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Review

Fundamentals and literature review of Fourier transform in power quality issues

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Economic impact of power quality (PQ) disturbances on industry is increasing day by day with widespread use of power electronics based devices. These disturbances should be detected quickly and accurately. Fourier transform (FT) has become a powerful tool to detect, classify and analyze various types of PQ disturbances in power systems and signal processing. In this paper, the fundamentals of Fourier series and FT with their types and distinctions over other disturbance detection methods are clearly discussed. An extensive literature review of more than 50 research papers on FT is also presented.

Key words: Power quality disturbances, literature review, harmonics, Fourier transform.

INTRODUCTION

Overview of Fourier series and Fourier transform

Fourier series

Fourier analysis is named after Jean Baptiste Joseph Fourier (1768 to 1830), a French mathematician and physicist. Joseph Fourier, while studying the propagation of heat in the early 1800's, introduced the idea of a harmonic series that can describe any periodic motion regardless of its complexity. Later, the spelling of Fourier analysis gave place to Fourier transform (FT) and many methods derived from FT are proposed by researchers. In general, FT is a mathematical process that relates the measured signal to its frequency content Heideman et al. (1985). The Fourier series is described in theory and problems of advanced calculus as follows:

Let, $f(x)$ is the periodic function. $f(x)$ can be defined in an interval $(-T, T)$ and outside this interval by $f(x+2T)=f(x)$, that is, assume that $f(x)$ has $2T$. The Fourier series or Fourier expansion corresponding to $f(x)$ is given in the following.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{T} + b_n \sin\frac{n\pi x}{T}\right) \quad (1)$$

where a_n and b_n are the coefficients of Fourier series, a_0 is the first term of a_n for n=0.

$$a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos\frac{n\pi x}{T}dx, \qquad 0 \le n \le \infty \quad (2)$$

$$b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\frac{n\pi x}{T}dx, \qquad 1 \le n \le \infty \quad (3)$$

A simple example for Fourier series is given in the following.

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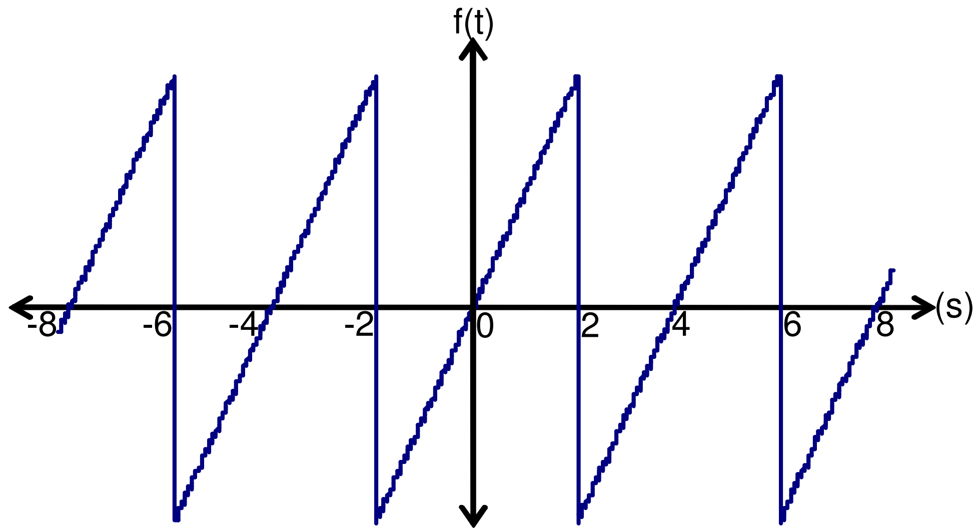

Figure 1. The waveform of a periodic triangular signal.

Example 1: Calculate the Fourier series of a triangular periodic signal given in Figure 1**.** The equation of signal is given as $f(t)=t$ in the interval $-2 \leq t < 2$.

Solution 1: The period (T) of triangular periodic signal is calculated as 4 (T=4), then $\omega = (2\pi/4) = 1/2\pi$ is obtained. The function $f(t)$ is an odd function by considering the situations $f(-t)=(-t)$ and $-f(t)=(-t)$. Because of this situation, the coefficients of a_n are calculated as zero. The coefficients of b_n can be calculated as:

$$b_n = 2\frac{1}{2}\int_0^2 t\sin\frac{n\pi t}{2}dt = -\frac{2}{n\pi}\left[t\cos\frac{n\pi t}{2}\right]_0^2 + \left(-\frac{2}{n\pi}\right)^2\left[\sin\frac{n\pi t}{2}\right]_0^2$$
$$= -\frac{2}{n\pi}(2\cos n\pi - 0) = -\frac{4}{n\pi}\cos n\pi$$

The coefficient values of b_n are calculated for the first five terms of positive integers given in the following.

$$b_1 = \frac{4}{\pi}, \quad b_2 = -\frac{4}{2\pi}, \quad b_3 = \frac{4}{3\pi}, \quad b_4 = -\frac{4}{4\pi}, \quad b_5 = \frac{4}{5\pi}$$

The last statement of $f(t)$ is written by using Fourier explanation in the series form and it is given in the following.

$$f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t = b_1\sin\omega t + b_2\sin 2\omega t + b_3\sin 3\omega t + b_4\sin 4\omega t + b_5\sin 5\omega t + ...$$
$$= \frac{4}{\pi}\left(\frac{1}{1}\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \frac{1}{4}\sin 4\omega t + \frac{1}{5}\sin 5\omega t + ...\right)$$
$$= \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin n\omega t$$

Fourier transform

The Fourier transform is the one of the several mathematical tools for analyzing the signals. It involves the decomposition of the signals in the frequency-domain in terms of sinusoidal or cosinusoidal components. The mathematical definition of a continuous FT (CFT) is given in the following.

$$X(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \qquad (4)$$

where $x(t)$ is the original signal, $X(\omega)$ is the representation of signal in the frequency-domain, j is the imaginary number, ω is the angular frequency and t is the time index.

The inverse of the continuous Fourier transform (ICFT) is defined as;

$$x(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}dt \qquad (5)$$

A simple example for Fourier transform is given in the following.

Example 2: Calculate the continuous Fourier transform of a single rectangular pulse given in Figure 2. The equation of signal is given as $x(t) = \begin{cases} 1, & \frac{-\tau}{2} < t < \frac{\tau}{2} \\ 0, & elsewhere \end{cases}$ and it is calculated by regarding the width of τ seconds.

Solution 2: The Fourier transform ($X(w)$) of $x(t)$ is

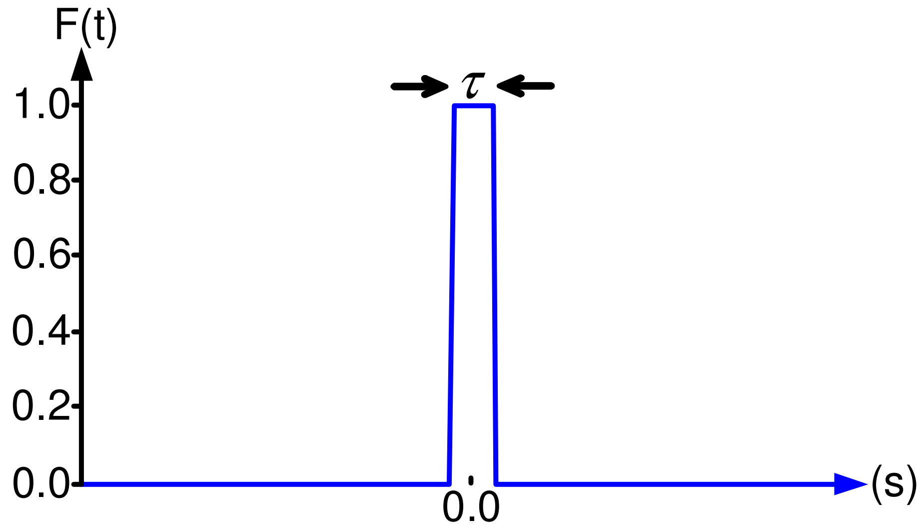

Figure 2. A single rectangular pulse.

