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Review

Underlying mathematical structures behind *David Blamey's* film "Rice"

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Images and texts are imprints of the ideas expressed by their creators. A viewer or a reader may decode such imprints differently, each time forming a scenario of a particular and unique structure. This process is determined by the educational and cultural stimuli as well as the conceptual formation of each viewer or reader. Thus, in this respect, both an image and a text act as a mind challenge to the production of subjective creations. The aim of this article is to discuss David Blamey's remarkably successful film, highlighting the following: (a) the geometrical structures forming the scene where the story of rice processing takes place; (b) the geometry of the harmonious movements of the persons involved and their projections (shades) in the horizontal plane of the soil; and c) the natural interpretation of the dropping of the rice from the basket to the horizontal plane of the soil.

Key words: Culture; image; geometrical structures; natural interpretation; geometry of the harmonious movements.

INTRODUCTION

David Blamey, inspired by his experience in India, where he has lived and occasionally worked for more than 30 years, has created an animation project on the traditional way of rice processing (Blamey, 2017). The backdrop of the whole process, which reminds of a ritual, is set in a forest region in India.

At sunrise, workers load rice quantities onto wicker baskets and transport them to specially shaped paved flat surfaces. With moves that remind of a theatrical performance, the workers reach the rice-loaded baskets, which are horizontally placed on their heads. They lean the baskets sideways and rotate them. That rotation makes the rice grains leave the basket, forming an impressive cloud, and thereafter scatter on the paved surface of the soil. This process continues until the flat rectangular pavement is completely covered with an even layer.

Then, throughout the day, two or three people move horizontally and vertically on the paved surface, spinning rice grains with their feet, for the rice to dry with the heat of the sun. To end the procedure, using improvised tools – the origins of which are traced back to centuries ago– the workers gather the scattered rice grains and place them on rectilinear layers. Then, they linearly load the

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Figure 1. The natural world.

Figure 2. The linear path of the light beam.

grains in rice sacks that they take to the market.

In addition to cultural, social and visual features, the rice processing method in this film has a sublime structure of Euclidean Geometry, theoretical algebra and engineering. The impressive images of the film highlight mathematical concepts such as parallelism, per-pendicularity, similitude, symmetry, as well as algebraic concepts, like the concept of group theory. The process of casting rice grains provides ground for an interpretation in terms of physics.

NATURAL WORLD AND REGULARITIES

The natural world Figure 1 is complex and people are very curious to comprehend it. However, this complexity prevents them from having full knowledge of its content. To overcome the difficulties created by complexity, the human mind imagined a world of regularities, via which it would be able to approach complexity. These regularities arise as a product of intellectual removal of the appreciable space. They will stimulate the mind to build philosophical theories such as Plato's world of Ideas (Taylor, 1804) as well as axiomatic systems such as Euclidean Geometry ((Heath, 1956), in which the mind will look for the "Truth" of their assumptions about how the natural world works.

These systems will be the spiritual foundation on which other systems will later be developed and will determine the basic scope of study and conceptual structure of what it nowadays call "Culture". When the Socratic philosopher Aristippus was shipwrecked on the coast of Rhodes, he observed geometrical figures engraved on the sand and cried out to his companions: "Let us be of good cheer, for I see the traces of man". He went straight to the city of Rhodes (Morgan, 1899).

By observing Figure 2 it is easy to see that the abstract object with which it could be simpler to represent the light

Figure 3. Parallel direction of the sun's rays.

beam passing through the hole of the laboratory's ceiling is the straight line. The straight line is a convention created by the human mind. There are no straight lines in the natural world.

In the 6th century BC, Thales of Miletus (Van Der Waerden, 1975) who was a pre-Socratic philosopher and mathematician, measured the height of the pyramid of Cheops in Egypt, relying on the assumption that the rays of the sun are directed to the earth on a rectilinear motion, and they are parallel to each other. Observing Figure 3, the authors' perception was very easily to accept, the parallel direction of the sun's rays. This perception could be questioned if they attempted to study the relationship between two light beams emitted by a light source near the projection level. In this case it is evident that the light rays show a relative deviation from each other. However, as the authors move the light source away from the projection plane, the light rays tend to become parallel to each other. Given that the sun, as a light source, is very far from the projection level that is the Earth's surface, the deviation from reality is not something possibly noticeable by the human senses. Therefore, the parallelism of the sun rays becomes almost certain to the human visual perception.

