Review

An evaluation of exchange rate models by carry trade

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We evaluate the effectiveness of economic fundamentals in enhancing profit of carry trades. We simulate carry trades in Japanese yen and Swiss franc against six target currencies based on forecasts of exchange rate models of economic fundamentals. The performance results are compared against those of the benchmark random walk and AR (1) models. We find that carry trades perform better in risk-adjusted returns and produce less downside risk in the exchange rate models of economic fundamentals. Particularly, the best improvement in profit to carry trades comes when the economic fundamentals are used in a factor-augmented regression framework where the factor is time-varying and derived from the fundamentals. The result is robust for different time periods and after controlling for transaction costs.

Key words: Exchange rate models, Japanese yen and Swiss franc, carry trade, currency risk premium.

INTRODUCTION

Are exchange rate models of economic fundamentals useful? The answer to this question had been controversial in the field of international finance. Meese and Rogoff (1983a, 1983b) first show that the forecasting ability of exchange rate models is worse than that of a simple random walk process. Since then, there have been a large number of papers subsequently documenting success for several economic fundamentals based models, but these results are not robust. Some of them are good at longer horizon, while others at shorter periods. For example, Evans and Lyons (2002: 170-171) states, "Macroeconomic models of exchange rates perform poorly at frequencies higher than one year. Indeed, the explanatory power of these models is essentially zero." In the words of Frankel and Rose (1995: 1704), the negative results has had a "pessimistic effect on the field of empirical exchange rate modeling in particular and international finance in general." And this pessimistic effect has been around us for 20 years.

Motivated by Engel and West (2005) who claim that an exchange rate model should not be evaluated simply by whether it can beat the random walk in the out-of-sample forecast because the exchange rates themselves are close to the random walk when the discount factor is close to one, this study evaluates exchange rate models of economic fundamentals from a practitioner's point of view. This practitioner is specializing in one particular currency investment: The carry trade. Particularly, we focus on economic fundamentals used in the Taylor rule.

The Taylor rule specifies how the interest rate responds to economic fundamentals. According to Engel and West (2005), the Taylor-rule models appear to be the potential candidate to "beat the random walk model". Many studies have found improvement in the forecasting ability of the exchange rate models when they include the economic fundamentals related to the Taylor rule (Chinn and Pascual, 2005; Choi et al., 2006; Engel and West, 2005; 2006; Engel et al., 2007; Molodtsova and Papell, 2008; Murray and Papell, 2002; Taylor et al., 2001). In this paper, we examine whether these economic fundamentals can increase the profitability of carry trades, hence, to some extent, clarifying whether exchange rate models of fundamentals can beat the random walk in this aspect.

Since the mid-1990s, the carry trade has become one of the major currency trading strategies. A plain-vanilla carry trade involves borrowing low-interest-rate currency and investing in high-interest-rate currency, where the low-interest-rate currency is called the funding currency and the high-interest-rate currency is called the target currency. The funding currency of the carry trade is mostly the Japanese yen or Swiss franc. Profit from a carry trade is the sum of the interest rate differential between the target currency and the funding currency, and change in the exchange rate of the target currency against the funding currency. According to the uncovered interest rate parity (UIRP), the carry trade is not profitable on average because the interest rate differential would be

offset by the relative depreciation of the target currency against the funding currency. However, almost all empirical studies point to the opposite conclusion (for instance, Cheung et al., 2002; Engel, 1996; Mark and Sul, 2001; Lewis, 1995; Meese and Rogoff, 1983a, 1983b). This implies that the carry trader can pocket both the interest rate differential and the appreciation of the target currency, with zero capital.

We simulate carry trades in two funding currencies, Japanese yen and Swiss franc, separately against the following major target currencies: Australian dollar, New Zealand dollar, British pound, Canadian dollar, euro, and U.S. dollar. Since each funding currency corresponds to six target currencies, we have a total of 12 carry trades with each involving one target currency and one funding currency. For each carry trade, the process follows a go or no-go binary decision at a monthly frequency. When the forecast return is positive, the carry trade is executed; otherwise, the trade is skipped. The out-of-sample return from each carry trade is obtained based on forecasts of fundamental exchange models. We experimented with five different specifications of the exchange rate models of fundamentals. Among them, two have a derived factor. Two other non-fundamental models, Random Walk and AR (1), serve as the benchmark.

To evaluate exchange rate models, both risk-adjusted returns (Sharpe ratios) and measures of downside risk are presented. Since a carry trade is highly leveraged and is subject to sudden crashes in the target currency, return skewness and the maximum drawdown is an important indicator of performance as well.

We find that exchange rate models of economic fundamentals perform better not only in the Sharpe ratio, but also in skewness and the maximum drawdown unanimously. This result is more prominent under a factor-augmented regression framework (FAR) where a time-varying factor is derived from the fundamentals and is used as an explanatory variable. Such use of fundamentals in a FAR is inspired by Engel et al. (2007) who indicate that an unobservable factor in the exchange rates themselves may contain useful information for prediction. Applying the Kalman filtering technique, we use a dynamic factor model to extract information in the fundamentals. This method is appealing in that the factor is derived from the interest rate parity and has economic meaning, it approximates the risk premium in exchange rates. The factor is then combined with the economic fundamental variables to forecast exchange rates in the FAR model. The OLS regression is the estimation method in the FAR framework.

FOREIGN EXCHANGE RATE MODEL AND ESTIMATION METHOD

Throughout this paper, we use the following predictive regression model of exchange rate

$$\Delta s_{t+1} = \beta_F \hat{F}_t + z_t + \varepsilon_{t+1} \text{ where } \varepsilon_{t+1} \sim NID\left(0,\sigma^2\right) \quad (1)$$

where s_t is the log exchange rate expressed as units of funding (base) currency (yen or Swiss franc) per unit of target currency and $\Delta s_{t+1} = s_{t+1} - s_t$. An increase in s_t indicates appreciation of the target currency and depreciation of the funding currency, and vice versa.

Depending on the form of z_t and the choice of β_F, we have different specifications of the exchange rate models studied in this paper. When we set $\beta_F = 0$, $z_t = 0$, the regression is the random walk (RW) model. This is the standard benchmark in the exchange rate prediction literature and practice. By setting $\beta_F = 0$, and $z_t = \alpha + \beta_s \Delta s_t$, we obtains an AR (1) model. The term z_t will include the economic fundamentals.

$$z_t = i_t - i_t^* + O\left(i_t - i_t^*\right) \quad (2)$$

where i_t is the interest rate of the funding (base) currency and i_t^* is the interest rate of the target (foreign) currency.

In this paper, an asterisk denotes variables in the target-currency country. The term $O\left(i_t - i_t^*\right)$ is the polynomial of higher orders of the interest rate differentials. If $O\left(i_t - i_t^*\right)$ is zero, the regression Equation (1) is the UIRP. According to the Taylor rule, the interest rate is determined by economic growth rate (y), inflation rate (π) and one-lag interest rate. Hence, we set the interest rate as a linear function of economic variables:

$$i_t = \beta_0 + \beta_y y_t + \beta_\pi \pi_t + \beta_{i1} i_{t-1}$$

for a funding-currency country and

$$i_t^* = \beta_y y_t^* + \beta_\pi \pi_t^* + \beta_{i1} i_{t-1}^*$$

for a target-currency country.

Therefore, by the Taylor rule, the interest rate differential is

$$i_t - i_t^* = \beta_0 + \beta_y \left(y_t - y_t^*\right) + \beta_\pi \left(\pi_t - \pi_t^*\right) + \beta_{i1} \left(i_{t-1} - i_{t-1}^*\right) \quad (3)$$

Since many studies have shown that the exchange rate possibly depends on nonlinear terms of fundamentals (Chinn 1991, 2008; Taylor et al., 2001; Kilian and Taylor, 2003; Rossi, 2005), a type of nonlinearity is considered as follows:

$$O\left(i_t - i_t^*\right) = \beta_{i2}\left(i_t - i_t^*\right)^2 + \beta_{i3}\left(i_t - i_t^*\right)^3 \tag{4}$$

The regression equation has a generated regressor $\hat{F}_t$, which is called the factor. The factor is estimated from the UIRP,

$$\Delta s_{t+1} = F_t + \left(i_t - i_t^*\right) + \varepsilon_{t+1} \tag{5}$$

The unobservable F_t is the persistent component of risk premium of exchange rates. The UIRP simply states that the current spot rate is expected to depreciate/appreciate by the amount of the interest rate differential ex ante. If the UIRP holds, a linear regression of the change in exchange rate onto the interest rate differential should yield a coefficient of one. Unfortunately, most empirical studies have found the coefficient to be near zero or negative. Engel and West (2005) argued that the failure of the UIRP has to do with the unobservable component F_t and this component contains useful information for predicting exchange rates. For this reason, we will apply the UIRP to estimating the factor. In addition, most studies find that the risk-premium process is persistent and time-varying (Engel and West, 2005). By this notion, combing the UIRP, we assume that the factor follows the dynamic process

$$\Delta s_{t+1} - (i_t - i_t^*) = F_t + \varepsilon_{t+1}$$
$$F_t = aF_{t-1} + w_t \tag{6}$$

where ε_t and v_t are i.i.d. white noises. The parameter a measures the persistence of the factor and is between 0 and 1. Since the regression Equation (1) contains the estimate of such an unobservable factor F, we called the type of regression as the factor augmented regression (FAR) (Bai and Ng, 2008).

