Full Length Research Paper

Analytical and experimental study of velocity shear instability in the presence of inhomogeneous perpendicular D.C electric field

R. K. Tyagi¹, K. K. Srivastava¹ and R. S. Pandey^{2*}

¹Department of Mechanical Engineering, Birla Institute of Technology, Mesra Ranchi, Jharkhand, India. ²Department of Applied Physics, Amity School of Engineering and Technology, Amity University, Uatter Pradesh, Noida, India.

Accepted 19 January, 2011

Analysis and experimental study of parallel flow velocity shear electrostatic ion cyclotron (EIC) instability has been done in plasma containing massive positive ions and electron by using the method of characteristics solution and kinetic theory in the presence of inhomogeneous direct current (DC) electric field perpendicular to ambient magnetic field. The effect of many parameters on growth rate and real frequency has been discussed by using the experimental data. Applications to possible laboratory plasmas and industries are also discussed.

Key words: Velocity shear instability with inhomogeneous D.C electric field.

INTRODUCTION

A parallel flow velocity shear which means a magnetic field (B) aligned flow with the velocity gradient perpendicular to B is known to be an origin of low frequency instabilities occurring in space plasmas. For example, ion acoustic instabilities observed in the ionosphere have recently been explained in terms of parallel flow velocity shear (lchiki et al., 2005).

Many characteristics of EIC instability that have been investigated experimentally were in agreement with the local theory of Drummond and Resenbluth (1962) appropriate to uniform magnetized plasma. Their treatment however could not account for certain aspects of instability related to in homogeneities present in laboratory plasmas. Bakshi et al. (1983) found that if width of the current channel was reduced to just a few ion Larmor radii, the EIC instability was quenched. The effects of a transverse gradient in the plasma flow velocity parallel to a magnetic field on the excitation of EIC waves has also been analyzed by Ganguli et al. (2002). They showed that ion flow gradients (parallel velocity shear) can give rise to a new class of ion cyclotron waves via inverse cyclotron damping with a resulting multiple cyclotron harmonic spectrum.

The effect of parallel velocity shear on EIC wave excitation was studied experimentally by many workers. These experiments provided clear evidence. That parallel velocity shear does play a role in the excitation of EIC waves. Experimental evidence have also been persecuted by Agrimson et al. (2003) to show that the filament quenching of EIC instability may be rendered ineffective if, in addition to field aligned current, parallel velocity shear is present in ion flow. Observation shows that the EIC instability can be excited in narrow current channels if a sufficient amount of parallel velocity shear is also present (Agrimson et al., 2003).

An examination of the EIC instability in the light of recent theoretical work of Gaungli et al. (2002) and Gavrishchaka et al. (2000) suggest that an ion flow along the magnetic field with a flow gradient (shear) transverse to the magnetic field can have an additional destabilizing effect on EIC waves in the presence of magnetic field aligned electron drift (current). A related theoretical prediction by Gavrishchaka et al. (1998) that perpendicular shear in the flow of ions parallel to a magnetic field substantially reduces the electron drift necessary to excite ion acoustic-like waves was a subject of previous

^{*}Corresponding author. E-mail: rspandey@amity.edu

experiment by Agrimson et al. (2001), since these predictions have important consequences for the inter prediction of low frequency plasma oscillations observations in the ionosphere.

Some experiments in Q-machines have been performed using Cesiveu plasma to test these in the laboratory. The purpose of these experiments was to investigate the possible influence of parallel velocity shear on the excitation of EIC waves. Experiment was performed in a single-ended Q machine using essentially the identical set up employed by many researches studying EIC waves. Addition evidence for the role of parallel velocity shear was obtained in the other experiment which employs a double ended Q machine Agrimson et al. (2002). Drift wave is observed to be destabilized by a magnetic-field-aligned ion flow velocity shear in the absence of field-aligned electron drift flow in laboratory experiments using a concentrically three segmented plasma source. The fluctuation amplitude increases with increasing a shear length, but the instability is found to be gradually stabilized where the shear strength exceeds a critical value (Kaneko et al., 2003). The stabilizing and destabilizing mechanisms are well explained by a plasma kinetic theory including the effect of radial density gradient (Kaneko et al., 2003).

Experimental demonstration of the certification of electrostatic ion cyclotron waves in plasma with a parallel ion flow shear but no magnetic field aligned current has already being discussed (Kim et al., 2004). Waves with frequencies near the ion cyclotron and multiple harmonics launched from an antenna are observed to grow in amplitude in a region of ion flow shear.

Particularly in the field of space plasma, the velocity shear of magnetic field-aligned flows has been studied to understand the physical properties of fluctuations occurring in space plasmas, which are observed by radars, rockets and satellites. The reason for predominant excitation of the ion acoustic instability in the Earth's ionosphere is explained by the fact that the parallel shear degrades the ion Landau damping rate of the ion acoustic mode to reduce this mode prominence (Gavrishchaka, 1998). This hypothesis was bench marked by several laboratory experiments. The reason for parallel shear modified instabilities has recently been extended to the drift wave instability (Teodorescu, 2002).

