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# Regimes Markov models with endogenous transition probabilities: Modeling fluctuations in Tunisia

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**This paper is an advanced analysis of the cyclical industry in Tunisia by taking the transition probabilities as endogenous in a Markov switching framework. Using Matlab programming of the Gibbs algorithm, Bayesian analysis allowed us to deal with the hidden Markov process with variable transition probabilities. Showing a persistent state, we obtained a positive relationship between previous and current regimes. These are presented as information leading to the variability of probabilities transition cycles. Furthermore, an anticipated increase in France would have a delayed effect on the business cycle in Tunisia because of domestic rigidities and institutional constraints as to depolarization. During recession in France, the persistence of expansion phases compared to recession seems to be verified in the Tunisian context. This type of application is not abundant in the empirical literature in Tunisia. Based on the various robustness tests (Vuong, 1989; Ang and Bekeart, 2002), the supremacy of MS-TVTP models over FTP in the treatment of cyclical fluctuations in Tunisia is shown.**

**Key words:** Markov switching, MS-FTP, MS-TVTP, Gibbs sampling, robustness test.

## INTRODUCTION

The statistical analysis of economic cycles is usually done through the identification and distinction between periods of prosperity "expansion" and periods of crisis "recession". A recent literature emerged highlighting the asymmetry in the economic cycles. It considers that non-linear models are more suitable than the linear representation to treat the fluctuations

Applied to the U.S. business cycles, the model of Hamilton (1989) using the ergodic Markov processes, expansions and contractions can be considered as the two regimes, each with specific characteristics

In this framework, many applications in the empirical literature consider not only the number of exogenous regimes as ad hoc, but also the constant probability of transition between regimes. However, in the process of economic openness, a dependency between the economic fluctuations of different economies is produced. Therefore, external shocks and domestic economic

policies can be introduced to explain the internal fluctuations of the economy

Therefore, external shocks and domestic economic policies can be introduced to explain the internal fluctuations of the economy. This limits the scope of the modeling cycles with constant transition probabilities and a predetermined number of regimes.

Hence, we need for models varying probabilities that highlight the determinants of transition probabilities change from one regime to another and detect the relationship between the national economy and external fluctuations.

Several empirical studies highlight the importance of the TVTP model (Time Varying Transition Probabilities) compared to Markov model with fixed probability. Diebold et al. (1994), Filardo (1994) and later Filardo and Gordon (1998) developed the model with Markov transition probabilities variables (TVTP). Diebold et al. (1994) argues that the transition probabilities of the exchange

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rate depends on several fundamentals economic variables. Filardo (1994) showed a performance of economic indicators to better explain the variation in economic cycles introduced in the TVTP model. In addition, Filardo and Gordon (1998) showed that TVTP models are more efficient than the FTP model to characterize the economic cycles of the United States. Indeed, the assumption that the transition probabilities are time-invariant may be costly from an empirical point of view. With fixed transition probabilities, the conditional expected durations do not vary over the cycle. This implies that exogenous shocks, macroeconomic policies and an economy's own internal propagation mechanisms are omitted to affect the expectation of how long an expansion or recession will last.

This paper is an advanced analysis of the cyclical opened industry in Tunisia by taking the transition probabilities as endogenous in a Markov switching framework. The objective is to study the economic cycle Tunisian by checking if the change of the turning points of the Tunisian economy is explained by those of its main European partner, that is, the French economy. We consider in this case that the probabilities of transition from one regime to another depends on the index of industrial production French. This dependence is studied through a comparison between the FTP regime-switching models and those of TVTP type. Thereby, we proceed to the filtering of the series of the index of industrial production in Tunisia and France on monthly data for the period 01-1993 to 12-2010. A comparison between the two types of models in terms of robustness is then considered.

The rest of the paper is organized as follows. The regime Markov model with endogenous transition probabilities and the method of Gibbs sampling are presented. Empirical results obtained are discussed. The following section is devoted to robustness tests. Final, conclusion.

## METHODOLOGY

### The MS model with TVTP

In his seminal paper, Hamilton (1989) has developed its new modeling nonlinear time series applied to the quarterly growth rate of real GNP U.S. According to Hamilton, the series follows an autoregressive process of order 4 with a regime change in average. He proposed the following model:

$$\Delta y_t - \mu_{s_t} = \beta_1(\Delta y_{t-1} - \mu_{s_{t-1}}) + \dots + \beta_4(\Delta y_{t-4} - \mu_{s_{t-4}}) + \varepsilon_t \quad (1)$$

$$\mu_{s_t} = (1 - S_t)\mu_0 + S_t\mu_1, \quad \mu_1 > \mu_0$$

The state variable  $S_t$  is a stochastic process that generates the unobservable state of the economy. Hamilton (1989) assumes that  $S_t$  is a first-order Markov-process, which means that the current regime ( $S_t$ ) depends only on the regime in the preceding period  $S_{t-1}$ . This is due to the ergodic feature of Markov chain.