The procedure to estimate the unobservable factor F_t involves the Kalman filtering technique and the maximum likelihood estimation. The estimation procedure is presented in the Appendix. Readers are referred to Hamilton (1994) and Green (2003) for standard treatment of Equation (6) on estimating the dynamic factor. See Bai (2003) and Stock and Watson (2005) for more discussion on statistical inference of the dynamic factor model.

In summary, the following is a complete list of models that are evaluated in this paper. They are all derived from Equation (1).

Model 1. Random Walk: $\beta_F = 0, z_t = \Delta s_{t-1}$

Model 2. AR(1): $\beta_F = 0, z_t = \alpha + \beta_s \Delta s_{t-1}$

Model 3. AR(1) + Taylor rule:
$$\beta_F = 0, \quad z_t = \beta_0 + \beta_y\left(y_t - y_t^*\right) + \beta$$
$$'\left(\pi_t - \pi_t^*\right) + \beta_{i1}\left(i_{t-1} - i_{t-1}^*\right) + \beta_s \Delta s_{t-1}$$

Model 4. Taylor rule:
$$\beta_F = 0, \quad z_t = \beta_0 + \beta_y\left(y_t - y_t^*\right) + \beta_\pi\left(\pi_t - \pi_t^*\right) + \beta_{i1}\left(i_{t-1} - i_{t-1}^*\right)$$

Model 5 Taylor rule + Nonlinear:
$$\beta_F = 0, \quad z_t = \beta_0 + \beta_y\left(y_t - y_t^*\right) + \beta_\pi\left(\pi_t - \pi_t^*\right)$$
$$\tau_t^*\right) + \beta_{i1}\left(i_{t-1} - i_{t-1}^*\right) + \beta_{i2}\left(i_t - i_t^*\right)^2 + \beta_{i3}\left(i_t - i_t^*\right)^3$$

Model 6. Factor + Taylor rule:
$$z_t = \beta_0 + \beta_y\left(y_t - y_t^*\right) + \beta_\pi\left(\pi_t - \pi_t^*\right) + \beta_{i1}\left(i_{t-1} - i_{t-1}^*\right)$$

Model 6. Factor + Taylor rule + Nonlinear:
$$z_t = \beta_0 + \beta_y\left(y_t - y_t^*\right) + \beta_\pi\left(\pi_t - \pi_t^*\right) + \beta_{i1}$$
$$\beta_{i1}\left(i_{t-1} - i_{t-1}^*\right) + \beta_{i2}\left(i_t - i_t^*\right)^2 + \beta_{i3}\left(i_t - i_t^*\right)^3$$

Carry trade

In the simulation, a carry trade is a binary trading strategy in spot markets that is based on projected return. The trading rule is that if $i_t^* - i_t > 0$ and the expected return is positive as predicted by the model, there is carry trade between a target foreign currency and a funding currency. We use $c=1$ to denote an execution of carry trade.

$$c_t = \begin{cases} 1 & i_t^* - i_t + E_t\left(\Delta s_{t+1}\right) > 0 \quad \text{and} \quad i_t^* - i_t > 0 \\ 0 & \text{otherwise} \end{cases}$$

Note that under the random walk theory, the change of expected exchange rate is zero, that is, $E_t\left(\Delta s_{t+1}\right) = 0$, so the carry trade decision depends solely on the interest rate differential. This type of carry trade is referred as the naïve carry trade. The size of the carry trade is the amount of borrowed funding currency. The profit to carry trade can be scaled by its size. For one unit of borrowed funding currency, the return to carry trade is calculated as

$$r_t = \begin{cases} i_t^* - i_t + \Delta s_{t+1} & \text{if} \quad c_t = 1 \\ 0 & \text{if} \quad c_t = 0 \end{cases}$$

In periods without a carry trade, the factor model predicts relatively large depreciation of the target currency. This

Table 1. Basic statistics of target currencies against funding currencies.

	Australia	Canada	Euro Zone	New Zealand	United Kingdom	U.S.
			Panel A funding currency: Japanese Yen			
1.Δs	-0.003 (0.034)	-0.003 (0.031)	-0.001 (0.027)	-0.003 (0.034)	-0.004 (0.029)	-0.003 (0.033)
2.i^*-i	0.044 (0.035)	0.036 (0.026)	0.012 (0.026)	0.073 (0.040)	0.035 (0.038)	0.025 (0.030)
3.$\Delta(y^*$-$y)$	0.002(0.018)	0.000 (0.018)	0.000 (0.021)		-0.001 (0.020)	0.000 (0.016)
4.π^*-π		0.002(0.006)	-0.001(0.008)		0.003(0.007)	0.001(0.006)
			Panel B funding currency: Swiss Franc			
1.Δs	-0.004(0.041)	-0.002(0.037)	-0.001(0.015)	-0.002(0.039)	-0.001(0.015)	-0.003(0.036)
2.i^*-i	0.054(0.031)	0.044(0.033)	0.022(0.016)	0.076(0.046)	0.045(0.033)	0.034(0.032)
3.$\Delta(y^*$-$y)$	-0.001(0.033)	0.003(0.370)	0.002(0.361)		0.001(0.360)	0.002(0.360)
4.π^*-π		0.002(0.005)	0.000(0.004)		0.003(0.007)	0.002(0.004

The number in parentheses under each variable is the standard deviation of the indicated variable. An asterisk indicates a non-Japan value, and the absence of an asterisk indicates a Japan value; Δs is the percentage change in the yen exchange rate (a higher value indicates appreciation against the yen); i is the money market rate or government bond yield; Δy is the growth rate of the industrial production index; π is the rate of inflation. Data are monthly, mostly 1973.1–2010.1. Exceptions include a beginning date of 1975.1 for Canada and 1978.5 for New Zealand. Monthly data of CPI for Australia and New Zealand are not available.

implies that shorting the target currency would generate additional profit. We enhance the carry trade by reversing the currency trade in the spot market during these periods. We term this strategy, the enhanced carry trade (ECT). The return to ECT is calculated as

$$r_t = \begin{cases} i_t^* - i_t + \Delta s_{t+1} & \text{if } c_t = 1 \\ -\Delta s_{t+1} & \text{if } c_t = 0 \end{cases}$$

**EMPIRICAL RESULTS**

Data

The monthly series of all variables are collected for the period of January, 1973 to January, 2010 depending on availability. The sample size is 444 (with exception noted below), due to the loss of one observation to differencing. Monthly exchange rates are sampled from the month-end daily exchange rates from the Federal Reserve's FRED. Exchanges rates of the target currency measured in the funding currency are computed as cross rates from their original dollar values. The six target currencies are Australian dollar, Canada dollar, euro, New Zealand dollar, the British pound, and the U.S. dollar. The International Financial Statistics (IFS) CD-ROM is the source for all the fundamental economic variables: industrial production as a proxy for economic growth, consumer prices for inflations, and interest rates. Since CPI and industrial production data for Canada and New Zealand are missing for earlier years, data for January, 1975 to January, 2010 and May, 1978 to January, 2010 are used, respectively. German exchange rates and fundamentals are substituted for those of Euro zone before January, 1999.