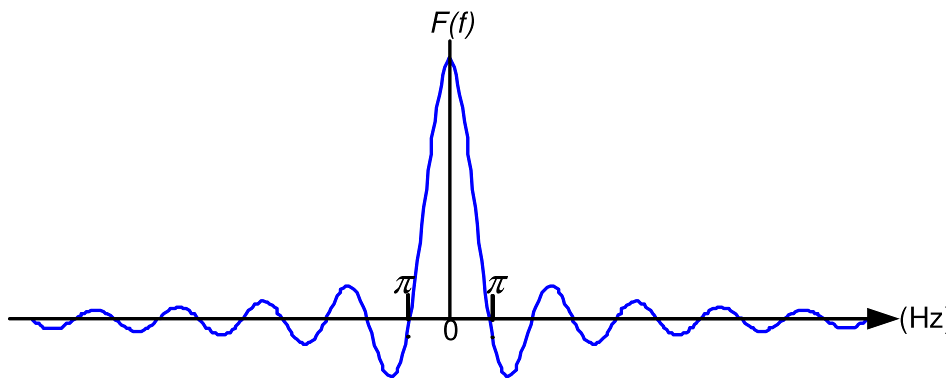

Figure 3. Fourier transform of a rectangular pulse.

calculated as:

$$X(\omega)=\int_{-\tau/2}^{\tau/2} e^{-j\omega t}\,dt=\frac{-1}{j\omega}\left[e^{-j\omega t}\right]_{-\tau/2}^{\tau/2}=\frac{1}{j\omega}\left(e^{j\omega\tau/2}-e^{-j\omega\tau/2}\right)$$
$$=\frac{2}{\omega}\sin(\omega\tau/2)=\frac{2\tau}{\omega\tau}\sin(\omega\tau/2)=\tau\frac{\sin(\omega\tau/2)}{\omega\tau/2}$$

The function $\frac{\sin(\omega\tau/2)}{\omega\tau/2}$ is named as *sinc* function by the mathematicians. In addition, the mathematical equation of *sinc* function can be rewritten as $X(\omega)=\tau\,\mathrm{sinc}(\omega\tau/2)$. Then, Fourier transform of the rectangular pulse can be given in Figure 3.

DISCRETE FOURIER TRANSFORM

The continuous-time periodic signals and finite-energy non-periodic signals can be defined with Fourier series representation. In addition, the discrete-time signals can be represented within finite duration in practice. An alternative transformation is called Discrete Fourier Transform (DFT) for a finite-length signal, which is discretized in frequency.

The frequency range of discrete-time signals is defined over the interval between –π to π. A periodic digital signal consisted of *N* samples is to separate the frequency components into 2π/*N* radians intervals by dividing the frequency-domain. Then, Fourier series representation of the discrete-time signal will consist of *N* frequency components Kuo et al. (2001). The general Fourier series representation of a periodic signal ($x(n)$) is expressed as:

$$x(n) = \sum_{k=0}^{N-1} c_k e^{jk(2\pi/N)n} \tag{6}$$

where N is the harmonic index related with the exponentials function ($e^{jk(2n/N)n}$) for k=0, 1,…,N-1, c_k is the coefficients of the Fourier series.

The coefficients c_k are calculated as:

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-jk(2\pi/N)n} \tag{7}$$

The coefficients c_k of Fourier series are the form of a periodic sequence of fundamental period N. The time-domain spectrum of a periodic signal can be represented as periodic sequence with DFT. The frequency analysis of discrete-time periodic signals ($sin(nt)$ and $cos(nt)$) involves Fourier transform of the time-domain signal. DFT is defined as multiplication of N samples $x(n)$ with N discrete frequencies. These samples are taken at discrete frequencies (ω_k=2πk/N), where k=0,1...N-1, between 0≤ω≤2π. This means that $X(\omega)$ is evaluated at the successive samples by equally spaced frequencies. $X(\omega)$ is given in the following.

$$X(\omega_k) = \sum_{n=-\infty}^{\infty} x(n) e^{-j(2\pi/N)kn} \tag{8}$$

DFT is a mapping between N samples $x(n)$ of the time-domain into N samples $X(\omega)$ of the frequency-domain. This gives the opportunity to compute DFT of the periodic and the finite-length signals. The frequency-domain spectrum of a periodic sequence can be re-obtained as the periodic signal by using inverse discrete-time Fourier transform (IDFT). IDFT can be defined by using the frequency samples of $X(k)$. It is given in the following.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-kn}, \quad n=0,\ 1,\ ...,\ N\text{-}1 \tag{9}$$

IDFT shows that there is no loss information by transforming the frequency spectrum of $X(k)$ back into the original time sequence of $x(n)$. A simple example for DFT is given in the following.

Example 3: Calculate an efficient algorithm for four samples DFT given in the following form.

$$X(k) = \sum_{n=0}^{3} x(n) W_N^{kn}, \ k = 0,1,2,3$$

Solution 3: The four-point DFT includes 16 complex multiplications. These multiplications can be given in the matrix form.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

The mathematical calculations of four-point DFT can be decreased by using the periodicity property ($W_N^{kn} = W_N^{k(n+N)} = W_N^{(k+N)n}$) and symmetry property ($W_N^{kn+N/2} = -W_N^{kn}$) of DFT. With the periodicity property ($W_4^0 = W_4^4 = 1, \quad W_4^1 = W_4^9 = -j$, $W_4^2 = W_4^6 = -1$ and $W_4^3 = j$), the matrix form of four-point DFT can be rewritten given in the following.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

After the using of periodicity property, the equations given above are also rewritten by using the symmetry property and the last equations can be given in the following form.

$$X(0) = x(0) + x(1) + x(2) + x(3) = \underbrace{[x(0)+x(2)]}_{g1} + \underbrace{[x(1)+x(3)]}_{g2}$$
$$X(1) = x(0) - jx(1) + x(2) + jx(3) = \underbrace{[x(0)-x(2)]}_{h1} - j\underbrace{[x(1)-x(3)]}_{h2}$$
$$X(2) = x(0) - x(1) + x(2) - x(3) = \underbrace{[x(0)+x(2)]}_{g1} - \underbrace{[x(1)+x(3)]}_{g2}$$
$$X(3) = x(0) + jx(1) - x(2) - jx(3) = \underbrace{[x(0)-x(2)]}_{h1} + j\underbrace{[x(1)-x(3)]}_{h2}$$

Hence, the equations of new algorithm become as follows:

$$\begin{array}{l||l} \underline{Stage\ 1} & \underline{Stage\ 2} \\ g_1 = x(0) + x(2) & X(0) = g_1 + g_2 \\ g_2 = x(1) + x(3) & X(1) = h_1 - jh_2 \\ h_1 = x(0) - x(2) & X(0) = g_1 - g_2 \\ h_2 = x(1) - x(3) & X(1) = h_1 + jh_2 \end{array}$$

The new algorithm includes only the two complex multiplications. The simple structure of this algorithm is illustrated as graphically in Figure 4.

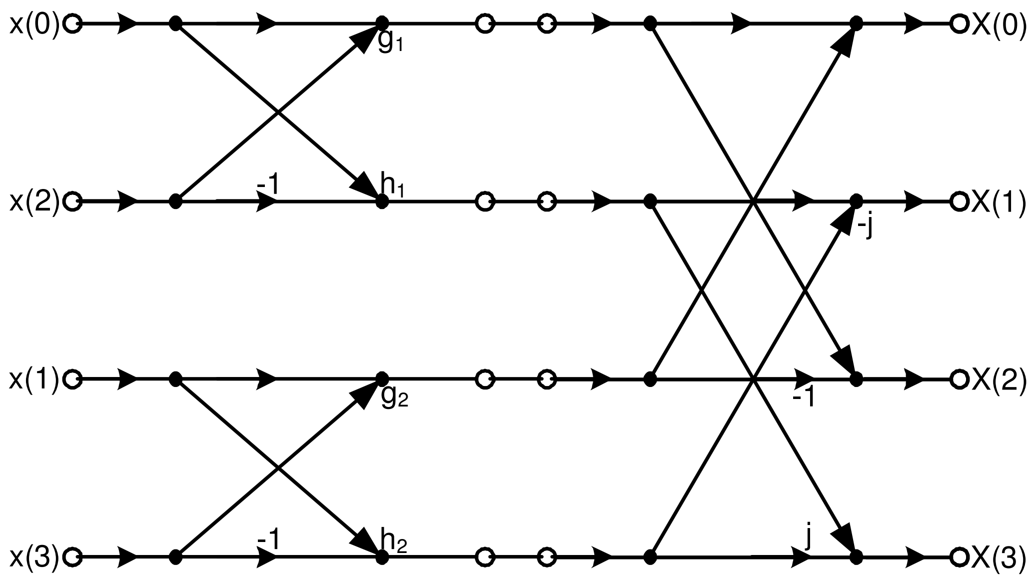

Figure 4. DFT algorithm for four samples.

