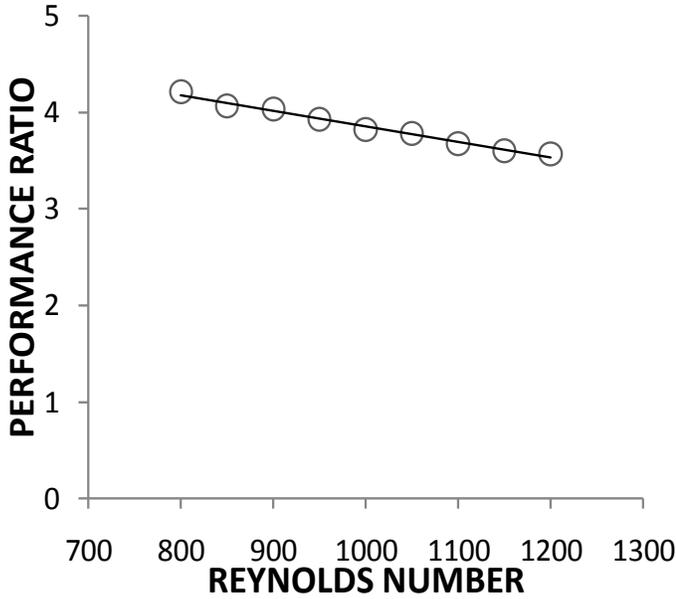


Figure 2. Variation of performance ratio with Reynolds number.

equations (8) and (16).

Step- 5: Finally,  $C_p(i)$  is computed from equation (17).

Step- 6: It is now checked whether the above – computed value of  $C_p(i)$  agrees closely with that assumed / computed earlier. If not, computations are repeated starting from step (2) with the newly computed value of  $C_p(i)$ .

The described iterative procedure is repeated for all segments starting from  $i = 1$  (first segment from module inlet) to  $i = n$  ( the  $n - th$  segment ). The overall permeate recovery (PR) and the permeate rate (P) are now obtained as

$$PR = 1 - \prod_{i=1}^n [1 - PR(i)] \tag{27}$$

$$P = Q_0(PR) \tag{28}$$

To compute the protein concentration in the concentrated cheese whey ( $C_b$ ) and that in permeate solution ( $C_p$ ), let us define two parameters  $p(i)$  and  $q(i)$  such that

$$q(i) = C_F(i) / C_p(i) \tag{29}$$

$$p(i) = PR(i) + q(i)[1 - PR(i)] \tag{30}$$

Now,

$$C_b = (q/p) C_{F0} \tag{31}$$

$$C_p = C_{F0} / p \tag{32}$$

$$\text{Where } q/p = \prod_{i=1}^n [q(i)/p(i)] \tag{33}$$

$$p = PR / [1 - (q/p)(1 - PR)] \tag{34}$$

The fractional solute rejection ( $S_R$ ) is now obtained as

$$S_R = 1 - [C_p / C_{F0}] \tag{35}$$

The discussed numerical procedure, no doubt, demands a large computational load since it involves segment to segment computations and in each segment, the process parameters are to be evaluated through a trial error procedure. However, the algorithm provides reliable predictions of the performance characteristics of the system, as is evident from the good agreement between computed and experimental values of system or process parameters (illustrated subsequently).

**EXPERIMENTAL WORK**

Two test modules were used for the experimental work. The first one consisted of a single constricted tubular module (made of treated polyamide, polyamide composites) enclosed in a 20 mm ID fibre glass tube, while the second consisted of ten constricted tubular modules enclosed in a fibre glass shell. In each case, the dimensions of each tubular membrane module were:

- Maximum diameter ( $D_2$ ) = 10 mm
- Minimum diameter ( $D_1$ ) = 5 mm
- Segment length ( $L_S$ ) = 60 mm
- Tan ( $\theta$ ) = 1 / 12
- Number of segments per tube = 08

The surface diameter ( $D_S$ ) of each constricted tubular module is

$$D_S [\text{from equation (25)}] = 8.0 \text{ mm}$$

The feed solution (cheese whey) is pumped by means of a high pressure pump into the test module. Pressure shutdown switches were used in feed line and also in the permeate line to protect the permeator.