The testing of out-of-sample performance starts from

January 1999, when the euro became official. At each month, we use data only up to the prior month for forecasting. Any missing data will be imputed with the "cubic spline" method in Matlab. We then estimate the unobservable factor. After obtaining the sequence of factors, we estimate coefficients of model 6 and model 7 using the OLS method and forecast exchange rates for that month. The forecasting equation is the conditional expectation form of Equation (1):

$$\Delta \hat{s}_{t+1} = E_t(\alpha + \beta_F \hat{F}_t + z_t)$$

The out-of-sample forecast is then used to determine the value of c_t, the decision making of carry trade at time t. Performance is computed using realized exchange rates. Under each funding currency, the process is performed for each of the six nations. The following diagram illustrates how data are used at January 1999. The same process is repeated as we move on to the next period, until January 2010. Therefore, 145 months of trade decision take place in total. Since volatility is not estimated, we can not construct a mean-variance optimal portfolio. Therefore, we present performance of an equally-weighted portfolio of the six carry trades in each funding currency.
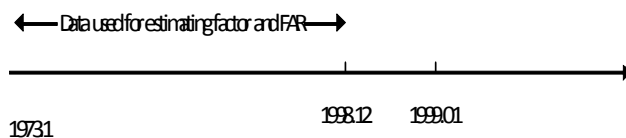
1973.1: January, 1973; 1998.12: December, 1998; 1999.01: January, 1999.

Table 1 presents some basic statistics of the whole sample constructed. The statistics are very much alike under each funding currency. Variables are all volatile because of their relatively large sample deviations compared to their sample means. All countries have

Table 2. Unit root tests.

	Australia	Canada	Euro Zone	New Zealand	United Kingdom	U.S.
			Panel A base currency: Yen			
1.Δs	1,1,1	1,1,1	1,1,1	1,1,1	1,1,1	1,1,1
2.i*-i	0,0,1	1,1,1	1,1,0	0,0,0	1,1,0	0,0,0
3.Δs-(i*-i)	1,1,1	1,1,1	1,1,1	1,1,1	1,1,1	1,1,1
			Panel B base currency: Swiss Franc			
1.Δs	1,1,1	1,1,1	1,1,1	1,1,1	1,1,1	1,1,1
2.i*-i	0,1,0	1,1,0	1,1,1	0,0,0	1,1,0	0,1,0
3.Δs-(i*-i)	1,1,1	1,1,1	0,1,1	1,1,1	1,1,1	1,1,1

H0=Unit root; 1=Accept; 0=Reject. The first number in a cell indicates the result of the augmented Dickey-Fuller test. The second number indicates the result of the Phillips-Perron (PP) test. The third number indicates the result of the Kim-Perron (PP).

higher interest rates on average than each funding-currency country.

Except that the euro zone has a lower average inflation rate then Japan, all countries experienced a higher inflation rate than the funding-currency country. On average, during the period of January, 1973 to 2010, most of the target-currency countries experience higher industrial growth than both funding-currency countries.

Table 2 reports the unit-root tests for three time series: Change in exchange rates (Δs), interest rate differentials (i^*-i), and the risk premium ($\Delta s+(i^*-i)$). The augmented Dickey-Fuller (ADF) test was performed as the benchmark test. The Phillips-Perron (PP) test (1988) was also conducted to increase the robustness of the ADF test results. The null hypothesis is that there is unit root in the time series. To determine whether a constant or a time trend or both exist in the alternative model, we visually examine the time series. Figures 1 to 6 plot the three variables for each funding currency. All changes in exchange rates seem trendless and center on zero so no constant and trend present in the alternative model. For the risk premium, a constant term is added for every target currency for each funding currency; there is no trend except the British pound against yen (or Swiss franc). Interest rate differentials do show some kind of trend for some periods and nonzero mean. So both a constant and a trend are added into the alternative models. To accommodate possible structural breaks during the sample period, we perform a Kim-Perron (KP) (2009) unit root test too. For each test, the result is reported as either 1 (acceptance) or 0 (rejection). The first number in each cell represents result of the ADF test; the second number the PP test; the third number the KP test. Both Δs and $\Delta s-(i^*-i)$ seem to be non-stationary time series because we are unable to reject the null of unit roots in them from any of the tests. For the interest rate differentials, we get some conflicting results from the tests. Although the risk premium process $\Delta s-(i^*-i)$ seems to have a unit root, Kalman filtering still fits to estimate

the dynamic factor because generally the Bayesian method is relatively robust to non-stationary data.

Performance of carry trades

Table 3 reports performance statistics of the equally-weighted portfolio of six carry trades for yen and franc separately. One panel is for one funding currency: The Japanese yen and Swiss franc. There are seven models in simulations. Therefore for each funding currency, we compare the performance of the portfolio among these seven models. Panel A reports the performance for yen carry trades while Panel B is for franc carry trades. Performance statistics include the annualized return, Sharpe ratio; return skewness and the maximum drawdown of returns for the period January, 1999 to January, 2010. Models with fundamentals (Models 3 to 7) generally outperform the RW model or AR (1) model. More importantly, models (6 or 7) with factors (FAR) generate better returns uniformly for yen carry trades and mostly for franc carry trades. For instance, the Sharpe ratio rises to 0.54 (Model 6) or 0.55 (Model 7) from 0.30 (RW) or 0.25 (AR (1)) for the portfolio of yen carry trades.

The maximum drawdown and Skewness are also important performance statistic. The maximum drawdown measures the largest possible loss during the life of the portfolio. Large negative skewness implies the high probability of large losses such as market crashes. The naïve yen carry trade (model 1) has a terrible skewness of -1.30 and maximum drawdown of 38%, while the naïve franc carry trade has a comparable skewness of -0.98 and maximum drawdown of 19%. The reason for such a large downside risk is simple: There are several episodes of target-currency collapses during the simulation period. Every crash in a target currency against the funding currency significantly increases the downside risk in the carry trade.

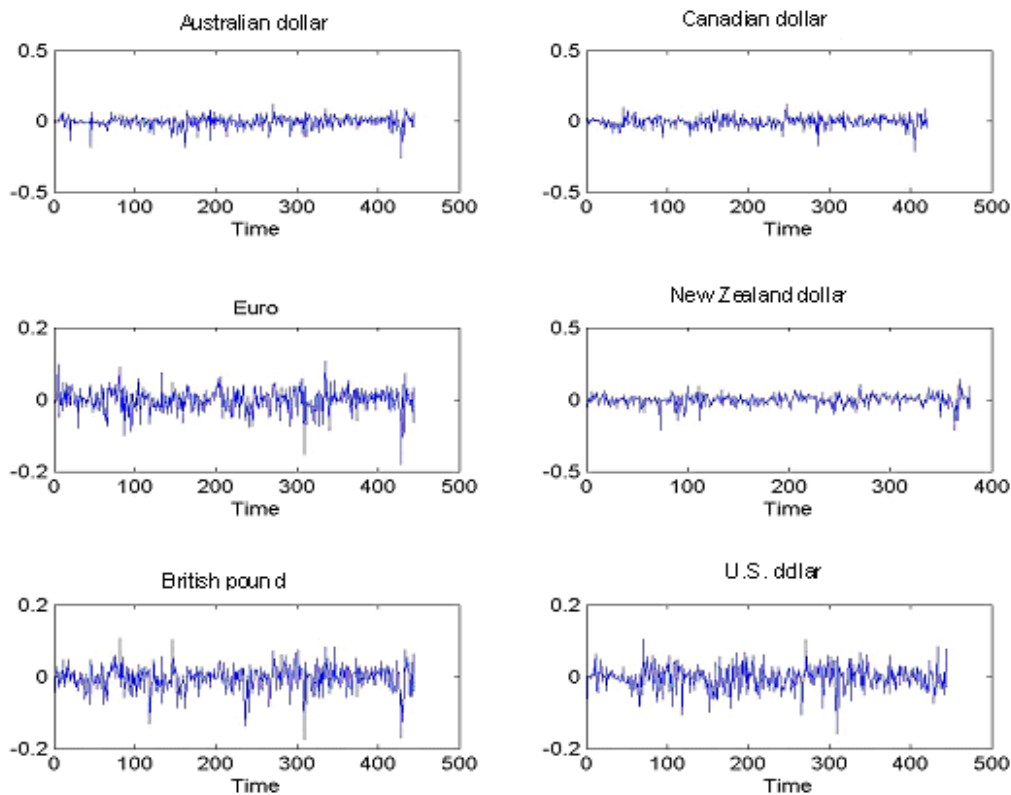We find that FAR models impressively reduce the