Fast Fourier transform

Fast Fourier transform (FFT) is an efficient algorithm to compute the discrete Fourier transform and its inverse. A DFT decomposes a sequence of values to the components of different frequencies. This operation is useful in many fields but computing it directly from the definition is often too slow to be practical. In other words, DFT is a native way to compute the Fourier transform of *N* samples by using the general definition. FFT is a fast way to compute the same result with DFT. DFT takes N^2 complex multiplications and $N^{2-}N$ complex additions are required operations, while FFT makes it possible to calculate DFT with $N\log_2 N$ operations instead of N^2 operations (Wikipedia, 2013).

The most common FFT algorithm is the Cooley–Tukey algorithm (Cooley and Tukey, 1965). This algorithm divides the transform into two pieces of size N/2 at each step. Therefore, it is limited to power-of-two sizes. These are called radix-2 and mixed-radix cases, respectively. In other words, Cooley–Tukey algorithm breaks DFT into smaller DFTs. It can be combined arbitrarily with any other algorithm for the DFT.

The two FFT algorithms are proposed.

(i) Decimation in time (DIT)
(ii) Decimation in frequency (DIF)

In DIT and DIF algorithms, the input time sequence is successively divided into smaller sequences and DFTs of these subsequences are combined in a certain pattern to yield the required DFT of the entire sequence with fewer operations. Since this algorithm was derived by separating the time-domain and frequency-domain sequences successively into the smaller sets, the resulting algorithm is referred to as decimation in time algorithm (Kuo et al., 2001).

i. Radix-2 Decimation in Time

Firstly, DIT computes Fourier transform of even-indexed numbers *x*(*0*), *x*(*2*), ...,*x*(*n-2*) and odd-indexed numbers *x*(*1*), *x*(*3*),...,*x*(*n-1*). The sets of even and odd sequences can be defined as:

$$x_1(m) = x(2m),\ m = 0,1,...,(N/2)-1 \tag{11}$$

$$x_2(m) = x(2m+1),\ m = 0,1,...,(N/2)-1 \tag{12}$$

These two results are combined to produce the Fourier transform of the whole sequence. Then, this idea can be performed recursively to reduce the overall runtime. This simplified form assumes that *N* is a power of two, since the number of *N* samples can be usually chosen freely by the application. This is often not an important restriction (Sundararajan, 2001). The name "butterfly" comes from the shape of the data-flow diagram in the radix-2 case. A basic butterfly operation of DIT algorithm required only twiddle-factor multiplies per stage is given in Figure 5.

Each butterfly involves just a single complex multiplication by a twiddle factor W_N^k. The twiddle factor converts *N* points DFT into an *N*/2-points DFTs. In other words, *N* points DFT sequence are divided into *N*/2 point subsequences, *X*(2*k*), *X*(2*k*+1) where *k* = 0,1,...,*N*/2-1.

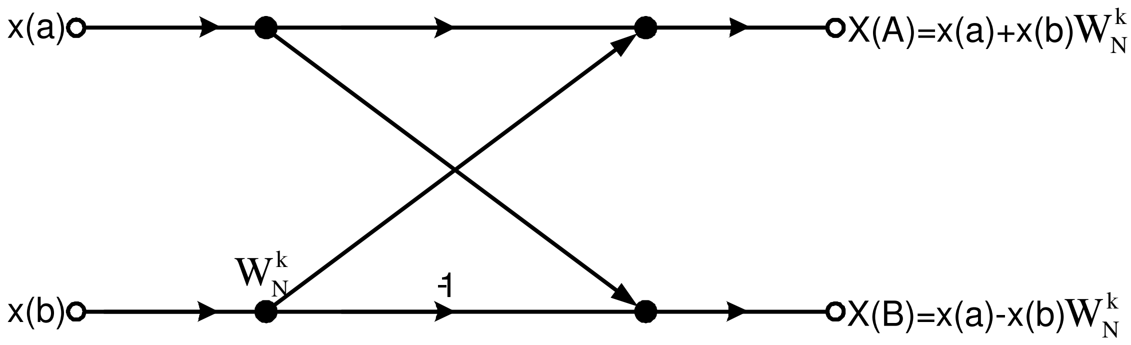

Figure 5. Twiddle-factor of 2-point DIT algorithm of FFT.

ii. Radix-2 decimation in frequency

A second variant of the radix-2 FFT is the decimation-in-frequency algorithm. In order to derive this algorithm, the input sequence is divided into the first and second halves of *N*/2 samples. The process of decomposition is continued until the last stage is made up of two-point DFTs. The decomposition proceeds from left to right for the decimation-in-frequency algorithm and the symmetry relationships are reversed from the decimation-in-time algorithm Mertins (1996). A basic butterfly operation of DIF algorithm is given in Figure 6.

A simple example for DIF algorithm of FFT is given in the following.

Example 4: Calculate the fast Fourier transform of discrete signal *x*(*n*) by regarding the decimation in time algorithm. It is given in the following.

$$x(n)=\{1,3,0,2,4,1,0,2\}$$

Solution 4: The sequence of *x*(*n*) is divided into subsequences included the odd (*o*) and even (*e*) index terms until at least a group had two terms left. The division of signal is given in the following.

1st division: $x_o(n)=\{1,0,4,0\} \qquad x_e(n)=\{3,2,1,2\}$

2nd division: $x_{oo}(n)=\{1,4\} \quad x_{oe}(n)=\{0,0\} \quad x_{eo}(n)=\{3,1\} \quad x_{ee}(n)=\{2,2\}$

In the first step, 2-Radix FFT of subsequences is taken by using the definition of FFT.

$$\text{For,}\ X(k)=\sum_{n=0}^{2-1}x(n)W_N^{kn}=\sum_{n=0}^{1}x(n)e^{-j(2\pi/N)kn}, \quad k=0,1,...,N-1$$

$$X_{oo}(0)=\sum_{n=0}^{1}x_{oo}(n)e^{-j(2\pi/8)0n}$$

$$=x_{oo}(0)+x_{oo}(1)=1+4=5$$

$$X_{oo}(1)=\sum_{n=0}^{1}x_{oe}(n)e^{-j(2\pi/8)1n}$$

$$=x_{oo}(0)e^{-j(2\pi/8)0n}+x_{oo}(1)e^{-j(2\pi/8)1n}=1+4(-1-0j)=-3,\ \text{then}$$

$$X_{oo}(k)=[5\ \ -3];\ X_{oe}(k)=[0\ \ 0]$$

$$X_{eo}(k)=[4\ \ 2];\ X_{ee}(k)=[4\ \ 0]$$

The second step is the 4-Radix FFT of other subsequences calculated by using DIT algorithm. The equations of 4-Radix FFT are used to calculate the odd terms of *X*(*k*) and the results are given in the following.

$$\left.\begin{aligned} X_o(k)&=X_{oo}(k)+W_4^kX_{oe}(k), \quad k=0,1\\ X_o(k+N/2)&=X_{oo}(k)-W_4^kX_{oe}(k), \quad k=0,1\end{aligned}\right\}\text{for odd terms, then}$$

$$X_o(k)=[5\ \ -3\ \ 5\ \ -3]$$

The equations of 4-Radix FFT are used to calculate the even terms of *X*(*k*) and the results are given in the following.

$$\left.\begin{aligned} X_e(k)&=X_{eo}(k)+W_4^kX_{ee}(k), \quad k=0,1\\ X_e(k+N/2)&=X_{eo}(k)-W_4^kX_{ee}(k), \quad k=0,1\end{aligned}\right\}\text{for even terms, then}$$

$$X_e(k)=[8\ \ 2\ \ 0\ \ 2]$$

The third step is the calculation of 8-Radix FFT for other subsequences. The equations of 8-Radix FFT are used to calculate FFT by utilizing the odd and even terms of *X*(*k*). The results of 8-Radix FFT are given in the following.