For simplicity it is assumed that there is no shear in the drift velocity. Generally speaking however, the velocity shear should be taken under consideration and may affect the system in two ways; on one half, it may lead to suppression or a complete stabilization of the perpendicular gradient instability; on the other hand, it may give rise to a traditional hydrodynamics instability of a shear flow. It has been focused that ion velocity shears can significantly lower the field aligned threshold current needed to trigger the instability, especially for wave vectors close to the perpendicular to the magnetic field. However the current density and shear requirements remain significantly higher than if collisions are neglected (Perron et al., 2009). The shear driven electrostatic ion cyclotron instability EIC has also been discussed using loss-cone distribution function by particle aspect analysis (Mishra et al., 2007).

In the present work, analytical and experimental study of velocity shear instability has been done in the presence of inhomogeneous perpendicular D.C electric field. The role of the electric field, magnetic field, shear scale length and temperature ratio has been discussed in detail on the growth rate of shear instability.

ANALYSIS

Spatially homogeneous anisotropic plasma subjected to an external magnetic field $B_0 = B_0 \hat{e}_z$ and an inhomogeneous DC electric field $E_0(x) = E_0(x)\hat{e}_x$ has been considered. In order to obtain the dispersion relation, the Vlasov–Maxwell equations are linearized for inhomogeneous plasma by small perturbations of E₁, B₁ and f_{s1}. These are perturbed quantities and are assumed to have harmonic dependence as exp i(kr- ω t). The linearized Vlasov equations obtained by separating the equilibrium and non equilibrium parts following the technique of Misra and Pandey (1995) and Pandey et al. (2001) in units of c=1 (c=velocity of light) are given as:

$$v\frac{\partial f_{s0}}{\partial r} + \frac{e_s}{m_s} \left[E_0(x) + (v \times B_0) \right] \left(\frac{\partial f_{s0}}{\partial v} \right) = 0$$
(1)

$$\frac{\partial f_{s1}}{\partial t} + v \frac{\partial f_{s1}}{\partial r} + \left(\frac{F}{m_s}\right) \left(\frac{\partial f_{s1}}{\partial v}\right) = S(r, v, t)$$
(2)

where force is defined as F= mdv/dt

$$F = e_s \left[E_0(x) + \left(\mathbf{v} \times B_0 \right) \right]$$
(3)

The particle trajectories are obtained by solving the equation of motion defined in Equation (3) and S(r,v,t) is defined as:

$$\mathbf{S}(\mathbf{r},\mathbf{v},\mathbf{t}) = \left(-\frac{\mathbf{e}_{s}}{\mathbf{m}_{s}}\right) \left[\mathbf{E}_{1} + \mathbf{v} \times \mathbf{B}_{1}\right] \left(\frac{\partial \mathbf{f}_{s0}}{\partial \mathbf{v}}\right)$$
(4)

The method of characteristics solution is used to determine the perturbed distribution function $f_{s1}.$ This is obtained from Equation (2) by

$$\mathbf{f}_{s1}(\mathbf{r},\mathbf{v},\mathbf{t}) = \int_0^\infty \mathbf{S}\left\{\mathbf{r}_0(\mathbf{r},\mathbf{v},\mathbf{t}), \mathbf{v}_0(\mathbf{r},\mathbf{v},\mathbf{t}), \mathbf{t}-\mathbf{t}\right\} dt'$$
(5)

where s denotes species. We transformed the phase space co-

ordinate system from (r,v,t) to (r₀,v₀,t-t) and t = t - t. The particle trajectories obtained by solving Equation (3) for the given external field configuration by Misra and Tiwari (1977) are given as

$$\mathbf{x}(t) = \mathbf{x}_{0} + \frac{\mathbf{v}_{\perp}}{\Omega_{s}} \left(1 + \frac{\overline{\mathbf{E}}'(\mathbf{x})}{4\Omega_{s}^{2}} \right) \left[\sin(\theta - \Omega_{s}t) - \sin\theta \right]$$
$$\mathbf{y}(t) = \mathbf{y}_{0} + \Delta' + \frac{\mathbf{v}_{\perp}}{\Omega_{s}} \left(1 + \frac{3}{4} \frac{\overline{\mathbf{E}}'(\mathbf{x})}{4\Omega_{s}^{2}} \right) \left[\sin(\theta - \Omega_{s}t) - \cos\theta \right]$$
$$\mathbf{z} = \mathbf{z}_{0} + \mathbf{v}_{\parallel}t$$
(6)

Where θ is the phase of v_{\perp} at t=0, $E^{'}(x),E^{''}(x)$ are the derivatives of E(x), and

$$\Delta = \frac{\overline{E}'(x)t}{\Omega_s^2} \left[1 + \frac{E''(x)}{E(x)} \cdot \frac{1}{4} \left(\frac{v_\perp}{\Omega_s} \right)^2 \dots \right],$$

$$\overline{E}(x) = \frac{e_s E(x)}{m_s},$$

$$E(x) = E_{0x} \left(1 - \frac{x^2}{a^2} \right), \quad \Omega_s = \frac{e_s B_0}{m_s}.$$

Where a is thought to be comparable to the mean ion gyro-radius but larger than the debye length when $x^2 \big/ a^2$ <1, E(x) becomes a constant uniform field.

