Journal of Electrical and Electronics Engineering Research Vol. 2(3), pp. 68-74, May 2010
Available online at http://www.academicjournals.org/jeeer
ISSN – 2141 – 2367 ©2010 Academic Journals

Full Length Research Paper

An application of FDTD using quadratic extrapolation

Vipul Sharma[1*], S. S Pattnaik[2], S. Devi[2], Shyam Kamal[1], Tanuj Garg[1], Ambarish Pathak[1] and Manu Smriti[1]

[1]Department of Electronics and Communication Engineering, Gurukul Kangri University, Haridwar-249404, India.
[2]Department of Electronics and Communication Engineering, NITTTR, Sector-26, Chandigarh-160019, India.

Accepted 20 March, 2010

Finite difference time domain (FDTD) has been the most obvious choice of researchers to simulate an electromagnetic environment which is quite complex in nature. But the amount of time consumed in running the simulation has always been the limiting factor. In this work, a time efficient FDTD tool that is, Quadratic FDTD is developed and used for simulation of a rectangular cavity resonator. Traditional FDTD simulation was run for 50 number of time steps and out of these, data of first 35 time steps are used to train quadratic extrapolation finite difference time domain method (Q-FDTD) program. The Q-FDTD program is then used to predict the rest of the data. The proposed Q-FDTD is applied to a rectangular cavity resonator to evaluate the performance. It is observed that Q-FDTD can save considerable amount of time (almost 30%). The results of this Q-FDTD are compared with the results of traditional FDTD and excellent performance in time is observed.

Key words: Quadratic extrapolation, FDTD, EM-CAD, air-filled rectangular cavity resonator.

INTRODUCTION

The finite difference time domain (FDTD) method has become the most widely used simulation tool of electromagnetic phenomena. It is powerful method extensively used for analysis of a wide range of EM problems (Shlager et al., 1995). The FDTD is characterized by the solution of Maxwell's curl equations in the time domain after replacement of the derivatives in them by finite differences. It has been applied to many problems of propagation, radiation and scattering of electromagnetic waves. The method owes its success to the power and simplicity it provides. The Finite Difference Time Domain (FDTD) method has been widely used to simulate various electrodynamics problems also (Shlager et al, 1995; Yee, 1966; Taflove et al., 2005) because of its flexibility and versatility. The main drawback of the FDTD versions is extremely high requirements of computer memory and storage specially in the resonance problem and in open-domain solution. The accuracy for fine resonance solution mainly in the cases like small size dielectric lenses or resonators is also another issue of concerned for FDTD simulation. Many variations and extensions of the FDTD exist, and the literature on the FDTD technique is extensive. NFDTD (Neural Network driven FDTD), PSOFDTD (PSO driven FDTD), PSO_LS-SVM FDTD (PSO driven Least Squares Support Vector Machine FDTD) are variants of FDTD to provide time efficient solutions (Panda et al., 2006; Jin et al., 2005; Yang et al., 2005). The proposed paper is an initiative to improve the computational time of FDTD method while enhancing the accuracy by using quadratic extrapolation tools. Quadratic extrapolation (Trefethen et al., 1997; Spencer et al., 2000) is a universal approximation tool for smooth functions, so it should be able to solve any smooth function approximation problem, whether, it is interpolation problem or it is extrapolation problem, given enough data. In this paper, the authors have proposed new variant of FDTD named as Q-FDTD. The proposed algorithm is tested in an air-filled rectangular cavity resonator. Comparison of the results with that of FDTD shows the potentiality of Q-FDTD in terms of computational time saving. Dispersion relation of Yee's algorithm can be

*Corresponding author. E-mail: vipul.s@rediffmail.com. Tel: +91-1334-241617.

used to predict precisely the frequencies of different modes at which a rectangular resonator will oscillate (Wagner et al., 2003).

