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Full Length Research Paper

Solving fuzzy linear programming problem as multi objective linear programming problem

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This paper proposes the method to the solution of fuzzy linear programming problem with the help of multi objective constrained linear programming problem when constraint matrix and the cost coefficients of an objective function are fuzzy in nature. Also proved is that the solutions are independent of weights.

Key words: Fuzzy linear programming problems, multi objective, fuzzy sets.

INTRODUCTION

Linear programming is a one of the most important operational research (OR) techniques. It has been applied to solve many real world problems but it fails to deals with imprecise data. So the many researchers succeed in capturing vague and imprecise information by fuzzy linear programming problem (FLPP) (Bellman and Zadeh, 1970; Campose and Verdegay, 1989; Mahmoud and Abo-Sinna, 2004; Negoita, 1970; Takashi, 2001; Takeshi et al., 1991).

The concept of a fuzzy decision making was first proposed by Bellman and Zadeh, 1970. Recently, much attention has been focused on FLPP (Campose and Verdegay, 1989; Mahmoud and Abo-Sinna, 2004; Negoita, 1970; Takashi, 2001; Takeshi et al., 1991) (Zimmermann, 1978).

An application of fuzzy optimization techniques to linear programming problems with multiple objectives has been presented by Zimmermann (1978) and Tanaka et al. presented a fuzzy approach to multi objective linear programming problems.

Negoita has formulated FLPP with fuzzy coefficient matrix, Zhang et al. (2003) formulated a FLPP as four objective constrained optimization problems where the cost coefficients are fuzzy and also presented its solution.

In this paper, we provided a method to solve FLPP where both the coefficient matrix of the constraints and cost coefficient are fuzzy in nature. Each problems, first converted into equivalent crisp linear problems, which are then solved by standard optimization methods.

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PRELIMINARIES

Definition 1: A subset A of a set X is said to be fuzzy set if;

$$\mu_A : X \to [0,1],$$

Where μ_A denote the degree of belongingness of A in X .

Definition 2: A fuzzy set A of a set X is said to be normal if;

$$\mu_A(x) = 1, \qquad \forall x \in X$$

Definition 3: The height of A is defined and denoted as;

$$h(A) = \sup_{x \in X} \mu_A(x) .$$

Definition 4: The $\alpha -$cut and strong $\alpha -$cut is defined and denoted as;

$$\alpha_A = \{x / \mu_A(x) \geq \alpha \}$$

$$\alpha^+{}_A = \{x / \mu_A(x) > \alpha \} \quad \text{respectively}$$

Definition 5: Let $\tilde{a}$, $\tilde{b}$ be two fuzzy numbers, their sum is defined and denoted as:

$\mu_{\tilde{a}+\tilde{b}}(z) = \sup \min_{z=u+v} \{, \mu_{\tilde{a}}(u) \ , \mu_{\tilde{a}}(v) \ \}$ When $0 \le \lambda \in R$

Definition 6: If a fuzzy number $\tilde{a}$ is fuzzy set A on R, it must possess at least following three properties:

i) $\mu_{\tilde{a}}(x) = 1$
ii) $\{x \in R / \mu_{\tilde{a}}(x) > \alpha \}$ is a closed interval for every $\alpha \in (0,1]$
iii) $\{x \in R / \mu_{\tilde{a}}(x) > 0 \}$ bounded and it is denoted by $[a_\lambda{}^L \ , a_\lambda{}^R]$

Theorem 1: A fuzzy set A on R is convex if and only if;

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \ge \min[\mu_A(x_1), \ \mu_A(x_2)],$$

For all $x_1, \ x_2 \in X$ and for all $\lambda \in [0,1]$, where min denotes the minimum operator.

Proof: Obvious.