Figure 1. Exchange rate returns (based currency: Yen).
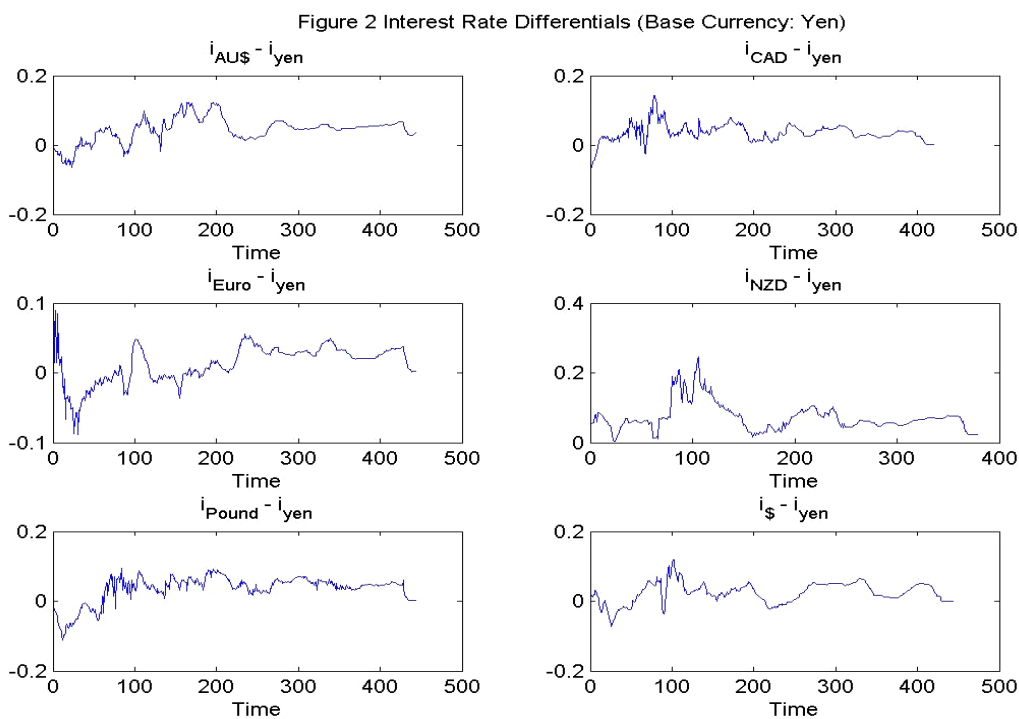

Figure 2. Interest rate differentials (base currency :Yen).
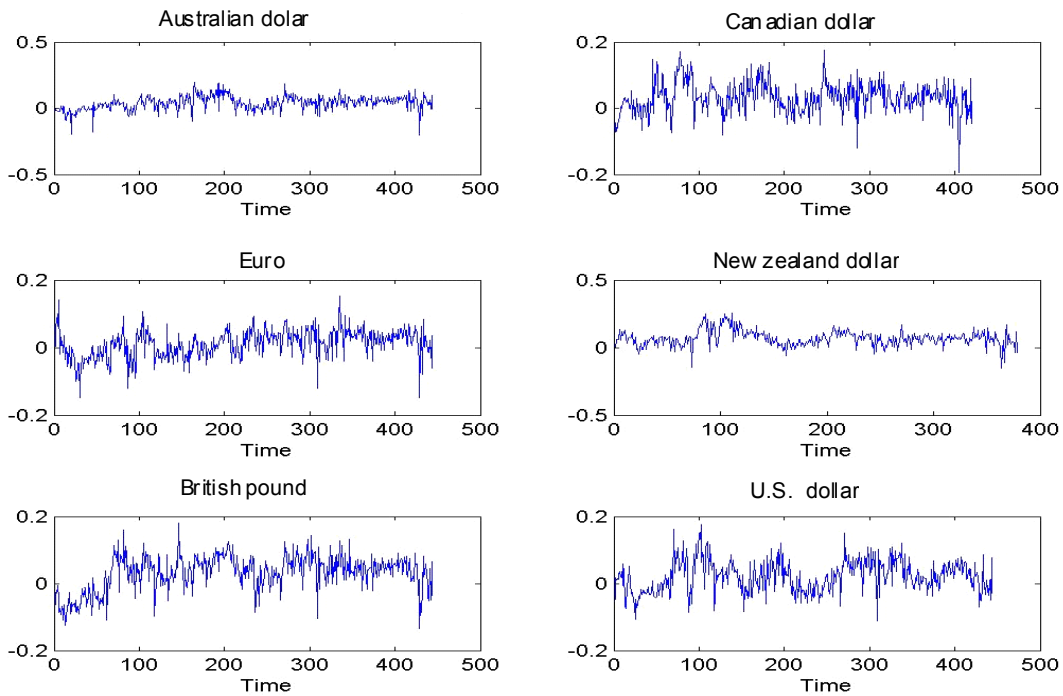
Figure 3. Exchange rate risk premium (Based currency: Yen).
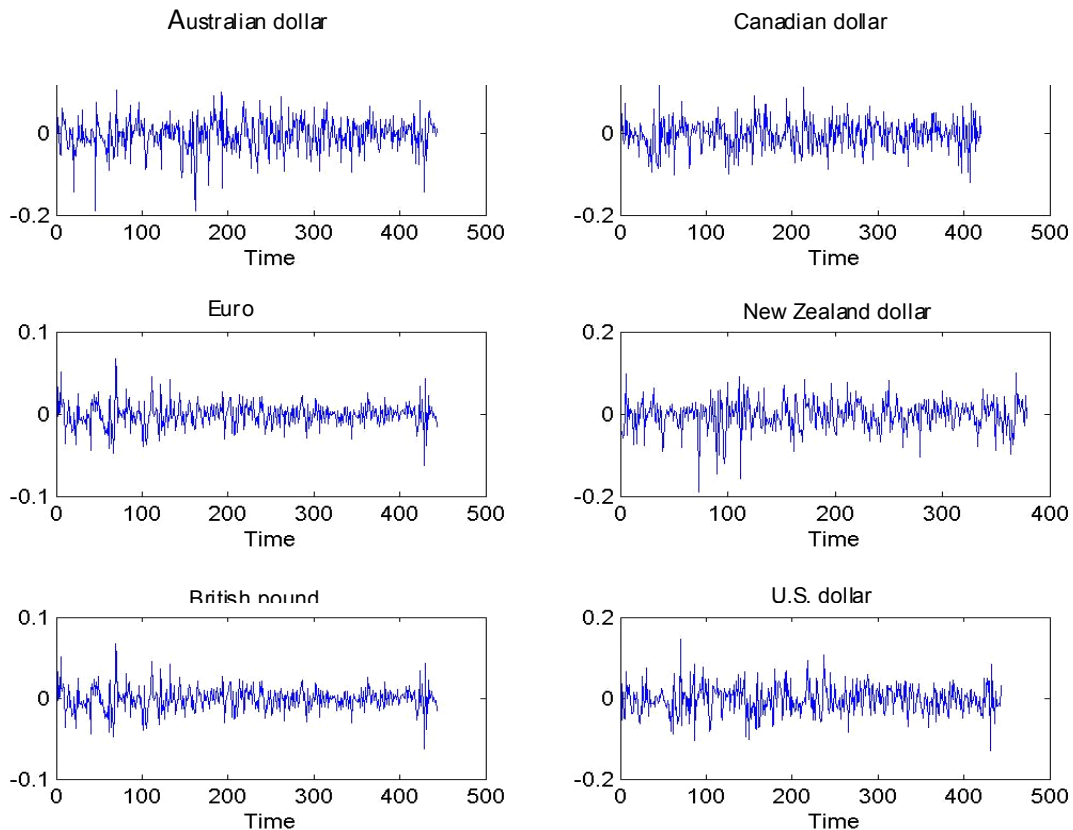
Figure   4.   Exchange   rate   returns   (Based   currency: Swiss   franc).

