# Full Length Research Paper

# Modelling daily value-at-risk using realized volatility, non-linear support vector machine and ARCH type models

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The aim of this paper is to compare the performance of the daily nonlinear support vector machines, the new semi-parametric tool for regression estimation, heterogeneous autoregressive (SVM-HAR)-ARCH type models based on the daily realized volatility (which uses intraday returns) with the performance of the classical HAR-ARCH type models by using different innovation distribution when the one-day ahead value-at-risk (VaR) is to be computed. The daily realized volatility is calculated using 5-, 15-min and optimally sampled intraday returns for Nikkei 225 index. This paper shows that the particular hybrid SVM-HAR-ARCH type model provides better performance when 15-min intraday returns are used. This paper also shows that the models based on a long memory skewed student distribution provide the better performance of one-day ahead value-at-risk forecasts.

**Key words:** Value-at-risk, HAR-RV model, nonlinear support vector machine-HAR-RV model, ARCH type models, Skewed student distribution, high frequency Nikkei 225 data.

## INTRODUCTION

Value-at-Risk (VaR), a measure of how the market value of an asset or a portfolio of assets is likely to decrease over a specific time period under typical conditions, has become the standard tool for measuring and reporting financial risk. Due to the recent availability of high-frequency intraday data for the financial variables (stock indexes, exchange rates, bonds etc) most of the financial researchers are mainly concerned with modeling and forecasting volatility, which is the key input to VaR modelling, in asset returns to quantify the risk of financial instruments over a particular time period, which is taken to be equal to one day in this paper.

Researches on time varying volatility and risk measure using the time series models have been active ever since Engle (1982) introduced the ARCH model. The GARCH model, generalized by Bollerslev (1986), has been extended in various directions and these extensions recognize

(based on the various researcher's empirical evidences) that there may be important nonlinearity, asymmetry, and long memory properties in the volatility process. The popular extensions can be referred to Nelson's (1991) EGARCH model, Glosten et al.'s (1993) GJR-GARCH which both account for the asymmetric relationship between stock returns and changes in variance (for e. g. Black 1976, the beginning study of the asymmetric effect and Engle and Ng 1993 for further discussion). Engle's (1990) AGARCH, Ding et al. (1993) APARCH; Zakoian's (1994) TGARCH; and Sentana's (1995) QGARCH models also have been developed for the flexibility of the models.

The recent literatures suggest the use of realized volatility, the square of intraday returns introduced in the literature by Taylor and Xu (1997) and Andersen and Bollerslev (1998) along with other researchers, as a measure of actual volatility and have quickly become popular after proposing the heterogeneous autoregressive realized volatility (HAR-RV) model of Corsi et al. (2001) and Corsi (2009) based on the HARCH (Heterogeneous ARCH)

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model of Müller et al. (1997b). Another recent development in the RV literature is the approach due to Barndorff-Nielsen and Shephard (2004, 2006), Andersen et al. (2003, 2007) of decomposing the RV into the contribution of continuous sample path variation and that of jumps. Extending the theory of quadratic variation of semi martingales, Barndorff-Nielsen et al. (2006) provided an asymptotic statistical foundation for this decomposition procedure under very general conditions.

This study is closer to Giot and Laurent (2004) and Clements et al. (2008). Giot and Laurent (2004) compare an ARCH-type model and a model using realized volatility in terms of forecasts of Value-at-Risk. They show better performance of Skewed student APARCH model. Clements et al. (2008) narrow their study to focus exclusively on models based on realized volatility and show comparatively better performance of HAR model for quantile forecasts. In this paper, the HAR-ARCH type models have been considered for one-day ahead Valueat Risk forecasts. The aim is to compare the performance of classical HAR-ARCH type models with the nonlinear support vector machine (SVM)-HAR-ARCH type models in terms of Value-at-Risk (VaR) forecasts, where the SVM is an efficient semi-parametric approach introduced by Vepnik, (1995) that guarantees to obtain globally optimal solution, (Cristianini and Shawe-Taylor, 2000), which solves the problems of multiple local optima in which the neural network usually get trapped into. The neural network (NN) and the SVM are being applied in different financial literatures. For example, Donaldson and Kamstra (1997) used neural network to model volatility based GJR-GARCH, Bildirici and Ersin (2009) fitted neural network based on nine different GARCH family models such as NN-GARCH, NN-EGARCH, NN-TGARCH, NN-GJR-GARCH, NN-SAGARCH. PGARCH, NN-NGARCH, NN-APGARCH and NPGARCH to forecast Istanbul stock volatility, McAleer and Medeiros (2009) proposed the NN-HAR model and estimated with Bayesian regul0061rization (BR) to forecast the daily volatility of the S&P 500 and FTSE 100 indexes, and more recently Dunis et al. (2010) applies higher order neural networks for modeling commodity Value-at-Risk of the Brent oil and gold bullion series with only autoregressive terms as inputs. In the very beginning, Müller et al. (1997a) applied SVM for time series forecasting to compare the performance of  $\epsilon$ insensitive loss and Huber's robust loss function. After that, Perez-Perez-Cruz et al. (2003) predicted GARCH (1,1) based volatility by SVM, Chen et al. (2008) proposed recurrent SVM as a dynamic process to model GARCH (1,1) based Volatility, Ou and Wang (2010) proposed GARCH-LSSVM, EGARCH-LSSVM and GJR-LSSVM hybrid models based on modification of Suykens and Vandewalle (1999) to forecast the leverage effect volatilities of ASEAN stock markets. These indicate that the hybrid models can also capture the stylized characteristics of time series. In this paper, the nonlinear SVM

and HAR are combined as nonlinear SVM-HAR model. To the author's knowledge, this paper is the first to apply the nonlinear SVM-HAR model to RV literature in the daily VaR modelling context.

When ARCH type models are used, a well established result in the financial time series literature is that the standardized returns do not have a Gaussian distribution. Andersen et al. (2000a, b, 2001, 2003) showed that the distribution of the standardized exchange rate series was almost Gaussian when the realized volatility (RV) was used. Furthermore, the logarithm of the realized volatilities was also nearly Gaussian. Other literatures on realized volatility can be refer along with many researchers to Ait-Sahalia and Mancini (2006), Ghysels and Sinko (2006), Ghysels et al. (2006), Corradi et al. (2006). According to Giot and Laurent (2004), "the key issue is to use a daily ARCH type model that clearly recognizes and fully takes into account the key features of the empirical data such as high kurtosis and skewness in the observed returns." In this paper, the RiskMetrics, GARCH, GJR, EGARCH and APARCH models are being considered along with Gaussian, Student's t and Skewed Student distributions to capture the features of the empirical data.

# REALIZED VOLATILITY, REALIZED BI-POWER VARIATION AND JUMP COMPONENT EXTRACTION

If we consider a simple diffusion process

$$dp(s) = \mu(s)dt + \sigma(s)dW(s)$$

where p(s)dt is the instantaneous log-price, W(s) is a standard Brownian process and  $\sigma(s)$  is the standard deviation of dp(s), which may be time-varying but is assumed to be independent of dW(s). Then the volatility for day t is defined as the integral of  $\sigma^2(s)$  over the interval (t,t+1) that is,  $\int_t^{t+1}\sigma^2(s)ds$ , which is known as integrated volatility and it is unobserved. Let the discretely sampled  $\Delta$ -period returns be denoted by,  $r_{t,\Delta}=p(t)-p(t-\Delta)$ . If the process (in our case the log of Nikkei 225 index level process) is a continuous semimartingale then under mild regularity conditions,

$$RV_t \equiv \sum_{j=1}^{1/\Delta} |r_{t+j\Delta,\Delta}|^2 \stackrel{p}{\to} \int_t^{t+1} \sigma^2(s) ds$$
 as  $\Delta \downarrow 0$ 

 $RV_t$  is the *t*-th day realized variance since *t* has the daily unit and  $\left(\frac{1}{\Delta}\right)$  is integer. We will hereafter use the terms realized volatility or realized variance interchangeably, or their common abbreviation RV.