Proposed Q-FDTD is based on Yee's algorithm. A rectangular cavity resonator is considered to test the performance. The paper is divided into five sections. Section I discusses the introduction. Basic FDTD method is discussed in Section II. Quadratic extrapolation and hybridization with FDTD is discussed in Section III. The results are presented in Section IV and Section V dedicated to conclusion.

FINITE DIFFERENCE TIME DOMAIN METHOD

In a pioneering paper, written in 1966 (Yee, 1966), Yee introduced a set of Finite-difference representations for the time-dependent Maxwell's curl equations. In FDTD method, Maxwell's equations are solved on a discrete spatial and temporal grid (Valcarce et al., 2009; Duan et al., 2009; Homsup et al., 2009; Xiao et al., 2010). In Yee's scheme, the computational domain is subdivided by using an orthogonal mesh in the Cartesian coordinate system. The electric fields are located along the edges of cells, while the magnetic fields are positioned at the centers of these cells. In isotropic medium, Maxwell's equations can be written as:

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad (1)$$

$$\nabla \times H - \sigma E + c \frac{\partial E}{\partial t} \quad (2)$$

The vector equations 1and 2 represent a system of six scalar equations which can be expressed in rectangular coordinate system as (Sadiku, 2001):

$$\frac{\partial Hx}{\partial t} = \frac{1}{\mu}\left(\frac{\partial Ey}{\partial z} - \frac{\partial Ez}{\partial y}\right) \quad (3a)$$

$$\frac{\partial Hy}{\partial t} = \frac{1}{\mu}\left(\frac{\partial Ez}{\partial x} - \frac{\partial Ex}{\partial z}\right) \quad (3b)$$

$$\frac{\partial Hz}{\partial t} = \frac{1}{\mu}\left(\frac{\partial Ex}{\partial y} - \frac{\partial Ey}{\partial x}\right) \quad (3c)$$

$$\frac{\partial Ex}{\partial t} = \frac{1}{\epsilon}\left(\frac{\partial Hz}{\partial y} - \frac{\partial Hy}{\partial z} - \sigma Ex\right) \quad (3d)$$

$$\frac{\partial Ey}{\partial t} = \frac{1}{\epsilon}\left(\frac{\partial Hx}{\partial z} - \frac{\partial Hz}{\partial x} - \sigma Ey\right) \quad (3e)$$

$$\frac{\partial Ez}{\partial t} = \frac{1}{\mu}\left(\frac{\partial Hy}{\partial x} - \frac{\partial Hx}{\partial y} - \sigma Ez\right) \quad (3f)$$

Maxwell's difference equations in the Yee scheme are written as (Sadiku, 2001):

$$E_x^{n+1}(i,j,k) = \frac{\epsilon_x(i,j,k) - 0.5*\Delta t*\sigma_x(i,j,k)}{\epsilon_x(i,j,k) + 0.5*\Delta t*\sigma_x(i,j,k)} E_x^n(i,j,k) + \frac{\Delta t}{\epsilon_x(i,j,k) + 0.5*\Delta t*\sigma_x(i,j,k)} \left[\frac{H_z^{n+1/2}(i,j+1,k) - H_z^{n+1/2}(i,j,k)}{\Delta y(j)} - \frac{H_y^{n+1/2}(i,j,k+1) - H_y^{n+1/2}(i,j,k)}{\Delta z(k)}\right] \quad (4)$$

$$E_y^{n+1}(i,j,k) = \frac{\epsilon_y(i,j,k) - 0.5*\Delta t*\sigma_y(i,j,k)}{\epsilon_y(i,j,k) + 0.5*\Delta t*\sigma_y(i,j,k)} E_y^n(i,j,k) + \frac{\Delta t}{\epsilon_y(i,j,k) + 0.5*\Delta t*\sigma_y(i,j,k)} \left[\frac{H_x^{n+1/2}(i,j,k+1) - H_x^{n+1/2}(i,j,k)}{\Delta z(k)} - \frac{H_z^{n+1/2}(i+1,j,k) - H_z^{n+1/2}(i,j,k)}{\Delta x(i)}\right] \quad (5)$$