$S_t = 0$  indicates that the economy is in recession phase where the conditional mean of  $y_t$  is  $\mu_0$  and  $S_t = 1$  explains the expansion period where  $\mu_1$  is the conditional mean of  $y_t$ . This model is completed by defining the transition probabilities of moving from one regime to another (so-called 'the transition probabilities'):

$$P(S_t = s_t / S_{t-1} = s_{t-1}) = \begin{bmatrix} q & 1-p \\ 1-q & p \end{bmatrix}$$

Where

$$p = P(S_t = 1 / S_{t-1} = 1), \text{ and } q = P(S_t = 0 / S_{t-1} = 0) \quad (2)$$

The conditional expected durations has a well-known geometric distribution with mean  $[1/(1-p)]$  in the expansion phase and  $[1/(1-q)]$  in the recessionary phase.

The MS-TVTP appears as an extension of MS model. This extension consists of the identification of the transition probabilities given in (2). In this model the transition probabilities become time-varying and depend on the relevant economic indicators that can inform on the state of the economy. In addition, this model with variable transition probabilities is more flexible and contains more information.

The MS-TVTP is given as follows:

$$(1 - \phi(L))(y_t - \alpha_0 - \alpha_1 S_t) = \varepsilon_t \quad (3)$$

Where,  $y_t$  is the specified series, assumed Auto-Regressive and stationary,  $\varepsilon_t$ , error terms, independent and identically distributed according to  $N(0, \sigma_y^2)$ , and  $S_t \in \{0, 1\}$  is governed by a first order Markov process with endogenous transition probabilities depend on a vector of indicators denoted  $Z_t$ . The equation for the probabilities matrix is then written as follows:

$$P(S_t = j / S_{t-1} = i, Z_{t-1}) = \begin{bmatrix} q(Z_{t-1}) & 1-p(Z_{t-1}) \\ 1-q(Z_{t-1}) & p(Z_{t-1}) \end{bmatrix} \quad (2)$$

In this model the probability of transition from one regime to another, dependent variables constitute the  $Z$  vector.

A univariate probit model can be estimated to measure the transition probability matrix at each. This model puts the transition probabilities in dependence with the information contained in the vector of indicators. In this case, a linear specification establishes a relationship between the probability of change cycles and other explanatory variables<sup>1</sup>. Following Filardo and Gordon (1998), the expression of the transition probability is given as follows:

$$P(S_t = 1) = P(S_t^* \geq 0) \text{ and } P(S_t = 0) = P(S_t^* < 0) \quad (3)$$

$$S_t^* = \mu_0 + \mu_z' Z_t + \mu_z S_{t-1} + u_t \quad (4)$$

Where,  $S_t^*$  is a latent variable,  $\mu_z$  is a vector of parameters, and  $Z_t$  is a vector of information. It's a variable that affects the

<sup>1</sup>See Filardo and Gordon (1998), Durland and McCurdy (1994), Bodman (1998) and Kim et al. (2008).

transition probabilities of business cycle phases. The random variable,  $u_t$ , is drawn from a process of independently distributed standard normal variables with mean equal to zero and variance equal to unity. This distributional assumption simplifies the calculation of the transition probabilities. Thus, the current economic cycle is explained by the state of the economy at time  $t-1$  ( $s_{t-1}$ ), and the vector of indicators,  $Z$ . The calculation of transition probabilities at time  $t$  is performed by evaluating the conditional cumulative distribution function for  $u$ , denoted  $\Phi_{U/Z}(u(z))$ . It is given by,

$$\Phi_{U/Z}(u(z)) = \int_{-\infty}^{u(z)} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right)dx \quad (7)$$

Thus, the expressions for the transition probabilities are determined by:

$$p = P(S_t = 1 / s_{t-1} = 1) = P(\mu_0 + \mu'_1 z_t + \mu_z + u_t \geq 0) = P(u_t \geq -\mu_0 - \mu'_1 z_t - \mu_z) \quad (8)$$

$$= 1 - \Phi_{U/Z}(-\mu_0 - \mu'_1 z_t - \mu_z)$$

$$q = P(S_t = 0 / s_{t-1} = 0) = P(\mu_0 + \mu'_1 z_t + u_t \leq 0) = P(u_t \leq -\mu_0 - \mu'_1 z_t - \mu_z) \quad (9)$$

$$= \Phi_{U/Z}(-\mu_0 - \mu'_1 z_t)$$

Transition probabilities evaluating the persistence of expansions and recessions period depends also on prior state of business cycles,  $S_{t-1}$ , and economic indicators,  $Z_t$ . The estimation of these probabilities is done using Gibbs method.