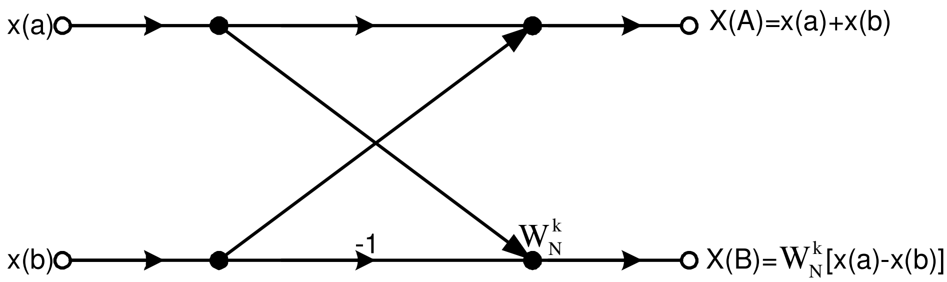

Figure 6. Twiddle-factor of 2-point DIF algorithm of FFT.

For, $X(k) = X_o(k) + W_4^k X_e(k), \quad k = 0,1,2,3$ and $X(k + N/2) = X_o(k) - W_4^k X_e(k), \quad k = 0,1,2,3,$ then

$$X(k) = [13 \quad (-1.5858 - j1.412) \quad 5 \quad (-4.4142 - j1.4142) \quad -3 \quad (-4.4142 - j1.4142) \quad 5 \quad (-1.5858 - j1.412)]$$

Inverse fast Fourier transform

The inverse FFT (IFFT) is calculated by using the equations based on FFT with simple operations (Kuo et al., 2001; Thede, 2004). Basically, FFT algorithm is used to compute IFFT of a sequence *X*(*k*). In the first step, the complex conjugating (*X*(*k*)*) of *X*(*k*) is used in FFT algorithm and then the results are divided by *N* (the length of FFT) to obtain $x^*(k)$. In the second step, FFT algorithm is also used to compute the inverse DFT by utilizing the complex conjugating of $x^*(k)$ for obtaining the sequence of *x*(*n*). If the signal is real-valued, the final conjugation operation is not required. The mathematical expression of IFFT is given in the following.

$$x^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X^*(k)W_N^{kn}, \quad n=0,\ 1,\ ...,\ N\text{-}1 \qquad (13)$$

All steps of FFT algorithms are based on two-input and two-output butterfly computations. These steps are classified as radix-2 complex FFT algorithms. Different radix butterflies can be combined with radix FFT algorithms.

Advantages and disadvantages of FFT

The advantages and disadvantages of FFT are summarized as given in the following.

(i) FFT is the easiest way to calculate the Fourier transform.
(ii) The fundamental harmonic and its multiply of whole number harmonics (except inter harmonics) are extracted easily by FFT.
(iii) FFT algorithm is insufficient to extract the inter-harmonics and variable-frequency harmonics of a signal. New FFT algorithms should be improved to calculate the inter-harmonics and variable-frequency harmonics.
(iv) To prevent the aliasing, the anti-aliasing filters are used before the sampling of the signal. It is resulted with the delaying at the response time of FFT.
(v) FFT can be calculated the harmonics after the one period of the signal left. Consequently, harmonics cannot be extracted simultaneous with reference signal.
(vi) To calculate the higher harmonics, the signal should be sampled at higher frequency.

LITERATURE REVIEW

Methods dependent on DFT and FFT

The references given in the following are related with the DFT and FFT based control algorithms in power quality issues.

A method for the calculation of bus voltage transients in an electric power system is presented (Heydt, 1989). The essence of the method is the Fourier transform of Ohm's law. The fast Fourier transform is used in order to give computational efficiency. Two approximations are found for the calculation of this transfer impedance and one of these is found to be applicable to the cited problem. Examples are used to illustrate the calculation of bus voltage transients and harmonic content. The Gibbs phenomenon appears in a discrete Fourier transform due to incomplete periodic, the waveform has not reached a full cycle within its period. Data flipping furnishes a complete periodic cycle to the waveform and thus suppresses the Gibbs phenomenon. This facilitates the design of a digital filter using fast Fourier transform without windowing. The filter does low-pass, band-pass, high-pass, band-stop, notch or single-frequency-pass simply by manipulating the band limits. The filter can be affected by spectral resolution and the slope discontinuity at the end data points.

The reduction of such effects and an alternative design are discussed (Pan, 1993). The evaluation of DFT spectra usually yields more detailed information than evaluation in the time domain is presented (Breitenbach, 1999). The evaluation of time-discrete spectra, hampered by leakage, which occurs if a non-integral number of periods, is present in the sampled data set. Minimization of spectral leakage is an important prerequisite for spectrum analysis. The use of non-rectangular time-domain windowing functions offers some improvement but also invites unwanted side-effects. Spectral leakage can be avoided entirely by ensuring that an integer number of periods falls into the sampling time (e.g. by the use of coherent sampling). The application of the windowed fast Fourier transform to electric power quality assessment is presented (Heydt et al., 1999). The windowed FFT is a time windowed version of the discrete time Fourier transform. The window width may be adjusted and shifted to scan through large volumes of power quality data. Narrow window widths are used for detailed analyses and wide window widths are used to move rapidly across archived power quality data measurements. The mathematics of the method is discussed and applications are illustrated.

A method for power signal harmonic analysis is proposed for the frequency and phasor estimating algorithm (Yang et al., 2005). The major components of the method are a frequency and phasor estimating algorithm, a finite-impulse-response comb filter and a correction factor. It also combines other methods to enhance our performance, such as discrete Fourier transform and least square error method. To verify proposed method, it compares FFT. An advantages and practical achievements of the frequency-domain qualification of dynamic properties based on the step response data obtained both experimentally and analytically are pointed out (Matyas et al., 2005). Measurement errors of the impulse peak value and time parameters have been evaluated via FFT, convolution and IFFT for a commercial divider. Direct determination of the high voltage impulse measurement range, no need for any additional parameters and illustrative graphical presentation of results are the main advantages of this approach. Limitations of different qualification approaches are discussed.

The problem of differentiating inrush currents from fault currents that are observed for a feeder at a distribution substation is discussed Baran et al. 2006. This paper shows that the two approaches the Fourier transform and the Wavelet transform can be adopted to extract features that make it possible to distinguish them from each other by using an artificial-neural-network-based classifier. The paper also illustrates how to address the issues for successful implementation of these schemes; such as prescreening of data, how to apply FFT and wavelet transform on the data and the training of the artificial neural network in order to maximize the performance of the classifier. These issues are illustrated using the actual field data. An approach to simplify the design of IFFT/FFT cores for OFDM applications is presented (Cortes et al., 2006). A novel software tool is proposed, called AFORE. It is able to generate efficient single and multiple mode IFFT/FFT processors. AFORE employs a parallel architecture, where the degree of parallelism can be varied. In order to assess the quality of the proposed approach, results are provided for some of the most widely used OFDM standards, such as, WLAN 802.11 a/g, WMAN 802.16a, DVB-T.