$$E_z^{n+1}(i,j,k) = \frac{\epsilon_z(i,j,k) - 0.5*\Delta t*\sigma_z(i,j,k)}{\epsilon_z(i,j,k) + 0.5*\Delta t*\sigma_z(i,j,k)} E_z^n(i,j,k) + \frac{\Delta t}{\epsilon_z(i,j,k) + 0.5*\Delta t*\sigma_z(i,j,k)} \left[\frac{H_y^{n+1/2}(i+1,j,k) - H_y^{n+1/2}(i,j,k)}{\Delta x(i)} - \frac{H_x^{n+1/2}(i,j+1,k) - H_x^{n+1/2}(i,j,k)}{\Delta y(j)}\right] \quad (6)$$

$$H_x^{n+1/2}(i,j,k) = H_x^{n-1/2}(i,j,k) + \frac{\Delta t}{\mu_x(i,j,k)} \left[\frac{E_y^n(i,j,k+1) - E_y^n(i,j,k)}{\Delta z(k)} - \frac{E_z^n(i,j+1,k) - E_z^n(i,j,k)}{\Delta y(j)}\right] \quad (7)$$

$$H_y^{n+1/2}(i,j,k) = H_y^{n-1/2}(i,j,k) + \frac{\Delta t}{\mu_y(i,j,k)} \left[\frac{E_z^n(i+1,j,k) - E_z^n(i,j,k)}{\Delta x(i)} - \frac{E_x^n(i,j,k+1) - E_x^n(i,j,k)}{\Delta z(k)}\right] \quad (8)$$

