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Review

Does a physically reasonable solution of the Navier Stokes equations exist?

Asya S. Skal

P. O. Box 1836, Ariel 44837, Israel. E-mail: asyaskal@yahoo.com

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According to the literature, the conservation of momentum equation needs to be coupled with the mass conservation equation. However, they cannot create a coupled system of equations of motion because they ignored third Newton law. The conservation of momentum equation is Newton's second law of motion, whereas conservation of mass belongs to kinematics that have no deal with forces at all. However, no one motion in nature can be described only by Newton's second law without Newton's third law (every action creates an equal and opposite reaction). Einstein (1905) referred to this as dynamic equilibrium. It is only half the task to construct the conservation of momentum equation of "action". The second equally important part is to find the equation of "reaction", which would satisfied flow problem. We will show that the system of the conservation of momentum and diffusion of momentum equations satisfies the dynamic equilibrium condition.

Key words: Static and dynamic equilibrium, momentum, Navier Stokes (NS) equations.

INTRODUCTION

The Navier Stokes (NS) equations were developed by Navier in 1831, and more rigorously by Stokes in 1845. According to the (literature) they can describe ocean currents, water flow in a pipe, flow around an airfoil (wing), motion of stars inside a galaxy, the analysis of the effects of pollution, and so on; we will show that they also deal with electrodynamics and plasma physics.

The NS equations are also of great interest in a purely mathematical sense; therefore the Clay Mathematics Institute calls them as one of the seven most important open problems in mathematics. They formulated the problem as follows (Constantin, 2001; Scheffer, 1976): The conservation of momentum coupled with supplemental equations (conservation of mass) and well formulated boundary conditions seem to model fluid motion accurately; even turbulent flows seem (on average) to agree with real world observations.

However, mathematicians cannot yet prove that in three dimensions, solutions always exist (existence), or that if they do exist they do not contain any infinities, singularities or discontinuities (smoothness). These are called the NS existence and smoothness problems. So the first of the entire question, which we present in title, needs to be answered: Does a physically reasonable solution of NS equations exist?

The main purpose of the present paper is to show that an answer for such formulated millennium problem does not exist. The NS equations consist of three foundational axioms of fluid dynamics, which are the conservation of mass, momentum and energy laws in physics. Such systems have five unknowns: scalar, pressure, temperature, vector, and velocity. However, it is convenient to view in fluid mechanics that conservation of momentum and mass equations can be uncoupled from the conservation of energy and solved for the four unknowns: a scalar pressure and a vector velocity without regards to the temperature variation. Such decoupling is possible if the viscosities are required to be constant.

The nonlinearity of NS equations are due to convective acceleration, which is an acceleration associated with the change in velocity over position. This was partly solved (equation with the one nonlinear term) in our previous papers (Skal, 1995, 2002).

However, there are two reasons why the momentum and mass conservation equations cannot be coupled. For incompressible fluid, the first reason is mathematical and the second more common reason is physical and valid for compressible and incompressible fluids.

THE MOMENTUM AND MASS CONSERVATION EQUATIONS CANNOT BE COUPLED TWICE FOR INCOMPRESSIBLE FLUID

For incompressible fluid the density for ρ = const. This reduces mass conservation equation for fluid with average velocity u:

$$\nabla(\rho u) = -\partial\rho/\partial t \quad (1)$$

To the equation of continuity:

$$\nabla \cdot u = 0 \quad (2)$$

The conservation of momentum equation of motion for fluid is a relation equating the rate of change of momentum and the sum of all volume and surface forces acting on a fluid. During the derivation of momentum equation, we used the sum of all volume and surface forces acting on fluid. There are the external forces which act at distantly like gravity, the internal force which is due to fact of shearing stress called the viscous force. The latter take a form in common case as:

$$e_{ij} = \mu\left(\frac{\partial^2 u_{ij}}{\partial x_i \partial x_j} + \frac{\partial u_{ij}}{3\partial x_i}\right) \quad (3)$$

A simplification of the resulting flow equations is obtained when considering an incompressible flow of a Newtonian fluid. The assumption of incompressibility rules out diagonal term of stress tensor, which reduced to $\mu \nabla 2(u)$ by using mass-conservation equation. So here we have coupled the momentum and mass conservation equations for the first time and have obtained well known equation ρ ∂(u):

$$\rho\, \partial(u)/\partial t + \nabla (\rho 2u2)/(2\rho) + \_ \times (\rho u) = v\nabla 2(\rho u) - \nabla [p + \rho\varphi] \quad (4a)$$

$$\Omega = \nabla \times u; \ \nabla \cdot \Omega = 0 \quad (4b)$$

where Ω [1/T] is the local vorticity, uρ is the momentum density, p [ML−1T−2] is pressure and φ[L2T−2] is the gravity potential, respectively.