Thales thought of the height of the pyramid as a straight stick placed vertically on the ground. He then placed another stick of known length also vertically on the ground. Then, he assumed that a sun ray hits the top of each stick, casting a flat shadow on the ground for each one. The corresponding shadow of each stick on the ground forms two triangles. Comparing the ratios that were generated by the respective sides of the triangles, Thales noticed that they were equal. This observation led to the calculation of the height of the pyramid. He also noticed that these triangles were not irrelevant to each other: the one triangle was the magnification of the other triangle. In other words, while retaining similarity in form,

they did not have the same size. The qualitative relation of the similarity of the forms of the triangles corresponded to the constant quantitative relationship of the proportions of their sides. This led Thales to discover a regularity which would have a universal validity for the human mind and would therefore later constitute an objective definition in the context of an axiomatically founded geometry: Euclidean Geometry.

INTUITIVE OBSERVATION AND MATHEMATICAL APPROACH- METHODOLOGY

Figure 4, looks like a theatrical scene where, on the background of the complex natural environment, the workers involved in the action follow the steady steps of a ritual dance and move towards achieving their purpose. Although the size of their bodies is different, their movements seem to be intuitively following a regularity imposed by the energy needed to carry out their work. Consequently, there is an attempt to confirm this intuitive observation through a mathematical approach.

A snapshot of the film has been selected, showing the course of the two workers towards the place they are going to empty the rice they carry in their baskets. The chosen image was placed in a geometric software program (The Geometer's Sketchpad, 2009). Making an estimation of the image data, the e, e_1, e_2, e_3, e_4 lines were plotted. With the horizontal line e, we mark the ground on which the workers walk.

The lines e_1 and e_2 are the straight lines passing vertically through the workers' bodies. As they are perpendicular to the horizontal line e of the ground, they shall be parallel to each other. The authors mark as e_3 and e_4 where the straight lines passing through the calves of the workers' legs, which are lateral to the ground. The process of approaching the stepping process of the two
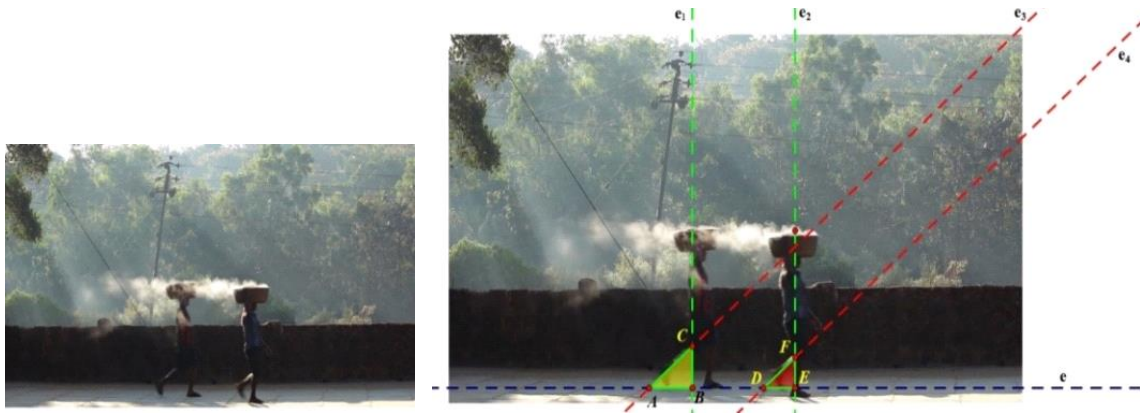
Figure 4. The similarity of the steps.

workers with these lines defines two triangles: ΔABC and ΔDEF. Thereafter, the authors measured the lengths of the sides of these triangles, and calculated the rations of their respective sides.

$$\frac{AB}{DE} \cong 1.38, \qquad \frac{BC}{EF} \cong 1.35, \qquad \frac{CA}{FD} \cong 1.36$$

The ratio calculations have resulted in values approximately equal, with an average approximate value of 1.36. The fact that the values of the ratios are approximately equal denotes the parallelism of the e_3 and e_4 lines and the similarity of the triangles ΔABC and ΔDEF. It can be inferred from this similarity that there is a regularity in the pacing process of the workers.