$$H_z^{n+1/2}(i,j,k) = H_z^{n-1/2}(i,j,k) + \frac{\Delta t}{\mu_z(i,j,k)}$$

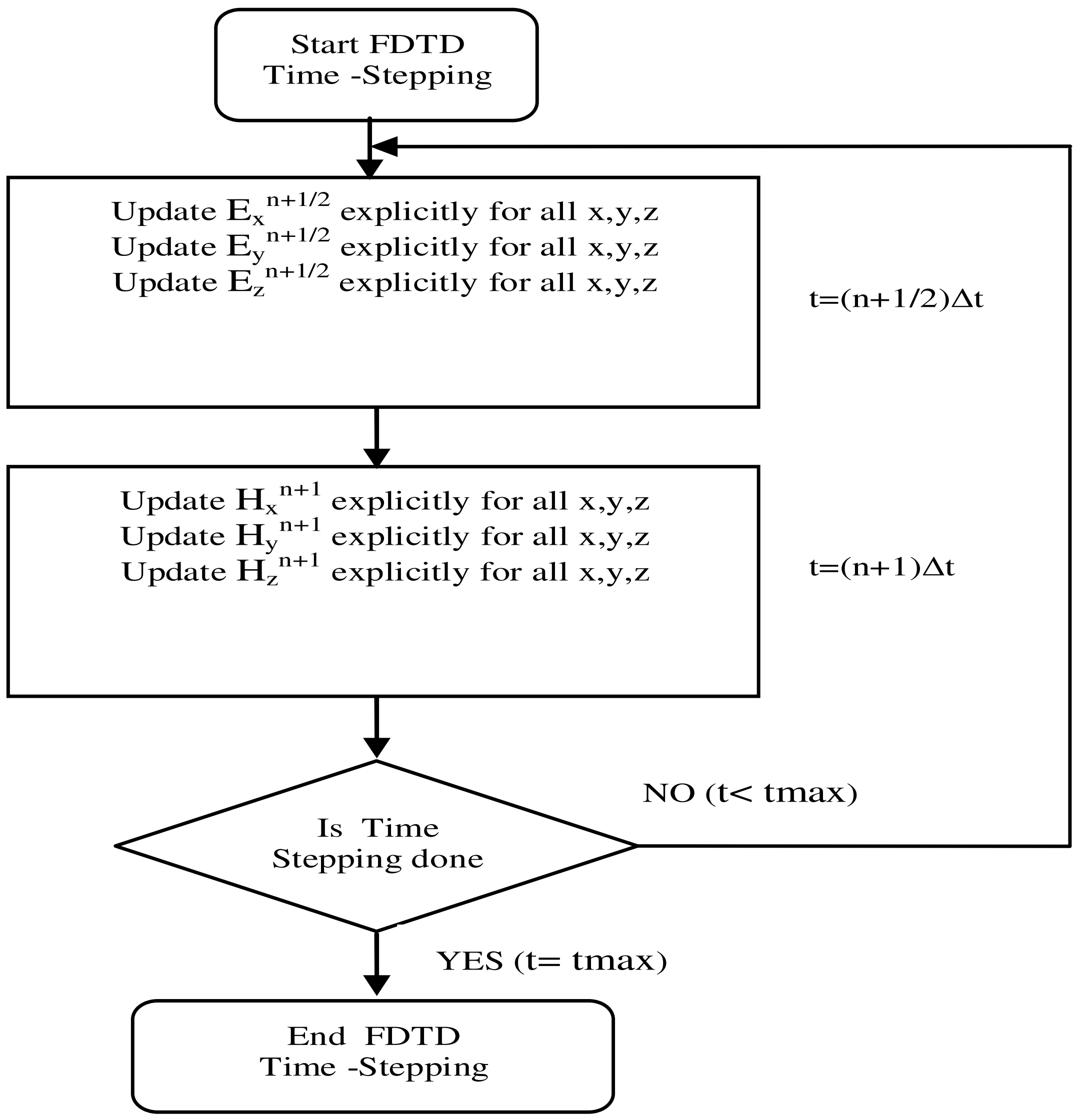

Figure 1. Flowchart for standard FDTD time stepping process.

$$\left[\frac{E_x^n(i,j+1,k)-E_x^n(i,j,k)}{\Delta y(j)} - \frac{E_y^n(i+1,j,k)-E_y^n(i,j,k)}{\Delta x(i)}\right] \quad (9)$$

where the superscripts represent the time index and the arguments correspond to spatial sampling locations. To avoid numerical instabilities, the time increment Δt must satisfy the Courant condition:

$$\Delta t \leq \frac{1}{c\sqrt{\left(\frac{1}{\Delta x}\right)^2+\left(\frac{1}{\Delta y}\right)^2+\left(\frac{1}{\Delta z}\right)^2}} \quad (10)$$

where, Δx, Δy, Δz are the lattice dimensions of the cells. Figure 1 shows the flow chart of the complete FDTD simulation process.

FDTD treats nonlinear behavior naturally (Taflove et al., 2005). Being a time-domain technique, FDTD directly calculates the non-linear response of an electromagnetic system. Recently, computer visualization capabilities have increasing rapidly. This is of particular advantage to FDTD methods, which generate time-marched arrays of field quantities suitable for use in color videos to illustrate the field dynamics (Taflove et al., 2005; Taflove, 2007).

QUADRATIC INTERPOLATION AND EXTRAPOLATION AND HYBRIDIZATION WITH FDTD

Quadratic interpolation and extrapolation are more accurate than linear because the quadratic polynomial $at^2 + bt + c$ can more easily fit curved functions than the linear polynomial $at + b$. Unfortunately, the formulas for quadratic fitting are more difficult to derive. For equally spaced data in t, Taylor's theorem, coupled with the approximations to the first and second derivatives discussed in this section, make it easy to derive and use quadratic interpolation and extrapolation (Spencer et al., 2000). Taylor's theorem says that an approximation to the function f(t) near the point t = a is given by(Spencer et al., 2000)

$$f(t) \approx f(a) + f'(a)(t-a) + \frac{1}{2} f''(a)(t-a)^2 + \ldots \quad (11)$$

