

Full Length Research Paper

Modelling extreme volatility in the daily exchange rates of the Kenya shilling against the U.S. dollarⁱ

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This study used extreme value theory to establish if the volatility witnessed in the exchange rate of the Kenya Shilling against the U.S. dollar in the period 1999 to 2013 could have been predicted and also determine if the long-term stability in the exchange rate was violated in the period. The peak over threshold model is applied to the tail of the volatility process of exchange rate returns of the Kenya Shilling against the U.S. dollar. The results showed that despite episodes of extreme volatility, the long-term stability of the exchange rate was maintained during the period. However, implementation of policies that will increase and sustain the level of foreign exchange inflows into the country is necessary to mitigate the vulnerability of the exchange rate to external and domestic shocks. Specifically, policies to promote the export sector and those to increase the level of foreign exchange reserves held by the Central Bank of Kenya should be encouraged. The comparably extreme volatility witnessed in the period 2008 to 2010 showed that political stability is a key component of foreign exchange market stability in Kenya.

Key words: Volatility, Extreme Value Theory, Peaks over Threshold model, GARCH model

INTRODUCTION

Extreme value theory (EVT) attracted considerable attention after volatility was witnessed in the financial markets during and after the global financial crisis in 2008/2009. Like other currencies in the East Africa region, the exchange rate of the Kenya Shilling against the U.S. dollar witnessed significant volatility in 2011 that was attributable to various factors including a wide current account deficit due to a high import bill, (Central Bank of

Kenya, 2012). The oil import bill, which peaked at 25 percent of total imports in 2011, exerted pressure on the current account deficit leading to exchange rate depreciation. The current account deficit widened from 4.6 percent of GDPⁱⁱ in 2009 to 7.9 percent in 2011 and remained high thereafter, averaging 9.2 percent in 2012 and 2013 (Kenya National Bureau of Statistics, 2014). The Eurozone crisis exacerbated pressure on the

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exchange rate with the U.S. dollar strengthening as the preferred reserve currency. Consequently, the maximum exchange rate of the Kenya Shilling against the U.S. dollar reached a historical level of 105.96 in 2011 (appendix 1). The annual standard deviation of 6.18 for the exchange rate in 2011 was also the highest since 1999.

Extreme volatility in exchange rates creates uncertainty about future returns and can affect long-term investment decisions of companies involved in international trade. Exchange rate volatility can stifle international trade through excessive import and export price variability. The uncertainty created by exchange rate volatility can prompt firms to add a risk premium to prices of internationally traded goods thereby reducing the demand for such goods and slowing down economic growth (Becketti et al. 1989). Exchange rate volatility can also alter international capital flows. It can promote short-term and speculative capital flows which can hinder the effectiveness of monetary policy. This can be costly as the Central Bank resorts to drawdown on its reserves to intervene in the foreign exchange market in order to stabilise the exchange rate. In this regard, the Central Bank of Kenya (CBK) sold a significant amount of foreign exchange in 2008 and 2009 to dampen extreme volatility in the exchange rate (Central Bank of Kenya, 2009). Extreme exchange rate volatility therefore raises concerns on the effectiveness of the Central Bank in performing its core mandate of maintaining price stability. Consequently, the long-term economic growth envisaged in Kenya's *Vision 2030* development plan is predicated on macroeconomic stability (Republic of Kenya, 2007).

Various studies have analyzed the extreme movements in the financial markets with respect to currency crises, stock market crashes and large credit defaults in recent times. The peaks over threshold (POT) model of EVT have been advocated in these studies (Embrechts *et al.* 1997). Given the episodes of extreme volatility such as that in 2011, this study applies the POT model to the volatility process of the daily exchange rate returns of the Kenya Shilling against the U.S. dollar using daily data from January 1999 to December 2013. The volatility process is generated from the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model introduced by Bollerslav (1986). The GARCH model is robust in modelling the volatility in financial data characterised by volatility clustering and heteroscedasticity. In order to describe the extreme volatility, the POT model is applied to the tail of the volatility process.

A comparative analysis of the extent of volatility in selected periods of the study period is undertaken to establish periods of extreme exchange rate volatility. The trend in the occurrence times and excesses of the extremes of the volatility process is then analysed. The return period of extreme exchange rate returns was computed to establish the approximate recurrence interval

of specific extreme observations. The main contribution of this study to the existing literature is the use of both the GARCH and EVT in the analysis of exchange rate volatility in Kenya.

The rest of the paper is divided into four sections. Section 2 provides the methodology adopted in the study; Section 3 describes the data used in the analyses; Section 4 reports the empirical results and Section 5 concludes the paper and provides policy recommendations.

LITERATURE REVIEW

The ability of macroeconomic models to predict volatility has been examined in literature. Although macroeconomic models have forecasting abilities, the most important factor in these studies was the lagged endogenous variable (Frankel et al., 2008). Recent studies on modelling volatility in financial data therefore focus mainly on time series models.

The Autoregressive Conditional Heteroscedasticity (ARCH) and GARCH models are important tools in describing the volatility in financial data (Engle, 1982; Bollerslav, 1986). The GARCH family models stand out in capturing heteroscedasticity and volatility clustering in financial data. Although comparatively long lags are required in ARCH models, the GARCH (1, 1) is adequate in describing most financial time series (Bollerslav et al., 1992). However, GARCH models have been criticised in that they do not provide a theoretical explanation of volatility or what information flows are in the volatility generating process (Tsay, 2005). The model also responds equally to asymmetric shocks, and cannot cope with significantly skewed time series which results in biased estimates. Other variations of the GARCH model such as Exponential GARCH, Threshold GARCH and Power GARCH have been proposed to address some of these weaknesses (Floros, 2008).