### Estimation probabilities transition by Gibbs sampling

The estimation of parameters of the flexible regime model is based on the method of Gibbs sampling introduced by Filardo and Gordon (1998). This approach is called MCMC technical (Markov Chain Monte Carlo). Its first use is for the application of Geman and Geman (1984). The main idea is to simulate draws artificial parameter estimates,  $\theta_j$ , from the probability density,

$$p(\theta_j / \theta_{-j}, x),$$

where  $\theta_{-j}$  represents parameters of the model other than  $\theta_j$ . With the feature of ergodicity of the Markov chain, the average of the artificial sample allows having a consistent estimator of expectation,  $E[g(\theta) / x]_2$ . The starting point of the Gibbs algorithm is to specify the prior distribution of the estimated parameters. Thus, the estimated model is given by two equations, the growth rate of industrial production index (3) and the transition probabilities (6). Parameters to be estimated are then as follows:

$$\theta = \{\alpha, \phi, \mu, \sigma_y^2, S_0, p_t, q_t\}$$

Given,  $\sigma_y^2$  the prior distribution of autoregressive parameter in Equation (1),  $\phi$  is a multivariate normal distribution  $\phi \sim N(\hat{\phi}, \hat{A}_\phi \sigma_y^2)$ . Where,  $\hat{\phi}$  and  $\hat{A}_\phi \sigma_y^2$  represent, respectively, the prior mean and prior variance for

$\phi$ . Similarly, prior beliefs about the state-dependent means,  $\alpha$ , given  $\sigma_y^2$  is represented by a truncated normal distribution in Equation (3),  $\alpha \sim I_{\alpha_i > 0} N(\hat{\alpha}, \hat{A}_\alpha \sigma_y^2)$ .  $I_{\alpha_i > 0}$  is an indicator function identifying  $s_t = 1$  as the high-growth state,  $\hat{\alpha}$  and  $\hat{A}_\alpha \sigma_y^2$  represent, respectively, the prior mean and prior variance for  $\alpha$ .

The prior distribution for the variance of output has the inverse-gamma form:

$$\sigma_y^2 \sim IG\left(\frac{\hat{v}_y}{2}, \frac{\hat{v}_y \hat{\sigma}_y^2}{2}\right)$$

Where,  $E[\sigma_y^2] = \sigma^2$  et  $V[\sigma_y^2] = 2 / \hat{v}_y \hat{\sigma}_y^4$ .

The prior distribution for  $\mu = (\mu_0, \mu'_z, \mu_z)$ , is a natural conjugate prior the normal form  $\mu \sim N(\hat{\mu}, \hat{A}_\mu)$ . Finally,  $s_0$ , which starts the Markov process on  $S_t$  has a Bernoulli prior distribution with  $p(s_0 = 1) = \hat{\pi}$ . The method of Gibbs sampling, following the Bayesian approach, consists to identify the prior distributions of the parameters, identify the posterior distributions and derive the point estimators<sup>3</sup>.

## EMPIRICAL RESULTS

The empirical analysis is for the time series of monthly industrial production index extracted from the database "International Financial Statistics" for the period January 1993 to December 2010. The choice of this period is motivated by several reasons. First, it is a period of expansion of the industry in Tunisia where the state encourages private investment<sup>4</sup>. Second, during this period Tunisia has signed several agreements such as a Free Trade with the European Union and even the United States.

The study is done on the growth rate of the time series of the index of industrial production following the modern business Cycle approach (Barro, 1989):

$$y_t = \ln(IP_t) - \ln(IP_{t-1}) \quad (5)$$

Figure 1 shows a first description of data representing a selection of original data " $IP_t$ " and the logarithmic transformations " $y_t$ " of the two economies for the period 1993:1 to 2010:12.

The stationarity of transformed series is checked by the KPSS test proposed by Kwiatkowski et al. (1992). Under the null hypothesis, the series is stationary against the existence of unit root as alternative hypothesis. The test of stationarity is given in Table 1.

<sup>2</sup>Gordon and Bélanger (1996) note that this concept is more broadly to include the posterior mean  $E[\theta / x]$  where  $g(\theta) = \theta$ .

<sup>3</sup> For a theoretical development of this technique, see Flood and Gordon (1998)