The permeate collects in the fibre glass tube and flows out into the collecting vessel. The protein concentrate is collected from the other end of the permeator. Pressure transducers with digital displays were used to record the fluid pressure in feed line and permeate line and also at the concentrate outlet. High pressure flow meters equipped with electronic recorders were employed to record fluid flow rates. The protein content of cheese whey (feed solution) and that of permeate were recorded using spectrophotometer. Compositions (of feed, concentrate and permeate) were also analyzed using liquid chromatograph (HPLC). Experiments were conducted at different feed concentrations and at each feed concentration, at different feed flow rates. Experiments were also conducted on a test module composed of straight, cylindrical tubular membranes of 8.0 mm in diameter (which is equal to the surface diameter of the constricted module of proposed configuration), for sake of comparison. This module was also made of the same polymer material and was of same wall thickness as the constricted tube module.

An uncertain analysis (Kleine and McClintock, 1953) was also performed and it showed  $\pm 1.5$  and  $\pm 1.65\%$  uncertainty, respectively in Reynolds number and the pressure dropped to 20:4

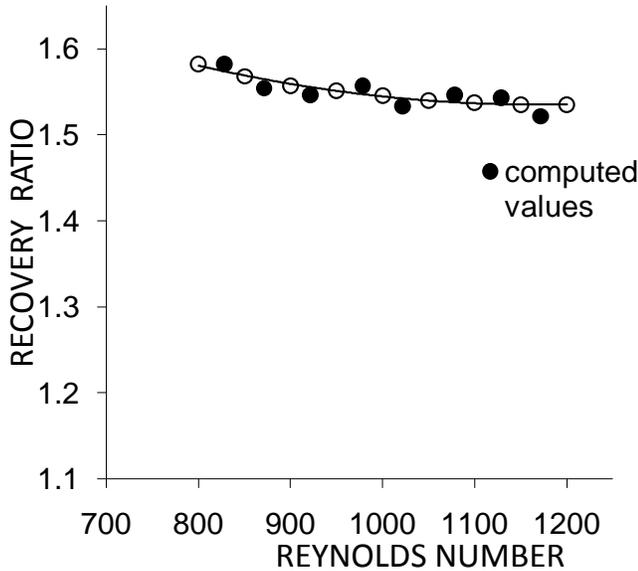


Figure 3. Variation of recovery ratio with Reynolds number.

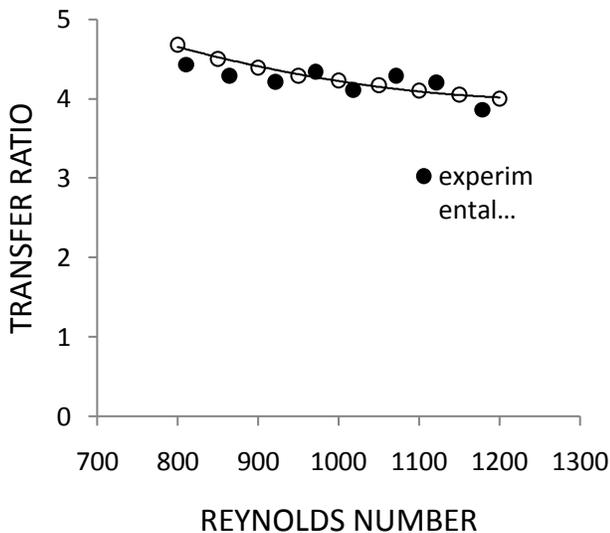


Figure 4. Variation of transfer ratio with Reynolds number.

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## RESULTS AND INFERENCES

The performance ratio (SR) has already been defined in equation (18) as the ratio of mass transfer coefficient for the tubular module of proposed configuration to that for a straight tube module of same membrane area per unit length. The plot of (SR) versus Reynolds number,  $Re_m$  [defined in equation (26)] is shown in Figure 2. It can be seen that even within the low range of  $Re_m$  (800 to 1200),

the mass transfer coefficient in the proposed construction is nearly four times that in the straight tubular module of the same membrane area per unit length.

The recovery ratio (RR) and transfer ratio ( $\Phi_R$ ) are defined as follows:

$$RR = S_R(\text{con}) / S_R(\text{st}) \quad (36)$$

$$\Phi_R = P(\text{con}) / P(\text{st}) \quad (37)$$

The fractional solute rejection ( $S_R$ ) is that defined in equation (35). The plots of (RR) versus  $Re_m$  and  $\Phi_R$  versus  $Re_m$  are shown in Figures 3 and 4, respectively. Two significant observations can be made from these figures, such as:

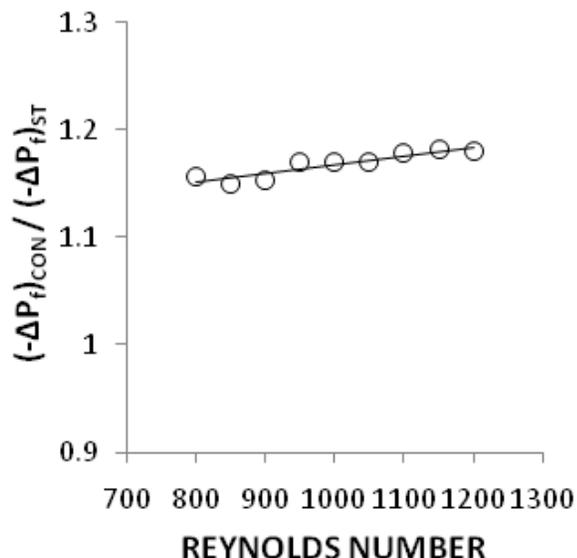
(a) The membrane module of proposed configuration provides substantially higher permeation rate (the augmentation is by a factor of 4.0) and markedly enhanced solute rejection. To note that  $S_R$  is 1.5 to 1.6 times that in the equivalent straight tube module.

(b) The experimental and computed values of both (RR) and  $\Phi_R$  agree closely with each other, the maximum deviation being  $\pm 12\%$ . This establishes the accuracy of the simulation model developed.

Figure 5 compares the frictional pressure drop in the proposed system with that in the equivalent straight tube module under the same operating conditions. It can be seen that the pressure drop in the proposed construction is only marginally higher than (115 to 118% higher than or 1.15 to 1.18 times) that in the equivalent straight tube module, thereby demonstrating that the operating cost of the present system shall not be materially different from that of the conventional tubular module of same membrane area per unit length.

Qualitative analysis of protein concentrate obtained from permeator of proposed design showed that it is of good quality containing little salt (sodium chloride), lactic acid and lactose and there is hardly any denaturation of proteins. This is further confirmed from the fact that analysis of permeate solution (by means of the liquid chromatograph) showed that its salt content, lactose content and lactic acid content are almost equal to those of the feed solution. This ensured that the obtained protein concentrate is uncontaminated. Comparison with standard protein solutions (spectrophotometer readings) demonstrated that there is no denaturation of proteins. This guarantees the product quality. The concentrated whey may be therefore considered suitable for recommendation for human consumption.

Another interesting observation obtained from repeated experimental tests is that scale deposition and consequent membrane fouling are at a lower degree in the module of proposed configuration. This observation is understandable in the light that the constricted wall geometry of the proposed design causes added turbulence at the wall and consequently, the deposited



**Figure 5.** Comparison between pressure drop in constricted tube module and that in equivalent straight tube module.

dirt gets re-entrained into the flowing fluid and gets carried away. This also helps in enhancing the useful life of membrane used.

The mentioned induced turbulence at the wall could also be responsible for increasing the permeate flux, by disturbing the concentrated boundary layer at the wall (which, in turn, tends to impede concentration polarization). The continuous change in the magnitude as well as in the direction of fluid velocity along the length of the module breaks the gel layer and thereby diminishes the resistance to permeate flow. Since this augmentation in performance efficiency of the permeator is thus not due to higher fluid shear, but is due to destruction of the stagnant fluid layer at the wall (caused by the diverging – converging nature of the wall geometry), the overall pressure drop penalty of the system does not increase materially [as is evident from Figure 5]. The fluid also takes a tortuous flow path through the module and this also helps in providing improved contacting between fluid elements and thereby enhancing mass transfer. Due to the same reason that the proteins are not subjected to higher fluid shear, denaturation of proteins does not occur. The life of the membrane is also accordingly improved since the chances of clogging the membrane pores by macrosolutes accumulated in the gel also get diminished

## Conclusions

(a) An improved design of tubular UF module that employs constricted (diverging – converging) tubes has been proposed for the production of protein concentrate

from cheese whey.

(b) The performance characteristics of modules of proposed design have been simulated mathematically and also studied experimentally. Satisfactory agreement has been observed between computed and experimental data, the maximum deviation being  $\pm 12\%$ .

(c) The UF module of proposed configuration provides significantly higher permeation rate and substantially enhanced solute rejection, as compared to the conventional tubular module of same membrane area per unit length. The performance efficiency of the proposed system is thus distinctly high. An additional advantage is that the frictional pressure drop (for fluid flow) in the proposed design and thereby its operating cost is only marginally higher than that in the case of the equivalent straight tube module.