The EVT approach is well established in literature, and provides a strong foundation to build statistical models to characterise extreme events (Resnick et al., 1996; Embrechts et al., 1997; McNeil et al., 2000; Smith, 2003). Andreev et al. 2012 argue that EVT and POT model are robust for estimating measures of tail risk under irregular volatility in market. The models are based on sound statistical theory and allow for extrapolation beyond the range of the data. McNeil et al. (2000) proposed a two stage approach where the GARCH model is fitted to return data, and EVT used to model the tail of the residuals from the estimated GARCH model. The approach, which is similar to the one used in this study, addresses the drawbacks of the previous EVT methods which failed to capture the stochastic volatility exhibited by most financial return data.

The implementation of EVT has various challenges including scarcity of extreme data, determining whether

the series is heavy-tailed, choosing the threshold or beginning of the tail, and choosing the methods of estimating the parameters (Resnick et al., 1996). Various diagnostic tools including QQ-plots, sample mean excess plots, scaled excesses and inter-arrival times, and the Hill plot have been suggested to address these challenges (Embrechts et al., 1997; Smith, 2003). The EVT approach may still be an accurate approximation of the actual distribution function of the extremes even if the independent and identically distributed (*i.i.d.*) assumptions on the data fails (McNeil, 1997).

METHODOLOGY

Generating the exchange rate return volatility process

The GARCH model is specified in line with Bollerslav (1986). Let (Z_n) be a sequence of *i.i.d.* random variables such that $Z_i \sim N(0,1)$. Then, ε_t is the *GARCH*(p, q) process if $\varepsilon_t = \sigma_t Z_t$, $t \in \mathbf{Z}$ with $\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_i \sigma_{t-i}^2$, $t \in \mathbf{Z}$

Where σ_t^2 is a non-negative process, and $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1, \dots, p$ while $\beta_i \geq 0$ for $i = 1, \dots, q$. The non-negativity restrictions on the parameters ensure positivity of the variance σ_t^2 .

The sizes of the parameters α_1 and β_1 determine the short-run dynamics of the resulting volatility process in the *GARCH* (1, 1) model. A large ARCH error coefficient α_1 implies that volatility reacts significantly to market movements. Similarly, a large GARCH coefficient β_1 indicates that volatility is persistent. A high α_1 coefficient relative to β_1 indicate that volatility tends to be more extreme. Bollerslav (1986) reports that a necessary and sufficient condition for the weak stationarity of the GARCH model

is $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j < 1$. However, strict stationarity of the *GARCH* (1,

1) model requires that $E(\log(\alpha_1 Z_t^2 + \beta_1)) < 0$ which allows for $\alpha_1 + \beta_1$ being equal to or slightly above 1 (Nelson, 1990). Stationarity of the GARCH model ensures that the behaviour and properties of the estimators do not change over time and that the persistence of shocks is not infinite. The *GARCH* (1, 1) model is estimated by maximum likelihood estimation (MLE) specifying the density of the error term ε as a generalised error distribution (GED), (Nelson, 1990). The positive shape parameter ν for the GED measures the thickness of the tails. The GED yields the normal distribution for $\nu = 2$, and the Laplace or double Exponential distribution for $\nu = 1$. The density has heavier tails than the normal distribution for $\nu < 2$, and thinner tails for $\nu > 2$. The GARCH effects in the models are examined using correlograms of the squares of the exchange rate returns. Autocorrelations larger than the critical values give evidence of presence of GARCH effects.

Modelling extremes in the volatility process of exchange rate returns

In the POT model, excess sizes and exceedance times of a threshold u are modelled as a two dimensional homogeneous Poisson process (Embrechts et al., 1997). The model is formulated such that, first, the corresponding excesses over the threshold u are independent and distributed as a Generalised Pareto Distribution (GPD). Denoting the threshold by u and the shape parameter by ξ , the conditional distribution of excess values of

X over u converges to the GPD as the threshold gets large. The GPD model is specified as follows:

$$G_{\xi, \beta}(x) = \begin{cases} 1 - \left(1 + \xi \frac{x}{\beta}\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{x}{\beta}} & \text{if } \xi = 0 \end{cases}$$

Where $x \geq 0$ and $\beta > 0$ is the scale parameter, and the estimate of the shape parameter ξ determines the weight of the tail.

Distributions for which $\xi > 0$, Frechet case, are called heavy-tailed and can be used to model large observations while distributions for which $\xi = 0$ are called thin tailed and correspond to all the common continuous distributions of statistics. The QQ-plot and sample mean excess plots can be used to determine the appropriate threshold for the GPD models. The QQ-plot also checks the validity of the distributional assumptions. Plots of the shape estimate for the GPD over a variety of thresholds are also used to reinforce the judgement in choosing the appropriate thresholds using the QQ-plot and sample mean excess plot (Embrechts et al., 1997). In addition, the Hill plot can also be used to find the optimal threshold for the GPD model. The *W*-statistic

$W_i = 1 / \xi \log(1 + \xi(X_i - u) / (\beta + \xi u))$ which refers to scaled excesses is analysed to avoid the subjectivity in the threshold selection using the mean excess over threshold plot (Smith, 2003). If all the assumptions on the GPD model are correct including the selected threshold u and the time span, the W_i 's are independent and exponentially distributed variables with mean 1. The GPD has finite mean for the shape parameter of $0 < \xi < 1$ and finite variance of $0 < \xi < 0.5$. Maximum likelihood regularity conditions are achieved and the maximum likelihood estimates are asymptotically normally distributed. The approximate standard errors for the estimators of β and ξ can therefore be obtained using *i.i.d.* from the Fisher Information Matrix.