<sup>4</sup> See Investment Incentives Code

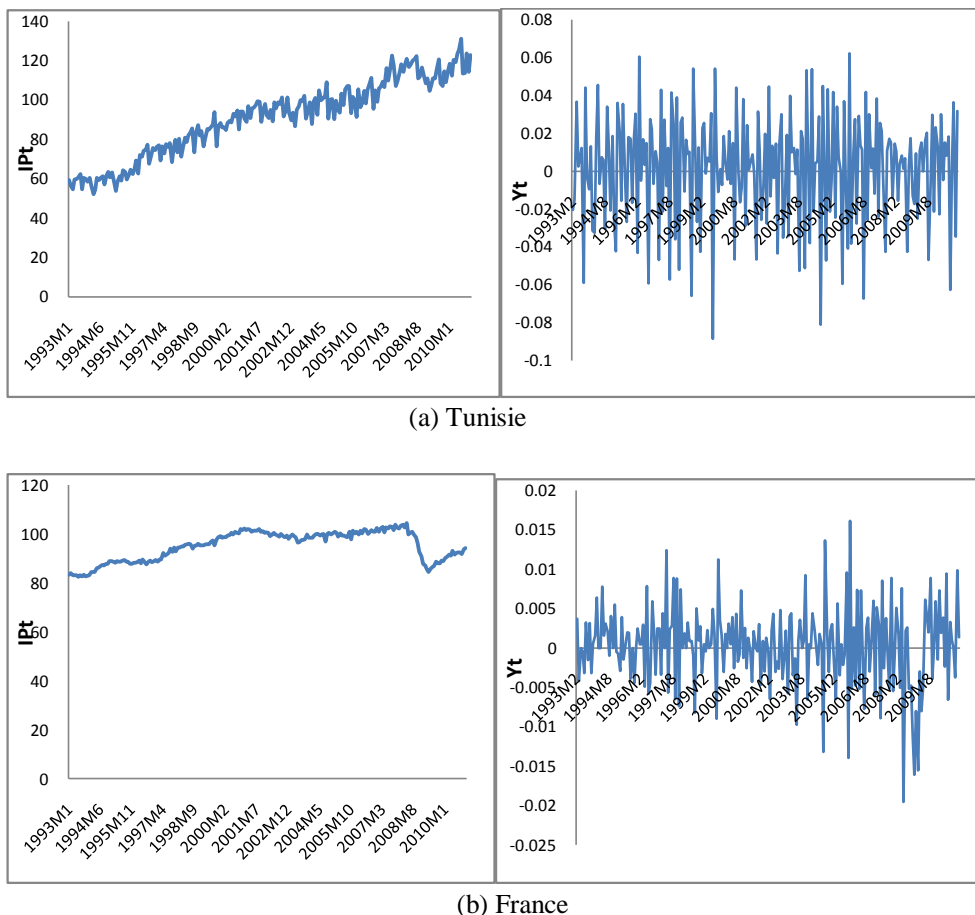


Figure 1. Level monthly observations (left panel) and logarithmic transformation (right panel) of (a) Tunisia and (b) France 1/1993 to 12/2010.

Table 1. Test for stationarity of the original series ( $IP_t$ ) and the differenced series ( $y_t$ ), and the statistical properties of  $y_t$ .

|   | Tunisia       | France        |
|---|---------------|---------------|
| $IP_t$  | 4.1906**      | 1.805**       |
| $y_t$   | 0.0291        | 0.287         |
| <b>Statistical characteristics <math>y_t</math></b> |               |               |
| Mean  | 0.00147       | 0.00015       |
| Standard-deviation                                  | 0.029         | 0.0052        |
| Kurtosis  | -0.0218       | 1.53          |
| Skewness  | -0.5227       | -0.398        |
| JB  | 9.677 (0.008) | 24.87 (0.000) |

\*\* significativity to 1%.

Table 1 shows the results of the stationarity test applied to the IP series as well as its growth rate, and descriptive

statistics. The first part of this table shows the stationarity of the growth rate. The lower part of the table is allocated to represent a statistical description of the distribution of the series stationary ( $y_t$ ). We measured the mean, the standard deviation, the Skewness, and the Kurtosis, so called "moments of the distribution", and the test statistic (Jarque and Bera, 1981).

The values of skewness and kurtosis indicate a significant deviation from the historical distribution compared to the normal distribution. JB statistics reject the null hypothesis of normality of the different series.

The endogenous regime-switching model estimated is given by the following system of equations:

$$\begin{cases} (1 - \phi(L))(y_t - \alpha_0 - \alpha_1 S_t) = \varepsilon_t \\ S_t^* = \gamma_0 + \gamma_1 y_{t-1} + \gamma_2 IPF_t + \gamma_3 S_{t-1} + \mu_t \end{cases} \quad (12)$$

Where,  $S_t^*$  is a latent variable that represents the state of the economy in the current period explained by  $y_{t-1}$  the value of the growth rate of the IP Tunisia to the previous

**Table 2.** Estimation of the model parameters  
 $y_t = \alpha_0 + \alpha_1 s_{t-1} + \phi(2)(y_{t-1} - \alpha_0 - \alpha_1 s_{t-1}) + \varepsilon_t$ .

| Final results: Total sample Gibbs 8000.0 |         |                    |
|--|---------|--------------------|
| Parameters                               | Mean    | Standard deviation |
| $\alpha_0$                               | -0.0096 | 0.0087             |
| $\alpha_1$                               | 0.0126  | 0.0092             |
| $\phi_1$                                 | -0.224  | 0.0312             |
| $\phi_2$                                 | -0.0051 | 0.0349             |
| Standard error                           | 0.0036  |                    |

date ( $t - 1$ ), the growth rate of IP French ( $IPF_t$ ) and the state of the economy in the previous period ( $S_{t-1}$ ). The residues  $\mu_t$  are  $i.i.d.N(0,1)$ .