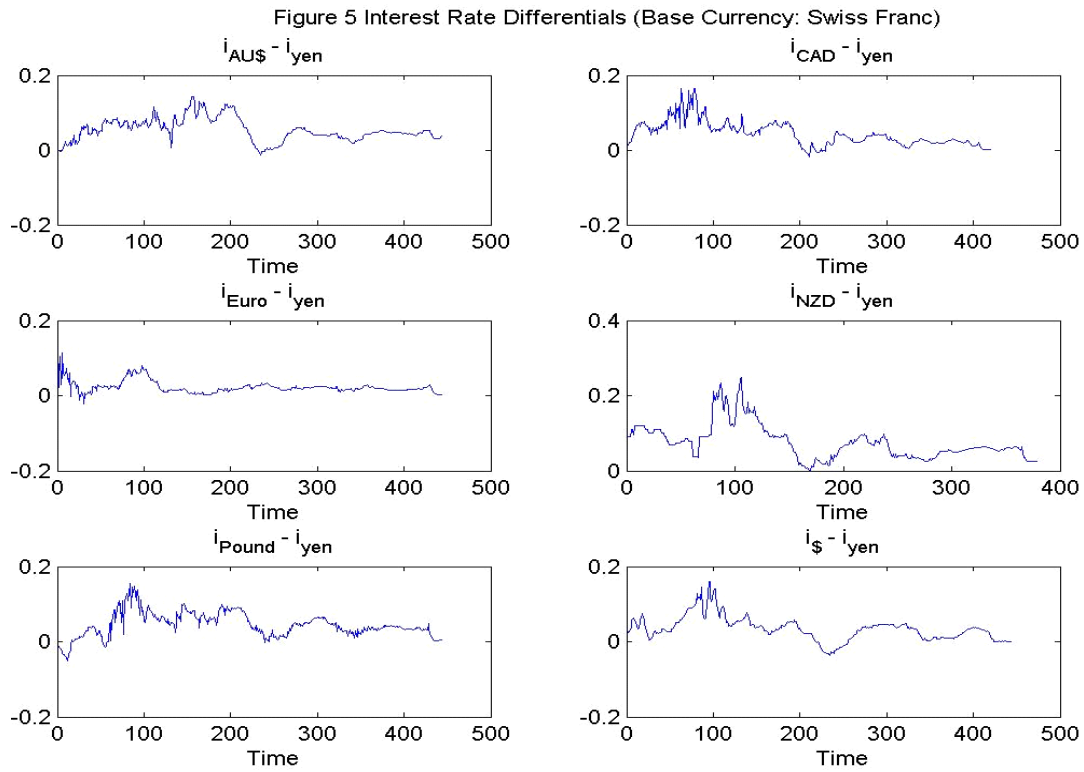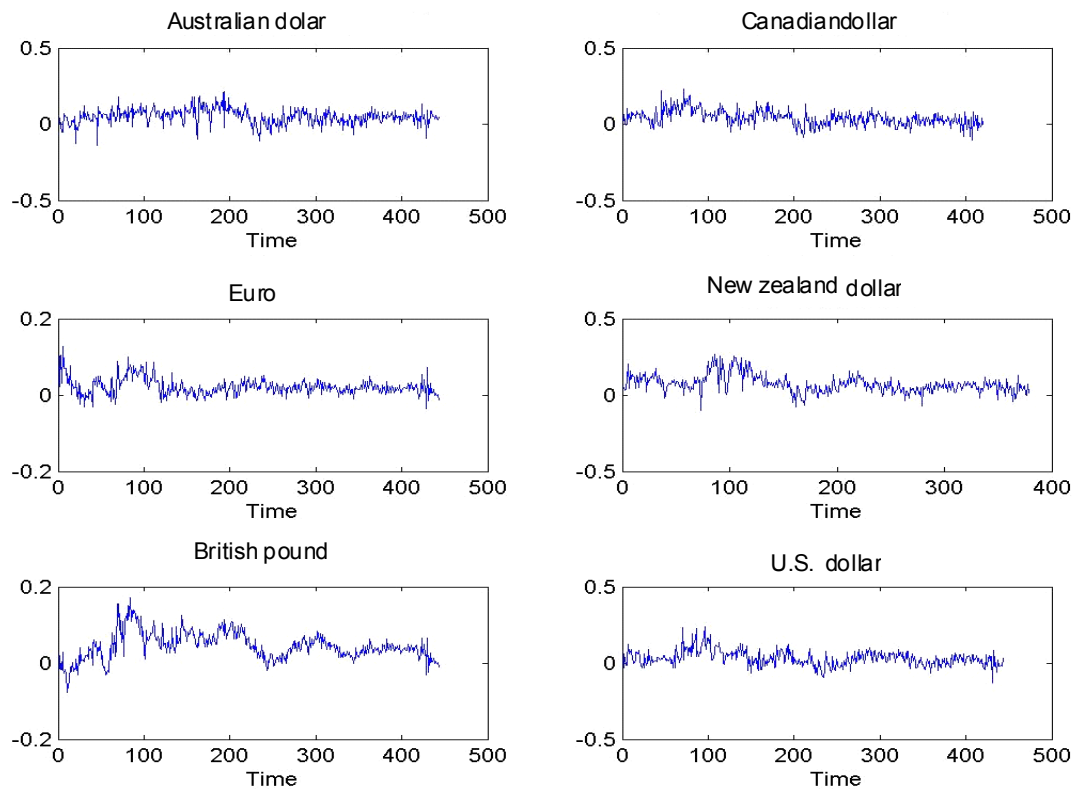Figure 5 Interest Rate Differentials (Base Currency: Swiss Franc)

Figure 5. Interest rate differentials (based currency:Swiss Fran).

Figure 6. Exchange rate risk premium (Base currency: Swiss Frans).

Table 3. Performance statistics of carry trade.

Model	1	2	3	4	5	6	7
Panel A			Funding Currency: Yen				
Mean return	0.03	0.02	0.03	0.03	0.03	0.04	0.04
Sharpe ratio	0.30	0.25	0.34	0.37	0.32	0.54	0.55
Skewness	-1.30	-0.75	-1.47	-1.45	-1.88	-0.11	-0.11
Max drawdown	0.38	0.24	0.30	0.31	0.30	0.13	0.13
Panel B			Funding Currency: Swiss Franc				
Mean return	0.02	0.00	0.03	0.02	0.03	0.02	0.02
Sharpe ratio	0.23	0.06	0.47	0.40	0.52	0.33	0.35
Skewness	-0.98	-1.43	-1.12	-1.02	-1.05	-0.65	-0.53
Max drawdown	0.19	0.15	0.14	0.16	0.12	0.12	0.12

All returns are annualized. The Sharpe ratio is defined as the ratio of mean excess return to the standard deviation.

Table 4. Performance Statistics of Enhanced Carry Trades.

Model	1	2	3	4	5	6	7
Panel A Funding Currency: Yen							
Mean Return	0.03	0.01	0.03	0.04	0.03	0.06	0.06
Sharpe Ratio	0.30	0.19	0.45	0.52	0.45	0.55	0.56
Skewness	-1.30	-0.91	-1.05	-1.18	-1.37	1.47	1.52
Max Drawdown	0.38	0.19	0.23	0.22	0.22	0.14	0.14
Panel B Funding Currency: Swiss Franc							
Mean Return	0.02	0.00	0.04	0.03	0.04	0.02	0.02
Sharpe Ratio	0.23	-0.07	0.67	0.54	0.83	0.42	0.48
Skewness	-0.98	0.22	-0.95	-0.85	-0.41	-0.32	0.22
Max Drawdown	0.19	0.19	0.10	0.15	0.09	0.09	0.08

downside risk. In the yen carry trades, the FAR models (6 or 7) improve the skewness to -0.11 and lower the maximum drawdown to 0.13. In the franc carry trades, similar improvement in the downside risk can be seen: Skewness rises to -0.65 and the maximum drawdown drops to 0.12 in either model 6 or 7.

Enhanced carry trades (ECTs) are generally better than the non-enhanced carry trades by any measure; hence certainly better than the naïve or AR (1) carry trades. The results are shown in Table 4. The results tell a similar story as in Table 3, but with much better performance for the FAR models. For yen carry trades, the FAR models generate a Sharpe ratio of 0.55 (Model 6) or 0.56 (Model 7), and a greatly improved skewness of 1.47 (model 6) or 1.52 (model 7) for the portfolio. The maximum drawdown is reduced to only 14% (model 6 and 7). For franc carry trades, the improvement is similar. The nonlinear terms in Models 5 or 7 does help the corresponding carry trades outperform their counterparts sometimes, but not significantly. A couple of reasons may cause this. First, the factor had already absorbed all relevant information that otherwise is contained in the nonlinear terms.

Second, the specification of nonlinearity may not hold up well simply because of lack of knowledge. We did not try other forms of nonlinearity, since we fear the arbitrary nature of such attempts.

Transaction cost

Transaction cost may change the decision making of carry trades that rely on fundamentals, so it will be helpful to see what its effect is. A typical transaction cost per foreign exchange trade is between 2 to 40 base points (Burnside et al., 2006). Using model 6 only, we simulate our carry trades for a transaction cost ranging from 2 to 40 bps per carry trade. For each target currency, we show the return to the equally weighted portfolio of six carry trades. The result in Table 5 indicates that transaction cost does not change the notion that carry trades based on FAR (model 6) outperforms those based on RW or AR (1) models. This is true for both yen and franc carry trades.