ISOMETRY AND SIMILARITY

In Figure 5, it was observe that at different time during the day, while walking on a flat surface evenly covered by rice grains, the workers are stirring rice grains with the soles of their feet so that these grains could be dry by solar heat. The way in which workers are placed on an orthogonal plane on the space they work is typical. They choose the most typical isometry position. Two workers stand at the edges and one worker is centered midway between the other two workers.
They all walk towards the same direction. The attached shadows of the workers on the ground define their own scene.

The sizes of the attached shadows, when compared to the corresponding sizes of the bodies that define them, also inform them about the time of the day when the task in question takes place. The positions of these shadows in relation to the bodies of the workers that define them, can also tell them about the position of the sun in relation

to the horizontal plane of the ground. Using the Dynamic Geometry software, the researchers superimposed the rectangular axle system on top of the image in such a way, so that the y-axis coincides with the body of the worker in the middle. Subsequently, they constructed the ΔABC and ΔDEF triangles defined by the workers' body directions, the directions of their shadows on the ground plane and the direction of the solar rays which pass through the heads of the two workers respectively and create their shadows. Then, using the length calculation tool of the dynamic geometry software, the authors measured the lengths of the sides of these triangles, and we calculated the ratios of their respective sides, with an approximation to the second decimal point.

$$\frac{AB}{DE} \cong 0.97, \qquad \frac{BC}{EF} \cong 0,98, \qquad \frac{CA}{FD} \cong 0,99$$

The ratio calculations have resulted in values approximately equal, with an average cutoff value of 0.98. They note that in both cases of Figures 4 and 5, the regularity of the similarity of the workers' movements is described by the one that Thales theorized at a different time and place, and in a different cultural and social context.