Second, excesses of *i.i.d.* observations over a threshold u occur at times of Poisson process. If the exceedances of the threshold occur at times of a homogeneous Poisson process with constant

intensity which is expressed as $\lambda_u = (1 + \xi u / \beta)^{-1/\xi}$, then the scaled inter-arrival times of exceedances given by $Z_k = \lambda \times (T_k - T_{k-1})$, should be *i.i.d.* exponential random variables with mean 1. Where T_k is the time of the k^{th} exceedance and $T_0 = 0$. Finally, excesses and exceedance times are independent of each other. The distribution of the *W* and *Z* values is

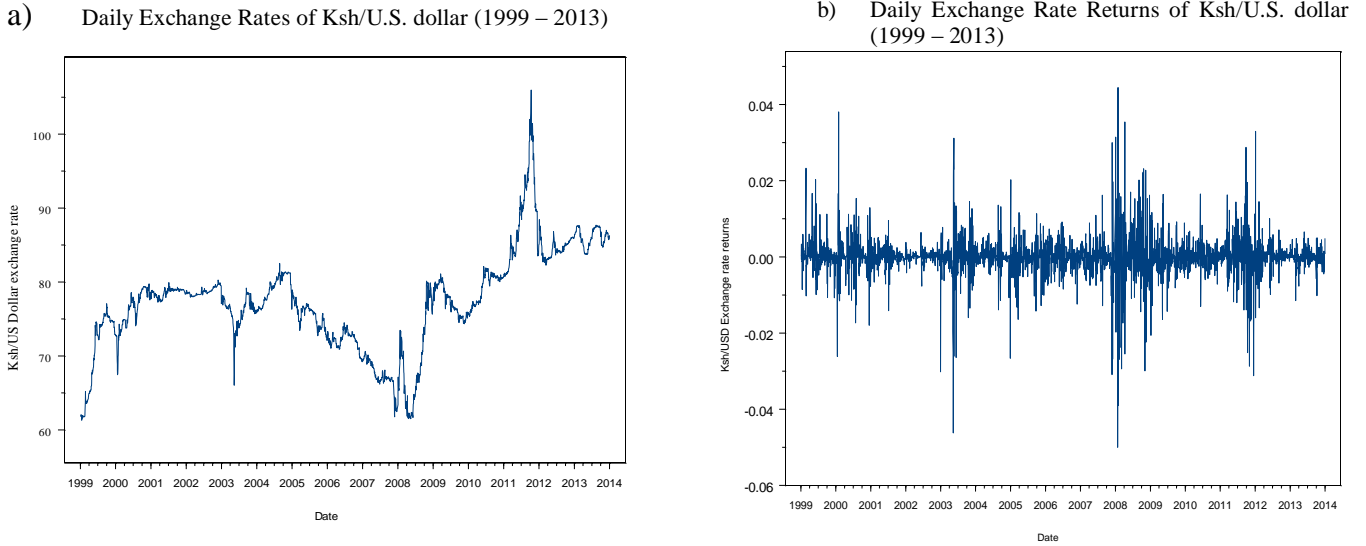


Figure 1. Trends in the daily exchange rate of the Kenya Shilling to the U.S. dollar (left panel), and exchange rate returns (right panel). Source: CBK

only approximately exponential since we do not know the true values of the GPD parameters and these have to be estimated. The diagnostic checks for the fitted POT model are based on Smith (2003). Scatter plots of W_i and Z_k values against the order of occurrence are used to check for identical distribution of these values. Any variation of these values with time would suggest a trend in the model. Smaller Z_k values indicate that exchange rate returns are becoming smaller. Plots of ordered W_i and Z_k values against expected exponential quantiles are used to check the exponential assumption on the distribution of these statistics. Approximate linearity of these plots would suggest that the exponential fit is good. The use of sample correlograms to check for independence of the W - values has a weakness in that the GPD has infinite variance for $\xi \geq 0.5$. Theoretical autocorrelations do not exist for this range of ξ .

Estimation of the parameters of the GPD is by numerical MLE method (Embrechts et al. 1997). The estimates for the parameters ξ and β are obtained by solving the simultaneous equations based on the log-likelihood function as below:

$$\frac{\partial L(\hat{\xi}, \hat{\beta})}{\partial \hat{\xi}} = \frac{1}{\hat{\xi}} \sum_{i=1}^n \log \left(1 + \hat{\xi} \frac{x_i}{\hat{\beta}} \right) - \left(\frac{1}{\hat{\xi}} + 1 \right) \sum_{i=1}^n \left(\frac{x_i}{\hat{\beta} + \hat{\xi} x_i} \right) = 0$$

$$\frac{\partial L(\hat{\xi}, \hat{\beta})}{\partial \hat{\beta}} = -\frac{n}{\hat{\beta}} + \left(\frac{1}{\hat{\xi}} + 1 \right) \sum_{i=1}^n \frac{\hat{\xi} x_i}{\hat{\beta}^2 + \hat{\beta} \hat{\xi} x_i} = 0$$

The return period of specific extreme volatility is determined by computing a quantile x_p at the tail for a given probability p . An estimate of the tail quantile $\hat{x}_p, 0 \leq p \leq 1$ gives the return level associated with the return period $1/p$ (Embrechts et al. 1997).

Data

The exchange rates data used in the study consist of daily data of the Kenya Shilling exchange rates against the U.S. dollar (Ksh/U.S. dollar). The exchange rate was derived by the CBK as an average of buying and selling rates of commercial banks spot exchange rates. The data comprises of 3,251 observations for the period from 4th January, 1999 to 31st December, 2013. The choice of the U.S. dollar was informed by its relative proportion in the CBK’s foreign exchange investment portfolio which comprised over 60 percent in U.S. dollar in December 2013 while the currency composition of imports was about 50 percent in U.S. dollars. The U.S. dollar is also the main reserve currency in the global currency markets. The data was obtained from the CBK website. The exchange rates are transformed and analysed as daily logarithmic changes or exchange rate returns, i.e. $X_t = \log(e_t / e_{t-1})$. The transformation makes prices independent of their unit and therefore comparable with each other (Mikosch, 2001). The plots of the exchange rate returns in Figure 1 reveals a characteristic dependence structure where large and small values tend to occur in clusters. This reflects volatility clustering in the data. The structure was consistent with empirical evidence as in Taylor (1986) in which financial data is heavy tailed, has changing volatility, and exhibits serial dependence.