Table 5. Returns to carry trade portfolio with transaction costs.

Transaction cost (bps per month)	2	5	10	20	30	40
			Panel A funding currency: Yen			
			Random walk			
Mean return	0.03	0.03	0.02	0.01	0.00	-0.01
Sharpe ratio	0.28	0.25	0.19	0.09	-0.01	-0.12
Skewness	-1.30	-1.30	-1.30	-1.30	-1.30	-1.30
Max drawdown	0.39	0.39	0.40	0.41	0.42	0.43
			Carry trade portfolio			
Mean return	0.04	0.03	0.03	0.02	0.01	0.01
Sharpe ratio	0.52	0.47	0.40	0.31	0.19	0.16
Skewness	-0.11	-0.30	-0.29	-0.01	-0.04	0.06
Max drawdown	0.13	0.13	0.13	0.13	0.12	0.10
			Enhanced carry trade portfolio			
Mean return	0.05	0.05	0.05	0.04	0.03	0.03
Sharpe ratio	0.53	0.49	0.45	0.41	0.34	0.31
Skewness	1.50	1.48	1.52	1.75	1.58	1.85
Max drawdown	0.14	0.14	0.15	0.15	0.14	0.12
			Panel B funding currency: Swiss Franc			
			Random walk			
Mean return	0.02	0.02	0.01	0.00	-0.01	-0.03
Sharpe ratio	0.28	0.23	0.14	-0.02	-0.19	-0.36
Skewness	-0.85	-0.85	-0.85	-0.85	-0.85	-0.85
Max drawdown	0.19	0.20	0.20	0.21	0.28	0.35
			Carry trade portfolio			
Mean return	0.02	0.02	0.01	0.00	0.00	0.00
Sharpe ratio	0.35	0.32	0.28	0.11	-0.06	-0.09
Skewness	-0.38	-0.34	-0.19	-0.25	-0.32	-0.47
Max drawdown	0.12	0.12	0.12	0.11	0.14	0.14
			Enhanced carry trade portfolio			
Mean return	0.02	0.02	0.02	0.01	0.00	0.00
Sharpe ratio	0.44	0.40	0.35	0.15	0.01	-0.05
Skewness	0.11	0.01	0.22	0.11	-0.08	0.06
Max drawdown	0.08	0.09	0.09	0.09	0.11	0.13

All returns are annualized. The Sharpe ratio is defined as the ratio of mean.

Alternative periods of carry trade

FAR-based carry trades are also tested in two other periods, January, 1996 to January, 2010 and January, 2006 to 2010. We test a non-FAR fundamental model 3 and a FAR model 6 only. Table 6 reports the result for both periods.

Panel A reports the performance statistics for January, 1991 to January, 2010 and Panel B reports those for January, 2006 to 2010. Again, performance statistics in Table 6 tell a similar story to those in Tables 3 and 4: Fundamental or FAR model-based carry trade outperforms naïve or AR (1) carry trades.

Why the factor helps

Why does factor help in the carry trades? We may find a little clue in the statistics about the factor. Table 7 list some of the statistics related to the estimated dynamic factor for each funding currency. In Table 7 (a) using yen as the base currency, Panel A reports the contemporaneous correlations between the factor and relevant variables. Panel B reports the correlation between time t-1 factor and time t variables. Table 7 (b) reports similar number for the franc as the base currency. One thing stands out: the factor that is correlated to the exchange rate changes in both ways with a high degree

Table 6. Returns to carry trades in different periods.

Model	1	3	6	6 (ECT)
Panel A 1996.1-2010.1				
Funding currency: Yen				
Mean return	0.04	0.03	0.04	0.05
Sharpe ratio	0.32	0.35	0.49	0.49
Skewness	-1.45	-1.74	-0.50	1.13
Max drawdown	0.38	0.31	0.20	0.16
Funding currency: Swiss Franc				
Mean return	0.03	0.03	0.02	0.03
Sharpe ratio	0.38	0.52	0.47	0.54
Skewness	-0.49	-0.66	-0.08	0.32
Max drawdown	0.25	0.16	0.13	0.10
Panel B 2006.1-2010.1				
Funding currency: Yen				
Mean return	-0.01	0.00	0.02	0.06
Sharpe ratio	-0.06	0.04	0.27	0.47
Skewness	-1.43	-1.56	-0.18	1.65
Max drawdown	0.38	0.31	0.16	0.18
Funding currency: Swiss Franc				
Mean return	0.00	0.00	0.00	0.00
Sharpe ratio	-0.02	0.04	0.06	0.04
Skewness	-1.10	-1.56	-1.18	-0.42
Max drawdown	0.19	0.31	0.16	0.15

All returns are annualized. The Sharpe ratio is defined as the ratio of mean excess return to the standard deviation.

of statistical significance. This is probably the reason that the factor has strong predictive power in carry trades. Another fact is that the factor is very persistent, since its autocorrelation does not die to zero, even at a time lag of 10. Bartholomew (1999), Diebold and Nerlove (1989), and Rossi (2005) show that a persistent factor has a strong predictive power if it is highly correlated with other variables.

REALITY CHECK

PowerShares DB G10 currency harvest (Ticker: DBV) is an exchange traded fund (ETF) that has a naïve carry trade strategy of longing currencies with high interest rates and shorting the three countries with the lowest interest rates. Since its inception in October 2006 until January 2010, PowerShares' average return is -2.10% and its Sharpe ratio is -15.6%, with an unpleasant skewness of -170%. The performance is no better than to that of naïve carry trades in yen or franc (Table 8 under the Yen RW or Swiss Franc RW). The FAR -based (Model 6) carry trade would have a better average return of 5.25% in the enhanced yen carry trade and -0.57% in the enhanced franc carry trade. Their corresponding Sharpe ratios are 35.24 and -8.57%, respectively. Considering the -15.6% Sharpe ratio in DBV and -20%

Sharpe ratio in the S&P500 during the same period, we see this as a stunning performance. The skewness of 155.37% in yen carry trades suggests the FAR model 6 has helped avoid some major crashes in foreign exchange rates. The cumulative return from the comparison group is also plotted in Figure 7. Note that the S&P500 has a *worse* return than any of the trading strategies for the same period.

Conclusion

In this paper, exchange rate models with Taylor-rule fundamental are evaluated by the profitability of carry trades. The results confirm that carry trades based on predictions of pure fundamental models or FAR models would indeed perform better than those based on random walk or AR (1) models.

But the FAR models generate much better results in the performance of carry trades. This implies that the virtue of the fundamentals is mainly in the form of the derived factors. A brief examination of factors shows that the unobservable factor contains very useful information for forecasting future exchange rates. It is highly persistent and correlated with future exchange rates. Given that we do not have a fully working model of exchange rate for prediction, these attributes make the