LINEAR PERSPECTIVE

In Figure 6, the straight lines q and r follow the two parallel sides of the rectangular surface, within which the rice grains have been laid. These straight lines, although they are in fact parallel, they converge and intersect at the point M. The straight line b also crosses this point, following the direction of the body of the worker in the middle. Point C corresponds to the head of the worker on the left. If from point C the authors draw the straight line l parallel to the line r, then they notice that it also runs
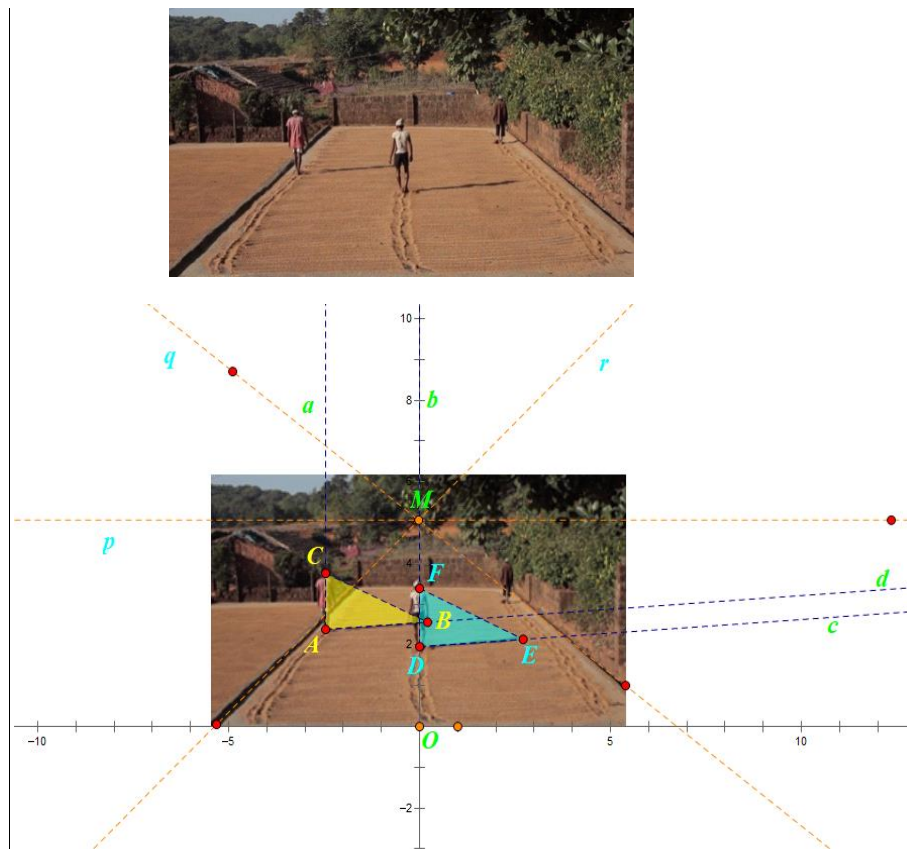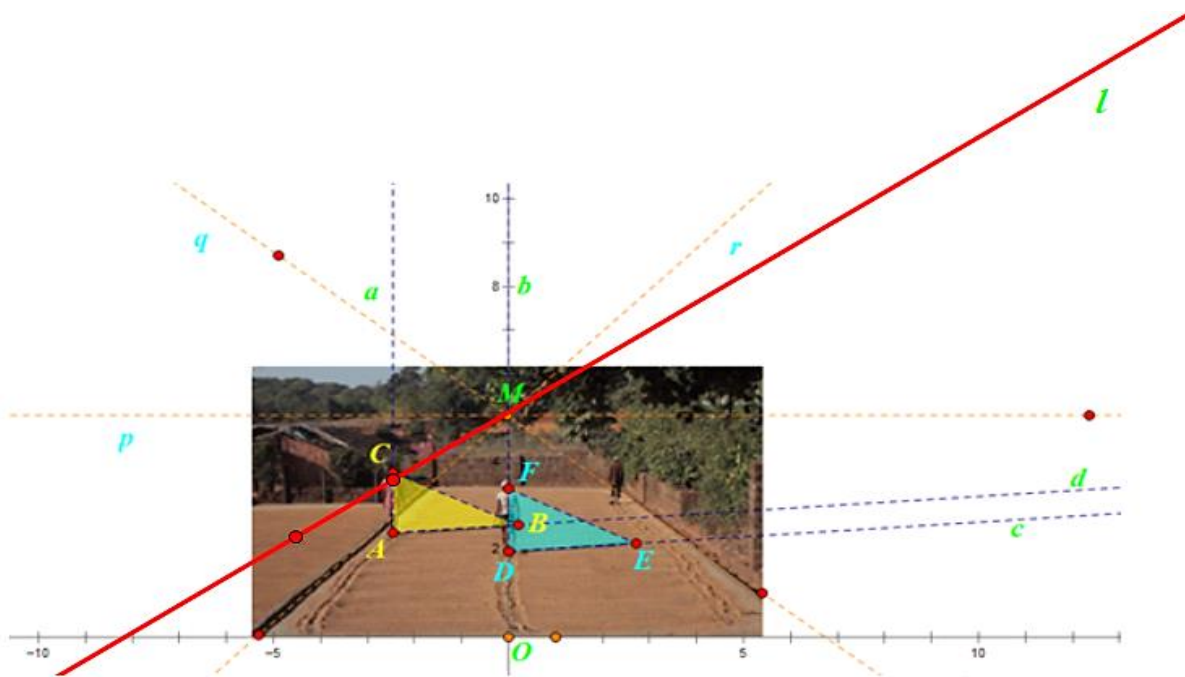
Figure 5. Isometry and similarity.

Figure 6. Linear perspective.

Figure 7. Empirical building structures.

through point M. Thus, the point M shall be defined as the vanishing point. The horizontal straight line p that runs through this point defines the horizon line which is in direct relation to the eye level of the worker who is standing up at ground level. The horizon line is at the eye height of this worker.

This figure highlights in a very characteristic way all the elements of a Renaissance painting, where the geometric analysis of the reality of the scene leads to the central linear perspective. The linear perspective as a method of visualizing the 3D reality on a two-dimensional canvas follows some geometric rules such as: a) all the lines of the scene that are perpendicular to the picture plane must be displayed at the picture plane in such a way, so that they meet at the designated vanishing point. In our case, such lines are q, p and l; b) all parallel horizontal lines of the real image which are parallel to the picture plane are constructed as horizontal and parallel. The vertical and parallel lines are constructed as vertical and parallel. In our case, such lines are a, b and all lines that follow the horizontal lines drawn by the feet of the workers.

