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Finding the growth rate during formation of South American community of Nations

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This article considers computation technique of the growth rate during formation of South American Community of Nations. Found values of growth are compared with the ones previously computed for Mexico and Canada under the formation of NAFTA in 1994, and much higher to the ones observed under formation of the EurAzEC in 2000 and the 2004 EU expansion. The method of computing the rate of growth in cases of economic integration requires statistics of the value added of sectors and domestic savings rates of member states willing to be economically integrated.

Key words: Modelling international economic integration, growth rate of the GDP, Latin American economic integration.

INTRODUCTION

The main purpose of economic integration is an increase of welfare, generally identical to economic growth, higher compared to what it could be without economic unification. Studies concerning economic integration (e.g., Badinger, 2001; Bretschger and Steger, 2004; Brodzicki, 2005; Caniels and Verspagen, 1999) were trying to reveal that growth were present in the dynamics of international economic integration. The growth has obvious condition to be higher enough to prove the need for integration.

For instance, it was previously shown (Dalimov and Umarov, 2008; Dalimov, 2009) that additional leap of the growth rate was present in cases of formation of NAFTA, EurAzEC and expansion of the EU in 2004. In this article, we implement the same type of technique in computations, previously used by authors, for the formation of South American Community of Nations (SACN) in 2004 by member states of MERCOSUR and Andean Commu-

nity, as well as by Chile, Guyana and Suriname.

THE MODEL

Basic equation previously obtained for the regional economic dynamics is as follows (Dalimov, 2008):

$$\ddot{Y}_k + (2+s_k)\dot{Y}_k + (1+s_k)Y_k - \mu\nabla^2 Y_k = \sum_{i=1}^{n}(P_{ik}Q_{ik} + \frac{d}{dt}P_{ik}Q_{ik}) = f \tag{1}$$

Where;
Y_k = the GDP of the country k;

$\dot{Y} \equiv \dfrac{dY}{dt}; \ddot{Y} \equiv \dfrac{d^2Y}{dt^2}$ Stand respectively for the first and second derivatives of the GDP; P_{ik} and Q_{ik} = price and output of the sector i of the country k; s = domestic sa-

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vings rate of the country k; n = number of sectors in country k,

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$ = the so called laplasian showing

spatial oscillations of the GDP;

x and y are geographic coordinates;

μ = the so called wave number.

Equation (1) is quite similar to several well known dynamic equations such as equation of attenuated oscillations, as well as to the telegraph equation both providing the tools for analysis of considered equation.

The concept of attenuated oscillations was used, showing in this case that the dynamics of the GDP is enforced by the dynamics of the value added standing in the right hand side of the equation (1), with the GDP also having spatial oscillations.

It can be infer that equation (1) has both temporal and spatial variables, while analysis of the GDP dynamics needs only temporal variables in (1) to be discussed. This problem may be resolved by separation of the variables using the Fourier substitution $Y = Y_t(t)Y_s(x, y)$ and for the sake of the modelling

assuming that $f = f_t(t) \cdot f_s(x, y)$.

Consider a case when $f_s(x, y) = aY_s(x, y)$ and $a = const$. It leads to the following:

$$\frac{\ddot{Y}_t}{Y_t} + (2 + s)\frac{\dot{Y}_t}{Y_t} + (1 + s) - \mu\frac{\nabla^2 Y_s}{Y_s} = \frac{f_t f_s}{Y_t Y_s} \qquad (2)$$

Or

$$\frac{\ddot{Y}_t}{Y_t} + (2 + s)\frac{\dot{Y}_t}{Y_t} + (1 + s) - \frac{f_t}{aY_t} = \mu\frac{\nabla^2 Y_s}{Y_s} = const \equiv \delta.$$

$$(3)$$

Since both parts of (3) are functions of independent variables then equation (3) is valid only in case if both its parts are equal to the constant. This leads to the two following equations (a = 1):

$$\begin{cases} \ddot{Y}_t + (2 + s_i)\dot{Y}_t + (1 + s_i + \delta_i)Y_t = \sum_{j=1}^{n}(P_jQ_j + \frac{\partial}{\partial t}P_jQ_j), \\ \\ \nabla^2 Y_s + \frac{\delta}{\mu}Y_s = 0. \end{cases}$$

$$(4)$$

From equation (4), it can be infer that the upper part has only temporal dependency, while the second part depends only from spatial coordinates, which is why it is neglected in further analysis.

The value added[1] terms standing at the right hand side of the first equation in (4) were previously shown (Dalimov, 2005) to have cyclic origin. Hence, we have the case identical to enforced attenuated oscillations for the temporal dynamics of the GDP.