Extreme volatility in the exchange rate data in the study period was attributed to various episodes. In order to rein in inflation and exchange rate volatility in 2011, the CBK adopted monetary policy responses through increases in the policy rate from 6.25 percent at the beginning of September 2011 to 18 percent in December 2011. The cash reserve ratio (CRR) for banks was also raised from 4.5 percent in May 2011 to 5.25 percent in December 2012. These measures, coupled with other regulatory announcements by the CBK in 2011 resulted in a rapid increase in interest rates. However, exchange rate stability was restored by the end of 2011 (appendix 1). Exchange rate stability in 2012 and 2013 was supported by increased foreign exchange inflows from diaspora remittances and foreign investment in equity through the stock market (Central Bank of Kenya, 2013). Previous episodes of extreme volatility in the exchange rate were witnessed in 2003 and 2008, Figure 1b. A significant amount of liquidity was injected in the banking system following the reduction of the CRR from 10 percent to 6 percent in

Table 1 . Summary statistics for exchange rate returns for various sample periods.

	1999-2013	1999-2007	2008-2010	2011	2012-2013
Mean	0.0000882	0.0000061	0.0003360	0.000206	0.0000289
Median	0.0000000	0.0000000	0.0000486	0.000771	0.0000993
Maximum	0.0445000	0.0380640	0.0444660	0.028788	0.032967
Minimum	-0.0500000	-0.0462420	-0.050001	-0.031210	-0.016004
Standard Deviation	0.0049070	0.0042200	0.006714	0.006881	0.002966
Skewness	-0.1818530	-0.448220	0.029662	-0.714779	2.105694
Kurtosis	22.482270	24.276050	17.05133	7.290145	36.90063
Jarque-Bera Statistic	59,390.01	42,418.70	6,211.24	215.27	24,360.38
Jarque-Bera Probability	0.0000	0.0000	0.0000	0.0000	0.0000
No. of Observations	3754	2245	755	253	501

June 2003. Interest rates declined drastically resulting in short-term capital outflows and consequent weakening of the exchange rate.

However, there were mixed trends as the exchange rate strengthened during the Safaricom Initial Public Offer in mid 2008 and privatisation of Telkom Kenya in 2007 that led to substantial foreign exchange inflows. However, the exchange rate weakened in August 2008 when the IPO started trading at the stock market following capital outflows (Central Bank of Kenya, 2008). The post poll jitters in the market in early 2008 and turbulence in the global economy following the global financial crisis contributed to the weakening of the Kenya Shilling against the U.S. dollar through October 2008 (Figure 1).

EMPIRICAL RESULTS AND DISCUSSION

The data analyses are conducted for the full data sample (1999 to 2013), and also across selected samples (1999 to 2007, 2008 to 2010, 2011, and 2012 to 2013). This facilitated comparison of the extent of extreme volatility across the periods and the impact of various domestic and global events in the selected periods on exchange rate volatility. The most notable events that had an impact on the exchange rate have already been highlighted in the sections 1 and 3. Descriptive statistics for the exchange rate returns X_t across various sample periods are presented in Table 1. The mean for exchange rate returns was higher in the period 2008 to 2010 compared to the other periods. The kurtosis coefficients are positive and higher than 3 across all samples while skewness coefficients are less than zero except for the periods 2008 to 2010 and 2012 to 2013. This indicates that the distributions of the exchange rate returns are not normal. The negative skewness coefficient for the entire sample indicates that the distribution of the returns is left skewed. This implies that appreciations in the exchange rate occurred more often in the study period. However, depreciations occurred more frequently in the periods 2008 to 2010 and 2012 to 2013 as depicted by the negative skewness coefficients.

Kurtosis coefficients are much higher than the 3 for a normal distribution indicating that the underlying distri-

butions of the returns are leptokurtic. The Jarque-Bera tests for normality indicated that the distribution of exchange rate returns has tails which are heavier than that of the normal distribution. The autocorrelation and partial autocorrelation coefficients of the squared exchange rate returns show presence of serial correlation which was an indication of GARCH type of heteroscedasticity (appendix 2).

Estimated volatility models

The parameter estimates and the value of the Akaike Information Criterion (AIC) for the fitted GARCH (1, 1) models for the exchange rate returns are shown in Table 2. The AIC was computed for comparison between the GARCH models for the different samples. All models produced almost similar AIC values across the different sample periods. The estimated GARCH (1, 1) models are significant at 5 percent significance level with a high persistence of shocks in the volatility. The estimated α_1 and β_1 parameters are positive while their sum was slightly above 1 across all the estimated models. A higher estimated GARCH coefficient β_1 shows that volatility was more persistent in 2011 relative to the other periods. However, the estimated ARCH coefficient α_1 is higher in the period 1999 to 2007, an indication that volatility in the exchange rate returns tended to be more extreme compared with that in the period 2008 to 2010, 2011 and 2012 to 2013. The quasi maximum likelihood estimates, corresponding to the estimated GED parameters of the exchange rate returns are highly significant and correspond to distributions with heavier tails than the normal distribution.