Again, if the process is semimartingale with finite-activity jumps, that is, only a finite number of jumps occurring in any finite time interval, such as Poisson

jumps, then the realized variance converges to the quadratic variation, which can be decomposed as,

$$RV_t \stackrel{p}{\to} \int_t^{t+1} \sigma^2(s) ds + \sum_{t < s \le t+1} k^2(s) \text{ as } \Delta \downarrow 0$$

where k(s) refers to the size of the jump occurring at time s. Andersen et al. (2007) proposed microstructure-noise-robust versions of the *bipower variation* as

$$BV_t \equiv \mu_1^{-2} (1 - 2\Delta)^{-1} \sum_{i=3}^{1/\Delta} |r_{t+i\Delta,\Delta}| |r_{t+(j-2)\Delta,\Delta}|$$

where  $\mu_1 \equiv \sqrt{2/\pi}$ , holds under mild conditions and proposed to use  $RV_t - BV_t \stackrel{p}{\to} \sum_{t < s \le t+1} k^2(s)$  or  $J_t \equiv max[(RV_t - BV_t), 0]$  as an estimator for  $\sum_{t < s \le t+1} k^2(s) . J_t$  is known to take non-zero, small values very frequently due to measurement and possibly due to the presence of jumps infinite-activity types.

## Data description and summary statistics

# Calculation of intraday returns and RV measures from minute-by-minute Nikkei 225 data

We measure the realized volatility of the Nikkei 225 index for the sample of the period 11 March 1996 to 30 September 2009. Nikkei (Nihon Keizai Shinbun, Inc) computes and disseminates the Nikkei 225 index once every minute during the trading hours of Tokyo Stock Exchange (TSE). In this paper, we contract a "five-minute (percentage) returns" series by taking the five-minute log differences multiplied by hundred from the minute-by-minute data. This choice is made to mitigate the effect of microstructure related noise and increase the precision of volatility measures. (Ishida and Watanabe, 2009; Watanabe and Yamaguchi, 2007).

The Tokyo Stock Exchange is open only for 9:00-11:00 (Morning Session) and 12:30-15:00 (Afternoon Session). Our database includes every minute prices of the Nikkei 225 stock index for both sessions. We first extract prices for 9:01, 9:05, 9:10,......,11:00 in the morning session and for 12:31, 12:35, 12:40,.....,15:00 in the afternoon session. Sometimes, the last transaction price for morning (and/or afternoon) session is observed slightly after 11:00 (and/or 15:00). In such cases, we use the last prices instead of prices at 11:00 (and/or 15:00). Next, by using these prices, we calculate the five-minute returns as previously mentioned. There are 54 five-minute returns for a typical trading day in total, 24 from the morning session and 30 from the afternoon session.

Given the recent literature on the market microstructure noise effect on realized volatility estimation, the optimal choice of sampling frequency as studied by Bandi and Russell (2003, 2008) has been considered here. The sampling frequency  $M^{opt}$  (the number of observations per day) is calculated as (Zhang et al., 2005; Clements et al., 2008)

$$M^{opt} = \left(\frac{\bar{Q}_i}{\hat{a}}\right)^{\frac{1}{3}}$$

Where,

$$\hat{\alpha} = \left(\frac{\sum_{t=1}^T \sum_{j=1}^M r_{j,t}}{r_M}\right)^2 \quad \text{and} \quad \bar{\hat{Q}} = \frac{1}{T} \sum_{t=1}^T \hat{Q}_t \,, \\ \hat{Q}_t = \frac{M_{15}}{3} \sum_{j=1}^{M_{15}} r_{j,t}^4 \,. \\ M_{15} \text{ is the 15-min returns and } M \text{ is the highest frequency at which data are available. In our case, it is 1-min returns.} \quad \text{The 15-min intraday returns are being considered to calculate realized volatility as well.}$$

We cannot calculate the 5-min, 15-min and optimally-sampled returns for the non-trading hours including lunch time and overnight period though we can calculate the lunch time and overnight returns by considering the last price of the morning session and the first prices of the afternoon session, and the last price of the afternoon session and the first price of the next morning session but following Hansen and Lunde (2005), we drop this idea and scale the realized volatility as follow,

$$RV_t \equiv C^* R V_t^{(0)}$$

Where.

$$C^* \equiv \frac{\sum_{t=1}^T (R_t - \bar{R})^2}{\sum_{t=1}^T RV_t^{(0)}}$$
, where  $\bar{R} \equiv \frac{\sum_{t=1}^T R_t}{T}$ 

and T is the number of complete trading days. In our sample period, the first trading in the second session from January 1, 2006 to April 21, 2006 observed at 13:01. Therefore, we remove these trading days along with the sessions from half trading days including the first and the last trading days of each year. The remaining number of complete trading days, T is 3279. We calculate  $RV_t$  and  $J_t$  by using this 3279 days data for the four series.

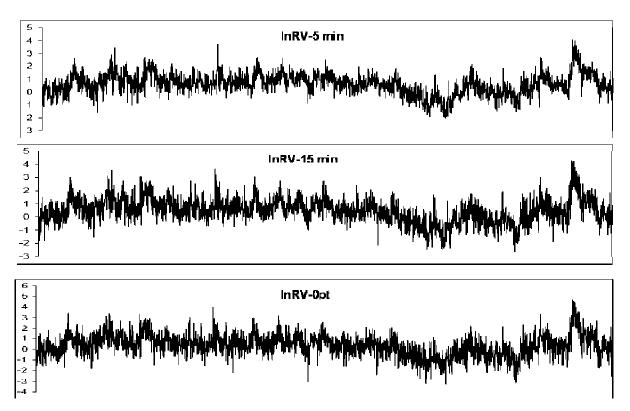
# Properties of the realized volatility and related measures

The average daily optimal sampling frequency was observed 7.828 ( $^{\infty}8$ ), that is,  $\frac{1}{\Delta}=8 \Rightarrow \Delta=33.75$ . 30-min returns are considered as optimally sampled returns here. The value of C for 5-, 15-min and optimally sampled returns were calculated as 2.02977, 1.78022 and 1.75065, respectively. Summary statistics of daily returns series, the daily logarithmic form of RV and jump series are presented on Table 1. In addition to the

Table 1. Summary statistics for Nikkei 225 daily returns, realized volatility and jumps logarithmic series.

Series		Mean	Std. dev.	Skew.	Kurt.	Min.	Max.	Jarque- Bera	LB(5)	LB(10)	LB(22)
Daily return		-0.024	1.621	-0.163	8.351	-11.953	12.912	3898.857	13.589	24.305	41.57
	5-min	0.609	0.817	0.030	3.719	-2.005	4.087	70.550	7710.639	13667.646	24950.93
InRV	15-min	0.496	0.927	0.117	3.615	-2.614	4.246	58.760	6210.035	11092.093	20348.31
	Optimal	0.385	1.027	0.087	3.635	-3.220	4.706	58.809	4620.977	8306.383	15354.54
	5-min	0.792	0.426	1.262	6.220	0.067	3.302	2270.538	6669.961	11674.345	20728.59
In(1+Jump)	15-min	0.724	0.496	1.558	6.509	0.035	3.631	2988.122	4549.871	7996.372	13955.17
	Optimal	0.673	0.550	1.923	8.672	0.009	4.623	6371.333	3082.669	5473.863	9818.06

The sample of the period 11 March 1996 to 30 September 2009, there are total 3279 daily observations. The 5% critical value for Jarque-Bera (that is, 5.991 (k = 2) and 5% critical values for LB (k) are 11.070 (5), 18.924 (10) and 33.924 (22), respectively.



**Figure 1**. Log-realized volatility series. The top plot shows the daily log-RV series using 5-min intraday returns, the middle plot shows the daily log-RV series using 5-min intraday returns and last plot shows the daily log-RV series using 5-min intraday returns optimally sampled intraday return data. The sample period is 11 March 1996 to 30 September 2009. There are 3279 complete trading days.

sample skewness and kurtosis, the Jarque-Bera (JB) statistic for testing normality and the Ljung-Box statistics of order 5, 10 and 22 (corresponding to roughly one week, two weeks and a month) for testing serial correlations up to their respective order are also presented on the Table 1. It was observed that the unconditional distribution of the daily return series is negatively skewed but highly significantly nonnormal with high positive kurtosis. The LB statistics also indicated that

the series are significantly serially correlated. It was also observed that the log transformation of RV's brings down the sample skewness and kurtosis values for all series but still significantly nonnormal. The value of skewness and kurtosis were close to zero (0.030) and three (3.719) for 5-min intraday returns while 0.117, 3.615 for 15-min and 0.087, 3.635 for optimally sampled intraday returns. All the transformed series remain highly significantly serially correlated. Figure 1 shows the daily InRV for

) is

three intraday returns series.