Theorem 2: Let $\tilde{a}$ be a fuzzy set on R , then $\tilde{a} \in f(R)$ if and only if $\mu_{\tilde{a}}$ satisfies

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 \ , & for \, x \in [m,n] \\ L(x), & for \, x < m \\ R(x), & for \ x > n. \end{cases}$$

Where $L(x)$ is the right continuous monotone increasing function, $0 \le L(x) \le 1$ and $\lim_{x\to\infty} L(x) = 0$, $R(x)$ is a left continuous monotone decreasing function, $0 \le R(x) \le 1$ and $\lim_{x\to\infty} R(x) = 0$.
Proof: Obvious.

FUZZY LINEAR PROGRAMMING PROBLEM

We consider the FLPP with cost of decision variables and coefficient matrix of constraints are in fuzzy nature.

$$\langle \tilde{c}, x \rangle = f_i(x_j) = f_i(x) = \text{Max } \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j \, x_j \ ... \qquad (1)$$

Subject to $\sum_{j=1}^{n} \tilde{A}_{ij\,j} x_j \le \tilde{B}_i$; $\qquad 1 \le i \le m$

Where at least one x_j > 0 and

$$\mu_{Bi}(x) = \begin{cases} 1 & for \ x \le b_{ij} \\ \dfrac{(b_i + p_i - x)}{p_i} & for \, b_i \le x \le b_i + p_i \\ 0 & for \, b_i + p_i \le x \end{cases}$$

Let's consider triangular fuzzy numbers i.e. any fuzzy number A can be represented by three crisp numbers s, l, r .

$$(1) \Rightarrow \qquad \langle \tilde{c}, x \rangle = f_i(x_j) = \text{Max} \sum_{j=1}^{n} \tilde{c}_j \, x_j$$

$$\text{such that} \sum_{x\ge 0} (s_{ij} \ , l_{ij} \ , r_{ij}) x_{ij} \le (\ t_i, \ u_i, \ v_i)$$

$$0 \le i \le m$$

$$0 \le j \le n$$

Where $\quad A_{ij} = \langle s_{ij} \ , \ l_{ij} \ , \ r_{ij} \rangle$
$B_{ij} = \langle \ t_i, \ u_i, \ v_i \rangle$ are fuzzy numbers.

Theorem 3: For any two fuzzy numbers, $A = \langle \ s_1, \ l_1, \ r_1 \rangle$ and $B = \langle s_2 \ , l_2 \ , r_2 \rangle$ and $A \le B$ if and only if $s_1 \le s_2$, $s_1 - l_1 \le s_2 - l_2$ and $s_1 + r_1 \le s_2 + r_2$

Proof: Obvious

Above problem can be rewritten as;

$$\langle \tilde{c}, x \rangle = f_i(x_j) = \text{ Max } \sum_{j=1}^{n} \tilde{c}_j \, x_j$$

Such that $\sum_{j=1}^{n} s_{ij} x_j \le t_i$

$$\sum_{j=1}^{n} (s_{ij} - l_{ij}) x_j \le t_i - u_i \qquad (2)$$

$$\sum_{j=1}^{n} (s_{ij} + r_{ij}) x_j \le t_i + v_i \ , x_i \ge 0$$

Where the membership functions of $\tilde{c}_j$ (x) is

$$\mu_{\tilde{c}_j(x)} = \begin{cases} 0 \; , & x < \alpha_j \\ x - \alpha_j \; , & \alpha_j \le x < \beta_j \\ 1 \; , & \beta_j \le x \le \gamma_j \\ \dfrac{\eta_i - x}{\eta_i - \gamma_j} , & \gamma_j < x \le \eta_j \\ 0 , & \eta_j < x \end{cases}$$

Definition 7: A point $x^* \in X$ is said to be an optimal solution to the FLPP if

$\langle \tilde{c}, x^* \rangle \ge \langle \tilde{c}, x \rangle$ for all $x \in X$

Fuzzy multiple objective optimization

Consider a multiple objective optimization problem with k fuzzy goals $f_1, f_2 ....f_k$ represented by fuzzy sets $\tilde{F}_i$, i = 1 . . . . . k, and m fuzzy constraints $g_1, g_2,.......g_m$, represented by fuzzy sets $\tilde{G}_j$ j = 1 . . . . . m. By generalizing the analogy from the single objective function, the resulting fuzzy decision is given as;

$$\tilde{F}_1 \cap \tilde{F}_2 \cap .....\tilde{F}_k \cap \tilde{G}_1 \cap \tilde{G}_2 \cap ......\tilde{G}_m$$