Assume that the price and output of a sector produce an oscillation such as $P = P_0 \cos \omega t$ and $Q = Q_0 \cos(\omega t + \varphi)$, where $P_0; Q_0$ are initial values of price and output for a considered time period and $\omega; \varphi$ are respectively frequency and phase shift of oscillations of price and output. Such presentation is proven by the fact that cycles of price and output of a sector are closely correlated, differing only by the phase shift φ: higher price stimulates growth in output and leads to saturation of the market, followed by downfall of price which consecutively lowers production output, finally leading to deficit and increase of price – according to well known cycle of the market.

For the case of economic integration, the solution of the first equation in (4) is given by the expression (Dalimov, 2008):

$$Y_k(t) = \frac{1}{2}\sum_{i,j}P_{ij0}Q_{ij0}\left(\frac{1}{1+s_k} + \frac{\omega_{ij}\sin(\omega_{ij}t + \varphi_{ijk}) - \cos(\omega_{ij}t + \varphi_{ijk})}{\sqrt{(\omega_{ij}^2 - 1 - s_k - \delta)^2 + 2(2+s_k)\omega_{ij}^2}}\right)$$

$$(5)$$

The summation by j is over sectors, while the summation by i is an input of economically unified regions, ω_{ij}, a frequency of oscillations of the value added of the sectors. Phase shift φ_{ijk} is computed using standard solution of enforced oscillations:

$$tg\varphi_{ijk} = \frac{\omega_{ij}(2 + s_k)}{\omega_{ij}^2 - 1 - s_k - \delta_k}. \qquad (6)$$

Constant magnitude δ in (4) - (5) stands for additional growth rate of the GDP due to integration because it is present within the natural frequency of oscillations of the GDP which is equal to [see 1st equation in (4)]:

$$\omega^2 = 1 + s + \delta \qquad (7)$$

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[1] Multiplication PQ of price per output is the value added of a sector or an entity.

It was previously shown (Dalimov, 2008) that the value of δ is positive. Therefore, the higher the value of δ, the higher the frequency of oscillations and so, the higher the growth of the GDP.

At the same time, we can always represent real time series of the GDP, Y in the form of Fourier series equal to $Y = \sum_{n=1}^{m} Y_n \sin \omega_n t$, with $Y_n; \omega_n$ as Fourier components and spectrum, respectively. If one main harmonic for the GDP is use such as $Y = Y_1 \sin \omega_1 t$ then productivity Pr, may be obtained as follow:

$$\mathrm{Pr} = \lim_{t \to 0} \frac{Y}{t} = \lim_{t \to 0} \frac{Y_1 \sin \omega_1 t}{t} = \lim_{t \to 0} \frac{Y_1 \omega_1 \sin \omega_1 t}{\omega_1 t} = Y_1 \omega_1 \qquad (8)$$

Using (7), we may see that the value δ makes an input to the growth of productivity which consequently increases the GDP. So the task is to compute values of δ which may show the rate of growth during cases of economic integration. Our consideration is limited by the formation of South American Community of Nations in 2004.

Computation technique and values of δ

Consider the case of Argentina, where statistically available data are domestic savings rate with the GDP: $s_k; Y_k$ (Addendum 2) as well as value added of sectors denoted as $P_i Q_i$.

Technique of computations was carried out as follows: Based on the data of the value added during 1994 - 2004[2], a time series curves was built, and an interval between two historically last peaks of the value added of sectors called the period of oscillations T_i was obtained (Table 1). This further allows finding out the respective frequency of oscillations ω_i $(\omega_i = \frac{2\pi}{T_i}; \pi \approx 3{,}14)$.

The values in Table 1 are substituted into the following expression of the phase shift following from (6):

$$\varphi_{ijk} = \arctan \left[\frac{\omega_i (2 + s_k)}{\omega_i^2 - 1 - s_k - \delta} \right]; \qquad (9)$$

Computed values for the phase's shifts of, e.g. Argentinean sectors are shown in Table 2.

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[2] OECD beta 1.0 database

It was use to calculate the value of growth δ for Argentina. Algorithm of calculations is described further below. Knowing that:

$$\sin \varphi_i = \frac{\omega_i (2 + s_k)}{(\omega_i^2 - 1 - s_k - \delta)\sqrt{1 + \frac{\omega_i^2 (2 + s_k)^2}{(\omega_i^2 - 1 - s_k - \delta)^2}}};$$
(10)

$$\cos \varphi_i = \frac{1}{\sqrt{1 + \frac{\omega_i^2 (2 + s_k)^2}{(\omega_i^2 - 1 - s_k - \delta)^2}}}, \qquad (11)$$

Available data values to expression (5) were replaced and an expression was obtained which generally may be represented in the functional form $Y_k = f(\delta)$.