In this, the approximation, ignoring all terms, is used beyond the quadratic term in (t – a) near a point (t_n, f_n) in an array of equally spaced t values. The grid spacing in t is dt. An approximation to Taylor's theorem that uses numerical derivatives in this array is then given by

$$f(t) \approx f_n + \frac{f_{n+1} - f_{n-1}}{2dt}(t-t_n) + \frac{f_{n-1} - 2f_n + f_{n+1}}{2dt^2}(t-t_n)^2 \quad (12)$$

This formula is very useful for getting function values that aren't in the array. For instance, it is easy to use this formula to obtain the interpolation approximation to f (t_n + dt/2).

$$f_{n+1/2} = -\frac{1}{8} f_{n-1} + \frac{3}{4} f_n + \frac{3}{8} f_{n+1} \quad (13)$$

and also to find the quadratic extrapolation rule.

$$f_{N+1} = 3f_N - 3f_{N-1} + f_{N-2} \quad (14)$$

Following the curve fitting approximation approach, the modified extrapolation formula is derived as:

$$f_{N+1} = 0.4*(3f_N - 3f_{N-1} + f_{N-2}) \quad (15)$$

As seen from equation 12, equation 13 and equation 14, an interpolation approximation to find function such as $f_{n+1/2}$ and extrapolation such as f_{N+1} are possible from function f_N. Therefore, in the proposed hybridization, the FDTD discretization is used to a limited time steps. Based on the data generated, the QFDTD algorithm is trained and the rest time steps are predicted using the quadratic approximation approach as per equation 12 to equation 15. The extrapolation characteristic of the quadratic approximation is used to predict the function value after running the algorithm up to a certain time step which is in our case is 35. The QFDTD algorithm is shown in Figure 2.

RESULTS

To illustrate the algorithm, an air-filled rectangular cavity resonator is modeled having PEC (perfect electric conductor) boundary condition. The length, width, and height of the cavity are 10.0 cm (x-direction), 4.8 cm (y-direction), and 2.0 cm (z-direction), respectively.

The computational domain is truncated using PEC boundary conditions. The PEC boundaries form the outer lossless walls of the cavity are explained as:

ex (i,j,k) = 0 on the j = 1, j = jb, k = 1, and k = kb planes.
ey (i,j,k) = 0 on the i=1, i = ib, k = 1, and k = kb planes.
ez(i,j,k)=0 on the i=1, i = ib, j = 1 and j = jb planes.

The cavity is excited by a line of current sources oriented along the z-direction and located in the center of the x-y plane. The source waveform is a differentiated Gaussian pulse given by J(t) = J0*(t-t0)*exp(-(t-t0)^2/tau^2), where tau=50 ps. The FWHM spectral bandwidth of this zero-dc-content pulse is approximately 7 GHz. The grid resolution (dx = 0.005 m) is chosen to provide at least 20 samples per wavelength up through 15 GHz.

Speed of light in free space = 2.99792458e8.
Permeability of free space = 4.0*pi*1.0e-7.
Max number of grid cells and Ex samples along x-direction = 50.
Max number of grid cells and Ey samples along y-direction = 24.
Max number of grid cells and Ez samples along z-direction = 10.
Location of z-directed current source = (26, 13, 6).
Number of actual simulation time step = 50.

Extrapolation

The extrapolation has been done in time domain on z-directed electric field values Ez, at 51 grid points in x-direction (since there are 50 grid cells in x-direction so there will be 51 grid points for electric field Ez). Therefore, at each of 50 time steps, there will be 51 values of Ez. Hence, simulated data is saved in 51*50 matrix.