In terms of corresponding membership values for the fuzzy goals and the fuzzy constraints, the resulting decision is;

$$\mu_{\tilde{D}}(X) = \min_{i,j} [\mu_{\tilde{F}_{iii}}(X), \mu_{\tilde{G}_j}(X)]$$

An optimum solution X* is one at which the membership function of the resulting decision $\tilde{D}$ is maximum, that is,

$$\mu_{\tilde{D}}(X^*) = \max \mu_{\tilde{D}}(X)$$

The shape of the membership functions such as a linear, concave, or convex function, for various objectives and constraints, can affect the optimum solution significantly. A linear approximation has been most commonly used because of simplicity and expediency.

Multi objective linear programming problem with fuzzy coefficients

In general, multi objective linear programming problem (MOLPP) refers to those LP problems of systems in which multiple objectives to be controlled. For above FLPP, the multi objective linear programming problem with fuzzy coefficients can be formulated as follow:

So multi objective fuzzy linear programming problem can be written as

$\underset{x \in X}{Max} \; \{ f_1(x), \;\; f_2(x),..........., f_k(x) \}$ Subject equation (2)

Where $f_i : R^n \to R^i$

Let R be the set of all real numbers, R^n be an n-dimensional Euclidean space, by considering the weighting factor, the MOLPP is defined as;

$$\underset{x \in X}{Max} \; \{ w_1 f_1(x), \;\; w_2 f_2(x),............, w_k f_k(x) \}$$

That is, $\underset{x \in X}{Max} \sum_{m=1}^{k} w_m f_m(x)$ subject to Equation (2)

Definition 8: A point $x^* \in X$ is said to be a complete optimal solution to the MOLPP if $(\langle c_0^L, x^* \rangle, \langle c_1^L, x^* \rangle, \langle c_0^R, x^* \rangle, \langle c_1^R, x^* \rangle)^T \le (\langle c_0^L, x \rangle, \langle c_1^L, x \rangle, \langle c_0^R, x \rangle, \langle c_1^R, x \rangle)^T$, for all $x \in X$

Numerical example

Example: We illustrate the method by numerical examples.
Solve the following FLPP

Max $f_i(x_1, x_2) = \tilde{c}_1 x + \tilde{c}_1 x_2$

Subject to the constraints;

$$(3, \; 2, 1) x_1 + (6 ,4 ,1) x_2 \le (13 ,5 ,2)$$
$$(4, \; 1, 2) x_1 + (6 ,5 ,4) x_2 \le (7 ,4 ,2)$$

Where the membership function of $\tilde{c}_1$ & $\tilde{c}_2$ are;

$$\mu_{\tilde{c}2}(x) = \begin{cases} 0, & x < 20 \\ x - 20, & 20 \le x < 25 \\ 1, & 25 \le x \le 35 \\ \dfrac{40 - x}{5} , & 35 < x \le 40 \\ 0, & 40 < x \end{cases}$$

Write the above FLPP as Max $f(x_1, x_2) = \tilde{c}_1 x + \tilde{c}_1 x_2$
Subject to the constraints

Table 1. lists of solution for MOLPP for various weights.