Aforementioned results are used to build an optimization curve for the function $F(\delta) = Y_k - f(\delta)$ (Addendum 1) and the value of δ from condition of $F(\delta) = 0$ was finding (Fikhtengolc, 1970).

Error estimation was done by calculating the GDP using equation (5) and values of δ obtained from calculations. Calculations of the growth rate for the other member states of SACN were conducted similarly using the same method applied for Argentina, with respective optimization curves shown in Addendum 1. Results of calculations are summated in Table 3.

RESULTS AND DISCUSSION

The article discusses the model and the mechanics in computing the GDP growth rate for the case of formation of South American Community of Nations. The basis of computations was the dynamic model previously developed by one of the authors (Dalimov, 2008).

The values of additional growth of the GDP were found to be mostly between 2 and 3, with a few exceptions showing higher growth in Bolivia, Paraguay and Suriname. The growth rate due to economic integration was also computed for the formation of NAFTA (Dalimov and Umarov, 2008), the Eurasian Economic Community in 2000 and expansion of EU in 2004 (Dalimov, 2009), where the values of growth rate due to economic integration were respectively equal to values shown in Table 4 for NAFTA and to 0.7525 for EurAzEC and 2004 EU expansion, respectively.

It can be inferred that the values of growth found for the formation of the SACN are similar for Canada and Mexico,

Table 1. Period of oscillations of the value added of SACN member states prior to 2004.

Name of sector	Argentina	Bolivia	Brazil	Chile	Colombia	Ecuador
Agriculture, hunting, forestry, fishing	10	4	10	12	18	14
Mining, manufacturing, utilities	10	10	10	10	10	16
Construction	8	8	12	18	8	16
Wholesale, retail trade, restaurants and hotels	14	4	12	12	16	14
Transport, storage and communication	12	14	12	12	10	16
Other activities	10	12	12	10	18	18

Name of sector	Guyana	Paraguay	Peru	Suriname	Uruguay	Venezuela
Agriculture, hunting, forestry, fishing	14	4	10	6	8	8
Mining, manufacturing, utilities	10	8	16	6	10	12
Construction	10	6	20	16	10	10
Wholesale, retail trade, restaurants and hotels	14	4	18	16	10	10
Transport, storage and communication	16	10	12	10	16	10
Other activities	12	12	18	10	12	10

Table 2. The value of the phase shift of the value added of sectors, Argentina.

Sector	Value of φ_i
Agriculture, hunting, forestry, fishing	$\varphi_1 = \arctan\left[\dfrac{1,4194684}{-0,0865916 - \delta}\right];$
Mining, manufacturing, utilities	$\varphi_2 = \arctan\left[\dfrac{1,4194684}{-0,865916 - \delta}\right];$
Construction	$\varphi_3 = \arctan\left[\dfrac{1,7743355}{-0,644075 - \delta}\right];$
Wholesale, retail trade, restaurants and hotels	$\varphi_4 = \arctan\left[\dfrac{1,013906}{-1,059084 - \delta}\right];$
Transport, storage and communication	$\varphi_5 = \arctan\left[\dfrac{1,18289033}{-0,986422 - \delta}\right];$
Other activities	$\varphi_6 = \arctan\left[\dfrac{1,4194684}{-0,865916 - \delta}\right];$

while the US growth rate has slightly less value. It may be described by the fact that the US has open economy already integrated to the world economy due to which the growth rate in case of formation of NAFTA is not so significant. At the same time, Mexico's same order of the growth rate magnitude seems justified since it geographically provides an isthmus between Latin American and North American economies. And the growth rate is quite higher compared to the ones of EurAzEC formation in 2000 and the 2004 expansion of European Union.

Comparison between real and forecasted values of the GDP shows statistically small errors, mostly less than 1% for the member states, while for union itself it is nearly 6.4%. It leads to the conclusion that a method itself may be used in forecasting and evaluating the benefits of potential and start-up economic unions such as East Asian Free Trade Area, Mediterranean Economic Union; a union between Russia, Turkey and three Caucuses states etc. In this case, it is advisable to calculate the magnitudes of δ within a period prior to economic inte-

Table 3. The growth rate under formation of SACN in 2004.