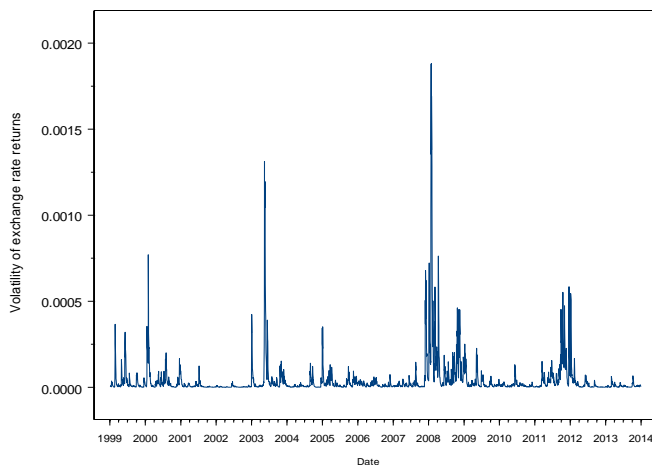
The volatility process of the exchange rate returns based on the full sample (1999 to 2013) model is plotted in Figure 2 (left panel). The volatility was comparably more extreme in the period 2008 to 2010 compared with that in 2011. This was mainly attributed to the uncertainty

Table 1. Estimated GARCH (1, 1) models for exchange rates returns.

<i>Sample</i>	<i>No. of observations</i>	α_0	α_1	β_1	<i>GED Parameter</i>	<i>AIC</i>
1999 – 2013	3754	0.000000166 (5.64189)	0.461117 (10.39986)	0.715223 (42.93854)	0.68939 (42.93854)	-9.0244
1999 – 2007	2245	0.000000116 (3.92816)	0.568871 (7.45706)	0.714094 (32.65388)	0.57697 (34.35977)	-9.3079
2008 – 2010	755	0.000000739 (3.16788)	0.266644 (4.08030)	0.746316 (19.40346)	0.746316 (16.74591)	-8.2190
2011	253	0.000000373 (1.31281)	0.29286 (3.25924)	0.772078 (13.47551)	1.147325 (7.874760)	-7.6103
2012 – 2013	501	0.000000253 (2.49826)	0.48100 (3.74136)	0.61493 (9.18273)	0.80833 (14.0850)	-9.8090

Note: t-statistics are in parentheses.

a) Volatility process of exchange rate returns from GARCH (1, 1) model



b) Standardised residuals of exchange rate returns from the GARCH (1, 1) model

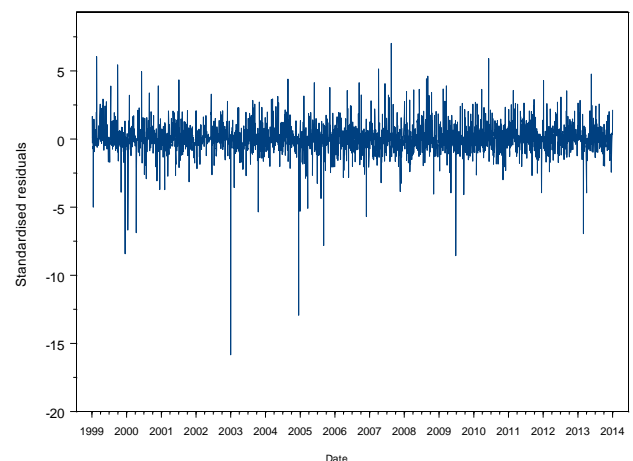


Figure 2. Volatility (conditional variance) process (left panel) and standardised residuals (right panel) of exchange rate returns derived from the GARCH (1, 1) model

in the foreign exchange market caused by the post poll crisis in Kenya, the impact of the global financial crisis and high oil prices which increased the import bill, Central Bank of Kenya (2008). The plot of the standardised residuals from the GARCH model of exchange rate returns (Figure 2 right panel) and the correlogram of squared standardised residuals in appendix 3 show that the model was well specified.

Extreme value models for the volatility of exchange rate returns

Data exploration

We first use the QQ-plot and sample mean excess plot to

determine if the distribution of the volatility process derived from the full sample GARCH (1, 1) model of the exchange rate returns is heavy tailed, and then determine appropriate thresholds above which the volatility can be modelled by a GPD. The QQ-plot in Figure 3 (left panel) shows a concave relationship between the quantiles of the empirical and the exponential distribution which indicates that the distribution of the volatility is fat tailed. This plot, interpreted together with the shape plot in Figure 4 (right panel), shows that the sample points start deviating from linear behaviour and forms a concave shape at around 0.0000773. Similarly, the sample mean excess plot (Figure 3 right panel) for the volatility process is approximately linear and positively sloped from the above threshold which indicates heavy tailed behaviour.