The estimated model parameters are defined by the following vector:

$$\theta = \{\alpha_0, \alpha_1, \phi_1, \phi_2, \mu_0, \mu_1, \mu_2, \mu_3, S_t^*, p_t, q_t\}$$

(6)

The estimate of  $\theta$  requires the initialization of some parameters namely:

$$\{\alpha_0^0, \alpha_1^0, \phi_1^0, \phi_2^0, \mu_0^0, \mu_1^0, \mu_2^0, \mu_3^0, S_t^0, p_0, q_0\}$$

To initialize the values of the state variable,  $S_t$ , we follow Filardo and Gordon (1998) who made the identification of the NBER (National Bureau of Economic Research). Otherwise, we use the Bry and Boschan method for dating turning points and thus identify states<sup>5</sup>.

The initial values of the parameters ( $\alpha_0, \alpha_1$ ) are given by a simple OLS estimation of the growth rate series of IP,  $y_t$ , depending to the initial values of the state variable,  $S_t$ , already identified. The estimate has allowed us to obtain the following values: ( $\alpha_0 = -0.038077, \alpha_1 = 0.048039$ ).

These initial values estimated of the vector  $\alpha^0$  allow us to estimate the initial values of the autoregressive parameters  $\phi^0$ . The values are as follows:  $\phi^0 : (\phi_0^0 = -0.038079, \phi_1^0 = 0.048042)$ . The initial state selected a priori as a recessionary period (Bry-Boschan method).

<sup>5</sup> This method assimilates a turning point to a change in means or variance of the study variable. This procedure has already been applied several times in the literature and is regarded as a practical and straightforward method to compute the turning points of a time series. The procedure was adopted by King and Plosser (1994), Watson (1994), Pedersen (1998) and Harding and Pagan (2002).

Thus, the unconditional probability of the initial state of recession is  $p(s_0 = 0) = \hat{\pi} = 0.221$ .<sup>6</sup>

To initialize the parameters of the vector  $\mu = (\mu_0, \mu_1, \mu_2, \mu_3)$ , Filardo and Gordon (1998) noted that  $\mu_0$  and  $\mu_3$  determine the unconditional mean durations of recession and expansion. Thus, to initialize these two parameters, we have taken the estimated Markov model with fixed transition probabilities and we determined the unconditional probabilities of recession and expansion. Parameters  $\mu_1$  and  $\mu_2$ , are assumed no-informative. First, it is assumed that the current state is neither explained by the previous value of  $y$  nor by the IPF. The authors have taken the zero values for these parameters. Hence, we have taken the initial values according to the authors. The gamma vector is initialized

as follows:

$$\begin{pmatrix} \mu_0 \\ \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} -0.992 \\ 0 \\ 0 \\ 1.984 \end{pmatrix}$$

The estimation results of the vector of parameters are presented in Tables 2 and 3. Following Filardo and Gordon (1998) initial values have been abandoned in the calculation of moment. Table 2 shows the parameters of the model, the growth rate of the index of production in Tunisia, and Table 3 incorporates gamma parameters of the equation of the latent variable.

In Table 2, the parameters of the equation of the output shows that  $\alpha_0 < 0$  and  $\alpha_1 > 0$ . This implies the presence of a period of recession and a period of expansion. The average growth rate of recession states is negative and that of the states of expansion is positive. Autoregressive parameters  $\phi_1$  and  $\phi_2$  represent the temporal dynamics of the cycle.

The estimation of the parameters of the equation of transition probabilities of the vector,  $\mu$ , is presented in Table 3. The results summarize the estimated values (on average) and its standard deviation. The results show that the value of the parameter,  $\mu_3$ , is the highest coefficient. This implies that the state of the previous period is the most explanatory variable of the economy to the present date. In addition,  $\mu_3 > \mu_0$ , which explains the persistence of periods of expansion compared to periods of recession (asymmetric effect evoked by the stylized facts established by Keynes (1936)).

The positive value of  $\mu_3$  implies that the state of the previous period explains highly that the present period. Intuitively, the negative sign of  $\mu_0$  (on the recession

<sup>6</sup>Markovien process  $S_t$  has a Bernoulli distribution.