Country	Value of δ in 2004	Statistic value of the GDP Y in 2005	Forecast of the GDP for 2005	Error, in (%)
Argentina	2.79679899051	183196	183140.3439	0.0303806305
Bolivia	3.61137652	9441	9504.899833	0.6768334287
	0.73269491		9146.776395	3.116445345
Brazil	2.72118884	795925	796165.2088	0.03017986619
Chile	2.765761501	118908	118884.1144	0.0200874625
Colombia	2.591619712	123120	123017.5976	0.0831730019
Ecuador	2.425382774	36489	36493.58323	0.0125606347
Guyana	2.60654973	826	826.0814254	0.00985783293
Paraguay	4.538110968	7473	7637.189264	2.197099706%
Peru	2.440007899	79382	79385.56261	0.004487856189
Suriname	3.311841803	1543	1536.024497	0.4520740112
Uruguay	2.7741144955	16615	16640.82045	0.155404514
Venezuela	2.762776991	143625	143422.9099	0.1407067015
SACN	2.7184486525	1425082	1516660.442	6.4261874

Table 4. The growth rate under formation of NAFTA in 1994.

Country	Canada	Mexico	USA	NAFTA
δ	2,224372823	3,06613875251	1,93354692	1,9871244775

integration in respective states and / or regions and then to forecast the GDP values which may show feasibility of the unification proposed.

Conclusion

The article considers computation technique of the growth rate during formation of South American Community of Nations. Found values of growth are comparable with the ones previously computed for the Mexico and Canada under formation of NAFTA in 1994 and much higher to the ones observed under formation of the EurAzEC in 2000 and the 2004 EU expansion. The method of computing the value of growth in cases of economic integration requires statistics of the value added of sectors and domestic savings rates of member states willing to be economically integrated.