(d) The proposed system also guarantees good product quality as the protein concentrate obtained contains little salt, lactose and lactic acid and there is no denaturation of proteins. A further improvement is that since chances of concentration polarization and membrane fouling has been observed to be at a lower degree, the useful life of membrane shall be much higher.

(e) Though more expensive to fabricate (not to forget that the angle of constriction is as low as  $5^\circ$ ), membrane modules of this kind must be recommended for industrial applications, since augmented performance efficiency is obtained at the cost of negligible increase in operating cost.

## NOMENCLATURE

**B**, parameter defined in equation (10),  $N/m^2$ ; **c**, molar density of solution,  $\text{kmoles}/m^3$ ; **C<sub>A</sub>**, protein concentration,  $\text{kmoles}/m^3$ ; **C<sub>b</sub>**, protein concentration in concentrate,  $\text{kmoles}/m^3$ ; **C<sub>F(i)</sub>**, protein concentration in concentrate solution leaving the *i* – th segment,  $\text{kmoles}/m^3$ ; **C<sub>F0</sub>**, protein concentration in feed to the module,  $\text{kmoles}/m^3$ ; **C<sub>P</sub>**, protein concentration in permeate,  $\text{kmoles}/m^3$ ; **C<sub>P(i)</sub>**, protein concentration in permeate from *i* – th segment,  $\text{kmoles}/m^3$ ; **D(z)**, diameter of constricted tube at a distance *z* from entrance, *m*; **D<sub>1</sub>**, **D<sub>2</sub>**, minimum diameter and maximum diameter of constricted tube, *m*; **D<sub>AS</sub>**, diffusivity of proteins in solution,  $m^2/s$ ; **D<sub>S</sub>**, surface diameter of constricted tube, *m*; **k<sub>CON(i)</sub>**, mass transfer coefficient for *i* – th segment of constricted tube, *m/s*; **k<sub>ST</sub>**, mass transfer coefficient for equivalent straight tube module of same membrane area per unit length, *m/s*; **K<sub>m</sub>**, membrane permeability coefficient,  $\text{kmoles}/(m^2 \cdot s \cdot \text{atm})$ ; **L<sub>s</sub>**, segment length of constricted tube, *m*; **n**, number of tubes in segment length of constricted tube, *m*; **n**, number of tubes in module; **p(i)**, parameter defined in equation (30), dimensionless; **P**, permeate rate,  $m^3/s$ ; **P(con)**, permeate rate in constricted tube module,  $m^3/s$ ; **P(st)**, permeate rate in equivalent straight tube module,  $m^3/s$ ; **PR**, overall permeate recovery, dimensionless; **PR(i)**, permeate recovery in *i* – th segment, dimensionless; **q**

**(i)**, dimensionless concentration ratio defined in equation (29); **Q (i)**, volume rate of solution (concentrated solution) leaving  $i$ -th segment,  $m^3/s$ ; **Q<sub>0</sub>**, volume rate of feed to the module,  $m^3/s$ ; **R**, universal gas constant, J/(kmole. K); **Re**, Reynolds number defined in equation (24), dimensionless; **Re<sub>m</sub>**, modified Reynolds number, dimensionless; **RR**, recovery ratio, dimensionless; **SR**, performance ratio, dimensionless; **Sh**, Sherwood number, dimensionless; **S<sub>R</sub>**, fractional solute rejection; **T**, absolute temperature, K; **U (i)**, average velocity of solution (concentrated solution) leaving  $i$ -th segment,  $m/s$ ; **α (i)**, parameter defined in equation (9), dimensionless; **α<sub>0</sub>**, parameter defined in equation (23), dimensionless; **B**, parameter defined in equation (22), dimensionless; **(ΔP)**, transmembrane pressure drop,  $N/m^2$ ; **ΔP<sub>f</sub>**, frictional pressure drop for flow through tubular module,  $N/m^2$ ; **η**, parameter defined in equation (5),  $m/s$ ; **θ**, angle of constriction, degrees; **θ<sub>0</sub>**, parameter defined in equation (7), dimensionless; **λ (i)**, parameter defined in equation (8), dimensionless; **μ<sub>f</sub>**, liquid viscosity,  $kg/(m.s)$ ; **ξ**, solute transport parameter,  $m/s$ ; **π**, osmotic pressure,  $N/m^2$  or atm **ρ<sub>f</sub>**, liquid density,  $kg/m^3$ ; **Ø**, parameter defined in equation (6),  $m^3/kmole$ ; **Ø<sub>C</sub>**, parameter defined in equation (16), dimensionless; **Φ<sub>R</sub>**, transfer ratio, dimensionless

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