The sliding DFT algorithm as an alternative for typical DFT used for spectrum analysis and synthesis is proposed Heriakian (2006). As an example, a control circuit for a three-phase 75kVA parallel Active Power Filter (APF) is used. The presented control APF algorithm allows selection of control parameters in mains currents: imbalance, reactive power or harmonics contents. In the proposed circuit transient performance of APF is improved using non-causal predictive current compensation. An improved FFT-based algorithm to measure harmonics and interharmonics accurately is proposed (Qian et al., 2007). In the proposed algorithm, a frequency-domain interpolation approach is adopted to determine the system fundamental frequency and the inter-polatory polynomial method is applied to reconstruct the sampled time-domain signal; it is followed by using the FFT to calculate the actual harmonic components. The performance of the proposed algorithm is validated by testing the actual measured waveforms. Results are compared with those obtained by directly applying a typical FFT algorithm and by the International Electrotechnical Commission (IEC) grouping method. The uncertainty analysis of the Root-Mean-Square (RMS) value and phase computed from the DFT spectrum of the non-coherently sampled signal using cosine windows are investigated (Novotny et al., 2007). The analysis is focused on investigating the influence of quantization noise.

A Direct Current-Space-Vector Control Scheme is presented for a three-level, neutral-point-clamped voltage source inverter, which is employed as an active power filter (Vodyakho et al., 2008). The proposed method generates the compensation current reference indirectly generating an equivalent ohmic conductance for the fundamental component by means of the APF’s dc-link voltage control. Based on the fast Fourier transform the compensation of the reactive fundamental current and selectable harmonics can be cancelled, confining the operation to only harmonic compensation and thus saving the APF’s apparent power. The sliding discrete Fourier transform is proposed (Sumathi et al., 2008). It splits periodic signals into selected harmonic components, as on-line time functions. Ordinarily, the sampling frequency is equal to the product of the nominal signal frequency and the window width. However, when the signal frequency drifts, to avoid the phase and magnitude errors, the sampling frequency can be adaptively adjusted using the phase-error itself. An integrated phase-locked loop scheme and its parameters like hold-in, pull-in ranges, lock time, steady-state errors are presented in this brief.

A strategy of group-harmonic weighting distribution is proposed for system-wide inter-harmonic evaluation in power systems (Lin, 2008). The proposed algorithm can restore the dispersing spectral leakage energy caused by the FFT and calculate the power distribution proportion around the adjacent frequencies at each harmonic to determine the inter-harmonic frequency. The numerical examples are presented to verify the performance of the proposed algorithm.

A precise and adaptive algorithm for interharmonics detection based on iterative DFT is proposed (Zhang et al., 2008). Practical formulas to calculate the parameters of harmonics and interharmonics in electric power systems are presented. An algorithm for fundamental frequency and harmonic components detection is presented (Lavopa et al., 2009). The technique is based on a real-time implementation of discrete Fourier transform and it allows fast and accurate estimation of fundamental frequency and harmonics of a distorted signal with variable fundamental frequency. The proposed is compared with a Phase Locked Loop algorithm for frequency detection and the synchronous DQ reference method for harmonic detection. A spectral correction-based algorithm for inter-harmonic computation is proposed (Salor, 2009) for especially highly fluctuating fundamental frequency cases in the power system. It has been observed and reported that fluctuating demands of some loads such as arc furnaces or disturbances and subsequent system transients make the fundamental frequency of the power system deviate and this causes non-existing interharmonics to appear in the spectrum due to grid-effect when a standard window length is used for the entire FFT process. The proposed method uses a synthetic waveform produced at the fundamental frequency and amplitude to determine the amount of the leakage due to the grid-effect at each frequency. Then the leakage is subtracted from the original FFT of the signal to correct the frequency spectrum. Both simulative and field data tests have been performed.

The technique of pre-calculation process for real-time FFT is presented by Yena et al. (2009). The real-time FFT algorithm simultaneously constructs and computes the butterfly modules while the incoming data is collected. The depending on the computing capability of the processor, different number of pre-calculation stages for better performance is also suggested. An algorithm for the estimation of the frequency of single-tone signals is presented by Radil et al. (2009). The algorithm works in the frequency domain and is based on best fitting a theoretical spectrum of a single-tone signal that is windowed using a rectangular window on the spectrum of the sampled signal. The influence of noise and harmonic and inter-harmonic distortions on the proposed algorithm was investigated and is reported. The algorithm's performance was compared with several other frequency-estimation algorithms (mostly those working in the frequency domain). A space vector (SV) current control scheme for shunt APFs is introduced with a three-level neutral point-clamped voltage source inverter (VSI) as well as a standard two-level VSI (Vodyakho et al., 2009). The proposed control can selectively choose harmonic current components by using a real-time fast Fourier transform to generate the compensation current. The proposed current control utilizes a rotating coordinate system, processing the information of the actual position of the grid-voltage SV, which is remarkably important in APF applications and chooses switching states from the switching table implemented in a field programmable gate array.

The transient performance and stability of a recurrent DFT based control methods for a series active filter integrated with a 12-pulse diode rectifier are considered by Le Roux et al. (2009). The control method targets specific harmonics and/or the negative sequence fundamental component of the supply current and is intended for use with non-sinusoidal/unbalanced supply voltages. The proposed control method is based on DFTs instead of the DQ-method and a simple approach is used to account for small frequency variations found in practical power systems. The three-phase four-wire APF based on a three-level neutral-point-clamped inverter is presented by Vodyakho and Kim (2009). To regulate and balance the split DC-capacitor voltages, a control using the sign cubical hysteresis controller is proposed. The control method discerns the harmonic currents by fast Fourier transform choosing switching states from a switching table based on the hysteresis control. An analysis for simultaneous sensing of multiple primary user activity in cognitive radios has been presented from a signal-processing perspective by Sheikh et al. (2009). It has been shown that a poly-phase DFT filter bank with an optimal window as prototype filter can lead to reliable as well as efficient sensing and detection architecture in cognitive radios with minimal overhead. A detailed account of challenges, performance limits and computational complexity comparisons for various spectrum-sensing solutions is also provided.

An analysis of the natural voltage balancing dynamics of a three-phase flying capacitor converter is presented when supplying an induction motor (McGrath et al., 2009). The approach substitutes double Fourier harmonic series for the pulse-width modulation switching waveforms and the frequency response of the motor, to create a linear state-space model of this type of load. Model parameters were measured by applying FFT analysis to variable frequency square waves injected into the motor terminals and fitting parameter values to these measurements using a least squares minimization method. Experimental verification results using a scaleddown flying capacitor converter drive are included. A partial cached FFT processor that accounts for the distribution of allocated resources to the users of the OFDMA system is designed by Chen et al. (2010). The constellation- and power-aware twiddle-factor multiplier for the variable FFT length and modulation order are designed. A 128 to 1024-point mixed pipelined/cached-FFT processor using a 0.18-μm 1P6M CMOS technology is implemented. The chip measurement results show that its energy dissipation ranges from 0.09 to 1.90nJ per FFT point and scales to the allocated resources in the OFDMA system.

A space-vector discrete-time Fourier transform for fast and precise detection of the fundamental-frequency and harmonic positive- and negative sequence vector components of three-phase input signals is proposed by De Souza et al. (2010). A recursive algorithm for low-effort online implementation is also presented. The detection performance for variable-frequency and inter-harmonic input signals is discussed. A closed-form analytical approximation of the output harmonic spectrum of a single-phase two-level inverter under the action of hysteresis current control is derived by Albanna et al. (2010). The analytical approach consists of first describing the error current as a triangular signal of variable duty cycle and frequency and subsequently, deriving the Fourier transform of the complex envelope of the modulated triangular signal. The spectrum of the error current is given in terms of Bessel functions of the first kind. An estimating the amplitude of harmonic components of a harmonically distorted sine wave by the Interpolated Discrete Fourier Transform (IPDFT) method with maximum side lobe decay windows is proposed by Belega et al. (2010). In addition, for a signal corrupted by stationary white noise, the statistical efficiency of the IPDFT method is investigated with respect to the single-tone unbiased Cramér–Rao Lower Bound. Finally, the performance of the IPDFT method is compared with that of the energy-based method on the basis of theoretical, simulation and experimental results and with that of a state-of-the-art method according to simulation and experimental results. An application of Fourier transform approach in modulation technique of experimental studies is considered by Khazimullin et al. (2010). The analysis has been confirmed by simulations and measurements of a quartz wedge birefringence by means of the photo elastic modulator. The obtained bias, noise level and measuring speed are comparable and even better than in lock-in amplifier technique.