**Table 3.** Estimated of the parameters in the equation of transition probabilities.

| Final results: Total sample Gibbs 8000 |         |                    |
|--|---------|--------------------|
| Parameters                             | Mean    | Standard deviation |
| $\mu_0$                                | -1.2482 | 0.1044             |
| $\mu_1$                                | -3.0299 | 0.7702             |
| $\mu_2$                                | -5.5205 | 4.0858             |
| $\mu_3$                                | 2.9736  | 0.1042             |
| Standard error                         |         | 0.0036             |

**Table 4.** Vuong Test: model selection.

| Model            | V      |
|------------------|--------|
| MS-TVTP / MS-FTP | 45.484 |

regime) and the positive sign of  $\mu_3$  (on the expansion regime) indicate the persistence of the Tunisian economy in the previous state. In contrast, industrial cycles of the French economy explain the regime of industrial cycles in Tunisia. It should be noted the synchronization delay seen by the negative sign of the parameter  $\mu_2$ . We can deduce a weak influence of fluctuations in the growth rate of the French IP on changing state growth rate of IP Tunisian economy (that is to say the effect earlier). The negative sign of the parameter indicates that an increase in the value of the index of French production increases the probability of changing state in Tunisia. Intuitively, an expansion planned at the date(t+1) in France leads an expansion in Tunisia at the date t. However, an anticipated increase in France would have a delayed effect on the business cycle in Tunisia because of domestic rigidities and institutional constraints as to depolarization. In the presence of recession in France, the persistence of expansion phases compared to the recession seems be verified in the Tunisian context. This explains the phase delay of the variation in industrial growth cycles between Tunisia and France.

A graphical representation of transition probabilities of recessions and expansions is given in Figure 2. We note that generally the probabilities are either close to zero (state of recession) or close to 1 (state expansion). In Figure 3, we superimposed the phases of expansion with the chronology dated by the Bry-Boschan method. A correlation was marked between the probabilities generated by the model and the dating of cycles by the Bry-Boschan method for some states of recession and expansion (Figure 4).

## ROBUSTNESS TEST

For the robustness of the approach of variability in the

transition probabilities, we evaluate the contribution of MS-TVTP model to explain changes of state growth rate of IP in Tunisia. We test whether the model MS-TVTP is better than the MS-FTP or vice versa. To do this, we refer to the test of Vuong (1989) and to the statistics of Ang and Bekeart (2002).

The Vuong' test, based on the likelihood ratio (LR), can select the most appropriate model. Under the null hypothesis, the two models are identical in terms of adjustment of the studied series. According to the alternative hypothesis, one of the two models is better than the other. Formally,  $F$  and  $F'$  are the likelihood functions whose parameters are associated with models MS-TVTP and MS-FTP, respectively. Vuong' test (1989) is defined as follows:

$$V = \frac{1}{w \times \sqrt{T}} LR_T(\theta, \theta') \sim N(0,1) \quad (14)$$

Where,  $LR_T(\theta, \theta') = F_T(\theta) - F'_T(\theta')$ , such as

$$w^2 = \frac{1}{T} \sum_{t=1}^T \left[ \log \frac{f(Y_t/I_t, \theta)}{f'(Y_t/I_t, \theta')} \right]^2 - \left[ \frac{1}{T} \sum_{t=1}^T \log \frac{f(Y_t/I_t, \theta)}{f'(Y_t/I_t, \theta')} \right]^2$$

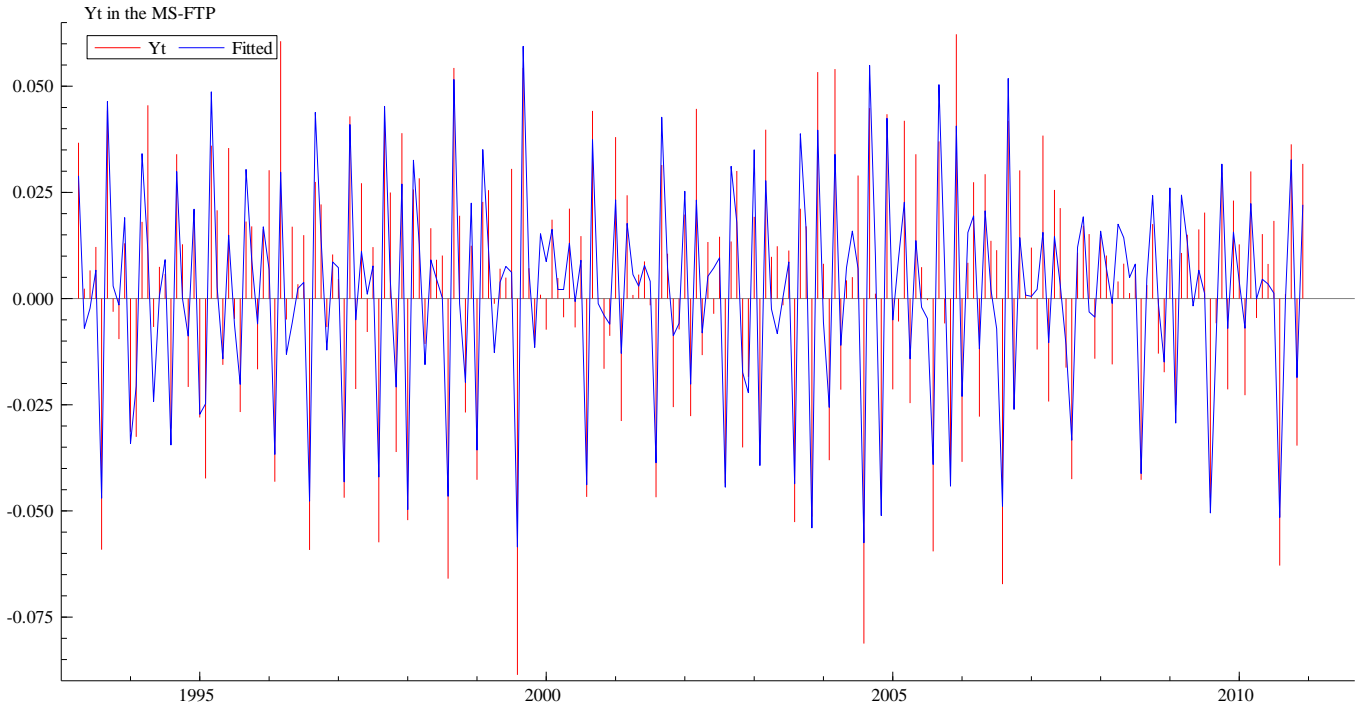
$f(\cdot)$  and  $f'(\cdot)$  are the density functions of the model MS-TVTP and MS-FTP, respectively. The critical value is chosen as the standard normal distribution for a given level of significance (for example for  $\alpha = 5\%$ ,  $C=1.65$ ). The MS-TVTP model is preferable to MS-FTP, under this alternative, if the statistics  $V > C$ . If  $V < -C$ , the MS-FTP model is better than the model with transition variable. Moreover, we cannot reject the null hypothesis if  $|V| \leq C$ , since  $V$  will be between the two critical values.