EMPIRICAL BUILDING STRUCTURES

In Figure 7, observing the wall of the workshop as well as the paved ground where the workers have spread the rice grains, the researchers notice that the stones and the slabs respectively have been placed in a common way. In other words, they would say that both the wall construction and the paved yard construction have a structure. In this case, both have approximately the same structure. The common structure of these two constructions is undoubtedly the product of long-standing human experience. Its purpose is to satisfy practical

needs such as the functionality and the durability of the construction, as well as the complete covering of the flat surfaces.

Since humans in different cultures across time (Marcus, 2008) started building houses and roads, the objective was, as is in the case of Escher or Muslim decorations, to find ways for jointing bricks, stones or slabs, in order to fully cover two-dimensional surfaces and three-dimensional spaces. Stone walls in rural areas resemble stone puzzles with irregular shapes. The layout of the stones looks entirely random; however, the sheepfold was certainly strong enough to keep farm animals inside and any potential raiders outside. The use of irregular shapes was certainly saving energy for the builder, as the builder did not need to carve the stones to give them a regular, square, rectangle or hexagonal shape. However, the lack of a model, that is a basic construction block of a regular shape, leads to a corresponding lack of efficiency in the construction of a wall or floor, as each new piece requires preparation by the craftsman to see how it will fit in the overall construction.

The process of covering flat surfaces without gaps and coatings is called flat paving. It is linked to the concept of symmetry and it is one of the most important geometrical and algebraic themes of mathematics. In Figure 7, the manufacturer of the paved yard was mainly interested in covering a limited part of the ground with the plates, in order to fulfill a particular purpose. Still, the manufacturer had to perform basic measurements and calculations. Making these measurements, the manufacturer arranged the plates in particular layouts and satisfied an underlying need for order. The image that derives from the harmonious arrangement of roughly equal rectangular plates confers an aesthetic content on the paved courtyard. A similar observation can be made for the stone wall of the workshop. What they perceive intuitively when observing both the paved plane and the surface of

Figure 8. Geometric interpretation of empirical building structures.

the stone wall is the repetition of a rectangular pattern, which indefinitely extends across the entire plane. This geometric configuration presents sufficient symmetry (Armstrong, 2002).

Plane symmetry is a transformation of the plane, which carries a pattern in such a way so that it is identical to the original pattern. A distance-preserving transformation is called isometry. There are four types of flat isometry: translation, rotation, reflection, and glide reflection.

In mathematics, the various patterns the authors encounter in nature are grouped based on the types of symmetries they present, that is the transformations that leave the patterns unchanged. For a given pattern, the set of symmetries that preserve it is called, in mathematics, a symmetry group. It is essentially a group of transformations. A group is based on a certain rule. This rule shall have the meaning of composition. In other words, if the authors have two elements of a group, the elements S and T, they can compose them in such a way to create a new element, the element ST, which will eventually belong to the group. It is therefore possible to compose plane transformations, which leave the plane unchanged. That is, they can first carry out the transformation S and then the transformation T. If each transformation is a symmetry for any given pattern, their composition will be a symmetry as well.   Considering only the translation symmetries of a pattern, we can categorize the different patterns of the plane. The points that are formed on a plane pattern, because of all possible translations of a point on the said plain pattern, shall form the so-called *lattice* of the pattern.

GEOMETRIC INTERPRETATION OF EMPIRICAL BUILDING STRUCTURES

At the bottom right end of Figure 8, where the authors see an apparent structure of the brick wall of the building, they marked the center of the orthogonal surface of each brick (or rectangular carved stone) with a point. In the context of mathematics and for the purpose of our discussion, this means that the role of the brick will be played by its corresponding center point.

Now let T_{AB} be a translation that is symbolized by the green arrow (vector) AB. As shown by the pattern in the Figure, T translates the green point to the right and slightly downwards. Let U_{AC} also be a translation, marked in red, which carries the green point upwards and slightly to the right by the AC vector. From the composition of T with U and their inverses, all other translation symmetries of the pattern can be constructed. What is more, each translation will be of the form $T^m U^n$, where m and n shall take integer values. Therefore, the lattice of the dots of the pattern corresponds to the translation symmetries that exist.