Let V (:,n) be the saved data of nth column.
Most recent three value that is, V(:,n-2), V(:,n-1), V(:,n) are used for extrapolating value V(:,n+1). Following expression obtained from equation 12 is applied for extrapolation:

V(:,n+1)=0.4*(3*V(:,n)-3*V(:,n-1)+V(:,n-2))

The simulation program is executed using traditional FDTD algorithm up to 35 time steps and using the results, the values of electric field at all the grid-points are obtained using quadratic extrapolation for up to 50 time steps, thus saving almost 30% of time. Figure 3 compares the results obtained with traditional FDTD and QFDTD. Excellent agreement is observed.

Simulation time of FDTD: 64.175 s.

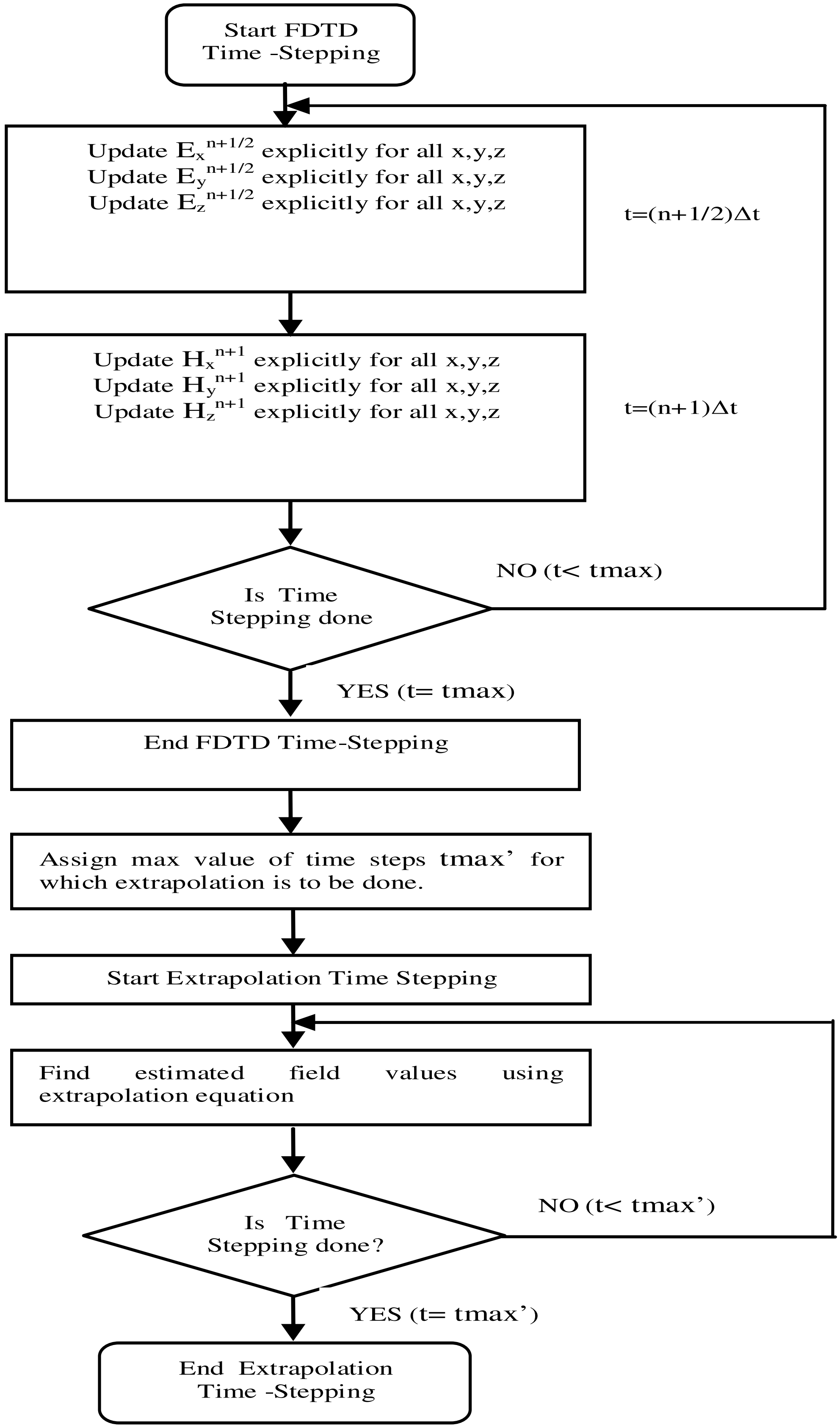

Figure 2. Flowchart for standard FDTD time stepping process.