Sr. No.	w_1	w_2	w_3	w_4	(x_1^*, x_2^*)
1	0	1	1	0	(0, 0.9)
2	0	1	0.5	0	(0, 0.9)
3	0.2	0.4	0.5	0.2	(0, 0.9)
4	0.1	0.2	0.3	0.4	(0, 0.9)
5	0	0.3	0	0.4	(0, 0.9)
6	0.2	0.4	0.6	0.8	(0, 0.9)
7	0.5	0	0.5	0	(0, 0.9)
8	0	1	1	0	(0, 0.9)
9	0	0	0	0.5	(0, 0.9)
10	0.3	0.1	1	1	(0, 0.9)
11	0.5	0.5	0.5	0.5	(0, 0.9)
12	0	0	0.5	0.5	(0, 0.9)
13	0.2	0.5	0.5	0.5	(0, 0.9)
14	0.1	0.2	0.3	0.4	(0, 0.9)
15	0	0.2	0	0.2	(0, 0.9)

$$\begin{aligned}
&3x_1 + 6x_2 \le 13 \\
&4x_1 + 6x_2 \le 7 \\
&x_1 + 2x_2 \le 8 \\
&3x_1 + x_2 \le 3 \\
&4x_1 + 7x_2 \le 15 \\
&6x_1 + 10x_2 \le 9 \\
&x_1, x_2 \ge 0
\end{aligned} \tag{3}$$

MOLPP Max

$$\left(7x_1 + 20x_2,\ 10x_1 + 25x_2,\ 14x_1 + 35x_2,\ 25x_1 + 40x_2\right)$$

subject to Equation (3)
MOLPP

$$Max(w) = w_1(7x_1 + 20x_2) + w_2(10x_1 + 25x_2) + w_3(14x_1 + 35x_2) + w_4(\ 25x_1 + 40x_2)$$

subject to Equation (3)

A standard optimization technique is used to solve the problem and found solution for different weights.

For example, $w_1 = 0 = w_4,\ w_2 = 1 = w_3$

MOLPP $Max(w)\ \mathrm{f}(\mathrm{x}_1, x_2) = 24x_1 + 60x_2$ subject to Equation (3)

$$(x_1, x_2) = (0,0.9)$$

$$Max(w)\ \mathrm{f}(\mathrm{x}_1{}^*, x^*{}_2). = f(0,0.9) = 0.9\tilde{c}_2$$

$$\mu_{(f(0,0.9))}(x) = \begin{cases} 0 & x \le 18 \\ \dfrac{x-18}{4.5} & 18 < x \le 22.5 \\ 1 & 22.5 < x \le 31.5 \\ \dfrac{36-x}{4.5} & 31.5 < x \le 36 \\ 0 & x > 36 \end{cases}$$

Following table lists the solution for above MOLPP for various weights and it also shows that the solutions are independent of weights $(w_i, i = 1,2,3,4)$ (Table 1).

Conclusion

We successfully discussed the solution of fuzzy linear programming problem with the help of multi objective constrained linear programming problem where constraint matrix and the cost coefficients are fuzzy quantities and also proved that the solutions are independent of weights.

REFERENCES

Bellman RE, Zadeh LA (1970). Decision making in fuzzy environment, Manage. Sci., 17B: 141-B164.

Campose L, Verdegay JL (1989). Linear programming problems and ranking of fuzzy numbers, Fuzzy Sets Syst., 32(1): 1-11.

Mahmoud A, Abo-Sinna (2004). Multiple objective (fuzzy) dynamic programming problems: a survey and some applications, Appl. Math. Comput., 157: 861-888.

Negoita CV (1970). Fuzziness in management, OPSA/TIMS Miami.

Takashi M (2001). A Fuzzy linear programming problems as bi-criteria optimization problem, Appl. Math. Comput., 120: 109-121.

Takeshi I, Hiroaki I, Teruaki N (1991). A model of crop planning under uncertainty in (1): 159-171.

Tanaka H, Ichihashi H, Asai K (1991). Formulation of fuzzy linear programming problem based on comparison of fuzzy numbers, Control Cybernetics 3 (3): 185-194.

Zhang G, Yong-Hong Wu, Remias M, Jie Lu (2003). Formulation of fuzzy linear programming problems as four-objective constrained optimization problems, Appl. Math. Comput., 139: 383-399.

Zimmermann HJ (1976). Description and optimization of fuzzy systems, Int. J. General Syst., 2/4: 209-215.

Zimmermann HJ (1978). Fuzzy programming and linear programming with several objective functions, Fuzzy Sets Syst., 1: 45-55.