Using Equations (2), (4) and (6) and doing some lengthy algebraic simplifications and time integration, we obtained the perturbed distribution function from Equation (5) as:

$$\mathbf{f}_{s1}(\mathbf{r},\mathbf{v},\mathbf{t}) = \frac{\mathbf{i}\mathbf{e}_{s}}{\mathbf{m}_{s}\boldsymbol{\omega}} \sum_{\mathbf{m},\mathbf{n},\mathbf{p},\mathbf{g}} \mathbf{J}_{\mathbf{n}}(\lambda_{1}) \mathbf{J}_{\mathbf{m}}(\lambda_{1}) \mathbf{J}_{\mathbf{p}}(\lambda_{2}) \mathbf{J}_{g}(\lambda_{2}) \mathbf{e}^{\mathbf{i}(\mathbf{m}-\mathbf{n})(\pi/2+\theta)} \mathbf{e}^{\mathbf{i}(g-p)(\pi/2+\theta)} \times \mathbf{e}^{\mathbf{i}(g-p)(\pi/2+\theta)} \mathbf{e}^{\mathbf{i}(g-p)} \mathbf{e}^{\mathbf{i}(g-p)($$

$$\left[E_{1x}U^{*} + V^{*}E_{1y} + W^{*}E_{1z}\right] \times \frac{1}{k_{\parallel}v_{\parallel} + n\Omega_{s} + p\Omega_{s} + k_{\perp}\Delta' - \omega}$$
(7)

Where;

$$\begin{split} \mathbf{U}^{*} &= \mathbf{C}\mathbf{v}_{\perp} \frac{\mathbf{n}}{\lambda_{1}} - \mathbf{C}\mathbf{v}_{\perp} \frac{\mathbf{n}}{\lambda_{1}} \frac{\overline{\mathbf{E}}'(\mathbf{x})}{4\Omega_{s}} + \mathbf{k}_{\perp}\mathbf{v}_{\perp}\boldsymbol{\xi}^{"} \frac{\mathbf{n}}{\lambda_{1}} + \mathbf{k}_{\perp}\mathbf{v}_{\perp}\boldsymbol{\xi}^{"} \frac{\overline{\mathbf{E}}'(\mathbf{x})}{4\Omega_{s}^{2}} \frac{\mathbf{n}}{\lambda_{1}} \\ \mathbf{V}^{*} &= \mathbf{i}\mathbf{C}\mathbf{v}_{\perp} \frac{\mathbf{J}_{n}'(\lambda_{1})}{\mathbf{J}_{n}(\lambda_{1})} + \frac{3}{4}\mathbf{C}\mathbf{v}_{\perp} \frac{\overline{\mathbf{E}}'(\mathbf{x})\mathbf{n}}{\Omega_{s}^{2}\lambda_{1}} - \Delta^{'}\mathbf{C} \\ \mathbf{W}^{*} &= \omega \frac{\partial \mathbf{f}_{s0}}{\partial \mathbf{v}_{\parallel}} + \mathbf{D}\mathbf{k}_{\perp}\mathbf{v}_{\perp} \frac{\mathbf{n}}{\lambda_{1}} + \frac{3\mathbf{D}\mathbf{k}_{\perp}\mathbf{v}_{\perp}}{4} \frac{\overline{\mathbf{E}}'(\mathbf{x})}{\Omega_{s}^{2}} \frac{\mathbf{n}}{\lambda_{1}} \\ \mathbf{C} &= \left(\omega - \mathbf{k}_{\parallel}\mathbf{v}_{\parallel}\right) \left(\frac{-2\mathbf{f}_{s0}}{\alpha_{\perp s}^{2}}\right) + \mathbf{k}_{\parallel} \frac{\partial \mathbf{f}_{s0}}{\partial \mathbf{v}_{\parallel}} \\ \mathbf{D} &= \mathbf{v}_{\parallel} \left(\frac{2\mathbf{f}_{s0}}{\alpha_{\perp s}^{2}}\right) - \frac{\partial \mathbf{f}_{s0}}{\partial \mathbf{v}_{\parallel}} \end{split}$$