In general, lattices are classified into five major categories. If the base area of a lattice is a square, then the lattice is called a square lattice. If its base area is a 60° rhombus, then it shall be called a hexagonal lattice. If the base area of a lattice is a rhombus with an angle different than 60^0, the lattice shall be called rhombic lattice. If the base area is rectangular, the lattice is called a rectangular lattice. In general, a lattice can simply be called a parallel grammatic lattice.
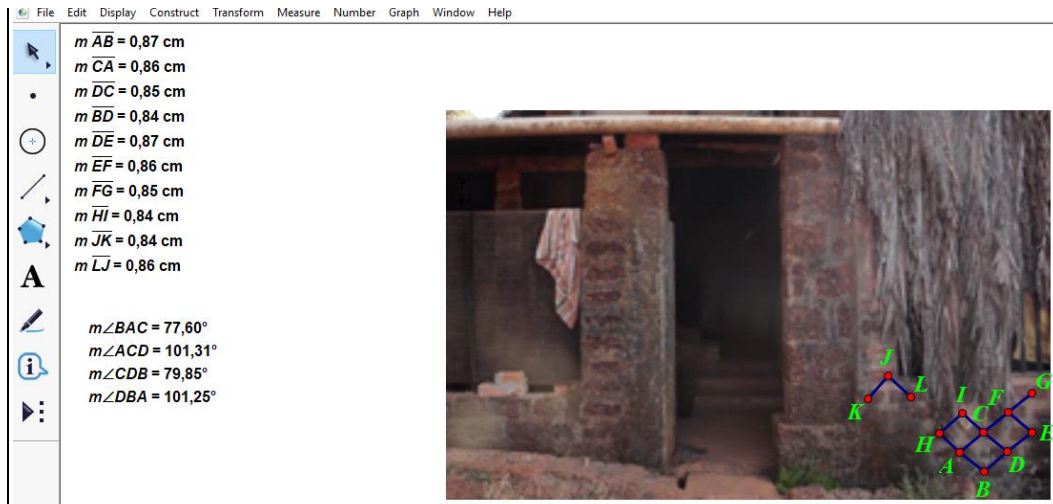
Figure 9. Rhombic lattice.

RHOMBIC LATTICE

Using the Dynamic Geometry software, they measured the straight segments of the lattice in Figure 8. As shown in Figure 9 the lengths of the aligned segments are approximately equal. The angles of the ABDC quadrilateral have also been measured to be approximately equal in opposite pairs, as it appears in Figure 9. As the aligned segments only deviate a little bit with each other in length, we can assume that they are ultimately all equal. Accordingly, the same conclusion can be made for the angles. This is the way we model physical reality. The ABDC quadrilateral, which is the basic building block of the lattice, has all its sides equal and all its consecutive angles different; therefore, the quadrilateral is a rhombus, and the lattice is a rhombic lattice. The conclusion of this discovery provides us with information about the type of symmetry that the creator of the construction selected among many other types of symmetry. The selection of the type or the types of symmetries by a social group allows us to draw conclusions about the development degree of the aesthetic perception of its members.

RICE CASTING

In Figure 10, the authors witness one of the most dynamic moments of the rice processing ritual. When the worker skillfully revolves the slightly tilted basket with the rice grains he carries on his head, a cloud of grain hits the space, creating an impressive formation. Movement, translation, rotation, and dispersion form the scene of a spiral, similar to those depicted in Vincent van Gogh's

artwork "The Starry Night". This shows that at a different place and time and taking the same form –which in our case is a spiral– two completely different stories are gleamed: a survival story that follows a solid algorithmic action process, verified by experiences of centuries-old traditions, and a story of unstable imagery and childhood memories of a troubled mind.

Hereon, the form of the spiral is presented as a symbol (signifier), the conceptual content of which (signified) is completely different in each case. However, in all the above-mentioned cases, the symbol itself does not require an interpretation of its own structural construction. Its only role is to symbolize. The way the spiral is formed and the dispersion of rice grains that follows is not among the priorities of a mind that simply thinks of survival. It is a need of a mind which thinks further. When this mind saw the spiral form, which was made in action and not on purpose, it ended up admiring the form and wondering: why? What is causing this feeling of admiration?.