#### **COMPUTING MODELS**

#### Realized volatility models

# Heterogeneous autoregressive realized volatility (HAR-RV) model

The HAR-RV class volatility models proposed by Corsi (2001, 2009) on the basis of a straightforward extension of the so-called heterogeneous ARCH (HARCH) class of models is analyzed by Müller et al. (1997).

To sketch the HAR-RV model, define the multi-period realized volatilities by the normalized sum of the one-period volatilities,

$$RV_{t,t+h} = h^{-1}(RV_{t+1} + RV_{t+2} + \dots + RV_{t+h})$$

Note that, by definition of the daily volatilities,  $RV_{t,t+1} \equiv RV_{t+1}$ . Also, provided the expectations exist,  $E(RV_{t,t+1}) \equiv E(RV_{t+1})$  for all h (Andersen et al., 2003, 2007). Also h = 5 and = 22 will produce the weekly and monthly volatilities, respectively. The daily HAR-RV model of Corsi (2001), Corsi (2009) may then be expressed as:

$$RV_{t+1} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \epsilon_{t+1}$$

where 
$$t = 1, 2, ..., T$$
.

Andersen et al. (2003, 2007) included the jump component, which has been explained previously, as an explanatory variable to the above model and introduced the new model as

$$RV_{t+1} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_I J_t + \epsilon_{t+1}$$

In this paper the logarithmic form of the above model is used as

$$log(RV_{t+1}) = \beta_0 + \beta_D log(RV_t) + \beta_W log(RV_{t-5,t}) + \beta_M log(RV_{t-22,t})$$
$$+ \beta_I log(1+I_t) + \epsilon_{t+1}$$

e.g., (Andersen et al., 2003, 2007)

#### The support vector machines (SVMs)-HAR

The support vector machines (SVMs) were introduced by Vapnik (1995) based on the statistical learning theory, which had been developed over the last three decades by Vapnik, Chervonenkis and others (Vapnik, 1982, 1995) from a nonlinear generalization of the Generalized Portrait algorithm. SVMs were developed to solve the

classification problem, but recently they have been extended to the domain of regression problems (Vapnik et al., 1997). The SVMs usually map data to a highdimensional feature space and apply a simple linear method to the data in that high-dimensional space nonlinearly related to the input space. Moreover, even though we can think of SVMs as a linear algorithm in a high-dimensional space, in practice, it does not involve any computations in that high-dimensional space (Karatzoglou and Meyer, 2006). The terminology for SVMs can be slightly confusing in the literature. In few literatures, SVM refers to both classification and regression with support vector methods. In this paper, the tern SVM will be used for the nonlinear support vector regression (NL-SVR). The mathematical formulation of SVM is as follows.

In the  $\epsilon$ -insensitive support vector regression of Vapnik (1995), our goal is to find a function f(x) that has an  $\epsilon$  deviation from the actually obtained targets  $y_t$  for all training data, and at the same time, is as flat as possible. Suppose f(x) takes the following form

$$f(x) = k\langle \omega, x \rangle + b$$
 with  $\omega \in X, b \in \mathbb{R}$ 

where X is the space of the input patterns and  $k\langle.,.\rangle$  denotes the kernel function. Flatness of the above model means need to find the small  $\omega$ . One way to ensure this is to minimize the Euclidean norm, that is, $\|\omega\|^2$  (Smola, 1998). By applying the *soft margin* formulation of Cortes and Vapnik (1995), and the Karush-Kuhn-Tucker (KKT) conditions (Karush, 1939; Kuhn and Tucker, 1951) one can estimate the above model as

$$f(x) = \sum_{t=1}^{T} (\alpha_t - \alpha_t^*) k \langle x_t, x \rangle + \hat{b}$$

where b can be computed as

$$\hat{b} = y_t - k\langle \omega, x_t \rangle - \epsilon \text{ for } \alpha_t \in (0, C)$$

$$\hat{b} = v_t - k\langle \omega, x_t \rangle + \epsilon \text{ for } \alpha_t^* \in (0, C)$$

where, C>0 determines the trade-off between the flatness of the f(x) and the amount up to which derivations larger than  $\epsilon$  are tolerated and  $\alpha_t, \alpha_t^* \geq 0$  (Smola and Schölkopf, 1998). A several numbers (Kernlab in R, MATLAB, etc) of statistical software are available to handle SVM method.

According to Cortes and Vepnik (1995), any symmetric positive semi-definite function that satisfies the Mercer's conditions can be used as a kernel function in the SVMs context. The Mercer's conditions are

$$\iint K(x,y) g(x)g(y)dxdy > 0$$
 and  $\iint g^2(x)dx < \infty$ ,

where 
$$K(x, y) \equiv \sum_{t=1}^{\infty} \alpha_t \psi(x) \psi(y), \alpha_t \ge 0$$

In this paper, the Laplacian kernel function  $K(x,y) \equiv exp\left(-\frac{\|x-y\|}{\sigma}\right)$  has been considered.

The models that combines SVM technique, previously discussed, and HAR-RV models are known as SVM-HAR-RV models.

Now, when  $\epsilon_{t+1} \sim N(0, \hat{\sigma}_{\epsilon}^2)$  then  $exp(\epsilon_{t+1}) \sim log N(0, \hat{\sigma}_{\epsilon}^2)$ , thus the conditional realized volatility can be computed according to

$$RV_{t+1|t} = exp\left(lnRV_{t+1} - \hat{\epsilon}_{t+1} + \frac{1}{2}\hat{\sigma}_{\epsilon}^2\right)$$

where  $\hat{\epsilon}_{t+1}$  and  $\hat{\sigma}^2_{\epsilon}$  are the estimated value and estimated variance of  $\epsilon_{t+1}$  respectively by HAR-RV-J model using both classical and SVM method separately. To compute the one-day ahead Value-at-Risk forecast, the ARCH type models will be applied on the conditional realized volatility series obtained from the aforementioned expression.

## **ARCH type models**

An ARCH type model can be define as

$$y_t = \mu_t + \epsilon_t$$

$$\epsilon_t = \sigma_t z_t$$

$$\mu_t = c(\eta | \Omega_{t-1})$$

$$\sigma_t = h(\eta | \Omega_{t-1})$$

where  $c(\eta|\Omega_{t-1})$  and  $h(\eta|\Omega_{t-1})$  are functions of  $\Omega_{t-1}$ , the information set at time t-1 and depend on an unknown vector parameters  $\eta$ ,  $z_t$  is a i.i.d. process, independent of  $\Omega_{t-1}$  with  $E(z_t)=0$  and  $Var(z_t)=1$ .  $\mu_t$  and  $\sigma_t$  are the conditional mean and conditional variance respectively.

In this paper, we fit an AR model for the conditional mean and five specifications for the conditional variance, which have been discussed as follows

#### **GARCH** model

Bollerslev (1986) proposed the GARCH model. We use GARCH (1,1) models as

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \omega > 0, \alpha, \beta \ge 0$$

where  $\omega$ ,  $\beta$  and  $\alpha$  are the parameters, which are assumed to be non-negative to guarantee that volatility is always positive. This model is able to capture the volatility clustering.  $|\beta + \alpha| < 1$ , implies that the volatility is stationary and the speed for which the shock to volatility decays becomes slower as  $\beta + \alpha$  approaches to

one.