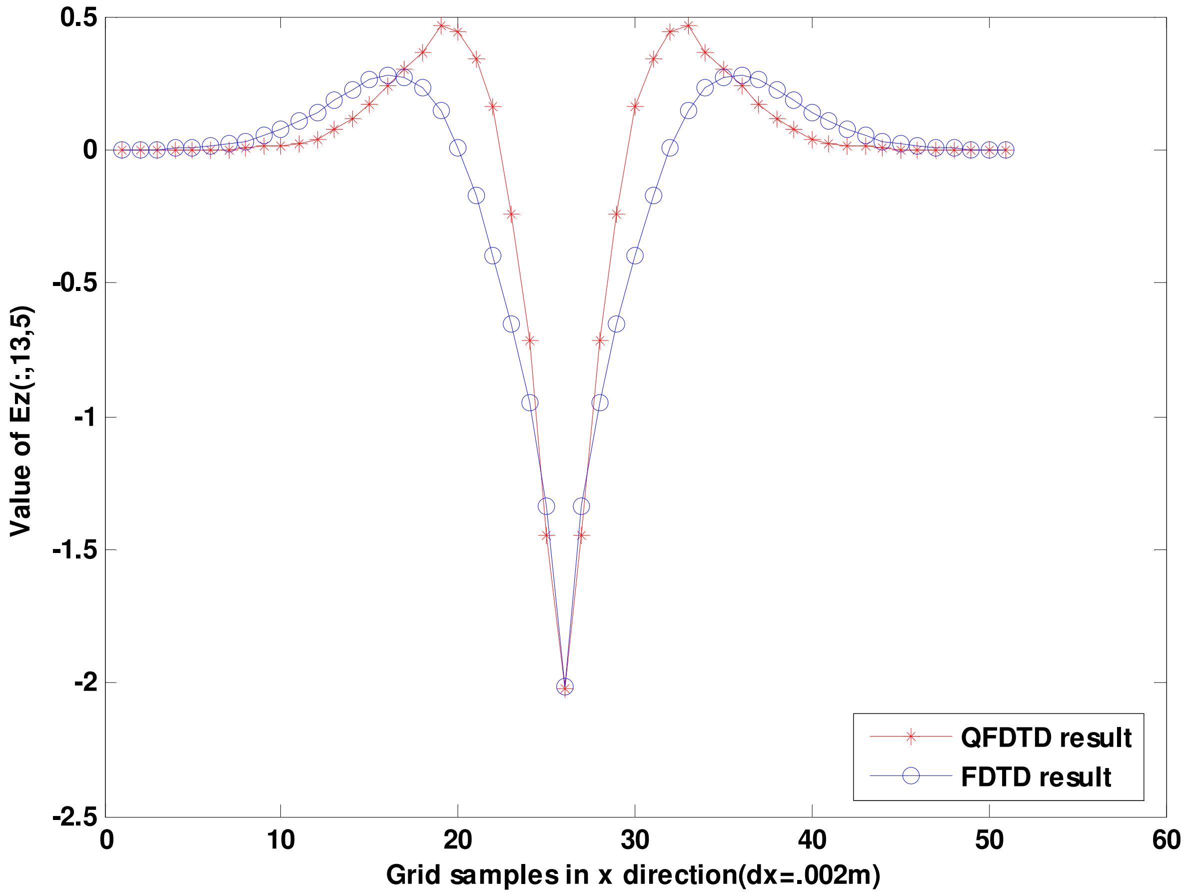

Figure 3. Comparison of electric field (QFDTD and FDTD simulation up to 35 and extrapolated upto 50 time step).