THE INTERPRETATION OF THE CASTING OF A GRAIN OF RICE IN TERMS OF PHYSICAL SCIENCE AND MATHEMATICS

In Figure 11, the authors try to answer this question. The formation of a spiral is a natural phenomenon whose mathematical interpretation is particularly difficult. However, the simplification of the phenomenon can be limited in an interpretation in terms of physics that explains the route of a rice grain. This interpretation can satisfy a demanding mind to a great extent. The red circle symbolizes the perimeter of the circular basket and the red dot is its center. As we see in the film, the worker performs a clockwise rotation. Point A, in yellow, denotes
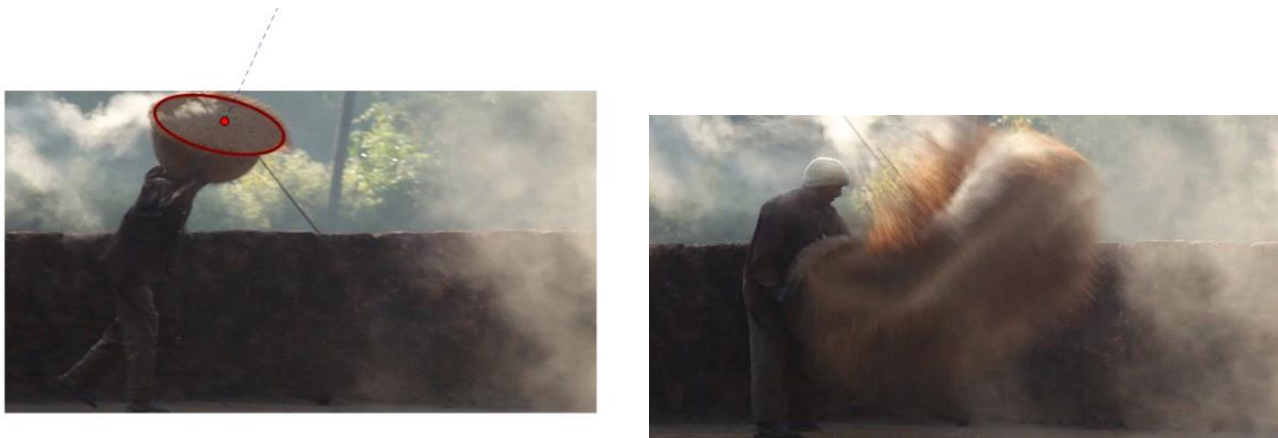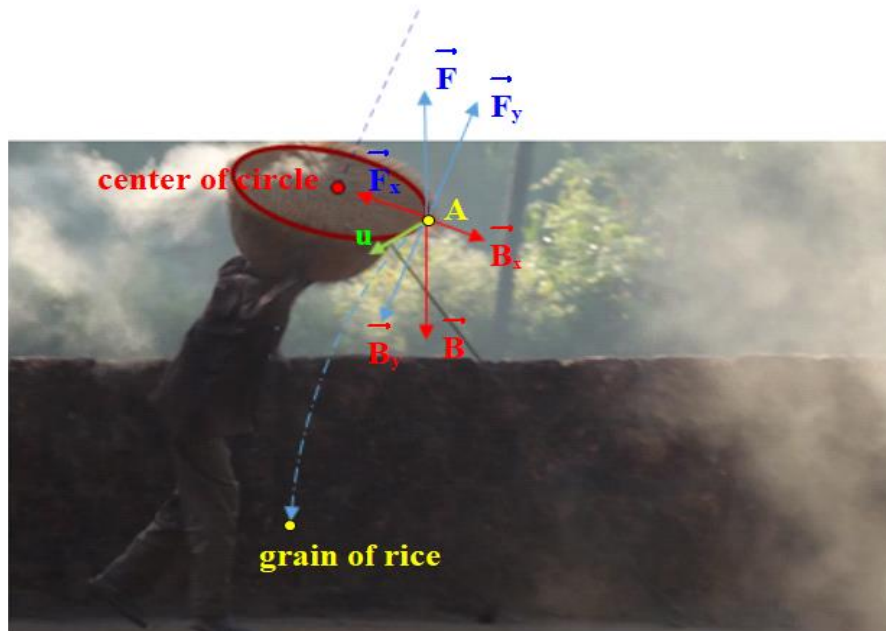
Figure 10. Rice casting.