The harmonic components of a carrier-based Pulse Width-Modulated (PWM) voltage source converter (VSC) output voltage are theoretically identified when the modulating wave includes fundamental and baseband harmonic components (Odavic et al., 2010). The analysis is based on a double Fourier series expansion in two variables. This approach to harmonic identification is evaluated by comparison with a fast Fourier transform analysis of simulated PWM waveforms. The Reference Signal Generator (RSG) for voltage-source converters (VSCs) that enables the maximization of its functionality is presented by Borisov et al. (2010). The proposed RSG is based on a combination of the Fortescue decomposition with recursive discrete Fourier transform. The method is characterized by computational efficiency, excellent detection accuracy and fast dynamic response. An experimental prototype of a multifunctional VSC with the proposed RSG has been built and experimental results are presented.

The feasibility and convenience of applying the efficient DFT techniques are analyzed for the computation of reference currents in APFs (Ortega et al., 2010). The two different implementations of the running DFT, including their recursive and non-recursive versions, are compared to the SRF approach in terms of steady-state performance, adequacy of transient response and computational effort. In addition to simulation results, an experimental setup is designed to prove the advantages of resorting to the DFT.

PWM of a sampled signal produces harmonic distortion is analyzed and proposed expressions are obtained for the Fourier transform of the modulated output, which are valid for any input signal (Colodro et al., 2010). In addition, simple expressions are given to compute the harmonic distortion factors for a sinusoidal input signal. Finally, these analytical results are validated by simulation for two data converters based on sigma–delta modulation, where the sampled output of a multibit-quantizer is pulse width modulated: one digital-to-analog and one analog-to-digital converter, respectively. The basics of two techniques, named here as the modified harmonic domain and the modified dynamic harmonic domain, proposed for calculating steady and dynamic states are presented, respectively (Ramirez, 2011). These techniques have their fundament in the harmonic domain with a substantial improvement: the inclusion of interharmonics in either steady or dynamic state. This is performed through the use of the discrete Fourier transform which allows an arbitrary frequency-domain discretization, thus permitting the representation of interharmonics.

The Cosine Self-Convolution Window (CSCW) is proposed by Zeng et al. (2011). The main-lobe and side-lobe behaviors of the first to the third order CSCWs are studied. A CSCW-based improved fast FFT for estimating power system signal parameters, such as frequency, phase and amplitude, is given. The effectiveness of the proposed method was analyzed by means of computer simulations and practical experiments for multi-frequency signals with the variations of the power system frequency as well as the presence of white noise and interharmonics. A method for transmission line protection using a variable window short-time Fourier is proposed by Samantaray et al. (2008). The fault detection, the impedance to the fault point is calculated using the estimated phasors of the faulted current and voltage signals which provide accurate results even with noisy conditions. The fault location is calculated using polynomial curve fitting technique with a devised index obtained from the ratio of spectral energy of the voltage and current signals, respectively.

Methods dependent on wavelet transform and FFT

The references given in the following are related with the Wavelet Transform and FFT based control algorithms in power quality issues. The comparison of Wavelet Transform and FFT is also investigated.

An algorithm based on the wavelet-packet transform is proposed for the analysis of harmonics in power systems (Barros et al., 2008). The paper studies the selection of the mother wavelet, the sampling frequency and the frequency characteristics of the wavelet filter bank for the two most common wavelet functions used for harmonic analysis and compares the performance of the proposed method with the results obtained using the DFT analysis and the harmonic-group concept introduced by the IEC under different measurement conditions. A modulation technique is based on a newly designed scaling function that is capable of supporting a non-uniform recurrent sampling process by Saleh et al. (2009). This scaling function generates sets of basic functions that span spaces, of which a collection constructs a non-dyadic-type multi-resolution analysis. Furthermore, the newly designed scaling function has a dual synthesis scaling function that is designed to reconstruct continuous-time signals from their non-uniform recurrent samples. Several performance tests are conducted for the proposed wavelet-modulated inverter, when supplying linear, nonlinear, static and dynamic loads.

A discrete wavelet transform is used as a conditioning tool to filter the motor current prior to its processing by the fractional FT (Sanchez et al., 2010). Experimental results that are obtained with a 1.1-kW three-phase squirrel-cage induction motor with broken bars are presented to validate the proposed method. A method of estimation of components above the harmonic frequency range up to 9 kHz by wavelet filtering is proposed by Tarasiuk (2011). The proposed method consists of the application of FFT for wavelet coefficients after input signal decomposition and partial synthesis for chosen frequency bands. The algorithm and its implementation in real device for power quality monitoring are presented. A method for the classification of the power system disturbances using support vector machines (SVMs) is proposed by Erişti et al. (2010). The input vector is started with the first best feature and incrementally added the chosen features. After the addition of each feature, the performance of the SVM is evaluated. The kernel and penalty parameters of the SVM are determined by cross-validation. The parameter set that gives the smallest misclassification error is retained. Both the noisy and noiseless signals are applied to the classifier.

Methods dependent on neural networks

The references given in the following are related with the Artificial Neural Network based control algorithms in power quality issues. The comparisons of Artificial Neural Network with FFT and Wavelet Transform are also investigated.

A strategy to estimate harmonic distortion from an AC line is presented for power electronic converters (Sekaran et al., 2008). An adaptive linear neural network (ADALINE) is used to determine precisely the necessary currents in order to cancel harmonics. The proposed strategy is based on an original decomposition of the measured currents to specify the neural network inputs. This new decomposition is based on the Fourier series analysis of the current signals and Least Mean Square training algorithm carries out the weights. The proposed strategy also allows extracting the harmonics individually. The method is based on the extraction of fundamental components of distorted line current using an ADALINE network. The output of the ADALINE is compared with distorted supply current to construct modulating signals and to generate. An alternative method based on artificial neural networks is presented to determine harmonic components in the load current of a single-phase electric power system with nonlinear loads, whose parameters can vary so much in reason of the loads characteristic behaviors as because of the human intervention (Oliveira et al., 2010). The effectiveness of this method is verified by using it in a single-phase active power filter with selective compensation of the current drained by an AC controller. The proposed method is compared with the fast Fourier transform. The radial-basis-function neural network is proposed to detect the harmonic amplitudes of the measured signal (Chang et al., 2010). The proposed method is compared with several commonly used methods (FFT, Wavelet ...etc.).

A real-time classification method of PQ disturbances is proposed by Zhanga et al. (2011). Five distinguished time frequency statistical features of PQ disturbances are extracted using RMS method and DFT. The proposed method is tested using the nine types simulated waveforms of PQ disturbances including voltage sag, swell, interruption, harmonic, notch, flicker, oscillatory transient, sag with harmonics and swell with harmonics. It compared with common solutions which are usually based on wavelet transform. The application of a Complex Adaptive Linear Neural Network (CADALINE) in tracking the fundamental power system frequency is proposed Sadinezhada et al. (2009). In this method, by using stationary-axes Park transformation in addition to producing a complex input measurement, the decaying DC offset is eliminated. This paper concludes with the presentation of the representative results obtained in numerical simulation and simulation in PSCAD/EMTDC software as well as in practical study.

Methods dependent on fuzzy logic

A fast fuzzy-logic-supported anytime frequency range-estimation procedure is proposed, which makes it possible to execute the frequency estimation after one quarter of the period of the unknown signal, that is, the adaptation and Fourier analysis can be performed without any delay (Varkonyi et al., 2009). The design of a tool to quantify power quality (PQ) parameters using wavelets and fuzzy sets theory is proposed by Mehera et al. (2010). The tool merges the best characteristics of these two theories in establishing a method to analyze PQ events. The proposed method addresses two issues, such as selection of discriminative features and classifies event classes with minimum error. Wavelet features (WF) of PQ events are extracted using wavelet transform (WT) and fuzzy classifiers classify events using these features. Often the captured signals are corrupted by noise. Varieties of PQ events including voltage sag, swell, momentary interruption, notch, oscillatory transient and spikes are considered for performance analysis.