**Table 7.** Statistics about the estimated factors (a) Yen (b) Swiss Franc.

	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.
(a) Yen						
			Correlation with factor			
Δs	0.39 (0.00)	0.42 (0.00)	0.46 (0.00)	0.38 (0.00)	0.48 (0.00)	0.44 (0.00)
i^*-i	0.11 (0.02)	0.39 (0.00)	0.20 (0.00)	0.24 (0.00)	0.31 (0.00)	0.41 (0.00)
$\Delta(y^*-y)$		-0.17 (0.00)	-0.03 (0.54)		-0.12 (0.01)	-0.04 (0.43)
$\pi^*-\pi$		0.03 (0.59)			0.11 (0.02)	0.10 (0.03)
			Correlation with one-lag factor (t-1)			
Δs	0.21 (0.00)	0.34 (0.00)	0.31 (0.00)	0.31 (0.00)	0.30 (0.00)	0.27 (0.00)
i^*-i	0.10 (0.04)	0.38 (0.00)	0.18 (0.00)	0.24 (0.00)	0.32 (0.00)	0.38 (0.00)
$\Delta(y^*-y)$		-0.19 (0.00)	-0.03 (0.58)		-0.17 (0.00)	-0.06 (0.22)
$\pi^*-\pi$		0.00 (0.94)			0.11 (0.02)	0.07 (0.12)
Lags		**Autocorrelation of factor**				
1	0.74	0.97	0.90	0.98	0.87	0.87
2	0.46	0.90	0.73	0.93	0.67	0.70
3	0.22	0.82	0.55	0.87	0.47	0.54
4	0.06	0.73	0.38	0.79	0.32	0.40
5	-0.03	0.64	0.26	0.72	0.21	0.29
6	-0.04	0.56	0.18	0.65	0.14	0.22
7	0.02	0.49	0.14	0.58	0.14	0.18
8	0.05	0.43	0.10	0.52	0.16	0.17
9	0.08	0.38	0.07	0.46	0.17	0.15
10	0.09	0.32	0.02	0.40	0.16	0.14
(b) Swiss Franc						
			Correlation with factor			
Δs	0.03 (0.54)	0.35 (0.00)	0.09 (0.05)	0.22 (0.00)	0.10 (0.03)	0.39 (0.00)
i^*-i	0.01 (0.87)	0.33 (0.00)	0.08 (0.10)	0.10 (0.06)	0.23(0.00)	0.32 (0.00)
$\Delta(y^*-y)$		0.01 (0.89)	0.01 (0.78)		0.02 (0.72)	-0.01 (0.77)
$\pi^*-\pi$		-0.03 (0.59)			-0.02 (0.65)	-0.03 (0.50)
			Correlation with one-lag factor			
Δs	0.07 (0.14)	0.30 (0.00)	0.02 (0.74)	0.07 (0.16)	0.03 (0.54)	0.23 (0.00)
i^*-i	0.00 (0.94)	0.33 (0.00)	0.07 (0.13)	0.10 (0.06)	0.20 (0.00)	0.30 (0.00)
$\Delta(y^*-y)$		0.01 (0.91)	0.01 (0.76)		0.01 (0.76)	-0.01 (0.76)
$\pi^*-\pi$		-0.04 (0.46)			0.04 (0.42)	-0.04 (0.46)
Lags			**Autocorrelation of factor**			
1	0.03	0.99	0.10	0.48	0.13	0.86
2	0.07	0.95	0.02	0.19	0.07	0.71
3	-0.02	0.91	0.01	0.02	0.06	0.58
4	-0.08	0.85	-0.01	-0.07	0.03	0.46
5	-0.01	0.79	-0.10	-0.07	-0.06	0.36
6	-0.07	0.73	-0.05	-0.10	-0.01	0.28
7	0.07	0.67	0.07	-0.02	0.10	0.22
8	-0.02	0.60	-0.04	-0.01	-0.01	0.19
9	0.08	0.54	0.01	0.03	0.05	0.16
10	0.00	0.47	-0.03	-0.01	0.01	0.14

Numbers in parentheses are the statistical p-value. A lower p-value indicates the correlation coefficient is more significantly different from zero.

Table 8. Performance compared to G10 ETF since October 2006.

	G10 (%)	Yen RW (%)	Yen ECT (%)	Swiss Franc RW (%)	Swiss Franc ECT (%)
Mean return	-2.10	-3.19	5.25	-0.23	-0.57
Sharpe ratio	-15.60	-19.72	35.24	-2.59	-8.57
Skewness	-170.00	-126.45	155.37	-108.63	-39.09
Max drawdown	23.55	38.32	18.71	19.06	15.07

Monthly returns are annualized. G10's return data is obtained from finance.yahoo.com.
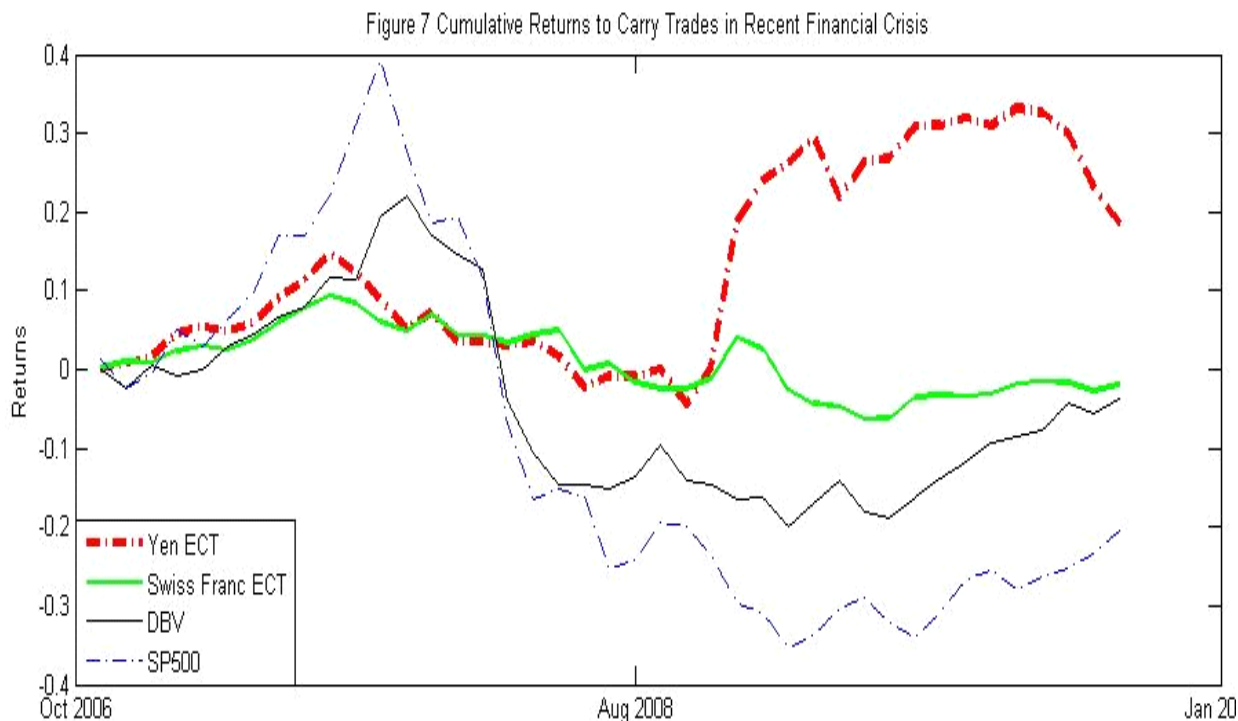
Figure 7. Cumulative returns to carry trades in recent financial crisis.

FAR model the attractive alternative for exchange rate predictability. We hope this study will contribute in this direction.

This study contributes a novel method to evaluate the exchange rate models too. It provides another angle to "beating" the random walk model. In recent years, many have found improvement in the forecasting ability of the exchange rate models when they incorporate the Taylor rule (Choiet al., 2006; Gali and Monacelli, 2005; Engel and West, 2006; Engel et al., 2007; MacDonald and Taylor, 1994; Mark, 1995; Murray and Papell, 2002; Taylor et al., 2001). But the improvement mostly shows up in the long-term forecasting in the mean root squared prediction errors (MRSPE). In this paper, fundamentals in the Taylor rule appear to boost the profits of carry trade in a monthly frequency over the naïve carry trade based on non-fundamentals forecasts. It suggests that fundamental models of exchange rates are superior in the forecasts of

carry trading. It is indispensable for better investment performance for practitioners.

REFERENCES

Bai J (2003). Inferential Theory for Factor Models of Large Dimensions. Econometrica, Econ. Soc., 71(1): 135-171.
Bai J, Serena N (2008). Large Dimensional Factor Analysis. Foundations Trends Econom., 3(2): 89-163
Bartholomew DJ, Knott M (1999). Latent variable models and factor analysis (2nd edition). Kendall's Library of Statistics 7. London: Arnold.
Burnside C, Martin E, Isaac K, Sergio R (2006). The Returns to Currency Speculation. National Bureau of Economic Research. Working, p. 12489.
Cheung YW, Menzie DC, Antonio GP (2002). Empirical Exchange Rate Models of the Nineties: Are Any Fit to Survive? NBER, Cambridge, MA. Working, p. 9393.
Chinn MD (1991). Some Linear and Nonlinear Thoughts on Exchange Rates. J. Int. Money Finan., 10: 214-230.

Chinn MD (2008). Nonlinearities, Business Cycles and Exchange Rates. Mimeo.

Diebold FX, Nerlove M (1989). The dynamics of exchange rate volatility: A multivariate latent ARCH model. J. Appl. Econ., 4:1-21.

Engel C (1996). The Forward Discount Anomaly and the Risk Premium: A Survey of Recent Evidence. J. Empir. Finan., 3: 123–191.

Engel C, Kenneth DW (2005). Exchange Rates and Fundamentals. J. Polit. Econ., 113: 485-517.

Engel C, Kenneth DW (2006). Taylor Rules and the Deutschemark-Dollar Real Exchange Rate. J. Money, Credit. Bank., 38: 1175-1194.

Engel C, Nelson CM, Kenneth D (2007). Exchange Rate Models Are Not as Bad as You Think. NBER Macroecon. Annu., pp. 381-441.