The test results obtained are given in Table 4. Thus, the Vuong' statistic shows that the MS-TVTP model is the model that detects better the change of regime in Tunisia growth rate during the period, 1993-2010.

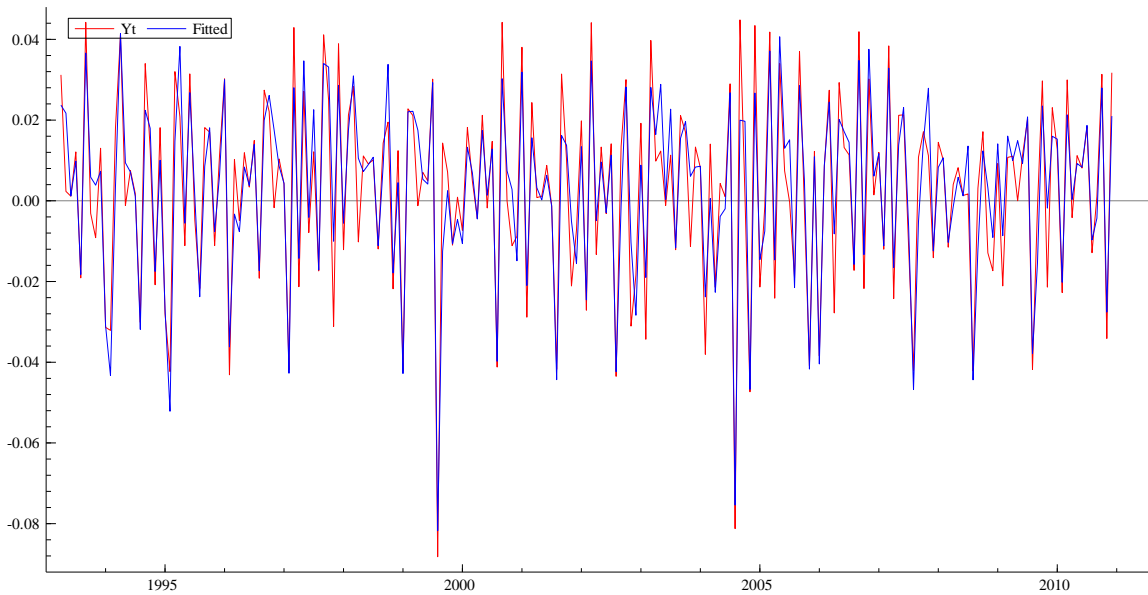
The statistics proposed by Ang and Bekeart (2002) checks whether the MS-TVTP model with two regimes is best identified than the MS-FTP model with two regimes. The statistic so called RCM (regime classification measure) is defined as follows:

$$RCM = 400 \times \frac{1}{T} \sum_{t=1}^T p_t(1-p_t). \quad (15)$$

Where  $p_t = P(s_t = 1 / F_{t-1})$  represents the probability of ex-ante regime (Ang and Bekeart, 2002). The constant used to normalize the RCM statistic as to be between 0 and 100 for the two states. Thus, the ex-ante probabilities are used as inferential values to verify the model capacity to capture regime changes. Indeed, probabilities close to zero or one indicates the model performance to classify regimes. However, the inadequate model would likely



**Figure 2.** Graphical representation between the current series  $y_t$  and the adjusted series after modeling MS-FTP.



**Figure 3.** Graphical representation between the current series  $y_t$  and the adjusted series after modeling MS-TVTP.

close to a half. A good classification of system (the most powerful model) is then associated with low values of the statistics RCM.