Methods dependent on other methods

Software sampling frequency adaptive algorithm that can obtain the actual signal frequency more accurately is presented and then adjusts sampling interval base on the frequency calculated by software algorithm and modifies sampling frequency adaptively (Pan et al., 2006). It can reduce synchronous error and impact of spectral leakage. This algorithm has high precision just like the simulations show and it can be practical methods in power system harmonic analysis since it can be implemented easily. A new modification of the least squares Prony's method for Prony's method for power-quality analysis in terms of estimation of harmonics and interharmonics in an electric power signal is proposed by Zygarlicki et al. (2010). The so-called reduced Prony's method can be competitive, in some specific case, to the Fourier transformation method and the classical LS Prony's method. The modification constitutes in a specific selection of a constant frequency vector in a Fourier-like manner leading to a remarkable reduction of the computational burden and enabling online real-time computations.

An efficient procedure that includes a high-resolution Prony-based method in conjunction with the down sampling technique for harmonics and interharmonics detection of the measured power signal is proposed by Chang and Chen (2010). The performance of the proposed method is validated by testing the simulated and actual measured power signals. Results are compared with those obtained by fast Fourier transform with and without synchronization, IEC sub-grouping method and other commonly used linear prediction approaches adopted in the Prony's method. An approach for the construction of a family of desired order continuous polynomial time window functions is presented without self-convolution of the parent window (Singla et al., 2010).

Published review papers on FFT

The higher order of continuity of the time window functions at the boundary of the observation window helps in suppressing the spectral leakage. Closed-form expressions for window functions in the time domain and their corresponding Fourier transform are derived. Several commonly used methods for time-varying harmonic and interharmonic detection of measured waveforms are reviewed and implemented in an integrated virtual instrumentation (Chen and Chang, 2010). Compared from the aspect of frequency identification for the reviewed methods, general guidelines for performing harmonic and inter-harmonic detection are also developed for the educational purpose. Major continuous-time, discrete-time and discrete Fourier related transforms as well as Fourier-related series are discussed both with real and complex kernels (Ersoy, 1994). The complex Fourier transforms, Fourier series, cosine, sine, Hartley, Mellin, Laplace transforms and Z-transforms are covered on a comparative basis. The short-time Fourier-related transforms are discussed for applications involving non-stationary signals. The one-dimensional transforms discussed are also extended to the two-dimensional transforms. Since fast algorithms for the DFT were first introduced thirty years ago, they have had a major impact on signal processing and are now a basic part of every electrical engineer's education. However, some of the options and particularly the recent advances are not as widely known as they deserve.

The fast algorithms for the DFT are reviewed by Blair (1995a). It considers why the DFT works and looks at the various fast algorithms for transforms whose orders are a power of two. The techniques for other orders, at adapting algorithms for purely real data and at the problems for fixed-point noise are discussed by Blair (1995b). The second of two which review the fast algorithms for the DFT, looks at algorithms for transforms whose orders are not a power of two. Also discussed are ways of adapting algorithms for purely real data, the problems of fixed-point noise and implementation options with existing hardware.

CONCLUSION

Fourier transform can be used to detect, classify and

Methods of Published Articles	Publication Years										Total
	1989	1993	1999	2005	2006	2007	2008	2009	2010	2011	
Fourier Transform (DFT, Recursive DFT, FFT,…, etc.)	1	1	2	2	2	3	5	9	9	2	36
FFT versus Wavelet Transform	-	-	-	-	1	-	1	1	2	1	6
FFT versus Neural Networks	-	-	-	-	-	-	1	-	2	2	5
FFT versus Fuzzy Logic	-	-	-	-	-	-	-	1	1	-	2
Other Methods (PLL, Prony's Method, Continous Polynomial)	-	-	-	1	-	-	-	1	3	-	5

Figure 7. Selected methods in the reviewed articles and publishing years.

analyze PQ disturbances with certain accuracy. An exhaustive review of Fourier transform in power quality issues is performed to provide a clear understanding on their applications. Most power quality analyzers also use FFT-based algorithm to identify the harmonics of the measured signals.

This paper presents the fundamentals of Fourier series, Fourier transform, discrete Fourier transform and fast Fourier transform with simple examples and review of Fourier transform to provide a clear understanding of its applications in power quality issues. Most of the selected articles were published in Scientific Index, Science Citation Index (SCI) or SCI extended between 2007 and 2011. It is concluded with the review of published papers on power quality disturbances that most of the selected methods are included, either Fourier transform or methods based on Fourier transform. The other methods are wavelet transform, neural networks and FUZZY logic, respectively. The selected methods in the reviewed articles and publishing years are given as shown in Figure 7.

In the literature, there are few review studies with published articles. Most of these studies are related to the explanation of DFT or FFT transforms. The presented paper is the first review study included both detail examination of published articles related to power quality in recent years and classification of these studies (Sarıbulut, 2012). The result of the reviewed published papers on FFT shows that Fourier transform can be used for many power quality issues detection and restoration. In addition, it can be used in image processing tasks, filtration, disturbance detection, modulation and demodulation.

REFERENCES

Albanna AZ, Hatziadoniu CJ (2010). Harmonic modeling of hysteresis inverters in frequency domain. IEEE T. Power. Electron. 25(5):1110-1114.

Baran ME, Jinsang K (2006). A classifier for distribution feeder over current analysis. IEEE T. Power Deliv. 21(1):456-462.

Barros J, Diego RI (2008). Analysis of harmonics in power systems using the wavelet-packet transform. IEEE T. Instrum. Meas. 57(1):63-69.

Belega D, Dallet D, Slepicka D (2010). Accurate amplitude estimation of harmonic components of incoherently sampled signals in the frequency domain. IEEE T. Instrum. Meas. 59(5):1158-1166.

Blair GM (1995a). A review of the discrete Fourier transform. 1. Manipulating the powers of two. Electron. Comm. Eng. J. 7(4):169-177.

Blair GM (1995b). A review of the discrete Fourier transform. 2. Non-radix algorithms, real transforms and noise. Electron. Comm. Eng. J. 7(5):187-194.

Borisov K, Ginn HL (2010). Multifunctional VSC based on a novel fortescue reference signal generator. IEEE T. Ind. Electr. 57(3):1002-1007.

Breitenbach A (1999). Against spectral leakage. Measurement 25(2):135-142.

Chang GW, Chen CI (2010). An accurate time-domain procedure for harmonics and interharmonics detection. IEEE T. Power Deliv. 25(3):1787-1795.

Chang GW, Chen CI, Teng YF (2010). Radial-basis-function-based neural network for harmonic detection. IEEE T. Ind. Electron. 57(6):2171-2179.

Chen CI, Chang GW (2010). Virtual instrumentation and educational platform for time-varying harmonic and interharmonic detection. IEEE T. Ind. Electron. 57(10):3334-3342.

Chen CM, Hung CC, Huang YH (2010). An energy-efficient partial FFT processor for the ofdma communication system. IEEE T. Circ. Syst. 57(2):136-140.

Colodro F, Torralba A (2010). Spectral analysis of pulse width-modulated sampled signals. IEEE T. Circuits II: Express Briefs 57(3):622-626.

Cooley JW, Tukey JW (1965). An algorithm for the machine calculation of complex Fourier series. Math. Comput. 19:297-301.

Cortes A, Velez I, Sevillano JF, Irizar A (2006). An approach to simplify the design of IFFT/FFT cores for ofdm systems. IEEE T. Consum. Electron. 52(1):26-32.

De Souza FAS, Bradaschia HEP, Cavalcanti F, Rizo MC, Rodriguez M, Neves FJ (2010). A space-vector discrete Fourier transform for unbalanced and distorted three-phase signals. IEEE T. Ind. Electron. 57(8):2858-2867.

Erişti H, Uçar A, Demir Y (2010). Wavelet-based feature extraction and selection for classification of power system disturbances using support vector machines. Elect. Power Syst. Res. 80(7):743-752.