#### **GJR-GARCH** model

Glosten et al. (1993) proposed the GJR-GARCH model to capture the asymmetry. In this study, we use the GJR-GARCH (1,1) model as

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \lambda \epsilon_{t-1}^2 I_{t-1}$$

where  $I_{t-1}$  is an indicator function with  $I_{t-1}=0$  if  $\epsilon_{t-1}\geq 0$  and with  $I_{t-1}=1$  if  $\epsilon_{t-1}<0$ .  $\omega$ ,  $\beta$ ,  $\alpha$  and  $\lambda$  are the parameters, which are assumed to be non-negative to guarantee that volatility is always positive.

#### **EGARCH** model

It has already established that the general GARCH model cannot capture the well-known volatility asymmetry phenomenon in stock markets. To capture this phenomenon, we use the Nelson (1991)'s EGARCH model. Specifically, we use the EGARCH (1,0) model as

$$ln(\sigma_t^2) = \omega + \phi[ln(\sigma_{t-1}^2) - \omega] + \theta z_{t-1} + \gamma(|z_{t-1}| - E|z_{t-1}|), |\phi| < 1$$

where  $E|z_{t-1}|=\sqrt{2/\pi}$ , since  $z_{t-1}$  is assumed to follow standard normal distribution. As EGARCH model specifies the logarithm of volatility, thus, it does not require any non-negativity constraints for the parameters.  $\theta<0$ , implies the consistency with the volatility asymmetry in stock markets. In this model,  $|\phi|<1$ , implies that the volatility is stationary and the speed for which the shock to volatility decays becomes slower as  $\phi$  approaches to one.

## **APARCH** model

The APARCH model is proposed by Ding et al. (1993) as an extension of Bollerslev's (1986) GARCH that nests at least seven GARCH specifications. In this paper, we use APARCH(1,1) model as

$$\sigma_t = \left[\omega + \alpha_1(|\epsilon_{t-1}| - \alpha_n \epsilon_{t-1})^{\delta} + \beta_1 \sigma_{t-1}^{\delta}\right]^{\frac{1}{\delta}}$$

where  $\omega, \alpha_1, \alpha_n, \beta_1$  and  $\delta$  are the parameters to be estimated.  $\delta > 0$ , plays the role a Box-Cox transformation of  $\sigma_t$ .  $\alpha_n, (-1 < \alpha_n < 1)$ , refers the so-called leverage effect.

Focusing on the recent studies, we observe that the long memory skewed student distribution has frequently been used in VaR forecasting, for example Giot and Laurent (2003, 2004), Wang and Hsu (2006). Following

Giot and Laurent (2003, 2004) or Giot (2003), the standard version of the skewed Student distribution, introduced by Fernández and Steel (1998), has been used as innovation distribution along with the Gaussian and Student t distributions in this paper. According to Lambert and Laurent (2001), the innovation process  $z_t$  is said to have (standard) skewed Student distribution if

$$f(z_t|\xi,\vartheta) = \begin{cases} \frac{2}{\xi + \frac{1}{\xi}} sg[\xi(sz_t + m)|v] \text{ if } z_t < -\frac{m}{s} \\ \frac{2}{\xi + \frac{1}{\xi}} sg[\xi(sz_t + m)/\xi|v] \text{ if } z_t \geq -\frac{m}{s} \end{cases}$$

where g(.|v), v>2 is a symmetric (unit variance) Student density and  $\xi$  is the asymmetry coefficient. Parameters  $m=\frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(\xi-\frac{1}{\xi}\right)$  and  $s^2=\left(\xi^2+\frac{1}{\xi^2}-1\right)-m^2$  are the mean and variance of the nonstandardized skewed Student (Lambert and Laurent, 2001)

## Assessment of the VaR performance

Here, the application of the volatility models previously discussed to the Value-at-Risk application is focused on. The framework of VaR forecasts is to apply the classical and SVM techniques to the HAR-RV-J models to obtain the realized volatility forecast and compute conditional realized volatility. Two conditional realized volatility series will be found. The ARCH specification is being applied on these two series as an AR(2), where the lag was selected using AIC, model to estimate the conditional mean along with four different models with three innovation distributions (Gaussian, student t and skewed Student) to estimate the conditional variances, that is,  $(3\times4) = 12$  models for the conditional variance. These results were used as inputs to compute one-day ahead VaR.

When assuming a Gaussian distribution for the innovation process, the VaR for long trading positions (that is, the left tail of the returns density distribution) is  $\mu_t + z_{\alpha} \sigma_t$ and for short trading positions (that is, the right tail of the returns density distribution) is  $\mu_t + z_{1-\alpha}\sigma_t$ , where  $z_{\alpha}$  and  $z_{1-\alpha}$  are the left and right quantile respectively at  $\alpha\%$  for the normal distribution. When the innovation distribution is Student t, the VaR for long and short trading positions are  $\mu_t + t_{\alpha,v}\sigma_t$  and  $\mu_t + t_{1-\alpha,v}\sigma_t$  respectively, where  $t_{\alpha,v}$ and  $t_{1-\alpha,y}$  are the left and right quantile, respectively at  $\alpha\%$  for the Student t distribution. And finally, for the skewed student innovation distribution, the VaR for long and short trading positions are respectively  $\mu_t$  +  $skst_{\alpha,\nu,\xi}\sigma_t$  and  $\mu_t + skst_{1-\alpha,\nu,\xi}\sigma_t$  with  $skst_{\alpha,\nu,\xi}$  $skst_{1-\alpha,\nu,\xi}$ , the left and right quantile respectively at  $\alpha\%$ for the skewed Student distribution.

The performances of classical HAR and SVM-HAR models are tested by using Kupiec's(1995) likelihood ratio test and Engle and Manganelli's (1999) dynamic

quantile test. Kupiec's likelihood ratio test is designed on computing the failure rate, the number of times realized returns exceed (in absolute value) the forecasted VaR, for the returns which is also known as the proportion if VaR violation (an occurrence of a market returns larger, in absolute value, than the forecasted VaR). If the VaR model is correctly specified, the failure rate should be equal to the prespecified VaR level,  $\alpha$ . That is,  $H_0$ :  $f = \alpha$  against  $H_1$ :  $f \neq \alpha$ . f, the failure rate, which is estimated as  $f = 1/T \sum_{t=1}^T I(y_t - VaR_t(\alpha))$  is the sequence of yes/no observations, where I(.) is an indicator function,  $y_t$  is realized return at time t and  $VaR_t(\alpha)$  is the VaR forecast at time t and at %.

The Kupiec's likelihood ratio test statistic to test the aforementioned hypothesis is  $LR = -2ln(\alpha^{T-N}(1-\alpha)^N) + 2ln\left(\left(1-\binom{N}{T}\right)^{T-N}\binom{N}{T}\right)^N$ , where N is the number of VaR violations, and T is the total number of observations. This LR test statistic is asymptotically distributed as  $\chi^2(1)$ .

Beside the failure rate, Engel and Manganelli (1999) consider the new variables as  $Hit_t(\alpha) = I(y_t < VaR_t(\alpha)) - \alpha$  and  $Hit_t(1-\alpha) = I(y_t > VaR_t(1-\alpha)) - \alpha$  on the basis of a relevant VaR model should feature a sequence of indicator functions that is not serially correlated and suggest to test jointly  $A1: E(Hit_t(\alpha)) = 0$  in case of long  $(E(Hit_t(1-\alpha)) = 0$  in case of short) trading positions and  $A2: Hit_t(\alpha)$  (or  $Hit_t(1-\alpha)$ ) is uncorrelated with the variables included in the information set.

Engel and Manganelli (1999) show that testing A1-A2 can be done using the artificial regression  $Hit_t=X\lambda+\epsilon_t$ , where X is a  $T\times k$  matrix whose first column is a column of one, the next q columns are  $Hit_{t-1},\cdots,Hit_{t-q}$  and k-q-1 remaining columns are additional independent variables (including the VaR itself). This test is known as dynamic quantile test. Under the null A1-A2, Engel and Manganelli (1999) show that dynamic quantile test statistic is  $\frac{\hat{\lambda}'X'X\hat{\lambda}}{\alpha(1-\alpha)}$ , which is asymptotically distributed as  $\chi^2(k)$ .