Engel C, Nelson CM, Kenneth DW (2008). Factor Model Forecasts of Exchange rates. Working Paper, Department of Economics, University of Wisconsin.

Green W (2003) Econometric Analysis, 5th ed,, Prentice Hall.

Hamilton J (1994). Time Series Analysis. Princeton University Press.

Jordà Ò, Alan MT (2009). The Carry Trade and Fundamentals: Nothing to Fear But FEER Itself. NBER Working National Bureau of Economic Research, p. 15518.

Kim D, Pierre P (2009). Unit root tests allowing for a break in the trend function at an unknown time under both the null and alternative hypotheses. J. Econ., 148: 1:1-13.

Lewis K (1995). Puzzles in International Financial Markets, in Gene Grossman and Kenneth Rogoff (eds.) Handbook of International Economics, Amsterdam: North-Holland.

Kilian L, Mark PT (2003) Why Is It So Difficult to Beat the Random Walk Forecast of Exchange Rates, J. Int. Econ., 60: 85–107.

MacDonald R, Taylor MP (1994). The monetary model of the exchange rate: Long-run relationships, short-run dynamics and how to beat a random walk. J. Int. Money Finan. Elsevier, 13(3): 276-290.

Mark N (1995). Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability. Am. Econ. Rev., 85: 201-218.

Mark NC, Donggyu Sul (2001). Nominal Exchange Rates and Monetary Fundamentals: Evidence from a Small Post–Bretton Woods Sample. J. Int. Econ., 53: 29–52.

Meese RA, Kenneth SR (1983a). Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample, J. Int. Econ., 14: 324.

Meese RA, Kenneth SR (1983b). The Out of Sample Failure of Empirical Exchange Models. In Exchange Rates and International Macroeconomics, edited by Jacob A. Frenkel. Chicago: Univ. Chicago Press (for NBER).

Molodtsova T, David P (2008). Out-of-Sample Exchange Rate Predictability with Taylor Rule Fundamentals, University of Houston.

Murray CJ, Papell DH (2002). The purchasing power parity persistence paradigm. J. Int. Econ. Elsevier, 56(1): 1-19.

Phillips PCB, Perron P (1988). Testing for a Unit Root in Time Series Regression. Biometrika, 75: 335–346.

Rossi B (2005). Testing Long-Horizon Predictive Ability with High Persistence, and the Meese-Rogoff Puzzle, Int. Econ. Rev., 46: 61-92.

Stock JH, Watson MW (2005). Implications of dynamic factor models for VAR analysis. NBER Work., p. 11467.

Taylor MP, David AP, Lucio S (2001). Nonlinear Mean-Reversion in Real Exchange Rates: Toward a Solution to the Purchasing Power Parity Puzzles. Int. Econ. Rev., 42: 1015–1042.

Vistesen C (1996). Carry Trade Fundamentals and the Financial Crisis 2007-2010, Working Paper.

APPENDIX: ALGORITHM FOR ESTIMATING THE DYNAMIC FACTOR

The Kalman filtering and the maximum likelihood (ML) method estimate the time-varying factor through two steps: (1) Construct an approximate estimate of the factor using the Kalman filtering. (2) The approximate factor is substituted into the likelihood function to estimate unknown parameters using the ML estimation. These two steps are repeated until the estimates of parameters converge. This algorithm is also called the expectation-maximization (EM) algorithm. Because the algorithm utilizes the Kalman filtering, we briefly introduce the Kalman filtering first.

Kalman filtering

The Kalman filtering is a set of mathematical equations that provides an efficient computational (recursive) solution to estimating unobservable factors. Readers are referred to Hamilton (1994). Hamilton (1994) gave an extensive discussion on the Kalman filtering for time series.

The Kalman filtering addresses the general problem of estimating the hidden factor $F \in R$ of a discrete-time process that is governed by the linear stochastic difference equation

$$F_t = aF_{t-1} + w_t$$

where a belongs to the interval of (0,1). The observed risk premium $y \equiv \Delta s + (i^* - i)$ follows

$$y_t = F_t + v_t$$

where a belongs to the interval of (0,1). The observed risk premium $y \equiv \Delta s + (i^* - i)$ follows

$$y_t = F_t + v_t$$

The random variables w_t and v_t represent the process and observation noise respectively. They are assumed to be independent of each other, Gaussian and with probability distribution:

$$w \sim N(0,Q), \quad v \sim N(0,R)$$

The factor F is assumed to start with the initial value $F_0 \sim N(\pi_0, V_0)$.

Suppose we have already observed a sequence of y at time t, denoted by $y^t = \{y_1, \ldots, y_t\}$. The best estimate of the factor F_t at time t is its conditional expectation on y^t, that is, $\hat{F}_t = E(F_t|y^t)$. Because noises are Gaussian, the conditional expectation is the same as the generalized least-squares estimate. Calculating $\hat{F}_t$ for each t is tedious if we apply the conditional expectation every time period. The Kalman filtering provides a very efficient way to calculate $\hat{F}_t$ by a set of recursive equations. The recursive formula is shown below.

$$\hat{F}_t = a\hat{F}_{t-1} + K_t(y_t - \hat{F}_{t-1})$$

$$K_t = \left(a^2 P_{t-1} + Q\right) \cdot \left[\left(a^2 \cdot P_{t-1} + Q\right) + R\right]^{-1}$$

$$P_t = \left(I - K_t\right)\left(a^2 P_{t-1} + Q\right)$$

with the initial values $P_0 = V_0$ and $\hat{F}_0 = \pi_0$.

Maximum likelihood estimation

Here we explain how the parameters are estimated. Assume we have observed a sample y^T. Let $f(y,F|F_0)$ denote the joint density of the observable $y^T = \{y_1, \ldots, y_T\}$ and unobservable factors $F^T = \{F_1, F_2, \ldots, F_T\}$ so that

$$f\left(y, F \mid F_0\right) = f(F_0)\Pi_{t=1}^T \{f(y_t \mid F_t) f(F_t \mid F_{t-1})\}$$

Where,

$$f\left(y_t \mid F_t\right) = \exp\left\{-\tfrac{1}{2}\left(y_t - F_t\right)^2 R^{-1}\right\}\left(2\pi R\right)^{-1/2}$$

and

$$f\left(F_t \mid F_{t-1}\right) = \exp\left\{-\tfrac{1}{2}\left(F_t - aF_{t-1}\right)^2 Q^{-1}\right\}\left(2\pi Q\right)^{-1/2}.$$

Then the log-likelihood function is given by

$$\ln L = \log\left(f\left(y, F \mid F_0\right)\right) = -\sum_{t=1}^T \left\{\frac{1}{2}\left(y_t - F\right)^2 R^{-1}\right\} - \frac{T}{2}\log|R|$$
$$-\sum_{t=1}^T \left\{-\frac{1}{2}\left(F_t - aF_{t-1}\right)^2 Q^{-1}\right\} - \frac{T}{2}\log|Q|$$
$$-\frac{1}{2}\left(F_0 - \pi_0\right)^2 V_0^{-1} - \frac{1}{2}\log|V_0| - T\log 2\pi$$

The parameters to be estimated are $\xi = \{a, Q, R, \pi_0, V_0\}$. Since F^T are not observable, the maximum likelihood method is practically impossible. A

way to get around this problem is to replace the factor with the Kalman filtering estimates $\hat{F}_t$.

The maximum likelihood estimation of ξ is carried out recursively. At iteration l, an estimate $\xi^{(l)}$ is obtained from the previous estimate $\xi^{(l-1)}$. The iterative process will stop until the new estimate cannot improve the log-likelihood. The following steps illustrate the iteration process.

Step 1: Set $l=0$ and choose $\xi^{(0)}$ with a good guess.
Step 2: Set $\xi = \xi^{(l)}$. Calculate the conditional expectation of the log-likelihood $\ln L$ on y^T, $E(\ln L|y^T)$. It involves calculating $E(F_t|y^T)$, $E(F_t^2|y^T)$ and $E(F_t F_{t-1}|y^T)$. They are

computed using the factor estimates $\hat{F}_t$, which are conveniently computed from the Kalman filtering.
Step 3: Maximize the log-likelihood function $E(\ln L|y^T)$ to obtain a new estimate $\xi^{(l+1)}$. In this step, we use the first order necessary condition or the generalized least squared to estimate the parameters $\xi^{(l+1)}$.
Step 4: Repeat Steps 2 and 3 until a stopping criterion is satisfied.