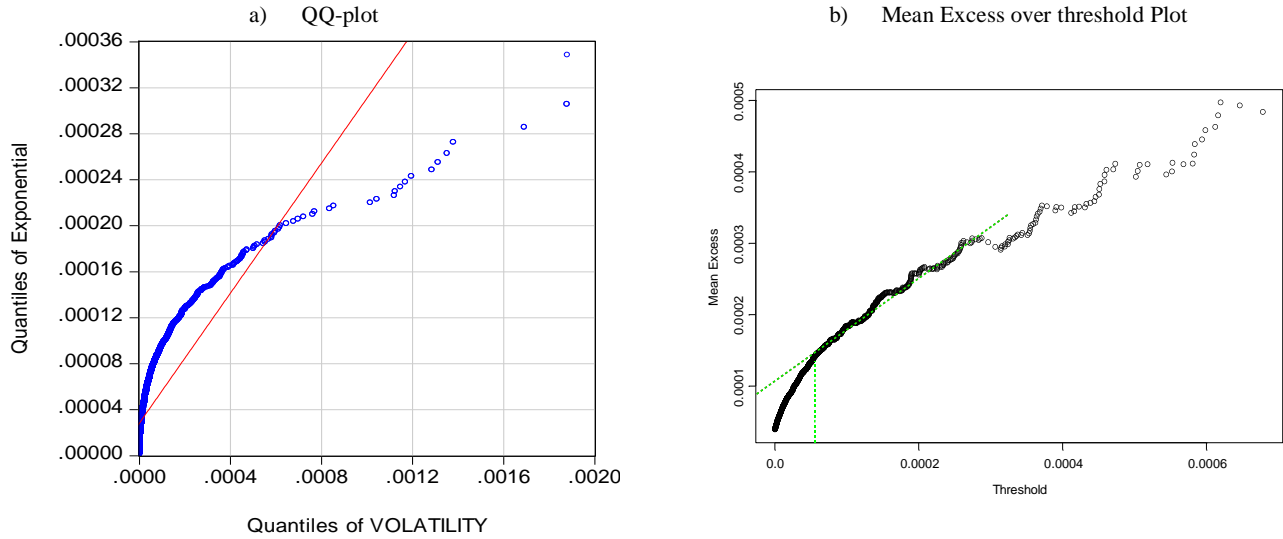


Figure 3. QQ-plot (left panel) and mean excess over threshold plot (right panel) for the volatility of exchange rate returns.

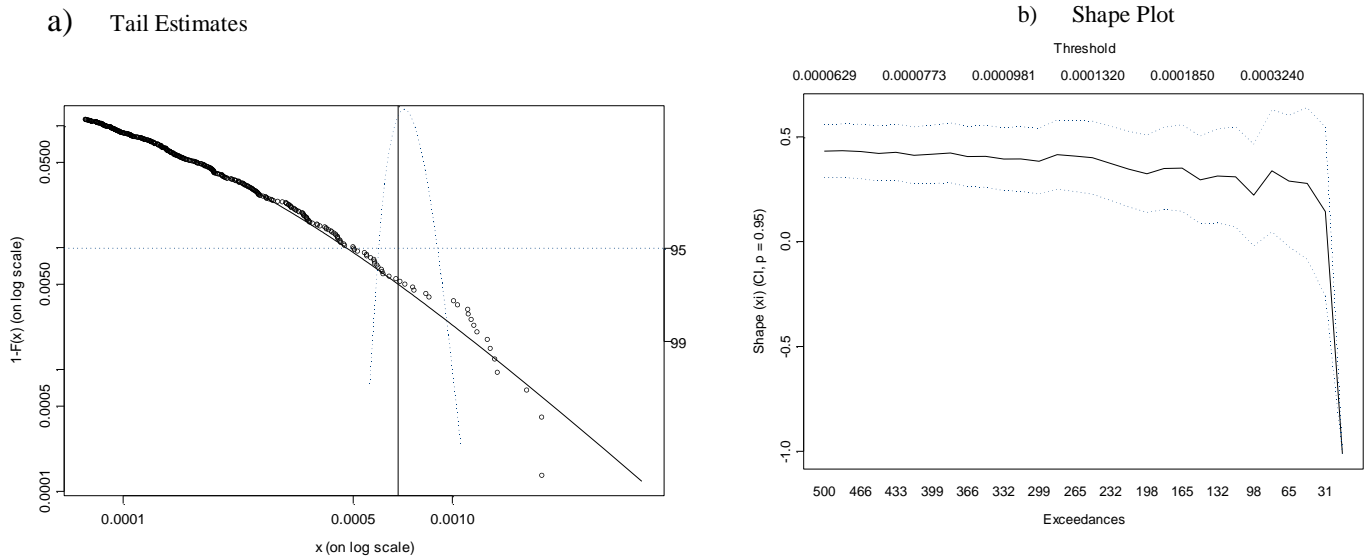


Figure 4. Tail estimates (left panel) and shape plot (right panel) of the volatility of exchange rate returns over various thresholds.

The shape plot provides estimates of the shape parameter over various thresholds. The distributions of excesses of the volatility process of exchange rate returns above this threshold can therefore be modelled by the GPD.

Estimated GPD models

As shown in Table 3, the fit of the GPD model to excesses of the volatility of exchange rate returns exceeding the selected threshold of 0.0000773 was highly significant for the full sample. The estimates for ξ and β

were statistically significant at 5 percent significance level. The shape estimates indicate heavy tailed distributions with finite mean and variance. The GPD models are estimated using the EVIS SPLUS Software provided by Prof. Alexander McNeil.

The GPD model was also fitted to the excesses of the volatility exceeding the selected threshold across various samples. The threshold for the entire samples was assumed for the shorter samples to facilitate comparability of the estimated models. A higher value of the estimated shape parameter in the GPD model for the volatility of exchange rate returns in the period 1999 to

Table 3. Estimated GPD models for volatility of exchange rate returns.

<i>Sample</i>	<i>Threshold (u)</i>	<i>No. of exceedances of the threshold u</i>	$\hat{\xi}$	$\hat{\beta}$
1999 – 2013	0.0000773	417	0.4136621 (6.046733)	0.0000964 (11.851358)
1999 – 2007	0.0000773	162	0.4509568 (3.889077)	0.0000873 (6.895858)
2008 – 2010	0.0000773	162	0.4308485 (3.999177)	0.0001155 (7.571914)
2011	0.0000773	81	0.1976621 (1.104454)	0.0000859 (5.350629)
2012 – 2013	0.0000773	12	0.1789681 (0.351431)	0.0001219 (1.714973)

Note: t-statistics for the respective coefficient estimates are in parenthesis.

Table 4. Estimated quantiles and return period for extreme volatility of exchange rate returns.

<i>Probability p</i>	<i>Estimated Quantile \hat{x}_p</i>	<i>Expected return period in Days</i>
0.990	0.0004752	100
0.999	0.0014797	1000

2007 and 2008 to 2010 compared with that in 2011 and 2012 to 2013 indicates that volatility was more extreme in the earlier periods.

The GPD model for the full sample fitted most of the data well in the tails of the distribution as shown in Figure 4 (left panel). However, the GPD fit does not capture extreme observations in the period 29th January, 2008 to 4th February, 2008 corresponding to the peak of the post poll crisis which caused extreme volatility in the foreign exchange market due to political uncertainty. Volatility in the exchange rate returns was more extreme on 30th January, 2008.