#### **EMPIRICAL RESULTS**

In this paper, the computation procedure is conducted in two steps. In the first step, the R-2.12.0-win32's and R 2.12.0-win32's Kernlab package for filling the classical HAR and SVM-HAR models and computed the conditional realized volatility forecasts for both models. In the second step, estimation involved in Value-at-Risk (VaR) approach is carried out by calling G@RCH 5.0 in Ox. The R and Ox code are available upon request. Both Kupiec's likelihood ratio test and dynamic quantile test were used to assess the performance of the classical HAR and SVM-HAR models through the 12 aforementioned types of VaR forecasts with a level  $\alpha$  that ranges

**Table 2.** VaR failure rate results for different models using 5-min intraday returns.

						In-sa	ample									Out-of	-sample				
			VaR fo	r short <sub>l</sub>	positions	1		VaR fo	or long p	ositions			VaR fo	r short p	ositions	i		VaR fo	or long p	ositions	
α		5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
GARCH-																					
Gaussian	HAR	0.189	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.047	0.522	0.528	0.031	0.000	0.000	0.000	0.000	0.000	0.001	0.089	0.000
Gaussian	SHAR	0.584	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.019	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.045	0.019	0.000
ST-t	HAR	0.000	0.000	0.000	0.003	0.006	0.000	0.000	0.000	0.000	0.000	0.010	0.020	0.000	0.002	0.086	0.000	0.000	0.000	0.000	0.000
0	SHAR	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.162	0.086	0.000	0.000	0.000	0.000	0.000
SKST	HAR	0.303	0.027	0.484	0.524	0.806	0.213	0.147	0.199	0.211	0.855	0.367	0.203	0.047	0.304	0.216	0.845	0.057	0.045	0.019	0.000
ONOT	SHAR	0.110	0.068	0.090	0.083	0.078	0.078	0.028	0.076	0.027	0.019	0.010	0.031	0.531	0.813	0.216	0.956	0.093	0.206	0.000	0.000
GJR-GARC	CH-																				
Gaussian	HAR	0.133	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.017	0.091	0.934	0.031	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
Gaassian	SHAR	0.875	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.019	0.000
ST-t	HAR	0.000	0.000	0.000	0.006	0.006	0.000	0.000	0.000	0.000	0.000	0.005	0.008	0.013	0.006	0.030	0.000	0.000	0.000	0.000	0.000
011	SHAR	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.002	0.216	0.000	0.000	0.000	0.000	0.000
SKST	HAR	0.403	0.433	0.778	0.728	0.522	0.524	0.657	0.423	0.107	0.855	0.825	0.729	0.084	0.304	0.216	0.825	0.402	0.016	0.019	0.000
ONOT	SHAR	0.653	0.294	0.059	0.142	0.160	0.812	0.147	0.294	0.211	0.255	0.068	0.147	0.474	0.162	0.470	0.720	0.143	0.045	0.000	0.000
EGARCH-																					
Gaussian	HAR	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.091	0.934	0.031	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Gaussian	SHAR	0.159	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.118	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
OT 1	HAR	0.000	0.000	0.000	0.001	0.015	0.000	0.000	0.000	0.000	0.000	0.019	0.031	0.025	0.015	0.086	0.000	0.000	0.000	0.000	0.000
ST-t	SHAR	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.002	0.216	0.000	0.000	0.000	0.000	0.000
SKST	HAR	0.953	0.978	0.778	0.728	0.806	0.890	0.703	0.294	0.211	0.855	0.934	0.470	0.363	0.079	0.086	0.720	0.297	0.045	0.089	0.000
2//21	SHAR	0.402	0.702	0.090	0.083	0.160	0.812	0.467	0.023	0.005	0.019	0.443	0.203	0.085	0.304	0.646	0.934	0.526	0.045	0.000	0.000
APARCH-																					
Gaussian	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Gaussiali	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ST-t	HAR	0.000	0.000	0.000	0.001	0.014	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.014	0.000	0.000	0.000	0.000	0.000

Table 2. Contd.

	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SKST	HAR SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

P-values for the dynamic quantile test statistic  $\frac{\hat{\lambda} \chi \chi \hat{\lambda}}{\alpha(1-\alpha)} \sim \chi^2(7)$  both for long and short trading with  $\alpha$  is successively equal to 5, 2.5, 1%, 0.5 and 0.25% of 15 previously explained ARCH specifications for classical HAR-ARCH and SVM-HAR-ARCH type models. The left part presenting the results for in-sample forecasts while the right part, the out-of-sample forecasts using 5-min intraday returns of Nikkei 225 index. The in-sample period is 11 March 1996 to 29 December 2004 and the out-of-sample period is 5 January 2005 to 30 September 2009.  $y_t = \mu_t + \sqrt{\sigma^2 R V_{t|t-1}, z_t}$ .

**Table 3.** VaR failure rate results for different models using 15 min intraday returns.

						In-sa	mple									Out-of-	sample				
α		١	/aR for s	hort pos	itions		,	VaR for I	ong pos	itions			VaR fo	r short p	ositions			VaR fo	r long p	ositions	
		5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
GARCH-																					
Gaussian	HAR SHAR	0.000 0.461	0.002 0.116	0.000 0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.019 0.000	0.068 0.934	0.001 0.003	0.000	0.000	0.000	0.000	0.000	0.000 0.001	0.000 0.019	0.000
ST-t	HAR SHAR	0.000	0.000 0.003	0.000 0.269	0.000 0.047	0.015 0.000	0.000	0.000 0.000	0.000	0.000	0.000	0.000	0.008 0.002	0.013 0.013	0.521 0.162	0.216 0.086	0.000	0.000	0.000	0.000	0.000 0.000
SKST	HAR SHAR	0.189 0.031	0.150 0.515	0.023 0.623	0.083 0.083	0.160 0.015	0.147 0.009	0.311 0.041	0.199 0.883	0.005 0.047	0.019 0.019	0.019 0.193	0.103 0.020	0.085 0.047	0.521 0.304	0.216 0.216	0.633 0.178	0.143 0.033	0.005 0.005	0.000 0.089	0.000 0.000
GJR-GAR	CH-																				
Gaussian	HAR SHAR	0.403 0.524	0.000 0.150	0.000 0.002	0.000	0.000 0.000	0.000	0.000	0.000	0.000 0.001	0.000	0.118 0.367	0.001 0.020	0.000	0.000 0.000	0.000	0.000	0.000	0.000 0.001	0.000 0.019	0.000
ST-t	HAR SHAR	0.000	0.000 0.002	0.000 0.133	0.000 0.047	0.006 0.001	0.000	0.000 0.000	0.000	0.000	0.000	0.000 0.001	0.008 0.003	0.006 0.003	0.080 0.162	0.216 0.086	0.000	0.000	0.000 0.000	0.000	0.000 0.000
SKST	HAR SHAR	0.799 0.351	0.238 0.978	0.059 0.945	0.524 0.357	0.300 0.035	0.736 0.348	0.558 0.311	0.043 0.199	0.005 0.211	0.091 0.091	0.152 0.193	0.729 0.203	0.363 0.363	0.162 0.521	0.867 0.216	0.737 0.934	0.210 0.142	0.016 0.045	0.000 0.019	0.000 0.000
EGARCH-																					
Gaussian	HAR SHAR	0.403 0.049	0.000 0.089	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.243 0.956	0.001 0.071	0.000	0.000	0.000 0.000	0.000	0.000	0.000	0.000 0.019	0.000 0.000
ST-t	HAR SHAR	0.000	0.000 0.007	0.000 0.023	0.000 0.003	0.015 0.006	0.000	0.000	0.000	0.000	0.000	0.000 0.003	0.008 0.003	0.235 0.047	0.521 0.080	0.216 0.468	0.000	0.000	0.000	0.000	0.000

Table 3. Contd.