Simulation time of QFDTD: 43.594 s.

The CPU specifications are: Pentium4 processor, 2.4 GHz with 640 MB RAM.

Conclusion

In this paper, a new variant of FDTD named as QFDTD is developed by hybridizing FDTD with quadratic extrapolation. The QFDTD is used for modeling of an air-filled rectangular cavity resonator having PEC (perfect electric conductor) boundary condition. Matlab platform is used to develop extrapolation and then to plot the results. Quadratic extrapolation method is simple, inexpensive and less time consuming than actual FDTD simulation. The proposed QFDTD can be used as a CAD tool for the extrapolation of electric field values for larger number of time steps first using less number of actual simulation time steps. The estimation principal can be extended to more complex applications in computational electromagnetism such as finding radiation patterns of antennas in space, cavity resonator filled with anisotropic materials and SAR study etc.

ACKNOWLEDGEMENT

The authors are thankful to the anonymous reviewers for their suggestions and comments that have helped to improve the quality of the paper.

REFERENCES

Duan X, Yang HW, Liu Han, An L (2009). Auxiliary Differential Equation FDTD Method of Plasma in Parallel Environment, J. Infrared, Millimeter and Terahertz Waves 30(8): 860-867.

Homsup N, Jariyanorawiss T, Homsup W (2009). FDTD Simulation of a Mobile Phone Operating Near a Metal Wall, J. Comput. 4(2): 168-170.

Jin N, Samii YR (2005). Parrallel Particle Swarm Optimization and Finite Difference Time Domain(PSO/FDTD)Algorithm for Multiband and Wide-Band Patch antenna Designs, IEEE Trans. Antennas Propag. 53(11): 3459-3468.

Panda DC, Pattnaik SS, Mishra RK, Bajpai OP (2006). Neuro FDTD to Calculate the Input Impedance of Microstrip Patch Antenna, Antennas and Propagation Society International Symposium pp. 3781-3784.

Sadiku MNO (2001). Numerical Techniques In Electromagnetics, Second Edition, CRC Press, N.W., pp. 159-162.

Shlager KL, Schneider JB (1995). A selective survey of the finite difference time domain literature. IEEE Antennas Propagation

Mag., 37(4): 39-57.
Spencer RL, Ware M (2000). Introduction to MATLAB, Matlab Tutor, Brigham Young University, Utah, 63-69. http://www.physics.byu.edu/Courses/Computational/phys330/matlab.pdf.
Taflove A, Hagness SC (2005). Computational Electrodynamics, Artech House Inc., Norwood pp. 2-16.
Taflove A (2007). A Perspective on the 40-Year History of FDTD Computational Electrodynamics, Applied Comput. Electromagnetic Society J. 22(1): pp. 1-21.
Trefethen LN, Bau D (1997). Numerical LinearAlgebra, SIAM Press, Philadelphia.
Valcarce A, Roche G, De L , Jüttner Á, Pérez Z, Jie DL (2009). Applying FDTD to the Coverage Prediction of WiMAX Femtocells, EURASIP J. Wireless Communications and Networking.. 2009.(308606) pp. 1-13.
Wagner CL, Schneider JB (2003). On the Analysis of Resonators Using Finite Difference Time Domain Techniques, IEEE Trans. Antennas and Propagation 51(10): 2885-2890.
Xiao SQ, Shao Z, Wang BZ (2010). Application of the improved matrix type fdtd method for active antenna analysis," Progress In Electromagnetics Research, PIER 100: 245-263.
Yang Y, Chen RS, Ye ZB, Liu Z (2005). FDTD time series extrapolation by the least squares support vector machine method with the particle swarm optimization technique, Asia-Pacific Conference Proceedings APMC 4: 1-3.
Yee KS (1966). Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media, IEEE Trans. Antennas and Propagation 14: 302-307.