The estimates of the shape parameters over various threshold levels with their 95 percent confidence levels are shown in Figure 4 (right panel). The plot shows that the shape estimates are all greater than zero, an indicator of heavy tails, have finite variance and quite stable at about 0.4 for thresholds in the range [0.0000629, 0.0000981]. The variability at the end of the plot is attributed to a high threshold resulting in fewer observations being used to estimate the GPD model.

Return periods for specific extreme volatility in exchange rate returns

Estimates of the 99 percent and 99.9 percent tail

quantiles for the estimated GPD model are shown in Table 4. The occurrence of these levels of volatility in exchange rate returns is expected every 100 days (99 percent quantile) and 1000 days (99.9 percent quantile), respectively. Although the probability of occurrence of these levels of exchange returns is small (0.01 and 0.001, respectively), their impact on the economy can be detrimental. Exchange rate returns for the Ksh/U.S. dollar on 28th January, 2008, 30th January, 2008 and 1st February, 2008 were way above the estimated 99.9 percent quantiles for the GPD models. The extreme volatility of the exchange rate returns observed during these dates may not have been in the nature of the economy when they occurred.

Diagnostic tests on exceedances of the threshold

In this section, we analyse the trends in the excesses and exceedance times in the volatility of exchange rate returns over the selected threshold. Graphical analyses were conducted to check whether the excess amounts of the volatility of exchange rate returns are i.i.d. from the GPD, and whether the threshold exceedance times occur as a homogeneous Poisson process with constant intensity. The scatter plots of the scaled inter-arrival times Z against the order of their occurrence (right hand

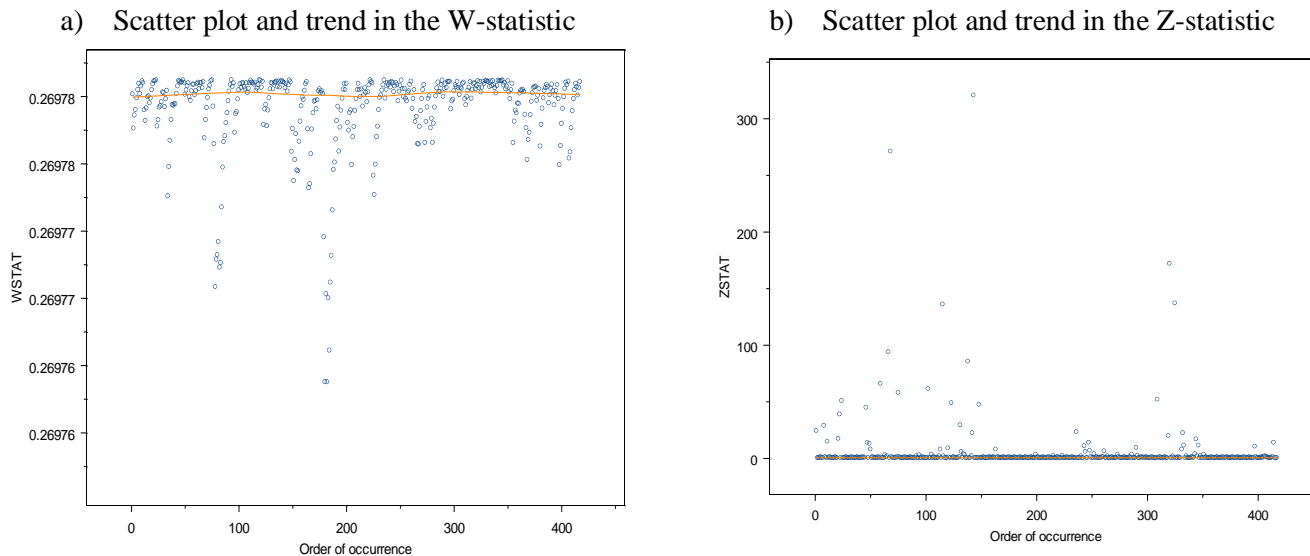


Figure 5. Diagnostic plots for the POT model of the volatility of exchange rate returns.

panel), and with superimposed Locally weighted scatter plot smoothing (Lowess) curves to capture the trend, indicate no significant trend (Figure 5). The Lowess is a smoothed mean value of the data and estimates the reciprocal of the intensity of the Poisson process (Embrechts et al., 1997).

The correlograms of the Z-statistics and further tests revealed that the inter-arrival times of the threshold exceedances are independent. The QQ-plots of the scaled inter-arrival times reveal that these are approximately exponentially distributed which indicates that the threshold exceedances occur as a homogeneous Poisson process. Scatter plots of the W-statistics against the occurrence time with superimposed smooth curves to capture the trend show no significant trend (left hand panel).

The correlograms of the W-statistics did not show any evidence of non-independence. We deduce that the excess sizes for the volatility of exchange rate returns of the U.S. dollar are *i.i.d.* from the GPD. The QQ-plots of the W-statistics show that they are approximately exponentially distributed which indicates that excesses are *i.i.d.* as GPD.

Conclusion

This study used EVT to establish if the extreme volatility witnessed in the daily exchange rate of the Kenya Shilling against the U.S. dollar in the period January 1999 to December 2013 could have been predicted, and also determined if the long-term stability in the exchange rate was affected during the period. The GARCH (1, 1) model was applied in estimating volatility of exchange rate

returns of the Kenya Shilling against the U.S. dollar and found to describe the volatility process well. The analysis revealed three key results for volatility of the exchange rate returns of the Kenya Shilling against the U.S. dollar in the study period. First, the quasi maximum likelihood estimates, corresponding to the estimated GED parameters of the exchange rate returns are highly significant and correspond to distributions with heavier tails than the normal distribution. Specifically, the volatility of exchange rate returns which exceed the threshold 0.0000773 can be modelled by the GPD. This threshold is therefore the indicative level for exchange rate returns above which the underlying depreciations in the exchange rate would be considered extreme which would require the CBK to intervene to stabilise the exchange rate. The estimated GARCH (1, 1) models show that the volatility in the daily exchange rates was comparatively extreme in the period 2008 to 2010.