Figure 11. The interpretation of the casting of a grain of rice in terms of physical science and mathematics.

the rice grain. The earth attracts the grain with a force $\vec{B}$. Given that the yellow grain remains on the basket and according to the law of action-reaction, an equal force $\vec{F}$ is being applied to balance it. A technique of mathematics to easily study the way these forces act on the rice grain is to analyze these forces in other forces, which will replace the original forces "equally". This means that they will have the same mechanical effects. Their analysis is always performed within a system of rectangular axes. As such, we select the Cartesian coordinate system defined by the plane of the red circle and the vertical straight line

at its center. In this coordinate system, the gravitational force $\vec{B}$ is analyzed in the component forces $\overrightarrow{B_x}$ and $\overrightarrow{B_y}$, while the force $\vec{F}$ is analyzed in the component forces $\overrightarrow{F_x}$ and $\overrightarrow{F_y}$. The net force of $\overrightarrow{B_y}$ and $\overrightarrow{F_y}$ is equal to 0. This means that they eliminate one another. The net force of $\overrightarrow{F_x}$ and $\overrightarrow{B_x}$ is a force called friction and it is directed towards the center of the circle. In the end, this is the force that keeps the grain on the basket. When the basket begins to rotate, this force coincides with another force called centripetal force. When the basket starts

rotating, it is this force that holds the grain on the basket. After a determining moment, the grain leaves the basket acquiring velocity $\vec{u}$. The rice grain then performs a shot, following the yellow dashed line and ending on the paved plane. What happens to one rice grain is also the case for the rest of the grains. However, the other grains follow different paths.

CONCLUSION

In conclusion, it can be said that the natural reality of a process of events, as reflected in a video, constitutes an area of intellectual development. In this framework, the viewers can interpret the story of the evolving events according to their own imaginary worlds. These events invoke the subjective experiences of a viewer and it is because of them that the viewer admires the whole work or part of it. However, each work, and in particular, the one we are discussing, is not limited to a simple projection of events. Instead, it is largely based on the presenting tools, techniques and infrastructures that represent centuries of life experiences. In the past, many generations have experimented with the rice drying process, until they found a more efficient method. The generations that followed also experimented to create a modern, automated processing method. In any case, however, what seems to be the case is the attempt of human nature to overcome Nature with the various tools it manufactures, and if possible, to manipulate it. In the framework of this endeavor, human nature manufactures tools and machines that go beyond mere experiences and require a mind that performs a more profound and complex thinking: a mind that thinks, in a way, as a scientist.

As a result, in this article, the authors sought to combine artistic creation with scientific exploratory interpretation of certain processes or real objects, using Mathematics.

CONFLICT OF INTERESTS

The authors have not declared any conflicts of interests.

REFERENCES

Armstrong AM (1988). Groups and Symmetry. Springer-Verlag, New York.

Blamey D (2017). Rice on Vimeo. https://vimeo.com/222856346.

Heath LT (1956). The Thirteen Books of Euclid's Elements, Vol. 1: Books 1-2. Dover Publications, INC, New York.

Marcus S (2008). Finding Moonshine: A Mathematician's Journey Through Symmetry. Harper Perennial, London, New York, Toronto, Sydney and New Delhi.

Morgan HM (1899). Vitruvius: The Ten Books on Architecture. Dover Publications, ING, New York.

Taylor T (1804). Plato: The Republic a Dialogue concerning Justice. Translated from the Greek by Thomas Taylor, Re-printed with new formatting, 2016. CreateSpace Independent Publishing Platform (May 30, 2016).

The Geometer's Sketchpad (2009). Dynamic Geometry Software for Exploring Mathematics, version 5.0. McGraw-Hill Education, https://www.dynamicgeometry.com/

Van Der Waerden BL (1975). Science Awakening I. Springer, Netherlands.