SKST	HAR	0.875	0.190	0.269	0.728	0.078	0.524	0.467	0.127	0.107	0.019	0.364	0.010	0.104	0.089	0.230	0.001	0.000	0.000	0.000	0.000
	SHAR	0.875	0.605	0.778	0,142	0.035	0.663	0.147	0.294	0.211	0.091	0.193	0.276	0.235	0.521	0.468	0.825	0.018	0.005	0.019	0.000
APARCH-	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Gaussian	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ST-t	HAR SHAR	0.000	0.000 0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000 0.000	0.000	0.000	0.000 0.000	0.000 0.000	0.000 0.000
SKST	HAR SHAR	0.000	0.000 0.000	0.000 0.000	0.000	0.000 0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000 0.000	0.000 0.000	0.000 0.000	0.000

*P*-values for the dynamic quantile test statistic  $\frac{\hat{\lambda} i X i X \hat{\lambda}}{\alpha(1-\alpha)} \sim \chi^2(7)$  both for long and short trading with  $\alpha$  is successively equal to 5, 2.5, 1%, 0.5 and 0.25% of 15 previously explained ARCH specifications for classical HAR-ARCH and SVM-HAR-ARCH type models. The left part presenting the results for in-sample forecasts while the right part, the out-of-sample forecasts using 5-min intraday returns of Nikkei 225 index. The in-sample period is 11 March 1996 to 29 December 2004 and the out-of-sample period is 5 January 2005 to 30 September 2009.  $y_t = \mu_t + \sqrt{\sigma^2 R V_{t|t-1}}$ ,  $z_t$ .

Table 4. VaR failure rate results for different models using optimally sampled intraday returns.

						In-sa	ample									Out-of-	-sample				
α			VaR fo	r short p	oositions			VaR fo	or long p	ositions			VaR fo	r short p	ositions	1		VaR fo	r long p	ositions	
		5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
GARCH-																					
Gaussian	HAR	0.890	0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.068	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Gaussian	SHAR	0.091	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.019	0.737	0.203	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
OT :	HAR	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.013	0.036	0.086	0.000	0.000	0.000	0.000	0.000
ST-t	SHAR	0.000	0.000	0.008	0.003	0.015	0.000	0.000	0.000	0.000	0.000	0.007	0.364	0.363	0.521	0.468	0.000	0.000	0.000	0.000	0.000
OLOT	HAR	0.133	0.294	0.192	0.025	0.035	0.298	0.081	0.043	0.017	0.091	0.118	0.203	0.531	0.162	0.216	0.178	0.093	0.016	0.019	0.000
SKST	SHAR	0.048	0.294	0.269	0.047	0.078	0.009	0.005	0.076	0.017	0.019	0.014	0.048	0.363	0.036	0.216	0.010	0.000	0.000	0.000	0.000
GJR-GAR	CH-																				
0	HAR	0.459	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.019	0.443	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Gaussian	SHAR	0.159	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.934	0.032	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
OT 4	HAR	0.000	0.000	0.000	0.000	0.035	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.025	0.079	0.030	0.000	0.000	0.000	0.000	0.000
ST-t	SHAR	0.000	0.000	0.000	0.003	0.015	0.000	0.000	0.000	0.000	0.000	0.007	0.203	0.737	0.848	0.468	0.000	0.000	0.000	0.000	0.000

Table 4. Contd.

-	HAR	0.159	0.433	0.090	0.142	0.300	0.890	0.311	0.076	0.107	0.019	0.300	0.203	0.235	0.303	0.216	0.535	0.057	0.045	0.019	0.000
SKST	SHAR				-												0.333				
	SHAR	0.519	0.868	0.623	0.357	0.078	0.348	0.028	0.294	0.364	0.522	0.019	0.147	0.145	0.080	0.216	0.100	0.000	0.005	0.000	0.000
EGARCH-																					
EGARCH-		0.400	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.450	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Gaussian	HAR	0.403	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.152	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	SHAR	0.519	0.019	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.019	0.737	0.013	0.000	0.000	0.000	0.000	0.000	0.005	0.019	0.230
ST-t	HAR	0.000	0.000	0.000	0.001	0.006	0.000	0.000	0.000	0.000	0.000	0.242	0.592	0.104	0.252	0.230	0.000	0.000	0.000	0.000	0.000
011	SHAR	0.000	0.000	0.003	0.025	0.006	0.000	0.000	0.000	0.000	0.000	0.007	0.147	0.786	0.848	0.468	0.000	0.000	0.000	0.000	0.000
SKST	HAR	0.653	0.605	0.366	0.524	0.300	0.953	0.193	0.043	0.005	0.019	0.068	0.103	0.145	0.162	0.216	0.135	0.010	0.045	0.019	0.000
SNST	SHAR	0.663	0.247	0.778	0.524	0.015	0.592	0.110	0.199	0.005	0.019	0.118	0.031	0.363	0.304	0.216	0.292	0.000	0.000	0.000	0.000
APARCH-																					
	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Gaussian	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ST-t	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	SHAN	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SKST		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

*P*-values for the dynamic quantile test statistic  $\frac{\hat{\lambda}'x'X\hat{\lambda}}{\alpha(1-\alpha)}\sim\chi^2(7)$  both for long and short trading with  $\alpha$  is successively equal to 5, 2.5, 1, 0.5 and 0.25% of 15 previously explained ARCH specifications for classical HAR-ARCH and SVM-HAR-ARCH type models. The left part presenting the results for in-sample forecasts while the right part, the out-of-sample forecasts using 5-min intraday returns of Nikkei 225 index. The in-sample period is 11 March 1996 to 29 December 2004 and the out-of-sample period is 5 January 2005 to 30 September 2009.  $y_t = \mu_t + \sqrt{\sigma^2 R V_{t|t-1}}$ ,  $z_t$ .

failure rate test while Tables 5 to 7 reported the P-values for Engel and Manganelli's (1999) dynamic quantile forecast test with q = 5 and k = 7 (that is, the contemporaneous VaR forecast is included as additional variable following Giot and Laurent (2004).

For both the tests, each table presented four panels successively the results for GARCH, GJR-GARCH, EGARCH and APARCH models. Each panel presented the results successively for Gaussian, Student t (ST-t) and skewed Student

(SKST) distributions for both the classical HAR (HAR) and SVM-HAR (SHAR) models. The left part of each table presented the in-sample VaR forecasts results while the right part presents the out-of-sample VaR forecasts results for both short and long trading positions.

#### DISCUSSION

We first compared the results of GARCH and APARCH models for each of the Tables, that is,

Tables 2 to 4 and Tables 5 to 7. The empirical results showed the same story for all intraday sampled returns and innovation distributions specifications. Almost all the *P*-values for both the tests are often smaller than 0.05. These three classes of models failed to forecast the one-day ahead VaR. Though we observe from Giot and Laurent (2004) that the Skewed-APARCH specification significantly improves the VaR forecasting performance where they use ARFIMAX model for volatility forecasts. The results for GARCH models

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**Table 5.** VaR quantile regression results for different models using 5-min intraday returns.