Where the Bessel identity

$$e^{i\lambda_{1}\sin\theta} = \sum_{n = -\infty}^{\infty} J_{n}(\lambda_{1}) e^{in\theta}$$

$$J_{n}(\lambda_{1}) = \text{Bessel function}$$

$$\begin{split} \lambda_{1} &= \frac{k_{\perp} v_{\perp}}{\Omega_{s}} , \lambda_{2} = \frac{3}{4} \frac{k_{\perp} v_{\perp} \overline{E'}(x)}{\Omega_{s}^{3}}, \Delta' = \frac{\partial \Delta}{\partial t} \\ J_{n}'(\lambda_{1}) &= \frac{J_{n}(\lambda_{1})}{d\lambda_{1}} \end{split}$$

The unperturbed bi-Maxwellian distribution function is written as

where ξ^{*} is being constant of motion and $\,n_{_{0}}^{}(x\,)$ =plasma particle density

$$\alpha_{\perp s} = \left(\frac{2k_{B}T_{\perp s}}{m_{s}}\right)^{1/2} , \alpha_{\parallel s} = \left(\frac{2k_{B}T_{\parallel s}}{m_{s}}\right)^{1/2}$$

Now simplifying m = n, g = p and using the definition of current density, conductivity and dielectric tensor, we get the dielectric tensor

$$\left\| \varepsilon(\mathbf{k}_{1}\omega) \right\| = 1 - \frac{4\pi \ \mathbf{e}_{s}^{2}}{m_{s}\omega^{2}} \int \frac{d^{3}v \sum J_{p}^{2}(\lambda_{2}) \| \mathbf{S}_{ij} \|}{\mathbf{k}_{\parallel} \mathbf{v}_{\parallel} + \mathbf{n}\Omega_{s} + \mathbf{p}\Omega_{s} + \mathbf{k}_{\perp}\Delta' - \omega}$$
⁽⁹⁾

Where;

$$\left\| \mathbf{S}_{ij} \right\| = \begin{vmatrix} \mathbf{J}_{n}^{2} \mathbf{U}^{*} \frac{\mathbf{n}}{\lambda_{1}} \mathbf{v}_{\perp} & \mathbf{J}_{n}^{2} \mathbf{V}^{*} \frac{\mathbf{n}}{\lambda_{1}} \mathbf{v}_{\perp} & \mathbf{J}_{n}^{2} \mathbf{W}^{*} \frac{\mathbf{n}}{\lambda_{1}} \mathbf{v}_{\perp} \\ -\mathbf{i} \mathbf{J}_{n} \mathbf{J}_{n}^{'} \mathbf{U}^{*} \mathbf{v}_{\perp} & -\mathbf{i} \mathbf{J}_{n} \mathbf{J}_{n}^{'} \mathbf{V}^{*} \mathbf{v}_{\perp} & -\mathbf{i} \mathbf{J}_{n} \mathbf{J}_{n}^{'} \mathbf{W}^{*} \mathbf{v}_{\perp} \\ \mathbf{J}_{n}^{2} \mathbf{U}^{*} \mathbf{v}_{\parallel} & \mathbf{J}_{n}^{2} \mathbf{V}^{*} \mathbf{v}_{\parallel} & \mathbf{J}_{n}^{2} \mathbf{W}^{*} \mathbf{v}_{\parallel} \end{vmatrix}$$

Now we consider electrostatic ion-cyclotron instability

$$\|\varepsilon_{\parallel}\| = \mathbf{N}^2 \tag{10}$$

Where N is refractive index

The required electrostatic dispersion relation can be obtained by using the approximation of Huba (1981) and from Equations (8-10).

$$D(\mathbf{k},\omega) = 1 + \frac{2\omega_{ps}^{2}}{\mathbf{k}_{\perp}^{2}\alpha_{\perp s}^{2}}\Gamma_{n}(\mu_{s})\sum J_{P}^{2}(\lambda_{2})\left[1 - \frac{\mathbf{E}'(\mathbf{x})}{4\Omega_{s}^{2}}\right]\frac{\mathbf{k}_{\perp}}{\mathbf{k}_{\parallel}}$$

$$\left[\left(\frac{-\omega}{\mathbf{k}_{\parallel}\alpha_{\parallel s}} - \frac{1}{2}\varepsilon_{n}\rho_{s}\frac{\alpha_{\perp s}}{\alpha_{\parallel s}}\right)Z(\xi) - \mathbf{A}_{s}\frac{\alpha_{\perp s}^{2}}{\alpha_{\parallel s}^{2}}(1 + \xi Z(\xi)) + \mathbf{A}_{T}\frac{\mathbf{k}_{\parallel}}{\mathbf{k}_{\perp}}(1 + \xi Z(\xi))\right]$$

$$(11)$$

Where $Z(\xi)$ is plasma dispersion function

$$\xi = \frac{\overline{\omega} - (n+p)\Omega_s - k_{\perp}\Delta'}{k_{\parallel}\alpha_{\parallel}}$$
$$A_s = \frac{1}{\Omega_s} \frac{\delta v_{oz}(x)}{\delta x}$$
$$\varepsilon_n = \frac{\delta \ln n_0(x)}{\delta x}$$
$$A_T = \frac{\alpha_{\perp s}^2}{\alpha_{\parallel s}^2} - 1$$

$$\overline{\omega} = \omega - k_{\parallel} v_{oz}(x)$$

$$\mu_{s} = \frac{k_{\perp}^{2} \rho_{i}^{2}}{2}, \lambda_{Ds}^{2} = \frac{\alpha_{\perp s}^{2}}{2\omega_{ps}^{2}}, \omega_{ps}^{2} = \text{Plasma frequency}$$

 $\Gamma_n(\mu_s) = \exp(-(\mu_s)I_n(\mu_s)), \quad I_n(\mu_s) = Modified Bessel$ function of order n.