Ersoy OK (1994). A comparative review of real and complex Fourier-related transforms. Proc. IEEE 82(3):429-447.

Heideman MT, Johnson DH, Burrus CS (1985). Gauss and the history of the fast Fourier transform, Archive for History of Exact Sciences. Arch. Hist. Exact Sci. 34(3):265-277.

Heriakian I (2006). Harmonic compensation using the sliding DFT algorithm for three-phase active power filter. Electr. Power. Qual. Utili. J. 12(2):15-20.

Heydt GT (1989). A new method for the calculation of subtransmission and distribution system transients based on the FFT. IEEE Power Eng. Rev. 9(7):73.

Heydt GT, Fjeld PS, Liu CC, Pierce D, Tu L, Hensley G (1999). Applications of the windowed FFT to electric power quality assessment. IEEE T. Power Deliv. 14(4):1411-1416.

Khazimullin MV, Lebedev YA (2010). Fourier transform approach in modulation technique of experimental measurements. Rev. Sci. Instr. 81(4):1-8.

Kuo SM, Lee BH (2001). Real-time digital signal processing implementations applications and experiments with the TMS320C55x. John Wiley & Sons, Inc.

Lavopa E, Zanchetta P, Sumner M, Cupertino F (2009). Real-time estimation of fundamental frequency and harmonics for active shunt power filters in aircraft electrical systems. IEEE T. Ind. Electron. 56(8):2875-2884.

Le Roux AD, Mouton H, Akagi H (2009). DFT-based repetitive control of a series active filter integrated with a 12-pulse diode rectifier. IEEE T. Power Electron. 24(6):1515-1521.

Lin HC (2008). Inter-harmonic identification using group-harmonic weighting approach based on the FFT. IEEE T. Power Electron. 23(3):1309-1319.

Matyas Z, Aro M (2005). HV impulse measuring systems analysis and qualification by estimation of measurement errors via FFT, convolution and IFFT. IEEE T. Instrum. Meas. 54(5):2013-2019.

McGrath BP, Holmes DG (2009). Natural capacitor voltage balancing for a flying capacitor converter induction motor drive. IEEE T. Power Electron. 24(6):1554-1561.

Mehera SK, Pradhanb AK (2010). Fuzzy classifiers for power quality events analysis. Elect. Power Syst. Res. 80(1):71-76.

Mertins A (1996). Signal analysis wavelets, filter banks, time-frequency transforms and applications. John Wiley & Sons, Inc.

Novotny M, Slepicka D, Sedlacek M (2007). Uncertainty analysis of the RMS value and phase in the frequency domain by non-coherent sampling. IEEE T. Instrum. Meas. 56(3):983-989.

Odavic M, Sumner M, Zanchetta P, Clare JC (2010). A theoretical analysis of the harmonic content of PWM waveforms for multiple-frequency modulators. IEEE T. Power Electr. 25(1):131-141.

Oliveira CF, Goedtel AA, Silva A, Serni IN, Nascimento PJA (2010). Neural network-based approach for identification of the harmonic content of a nonlinear load in a single-phase system. IEEE Latin Am. T. 8(1):65-73.

Ortega JMM, Macias JAR, Exposito AG, Mannozzi SC, Villarejo MB (2010). Reference current computation for active power filters by running DFT techniques. IEEE T. Power Deliv. 25(3):1986-1995.

Pan C (1993). Design of a windowless digital filter using FFT algorithm. IEEE Trans. Acoust. Speech Sig. Proc., pp. 6221.

Pan LD, Wang F (2006). A software sampling frequency adaptive algorithm for reducing spectral leakage. Frontiers Electr. Electron. Eng. China 1(4):375-379.

Qian H, Zhao R, Chen T (2007). Measuring power system harmonics and interharmonics by an improved fast Fourier transform-based algorithm. IEEE T. Power. Deliv. 2(2):1064-1069.

Radil T, Ramos PM, Serra AC (2009). New spectrum leakage correction algorithm for frequency estimation of power system signals. IEEE T. Instrum. Meas. 58(5):1670-1679.

Ramirez A (2011). The modified harmonic domain: interharmonics. IEEE T. Power Deliv. 26(1):235-241.

Sadinezhada I, Joorabian M (2009). A novel frequency tracking method based on complex adaptive linear neural network state vector in power systems. Elect. Power Syst. Res. 79(8):1216-1225.

Saleh SA, Moloney CR, Rahman MA (2009). Development and testing of wavelet modulation for single-phase inverters. IEEE T. Ind. Electron. 56(7):2588-2599.

Salor Ö (2009). Spectral correction-based method for interharmonics analysis of power signals with fundamental frequency deviation. Electr. Power Syst. Res. 79(7):1025-1031.

Samantaray SR, Dash PK (2008). Transmission line distance relaying using a variable window short-time Fourier transform. Elect. Power Syst. Res. 78(4):595-604.

Sanchez MP, Guasp MR, Daviu JAA, Folch JR, Cruz JP, Panadero RP (2010). Diagnosis of induction motor faults in the fractional Fourier domain. IEEE T. Instrum. Meas. 59(8):2065-2075.

Sarıbulut L (2012). Detection and mitigation methods of power quality disturbances. PhD Thesis, Çukurova University, Institute of Natural and Applied Science.

Sekaran EC, Anbalagan P (2008). Comparison of neural network and fast-Fourier transform based selective harmonic extraction and total harmonic reduction for power electronic converters. Asian Power Electron. J. 2(1):1-9.

Sheikh F, Masud S, Bing B (2009). Harmonic power detection in wideband cognitive radios. IET Sig. Proc., 3(1):40-50.

Singla P, Singh T (2010). Desired order continuous polynomial time window functions for harmonic analysis. IEEE T. Instrum. Meas. 59(9):2475-2481.

Sumathi P, Janakiraman PA (2008). Integrated phase-locking scheme for SDFT-based harmonic analysis of periodic signals. IEEE T. Circuits II: Express Briefs 55(1):51-55.

Sundararajan D (2001). The discrete Fourier transform: theory, algorithms and applications. World Scientific Publishing Co. Pte. Ltd.

Tarasiuk T (2011). Method, algorithm and device for estimation of components above the harmonic frequency range up to 9 kHz. Measurement 44(1):219-229.

Thede L (2004). Practical analog and digital filter design. Artech House Inc.

Varkonyi KAR (2009). Fast anytime fuzzy-Fourier estimation of multisine signals. IEEE T. Instrum. Meas. 58(5):1763-1770.

Vodyakho O, Hackstein D, Steimel A, Kim T (2008). Novel direct current-space-vector control for shunt active power filters based on the three-level inverter. IEEE T. Power Electron. 23(4):1668-1678.

Vodyakho O, Kim T (2009). Shunt active filter based on three-level inverter for three-phase four-wire systems. IET Power Electron. 2(3):216-226.

Vodyakho O, Kim T, Kwak S, Edrington CS (2009). Comparison of the space vector current controls for shunt active power filters. IET Power Electron. 2(6):653-664.

Wikipedia (2013). (http://en.wikipedia.org/wiki/Fast_Fourier_transform)

Yang JZ, Yu CS, Liu CW (2005). A new method for power signal harmonic analysis. IEEE T. Power Deliv. 20(2):1235-1239.

Yena WF, Shingchern DY, Changc YC (2009). Real-time FFT with pre-calculation. Comput. Electr. Eng., 35(3):435-440.

Zeng B, Teng Z (2011). Parameter estimation of power system signals based on cosine self-convolution window with desirable side-lobe behavior. IEEE T. Power Deliv. 26(1):250-257.

Zhang Q, Liu H., Chen H, Li Q, Zhang Z (2008). A precise and adaptive algorithm for interharmonics measurement based on iterative DFT. IEEE T. Power. Deliv. 23(4):1728-1735.

Zhanga M, Lib K, Hub Y (2011). A real-time classification method of power quality disturbances. Electric Power Syst. Res. 81(2):660-666.

Zygarlicki J, Zygarlicka M, Mroczka J, Latawiec KJ (2010). A reduced prony's method in power-quality analysis-parameters selection. IEEE T. Power Deliv. 25(2):979-986.