						In-s	sample									Out-of-	-sample				
α			VaR fo	or short	ositions	1		VaR	for long	positions			VaR fo	r short p	oositions			VaR fo	r long p	ositions	
		5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
GARCH-																					
Gaussian	HAR	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.121	0.785	0.999	0.902	0.365	0.000	0.000	0.000	0.001	0.017	0.292	0.901	0.842
Gaacolan	SHAR	0.989	0.046	0.000	0.000	0.000	0.000	0.000	0.012	0.175	0.729	0.139	0.000	0.000	0.000	0.000	0.001	0.049	0.775	0.725	0.842
0.7.1	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.043	0.175	0.490	0.079	0.281	0.000	0.014	0.631	0.000	0.001	0.089	0.485	0.842
ST-t	SHAR	0.000	0.000	0.000	0.000	0.022	0.000	0.000	0.003	0.092	0.490	0.012	0.000	0.012	0.810	0.631	0.001	0.017	0.089	0.485	0.842
OLICE	HAR	0.001	0.006	0.016	0.030	0.000	0.215	0.633	0.923	0.963	0.999	0.851	0.684	0.151	0.948	0.907	0.668	0.530	0.775	0.725	0.842
SKST	SHAR	0.643	0.414	0.385	0.594	0.634	0.091	0.410	0.802	0.629	0.729	0.113	0.102	0.945	0.999	0.907	0.309	0.150	0.956	0.485	0.842
GJR-GAR	CH-																				
	HAR	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.043	0.629	0.904	0.714	0.177	0.000	0.000	0.000	0.000	0.017	0.292	0.485	0.842
Gaussian	SHAR	0.828	0.084	0.000	0.000	0.000	0.000	0.000	0.012	0.0175	0.490	0.178	0.000	0.000	0.000	0.000	0.000	0.030	0.620	0.725	0.842
	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.175	0.490	0.031	0.076	0.074	0.051	0.289	0.000	0.001	0.089	0.485	0.842
ST-t	SHAR	0.001	0.000	0.000	0.001	0.005	0.000	0.000	0.001	0.092	0.490	0.001	0.000	0.011	0.014	0.907	0.000	0.001	0.089	0.485	0.842
	HAR	0.088	0.333	0.023	0.020	0.000	0.535	0.862	0.522	0.899	0.999	0.669	0.909	0.451	0.948	0.907	0.073	0.009	0.620	0.725	0.842
SKST	ПАП	0.719	0.648	0.023	0.760	0.847	0.699	0.742	0.435	0.963	0.982	0.228	0.317	0.451	0.810	0.994	0.262	0.554	0.020	0.725	0.842
EGARCH-																					
EGANCH-	HAR	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.043	0.299	0.904	0.660	0.165	0.000	0.000	0.000	0.000	0.017	0.089	0.485	0.842
Gaussian	SHAR	0.594	0.003	0.000	0.000	0.000	0.000	0.000	0.006	0.175	0.490	0.353	0.018	0.000	0.000	0.000	0.002	0.030	0.292	0.485	0.842
	LIAD	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.475	0.400	0.450	0.477	0.455	0.447	0.001	0.000	0.001	0.000	0.405	0.040
ST-t	HAR SHAR	0.000	0.000	0.000	0.000 0.001	0.000 0.005	0.000	0.000	0.024 0.001	0.175 0.092	0.490 0.490	0.158 0.000	0.177 0.000	0.155 0.025	0.147 0.000	0.631 0.907	0.000	0.001 0.000	0.089	0.485 0.485	0.842 0.842
SKST	HAR	0.165	0.813	0.007	0.020	0.000	0.289	0.484	0.955	0.963	0.999	0.704	0.890	0.904	0.578	0.631	0.819	0.059	0.775	0.901	0.842
	SHAR	0.930	0.485	0.385	0.594	0.847	0.779	0.560	0.602	0.457	0.729	0.286	0.788	0.216	0.948	0.999	0.011	0.874	0.001	0.485	0.842
APARCH-																					
Gaussian	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.485	0.842
Gaassian	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.485	0.842
ST-t	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.485	0.842
J1-l	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.485	0.842

Table 5. Contd.

OKOT	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.485	0.842
SKST	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.485	0.842

*P*-values for the dynamic quantile test statistic  $\frac{\hat{\lambda}' X' X \hat{\lambda}}{\alpha(1-\alpha)} \sim \chi^2(7)$  both for long and short trading with  $\alpha$  is successively equal to 5, 2.5, 1, 0.5 and 0.25% of 15 previously explained ARCH specifications for classical HAR-ARCH and SVM-HAR-ARCH type models. The left part presenting the results for in-sample forecasts while the right part, the out-of-sample forecasts using 5-min intraday returns of Nikkei 225 index. The insample period is 11 March 1996 to 29 December 2004 and the out-of-sample period is 5 January 2005 to 30 September 2009.  $y_t = \mu_t + \sqrt{\sigma^2 R V_{t|t-1}}$ ,  $z_t$ .

Table 6. VaR quantile regression results for different models using 15-min intraday returns.

						In-sa	ample									Out-of-	sample				
α			VaR fo	r short p	ositions	1		VaR fo	or long p	ositions			VaR fo	or short	positions			VaR fo	or long p	ositions	
		5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
GARCH-																					
Gaussian	HAR SHAR	0.010	0.003	0.000 0.020	0.000	0.000	0.000	0.000	0.043 0.006	0.175 0.092	0.729 0.490	0.005 0.213	0.006 0.016	0.000	0.000	0.000	0.000	0.009 0.001	0.089 0.292	0.490 0.725	0.842 0.842
	SHAR	0.801	0.174	0.020	0.000	0.000	0.000	0.000	0.006	0.092	0.490	0.213	0.016	0.000	0.000	0.000	0.000	0.001	0.292	0.725	0.842
ST-t	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.092	0.490	0.000	0.081	0.074	0.993	0.907	0.000	0.000	0.089	0.490	0.842
51-l	SHAR	0.000	0.001	0.733	0.417	0.001	0.000	0.003	0.006	0.092	0.490	0.000	0.014	0.053	0.810	0.630	0.017	0.005	0.089	0.490	0.842
SKST	HAR	0.006	0.334	0.011	0.001	0.847	0.165	0.559	0.923	0.457	0.729	0.023	0.696	0.451	0.993	0.907	0.381	0.745	0.449	0.490	0.842
3N31	SHAR	0.170	0.363	0.935	0.594	0.187	0.033	0.399	0.983	0.785	0.729	0.312	0.067	0.283	0.948	0.907	0.381	0.307	0.449	0.901	0.842
GJR-GAR	CH-																				
Gaussian	HAR	0.005	0.005	0.000	0.000	0.000	0.000	0.000	0.006	0.092	0.490	0.048	0.003	0.000	0.000	0.000	0.000	0.002	0.089	0.490	0.842
addolan	SHAR	0.862	0.188	0.004	0.000	0.000	0.000	0.000	0.012	0.299	0.490	0.164	0.146	0.000	0.000	0.000	0.000	0.005	0.292	0.725	0.842
ST-t	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.138	0.031	0.578	0.907	0.000	0.000	0.089	0.490	0.842
31-1	SHAR	0.002	0.029	0.505	0.417	0.005	0.000	0.000	0.006	0.092	0.490	0.001	0.010	0.011	0.810	0.631	0.004	0.005	0.089	0.490	0.842
SKST	HAR	0.182	0.702	0.017	0.030	0.963	0.147	0.702	0.108	0.457	0.904	0.108	0.540	0.904	0.810	0.999	0.268	0.638	0.620	0.490	0.842
ONOT	SHAR	0.963	0.783	0.976	0.957	0.386	0.292	0.314	0.343	0.963	0.904	0.043	0.407	0.904	0.993	0.907	0.881	0.610	0.775	0.725	0.842
EGARCH-																					
Gaussian	HAR	0.064	0.008	0.000	0.000	0.000	0.000	0.000	0.003	0.092	0.490	0.035	0.001	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
Gaassiaii	SHAR	0.581	0.554	0.000	0.000	0.000	0.000	0.000	0.024	0.175	0.490	0.361	0.046	0.000	0.000	0.000	0.000	0.002	0.170	0.725	0.842
ST-t	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.081	0.792	0.993	0.907	0.000	0.000	0.089	0.490	0.842
J. (	SHAR	0.000	0.096	0.127	0.030	0.072	0.000	0.000	0.001	0.092	0.490	0.000	0.002	0.065	0.578	0.994	0.003	0.002	0.089	0.490	0.842
SKST	HAR	0.294	0.608	0.017	0.000	0.000	0.054	0.708	0.873	0.899	0.729	0.180	0.396	0.889	0.901	0.982	0.040	0.001	0.089	0.490	0.842
J. 10 .	SHAR	0.875	0.980	0.962	0.760	0.386	0.561	0.591	0.955	0.963	0.903	0.223	0.710	0.328	0.993	0.994	0.480	0.338	0.449	0.725	0.842

Table 6. Contd.

APARCH-																					
Councien	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
Gaussian	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
CT+	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
ST-t	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
CVCT	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
SKST	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842

*P*-values for the dynamic quantile test statistic  $\frac{\hat{\lambda}' X' X \hat{\lambda}}{\alpha(1-\alpha)} \sim \chi^2(7)$  both for long and short trading with  $\alpha$  is successively equal to 5, 2.5, 1%, 0.5 and 0.25% of 15 previously explained ARCH specifications for classical HAR-ARCH and SVM-HAR-ARCH type models. The left part presenting the results for in-sample forecasts while the right part, the out-of-sample forecasts using 5-minute intraday returns of Nikkei 225 index. The in-sample period is 11 March 1996 to 29 December 2004 and the out-of-sample period is 5 January 2005 to 30 September 2009.  $y_t = \mu_t + \sqrt{\sigma^2 R V_{t|t-1}}$ ,  $z_t$ .