The estimated return period for specific extreme volatility in the exchange rate returns showed that once after about 3 years (1000 days), we expect to observe an extreme volatility of 0.00148 in the exchange rate returns. This finding seems to mimic reality as depicted in Figure 1b which shows recurrence of extreme volatility in the exchange rate returns almost every three years. This extreme volatility requires the Central Bank to have adequate foreign exchange reserves to intervene to stabilise the exchange rate. However, the results showed a stable trend in the occurrence times of the extreme volatility and excesses above the selected threshold indicating that the long-term stability of the exchange rate was maintained in the study period.

This study shows that implementation of policies that will increase and sustain the level of foreign exchange

inflows into the country is necessary to mitigate the vulnerability of the exchange rate to external and domestic shocks. These include policies to promote the export sector as well those to enhance the level of foreign exchange reserves held by the CBK. These measures will reduce the current account deficit and enhance the Central Bank's capacity to intervene in the foreign exchange market to stabilise the exchange rate during period of extreme volatility. In addition, given that exchange rate volatility was comparably more extreme following the post poll crisis in 2008; political stability is a key component of foreign exchange market stability in Kenya.

LIMITATIONS AND SUGGESTIONS

Given the importance of the Euro, Sterling Pound and Japanese Yen in the Kenyan economy, it may be of interest to replicate this study considering exchange rate returns of the Kenya Shilling against these international currencies. The study can also be extended to incorporate other variations of the GARCH model to generate the volatility process.

Conflict of Interests

The authors have not declared any conflict of interest.

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Appendix 1. Selected Statistics for Daily Exchange Rates of the Kenya Shilling against the U.S. dollar.

Year	Mean	Standard deviation	Minimum	Maximum
1999	70.44	5.17	61.31	77.07
2000	76.19	2.59	67.46	79.76
2001	78.57	0.53	77.28	79.93
2002	78.72	0.53	77.07	80.28
2003	75.94	1.96	66.04	79.19
2004	79.17	1.82	75.94	82.52
2005	75.57	1.48	72.09	78.93
2006	72.15	1.11	69.21	74.48
2007	67.42	1.76	61.77	70.63
2008	69.05	5.75	61.51	80.12
2009	77.34	1.90	74.33	81.11
2010	79.26	1.99	75.36	82.10
2011	88.87	6.18	80.74	105.96
2012	84.52	1.18	82.27	88.44
2013	86.13	1.21	83.72	87.70

Annual means, standard deviation, minimum and maximum statistics for the exchange rate of the Kenya Shilling to the U.S. dollar.

Appendix 2. Correlogram of Squared Exchange Rate Returns of Kenya Shilling against the U.S. dollar.

Lag	Autocorrelation Coefficient	Partial Autocorrelation Coefficient	Q-Stat	Prob
1	0.3300	0.3300	408.4000	0.0000
2	0.3470	0.2680	861.4300	0.0000
3	0.2390	0.0810	1075.9000	0.0000
4	0.2280	0.0770	1271.8000	0.0000
5	0.1900	0.0480	1407.9000	0.0000
6	0.1000	-0.0530	1445.5000	0.0000
7	0.1070	0.0090	1488.3000	0.0000
8	0.0690	-0.0040	1505.9000	0.0000
9	0.0890	0.0340	1535.8000	0.0000
10	0.0900	0.0440	1566.2000	0.0000
11	0.0780	0.0180	1589.1000	0.0000
12	0.0900	0.0280	1619.3000	0.0000
13	0.1140	0.0580	1668.1000	0.0000
14	0.1390	0.0640	1741.0000	0.0000
15	0.1340	0.0370	1808.9000	0.0000
16	0.1090	-0.0030	1853.7000	0.0000
17	0.1090	0.0070	1898.3000	0.0000
18	0.0890	-0.0060	1928.4000	0.0000
19	0.1070	0.0290	1971.3000	0.0000
20	0.0800	0.0050	1995.5000	0.0000

Appendix 3. Correlogram of Squared Standardised Residuals from GARCH (1, 1) Model.

Lag	AC	PAC	Q-Stat	Prob
1	0.0020	0.0020	0.0202	0.8870
2	-0.0100	-0.0100	0.4277	0.8070
3	-0.0150	-0.0150	1.2545	0.7400
4	-0.0100	-0.0100	1.6627	0.7970
5	-0.0100	-0.0100	2.0312	0.8450
6	-0.0160	-0.0160	2.9683	0.8130
7	-0.0090	-0.0090	3.2505	0.8610
8	-0.0060	-0.0070	3.4013	0.9070
9	0.0250	0.0240	5.7426	0.7650
10	-0.0080	-0.0090	5.9834	0.8170
11	-0.0070	-0.0070	6.1546	0.8630
12	-0.0080	-0.0080	6.3848	0.8950
13	-0.0090	-0.0100	6.7206	0.9160
14	-0.0060	-0.0060	6.8528	0.9400
15	-0.0100	-0.0100	7.2248	0.9510
16	-0.0070	-0.0070	7.4033	0.9650
17	0.0140	0.0130	8.1063	0.9640
18	0.0000	-0.0020	8.1066	0.9770
19	0.0090	0.0090	8.4054	0.9820
20	0.0060	0.0060	8.5565	0.9870

ⁱ The views expressed in this paper are the authors' and do not necessarily reflect those of the Central Bank of Kenya.
ⁱⁱ The nominal GDP data used in the computation is based on the rebased GDP series released in September 2014).