Since this vector equation represent the system of three scalar equations and has four unknown variable (u, p), it is usually coupled a second time with the mass conservation equation, which is against the mathematical rules regardless of a statement that the conservation of mass is generally necessary.

THE PHYSICAL REASON WHY THE MOMENTUM AND MASS CONSERVATION EQUATIONS CANNOT BE COUPLED

The law of conservation of mass says that the mass of a closed system will remain constant, regardless of the processes acting inside the system. Conservation of linear momentum (also known as Newton's second law of motion) are physical laws that form the basis for classical mechanics. Whereas the conservation of momentums are dynamical equations, the conservation of mass is a kinematical equation. Kinematics deals with mere geometry of motion without reference to the applied forces, whereas dynamics deals with the applied forces that produce changes in the motion of the fluid. No one motion in nature can be described only by Newton's second law without Newton's third law (action had aroused reaction), that Einstein (Einstein, 1905), named dynamic equilibrium. How can conservation of momentum and mass equations satisfy a dynamic equilibrium condition if they belong to different branches of fluid mechanics? Therefore, one can say that the conservation of momentum equation alone (or even supplemented by the conservation of mass equation) is incomplete, because liquid would never flow in the pipe or river by forces presented in these equations.

So the millennium problem of a system of momentum and mass conservation equations has no mathematical solution because it has no physically reasonable solutions. The second purpose of this paper is to find the supplemented dynamical equation which satisfies Newton's third law and the dynamic equilibrium.

THE DIFFUSION OF MOMENTUM EQUATION SATISFIES NEWTON'S THIRD LAW

Momentum diffusion refers to the diffusion, or spread of momentum between particles (atoms or molecules) of matter. The rate of transport is governed by the viscosity of the fluid and the momentum gradient. Momentum transport in a fluid depends on the macroscopic momentum of molecules of the system.

If fluid is in motion, the molecules will posses a microscopic momentum in the moving x-direction of flow. If there is a variation in flow velocity, the transfer moving molecules possesses a greater momentum in the z-direction of flow and can transfer the excess momentum to their slower moving neighbors perpendicular to the flow in the y-direction and x-direction.

There are two friction mechanisms in fluid transport: the internal molecular shear friction and the drag friction. The shear stresses in fluid can be thought of as the transfer of momentum in the fluid.. The shear force is parallel to flow z-direction and belong to the equation of conservation of momentum (Newton's second law, "action"). However, this friction force inside a fluid in addition to all other driving forces would not make a fluid to flow and so is not responsible for the third Newton law.

Momentum diffusion can be seen as an important process responsible for Newton third law. In the case of the laminar flow of a liquid past a solid surface, momentum diffuses across the boundary layer which forms at the boundary where the solid meets the liquid.

The gradient in this case occurs between the liquid in contact with the surface, which does not move at all and has zero momentum (see no-slip boundary condition), and the liquid far away from the wall, which has momentum proportional to the speed at which it is flowing. This gradient of a velocity gives rise to the transfer of momentum toward the boundary, which causes drag friction force. For example, the liquid in the pipe is moving only because the liquid in a pipe exerts the force on the boundary of pipe due to that gradient. According to Newton's third law, the force exerts from liquid balanced by a friction force exerts by the boundary of a pipe.

One has two momentum diffusion mechanisms, one for shearing force inside the fluid and second for friction force on boundaries. The first force belongs to Newton's second law whereas the second force belongs to Newton's third.

In laminar flow, momentum is generated uniformly throughout the fluid and is transferred to the boundaries perpendicular to flow direction by molecular diffusion. A total force balance around the volume element states that the sum of drag forces must equal zero at steady state. This balance means that pressure drop is equal to the sum of the shearing force and the drag friction force on boundary.

Thus the system of equations of conservation of momentum and diffusion of momentum satisfied dynamical equilibrium. Exists or does not exist solution of such system can be proved by comparison of the experimental data with theory.

CONCLUSION

Just as in hydrostatics, we need to satisfy static equilibrium condition for forces, in the hydrodynamics, we need to construct our system so as to satisfy the dynamic equilibrium condition, which cannot be satisfied by coupling NS and mass conservation equations. Therefore the millennium problem formulated in such a way by Clay Mathematical Institute needs be reformulated.

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