In order to get dispersion relation for electrons and ions, approximations for electrons are assumed as $\,k_{\perp}^{}\rho_{e}^{}<<\!1$ and for ions no such assumptions is done thus above equation for p=1 and species s= i, e becomes:

_

$$D(\mathbf{k},\omega) = 1 + \frac{1}{k_{\perp}^{2}\lambda_{De}^{2}} \eta_{e} \frac{T_{\perp e}}{T_{\parallel e}} + \frac{1}{k_{\perp}^{2}\lambda_{Di}^{2}} \eta_{e} \left[\frac{T_{\perp i}}{T_{\parallel i}} \Gamma_{n}(\boldsymbol{\mu}_{i}) \frac{k_{\perp}}{k_{\parallel}} \right] \left(\frac{\overline{\omega}}{k\alpha_{\parallel i}} \times \frac{T_{\perp i}}{T_{\parallel i}} - \frac{1}{2} \varepsilon_{n} \rho_{s} \frac{\alpha_{\perp i}}{\alpha_{\parallel i}} \right)$$
$$+ \frac{n\Omega_{i} + k_{\perp}\Delta'}{k_{\parallel}\alpha_{\parallel i}} \left(1 - \frac{T_{\perp i}}{T_{\parallel i}} \right) \left(2(\xi_{i}) - A_{i} \frac{T_{\perp i}}{T_{\parallel i}} (1 + \xi Z(\xi_{i}))) \right]$$
(12)

After substituting the asymptotic expansion of $Z(\xi_i) = -\frac{1}{\xi_i} - \frac{1}{2\xi_i^3} \text{ for ion i.e. } |\xi_i| <<1 \text{ Fried and Conte (1961)}$

and $n_{_{0i}} = n_{_{0e}}$ multiplying throughout by $\frac{k_{\perp}^2 \lambda_{_{Di}}^2}{\eta_i}$ the above equation becomes:

$$\int 0 = \frac{\lambda_{\text{Di}}^{2}}{\lambda_{\text{De}}^{2}} \frac{\eta_{\text{e}}}{\eta_{\text{i}}} \frac{T_{\perp\text{e}}}{T_{\text{He}}} + \frac{T_{\perp\text{i}}}{T_{\text{Hi}}} - \Gamma_{n}(\mu_{\text{i}}) \frac{T_{\perp\text{i}}}{T_{\text{Hi}}} + \frac{\Gamma_{n}(\mu_{\text{i}})k_{\perp}}{2k_{\text{H}}} \varepsilon_{n} \rho_{\text{i}} \frac{\alpha_{\perp\text{s}}}{\alpha_{\text{Hs}}} \frac{k_{\text{H}}\alpha_{\text{Hi}}}{\omega - n\Omega_{\text{i}} + k_{\perp}\Delta'} - \frac{\Gamma_{n}(\mu_{\text{i}})k_{\perp}}{2k_{\text{H}}} \varepsilon_{n} \rho_{\text{i}} \frac{\alpha_{\perp\text{s}}}{\alpha_{\text{Hs}}} \frac{k_{\text{H}}\alpha_{\text{Hi}}}{\omega - n\Omega_{\text{i}} + k_{\perp}\Delta'} - \frac{\Gamma_{n}(\mu_{\text{i}})k_{\perp}}{2(\omega - n\Omega_{\text{i}} - k_{\perp}\Delta)^{2}} \cdot \frac{T_{\perp\text{i}}}{T_{\text{Hi}}} (k_{\text{H}}\alpha_{\text{Hi}})^{2} \left(1 - \frac{k_{\perp}}{k_{\text{H}}}A_{\text{i}}\right)$$
(13)

Where

$$\begin{split} \eta_{i} &= 1 - \frac{\overline{E_{i}}'(x)}{4\Omega_{i}^{2}} \\ \eta_{e} &= 1 - \frac{\overline{E_{e}'}(x)}{4\Omega_{e}^{2}} \\ \text{Multiplying} & \text{Equation} \quad (13) \quad \text{throughout} \quad \text{by} \\ & \left(\frac{\overline{\omega} - n\Omega_{i} - k_{\perp}\Delta'}{k_{\parallel}\alpha_{\parallel i}}\right)^{2} \text{ we obtain a quadratic equationion as;} \end{split}$$

$$\frac{\overline{\omega'}}{\Omega_i} = -\frac{b_1}{2a_1} \left[1 \pm \sqrt{\left(1 - \frac{4a_1c_1}{b_1^2}\right)} \right]$$
(14)