The results obtained as shown in Table 5 indicate that the smallest value corresponds to the MS-TVTP model. Thus, the model with regime change with varying transi-

tion probabilities performs better to analyze the business cycles in Tunisia compared to model with fixed transition probability for the period, 1993-2010.

A graphical representation confirms our results obtained from the use of RCM statistics of Ang and Bekeart (2002). It shows in terms of comparison, the actual series

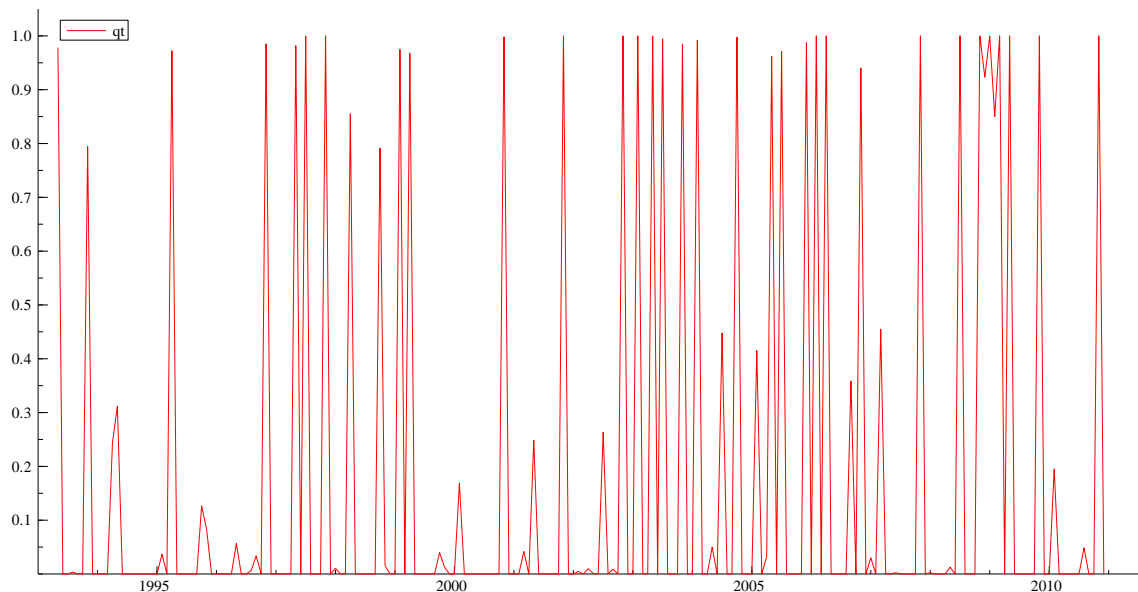


Figure 4. Probability of smoothing regime recession.

Table 5. RCM statistics of Ang and Bekeart.

| Model             | RCM         |
|-------------------|-------------|
| MS-FTP (EM)       | 16.601140   |
| MS-FTP (E. Gibbs) | 7.186       |
| MS-TVTP           | 6.991203723 |

of the growth rate of IP in Tunisia and the adjusted series by the MS-FTV models (Figure 2) and the MS-TVTP model (Figure 3).

## Conclusion

We deepened the study of fluctuations in industrial cycles in Tunisia by introducing explanatory variables in the calculation of transition probabilities. The MS-TVTP allows us to have more information on the phase in which the economy is, and the effects of exogenous variables introduced in regime change. This helps to identify the regime in which the economy is, and predict when the economy will change phases. The MS-TVTP model is preferable to MS-FTP model in so far as the transition probabilities change during the period of analysis, as well as additional information procured on state changes, are offered by the explanatory variables.

Using Matlab programming of the Gibbs algorithm, Bayesian analysis allowed us to deal with the hidden Markov process with variable transition probabilities. Showing a persistent state, we obtained a positive relationship between previous and current regimes. In addition, the estimated model identifies a negative

relationship between the state of the Tunisian economy and industry cycles in France. Indeed, an increase in the index of French production increases the probability of changing state in the Tunisian economy.

Robustness tests were conducted to verify the performance of the Markov model with varying transition probabilities from the model with fixed transition probabilities. We refer to the test of Vuong (1989) and the statistics of Ang and Bekeart (2002) based on the ex-ante probability. Our results show a considerable performance MS-TVTP model to detect regime changes in the growth rate of IP in Tunisia for the period, 1993-2010.

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