Table 7. VaR quantile regression results for different models using optimally sampled intraday returns.

						In-sa	ample									Out-of-	sample				
α			VaR fo	r short	positions	3		VaR f	or long p	ositions			VaR fo	r short p	ositions			VaR fo	or long p	ositions	
		5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
GARCH-																					
Gaussian	HAR	0.564	0.071	0.000	0.000	0.000	0.000	0.000	0.006	0.092	0.490	0.121	0.003	0.000	0.000	0.000	0.000	0.002	0.170	0.490	0.842
Gaussian	SHAR	0.173	0.003	0.000	0.000	0.000	0.000	0.000	0.024	0.175	0.729	0.389	0.480	0.000	0.000	0.000	0.000	0.001	0.089	0.490	0.842
o <del>-</del> .	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.001	0.052	0.330	0.631	0.000	0.001	0.089	0.490	0.842
ST-t	SHAR	0.000	0.000	0.000	0.004	0.187	0.000	0.000	0.003	0.092	0.490	0.002	0.625	0.904	0.993	0.994	0.000	0.000	0.089	0.490	0.842
	HAR	0.039	0.663	0.016	0.001	0.000	0.405	0.562	0.710	0.629	0.904	0.118	0.304	0.965	0.810	0.907	0.595	0.612	0.620	0.725	0.842
SKST	SHAR	0.010	0.158	0.014	0.032	0.634	0.164	0.172	0.802	0.629	0.729	0.007	0.167	0.904	0.330	0.907	0.125	0.049	0.089	0.490	0.842
GJR-GAR	CH-																				
	HAR	0.295	0.289	0.000	0.000	0.000	0.000	0.000	0.003	0.092	0.490	0.556	0.014	0.000	0.000	0.000	0.000	0.002	0.089	0.490	0.842
Gaussian	SHAR	0.330	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.457	0.729	0.212	0.049	0.000	0.000	0.000	0.000	0.001	0.170	0.490	0.842
	HAR	0.000	0.000	0.000	0.000	0.386	0.000	0.000	0.001	0.092	0.490	0.002	0.000	0.150	0.578	0.289	0.000	0.000	0.089	0.490	0.842
ST-t	SHAR	0.000	0.001	0.000	0.000	0.187	0.000	0.000	0.003	0.092	0.490	0.001	0.339	0.989	0.999	0.994	0.000	0.000	0.089	0.490	0.842
	HAR	0.167	0.844	0.253	0.044	0.962	0.061	0.581	0.171	0.899	0.729	0.549	0.593	0.793	0.948	0.907	0.668	0.636	0.775	0.725	0.842
SKST	SHAR	0.649	0.380	0.275	0.957	0.634	0.647	0.189	0.426	0.000	0.000	0.006	0.101	0.632	0.578	0.907	0.559	0.041	0.449	0.490	0.842

Table 7. Contd.

EGARCH-																					
Gaussian	HAR	0.433	0.002	0.000	0.000	0.000	0.000	0.000	0.003	0.092	0.490	0.215	0.086	0.000	0.000	0.000	0.000	0.001	0.089	0.490	0.842
	SHAR	0.403	0.023	0.000	0.000	0.000	0.000	0.000	0.012	0.175	0.729	0.790	0.207	0.000	0.000	0.000	0.000	0.005	0.449	0.725	0.982
ST-t	HAR	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.410	0.431	0.889	0.981	0.982	0.000	0.000	0.089	0.490	0.842
	SHAR	0.000	0.000	0.005	0.022	0.072	0.000	0.000	0.001	0.092	0.490	0.000	0.143	0.998	0.999	0.004	0.000	0.000	0.089	0.490	0.842
SKST	HAR	0.790	0.849	0.012	0.000	0.000	0.221	0.636	0.108	0.457	0.729	0.078	0.310	0.632	0.810	0.907	0.250	0.396	0.775	0.725	0.842
	SHAR	0.793	0.082	0.724	0.989	0.187	0.108	0.170	0.336	0.000	0.729	0.000	0.019	0.904	0.948	0.907	0.920	0.119	0.170	0.490	0.842
APARCH-																					
APARCH	LIAD	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.400	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.400	0.040
Gaussian	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
ST-t	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
SKST	HAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
	SHAR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.092	0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.490	0.842
	JIIAII	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.700	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.700	J.U72

*P*-values for the dynamic quantile test statistic  $\frac{\hat{\lambda}'x_t\hat{x}\hat{\lambda}}{\alpha(1-\alpha)}\sim\chi^2(7)$  both for long and short trading with  $\alpha$  is successively equal to 5, 2.5, 1, 0.5 and 0.25% of 15 previously explained ARCH specifications for classical HAR-ARCH and SVM-HAR-ARCH type models. The left part presenting the results for in-sample forecasts while the right part, the out-of-sample forecasts using 5-minute intraday returns of Nikkei 225 index. The in-sample period is 11 March 1996 to 29 December 2004 and the out-of-sample period is 5 January 2005 to 30 September 2009.  $y_t = \mu_t + \sqrt{\sigma^2 R V_{t|t-1}}$ ,  $z_t$ .

go with the results of Ebens (1999), where he concluded that the GARCH model underperforms (when volatility must be forecasted) with respect to the model based on the daily realized volatility. We then compared the results for GJR-GARCH and EGARCH models for each Table. All the Tables showed almost similar empirical results. The skewed Student distribution specification produced better forecasts for both the classical HAR and SMN-HAR models while the Gaussian and Student t specifications failed to forecast one day ahead VaR. We observed that the optimally sampled intraday returns could not compute with 5 and 15 min intraday returns to estimate the daily realized volatility for both the skewed Student

HAR and skewed Student SVM-HAR models. Finally, we consider the empirical results from 15-min intraday returns. We observed for the SVM-HAR-RV models that for the dynamic quantile test, all the *P*-values are greater than 0.05 for the SKST- EGARCH model and only one *P*-value is smaller than 0.05 for SKST- GJR-GARCH model out of 20 *P*-values (10 for in-sample and 10 for out-of-sample forecasts). Almost similar results were observed when 5-min intraday returns were used. The results for classical HAR-RV forecasts were also satisfactory but failed to compete with the SVM-HAR. The failure rate test results of SKST-GJR- GARCH and SKST-EGARCH were also satisfactory for both the classical HAR-RV

and SVM-HAR-RV forecasts. Maximum of the *P*-values were greater than 0.05. Comparing all the results, we can conclude that the SVM-HAR-RV model performs better to forecast one day ahead VaR.

#### **CONCLUDING REMARKS**

In this paper, the Support Vector Machine (SVM) regression was combined with Heterogeneous Autoregressive (HAR) model as a hybrid (SMV-HAR) model to improve the VaR forecasting ability. We examined the VaR forecasting ability of the realized volatility based models for Nikkei 225

stock returns. The empirical results presented here are suggestive for several interesting extensions. First, we set the values C=1 and  $\epsilon=0.1$  for the SVM-HAR-RV class models and observed better forecasting ability. The appropriate choice of the value C and  $\epsilon$  could be helpful to improve the forecasting ability.

Second, we consider only the Laplacian kernel for the SVMs and observe better performances. The choice of other existing kernels in SVM literature or an appropriate new kernel could improve the forecasting ability.

Third, 5- and 15-min intraday returns were considered along with optimally sampled intraday returns to estimate daily realized volatility. The optimally sampled intraday returns as studied by Bandi and Russell (2003, 2008) were considered to mitigate the market microstructure noise but it failed to produce better VaR forecasting performance compared to 5- and 15-min intraday returns. It would be interesting to consider the two- scale estimators of Zhang et al. (2005) and kernel estimator of Barndorff-Nielsen et al. (2009a, b).

Fourth, since the long memory skewed Student distribution produces better performance, therefore, the long memory ARCH type model for conditional variance could be interesting.

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