Where;

$$\begin{aligned} a_{1} &= a_{2} \left(\frac{\Omega_{i}}{k_{\parallel} \alpha_{\parallel}} \right)^{2}, \\ a_{2} &= \frac{\eta_{e}}{\eta_{i}} \frac{T_{\perp i}}{T_{\parallel i}} + \frac{T_{\perp i}}{T_{\parallel i}} - \Gamma_{n} \left(\mu_{i} \right)^{T_{\perp i}} \frac{1}{T_{\parallel i}} \\ b_{1} &= \frac{\Omega_{i}}{k_{\parallel} \alpha_{\parallel i}} b_{2} - \frac{2k_{\perp} \Delta'}{k_{\parallel}^{2} \alpha_{\parallel i}^{2}} a_{2} \Omega_{i} \\ b_{2} &= \frac{\Gamma_{n} \left(\mu_{i} \right) k_{\perp}}{2k_{\parallel}} \varepsilon_{n} \rho_{i} \frac{\alpha_{\perp i}}{\alpha_{\parallel i}} - \frac{\Gamma_{n} \left(\mu_{i} \right) k_{\perp}}{2k_{\parallel}} - \frac{\Gamma_{n} \left(\mu_{i} \right) k_{\perp} n \Omega_{i}}{2k_{\parallel}^{2} \alpha_{\parallel i}} \\ c_{1} &= \frac{\Gamma_{n} \left(\mu_{i} \right) T_{\perp i}}{2T_{\parallel i}} \left(1 - \frac{k_{\perp}}{k_{\parallel}} A_{i} \right) - \frac{b_{2} k_{\perp} \Delta'}{k_{\parallel} \alpha_{\parallel i}} + \frac{k_{\perp}^{2} \Delta'^{2}}{k_{\parallel}^{2} \alpha_{\parallel i}^{2}} \\ \overline{\omega}' &= \overline{\omega} - n \Omega_{i} \\ P &= \frac{x^{2}}{a} \\ E(x) &= E_{0x} \left(1 - \frac{x^{2}}{a^{2}} \right) \end{aligned}$$

The solution of Equation (13) is

$$\overline{\omega'} = -\frac{b_1}{2a_1} \left[1 \pm \left(1 - \frac{4a_1c_1}{b_1^2} \right) \right]$$
(15)

From this expression dimensionless growth rate has been calculated by computer technique when $b_1^2 < 4a_1c_1$.

RESULTS AND DISCUSSION

Here, we show the solution of Equation (15) using parameters may be representative of the laboratory by Kim and Merlino (2007) and Rosenberg and Merlino (2008). We consider plasma in which the heavy positive

ions are produced due to ionization of $K^{\scriptscriptstyle +}$ and light electron are produced from SF6 $^-$. It is further assumed that the plasma is immersed in magnetic field strengths, are varying like 0.24 to 0.32 T and homogeneous DC electric field strength 4 to 12 V/m which is perpendicular to magnetic field. In this case, the positive ions gyro radius $\rho{\sim}$ 0.095 cm and having a temperature anisotropy

$$A_{_T} = \frac{T_{\perp i}}{T_{\parallel i}} - 1$$
 = 0.5 to 1.5 with density gradient $\epsilon_{_n} \rho_{_i}$

=0.2 and inhomogeneity in DC electric field P/a= 0.5,0.55. In this case, we would except that the heavy positive ions electrostatic ion cyclotron instability could become excited by parallel velocity shear scale length like $A_i = 0.5$ to 0.55.

In Figure 1, variation of growth rate
$$\frac{\gamma}{\Omega_{i}}$$
 versus $k_{\perp}\rho_{i}$

for different values of velocity shear scale length A_i has been shown for other fixed plasma parameters. The growth rate increase with increasing the value of shear scale length A_i band width did not shift for lower and higher valued of $k_\perp \rho_i$. The maximum peak value of growth rate is 4.00×10^{-3} at $k_\perp \rho_i$ =1.75 for shear length A_i=0.55 and in homogeneity in DC electric field P/a=0.5. The mechanism for instability of this mode is due to coupling of regions of positive and negative wave energy. This coupling occurs if velocity shear is non-uniform and hence velocity shear is the source of instability.

Figure 2 shows the variation of growth rate
$$\frac{\gamma}{\Omega_i}$$
 versus

 $k_{\perp}\rho_{\rm i}$ for various value of electron ion temperature ratio ${\bf T}$

 $\frac{T_e}{T_i}$. In this Figure 2, the growth rate increases by

increasing the values of electron ion temperature ratio due to in-homogeneity in electron ion temperature that dependence on the applied voltage of electrodes. The maximum peak values of growth rate is 3.86×10^{-3} at the $k_\perp \rho_i$ =1.75 with inhomogeneous DC electric field. As velocity shear term is proportional to electron ion temperature ratio $\frac{T_e}{T_i}$.

Figure 3a and 3b shows the variation of real frequency

 $\frac{\omega_{\rm r}}{\Omega_{\rm i}}$ and growth rate $\frac{\gamma}{\Omega_{\it i}}\,$ versus $\,k_{\perp}\rho_{\rm i}\,$ for different values

of magnetic field strength B_0 with other fixed parameters listed in the figure caption. The growth rate and real frequency decrease with increasing the magnetic field strength. Due to change in magnetic field, gyro frequency has been changed and also modified Doppler shift



Figure 1. Variation of growth rate γ/Ω_i verses $k_\perp \rho_i$ for different values of A_i and other parameters are B₀ = 0.24 T, T_e/T_i = 2, E₀ = 8 V/m, x/a =0.5, $\theta_1 = 88.5^0$, A_T =1.5, $\epsilon_n \rho_i$ =0.2.



Figure 2. Variation of growth rate γ/Ω_i verses $k_{\perp}\rho_i$ for different values of T_e/T_i and other parameters are A_i = 0.5, B₀ = 0.24 T, E₀=8 V/m, x/a = 0.5, θ = 88.5^o, A_T =1.5, $\epsilon_n\rho_i$ = 0.2

frequency. The homogeneous magnetic field couples positive and negative energy waves and changes the growth of the wave. The magnetic field strength is a useful parameter for required velocity of EIC wave. Hence this is useful result for designing a machine for cold spray, metal cutting operations and micromachining. Figure 4a shows the variation of growth rate $\frac{\gamma}{\Omega_i}$ versus

 $k_{\perp}\rho_i$ for different values of magnitude of inhomogeneous DC electric field. The growth rate decreases with increasing the value of magnitude of inhomogeneous



Figure 3. (a) Variation of growth rate γ/Ω_i verses $k_\perp \rho_i$ for different values of B₀ and other parameters are $A_i = 0.5$, $T_e/T_i = 2$, $E_0 = 8$ V/m, x/a = 0.5, $\Theta = 88.5^\circ$, $A_T = 1.5$, $\epsilon_n \rho_i = 0.2$. (b) Variation of real frequency ω_r/Ω_i verses $k_\perp \rho_i$ for different values of B₀ and other parameters are $A_i = 0.5$, $T_e/T_i = 2$, $E_0 = 8$ V/m, x/a = 0.5, $\Theta = 88.5^\circ$, $A_T = 1.5$, $\epsilon_n \rho_i = 0.2$.

DC electric field like 4 to 12 V/m. It means the magnitude of inhomogeneous DC electric field shows the stabilizing effect on the growth rate. Figure 4b shows the variation of

real frequency $\frac{\omega_{\rm r}}{\Omega_{\rm i}}$ versus $k_{\perp}\rho_{\rm i}$ for different values of

real frequency. It is clear from the Figure 4a that the real

frequency increases with increasing the value of magnitude of inhomogeneous electric field. The above discussion shows that the magnitude of inhomogeneous DC electric field has a destabilizing effect on the real frequency. In general, the non-resonant particles support the oscillatory motion of the wave while the resonant particles participate in energy exchange with the wave.



Figure 4. (a) Variation of growth rate γ/Ω_i verses $k_{\perp}\rho_i$ for different values of E₀ and other parameters are A_i =0.5, B₀=0.24 T, T_e/T_i =2, x/a =0.5, θ = 88.5^0 , A_T =1.5, $\epsilon_n\rho_i$ =0.2. (b) Variation of real frequency ω_r/Ω_i verses $k_{\perp}\rho_i$ for different values of E₀ and other parameters are A_i =0.5, B₀=0.24 T, T_e/T_i =2, θ = 88.5^0 , x/a =0.5,A_T =1.5, $\epsilon_n\rho_i$ =0.2.

The velocity of EIC wave is 1000 m/s for the value of magnitude of inhomogeneous DC electric field 8 V/m and it is 2500 m/s for 20 V/m with other fixed parameters listed in figure caption. Figure 5 shows the variation of growth rate versus inhomogeneity in DC electric field x/a with others parameters listed in figure caption. The

maximum value of dimensionless growth rate is 3.895×10^{-3} at $k_{\perp} \rho_i = 1.75$ obtained for x/a =0.3. This is the effective parameter for group velocity of the generation and control modulation of the wave.

The parameters like density gradient $\epsilon_{n}\rho_{i}$ and



Figure 5. Variation of growth rate γ/Ω_i verses x/a for parameters like $k_\perp \rho_i = 1.75$, $A_i = 0.5$, $B_0 = 0.24T$, $T_e/T_i = 2$, $E_0 = 8$ V/m, $A_T = 1.5$, $\epsilon_n \rho_i = 0.2$.

temperature anisotropy $A_{_{T}} = \frac{T_{_{\perp i}}}{T_{_{\parallel i}}} - 1$ have less effect of

growth rate for the experimental data but the growth rate increases slightly with increasing the value of $\epsilon_{\rm n}\rho_{\rm i}$ and $A_{\rm T}$.

CONCLUSION

In this paper the effect of magnetic field, electric field electron ion temperature ratio, temperature anisotropy in ions, shear scale length, density gradient and inhomogeneity in DC electric field x/a on the growth rate and real frequency have been studied separately.

REFERENCES

- Agrimson E, Kim SH, Angelo ND, Merlino RL (2003). Effect of parallel velocity shear on the electron static ion-cyclotron instability in filamentary current, Phys. Plasmas, 10: 3850.
- Agrimson E, Angelo ND, Merlino RL (2001). Excitation of Ion-Acoustic-Like Waves by Subcritical Currents in a Plasma Having Equal Electron and Ion Temperatures, Phys. Rev. Lett., 86: 5282-5285.
- Agrimson EP, Angelo ND, Merlino RL (2002). Effect of Parallel Velocity Shear on the Excitation of electrostatic ion cyclotron waves, Phys. Lett., 293: 260.
- Bakshi P, Ganguli G, Palmadesso P (1983). Finite width currents magnetic shear and current driven ion-cyclotron instability Phys. Fluids. 26: 1808-1983.
- Drummond WE, Rosenbluth MN (1962). Anomalous diffusion arising from microinstabilities in a plasma, Phys. Fluids, 5: 1507.
- Fried BD, Conte SD (1961). The Plasma dispersion function, San Diego, Calif.
- Ganguli, G, Slinker S, Gavrishchaka V, Scales W (2002). Low frequency oscillation in a plasma with spatially variance field-aligned flow, Phys. Plasmas, 9: 2321.

- Gavrishchaka V, Ganguli GI, Scales WA (2000). Multiscale coherent structures and broadband waves due to parallel inhomogeneous flows Phys. Rev., 85: 4285-4288.
- Gavrishchaka VV, Ganguli SB, Ganguli GI (1998). Origin of low frequency oscillations in the ionosphere, Phys. Rev. Lett., 80: 728.
- Huba JD (1981). The Kelvin-Helmholtz instability in inhomogeneous plasmas, J. Geophys. Res., 86: 3653-3656.
- Ichiki R, Kaneko T, Hatakeyama R (2005). Eindhoven, Netherlands. pp. 18-22.
- Kaneko T, Tsunoyarna H, Hatakeyama R (2003). Drift-Wave Instability Excited by Field-Aligned Ion Flow Velocity Shear in the Absence of Electron Current, Phys. Rev., 90: 1205001.
- Kim SU, Agrimson E, Miller MJ, Angelo ND, Merlino RL (2004). Amplification of electrostatic ion-cyclotron waves in a plasma with magnetic-field-aligned ion flow shear and no electron current, Phys. Plasmas, 11: 4501-4505.
- Kim SH, Merlino RL (2007). Electron attachment to C_7F_{14} and SF_6 in a thermally ionized potassium plasma, Phys. Rev., 76 : 35401.
- Mishra R, Verma P, Tiwari MS (2007). Effect of general loss loss-cone distribution on shear- driven electrostatic ion cyclotron instability, J. Planetary Space Sci., 9: 4.
- Misra KD, Pandey RS (1995). Generation of Whistler Emissions by Injection of Hot Electrons in the Presence of a Perpendicular Ac Electric Field, J. Geophys. Res., 100: 19405-19411.
- Misra KD, Tiwari MS (1977). Particle aspect analysis of drift instability in the presence of non- uniform electric field, Phys. Scripta, 16: 142.
- Pandey RS, Misra KD, Tripathi AK (2001). Kelvin –Helmholtz instabilityin an Anisotropic Magneto-plasma in the presence of Inhomogeneous Perpendicular Electric Field and Parallel Flow Velocity Shear Indian J. Radio Space Phys., 30: 113.
- Pandey RS, Misra KD, Tripathi AK (2003). Generation of electrostatics Ion-Cyclotron like Wave by Parallel Flow Velocity Shear in the presence of Inhomogeneous Electric Field in an Anisotropic Magneto-plasma, Indian J. Radio Space Phys., 30: 75-80.
- Perron PJ, Noel G, St-Maurice JP (2009). Velocity shear and current driven instability in a collisional F-region, Ann. Geophys. 27: 381-394.
- Rosenberg M, Merlino RL (2008). Instability of higher harmonic electrostatic ion cyclotron waves in a negative ion plasma, J. Plasmas Phys., pp. 1-14.
- Teodorescu C, Reynolds EW, Koepke ME (2002). Observation of inverse lon-cyclotron damping induced by parallel velocity shear, Phys. Rev. Lett., 88: 185003.