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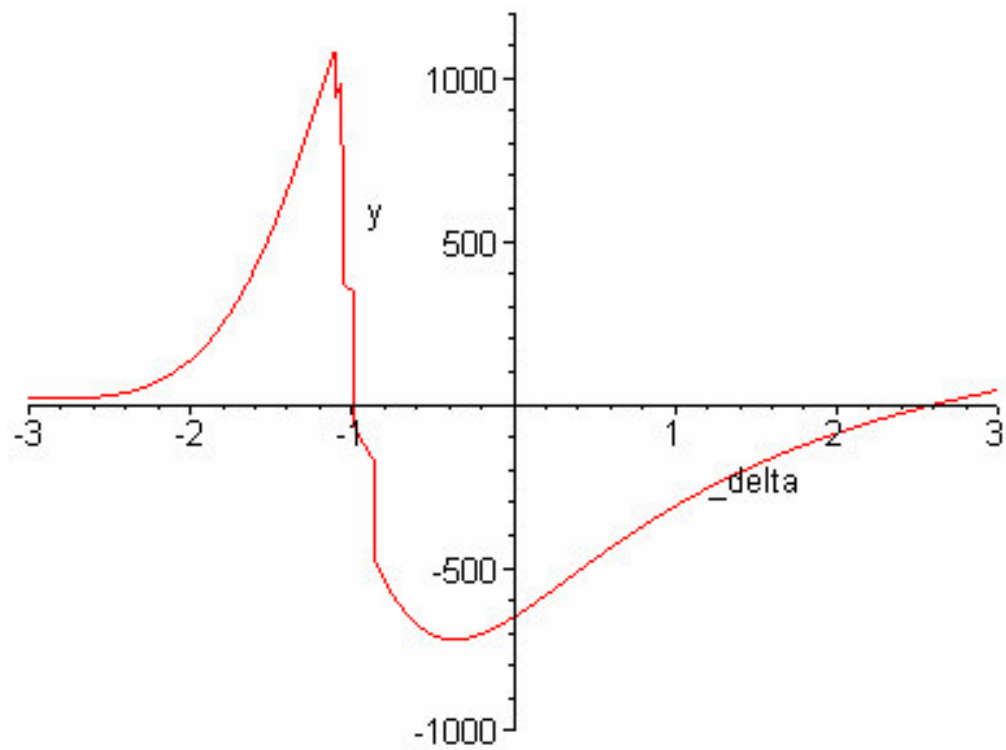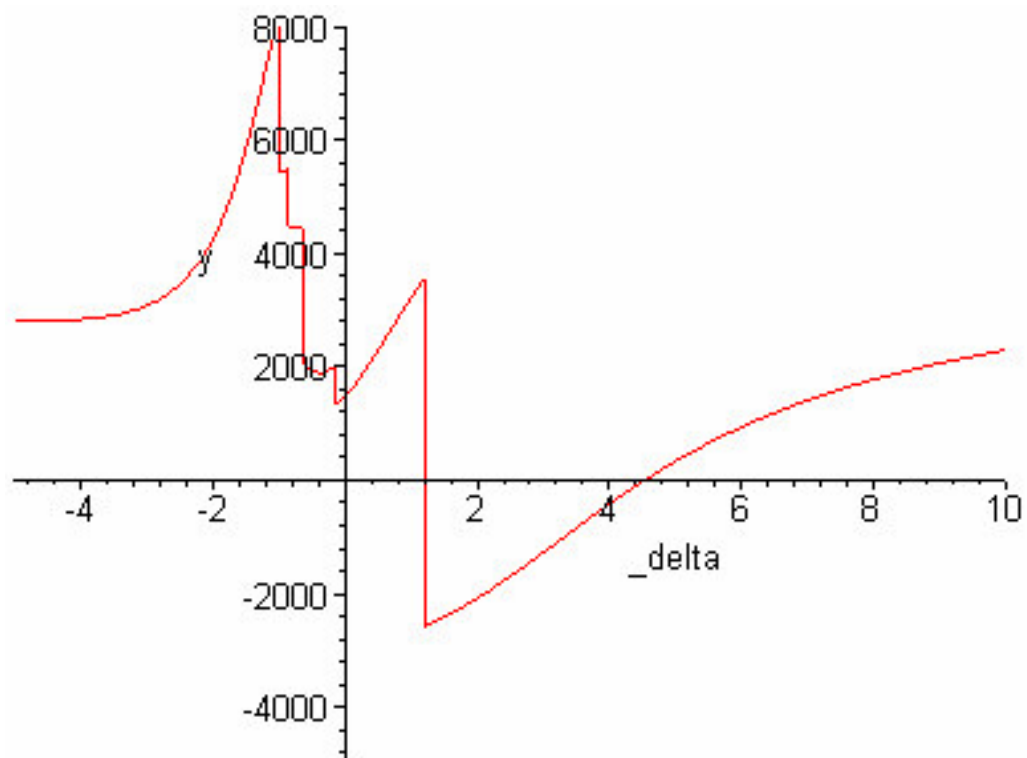
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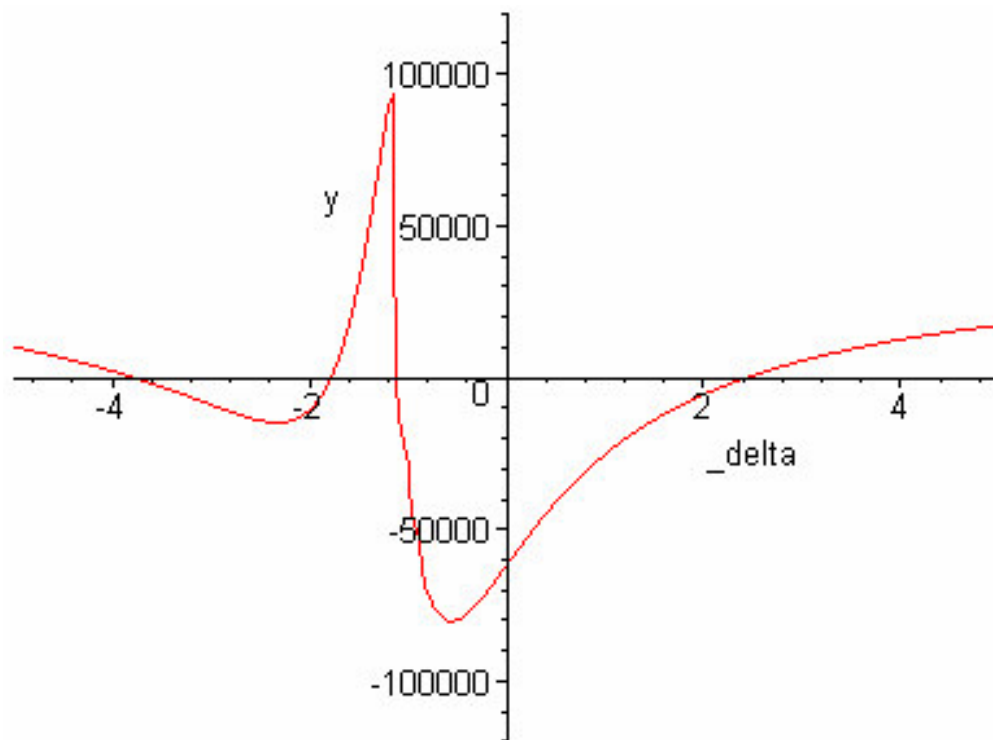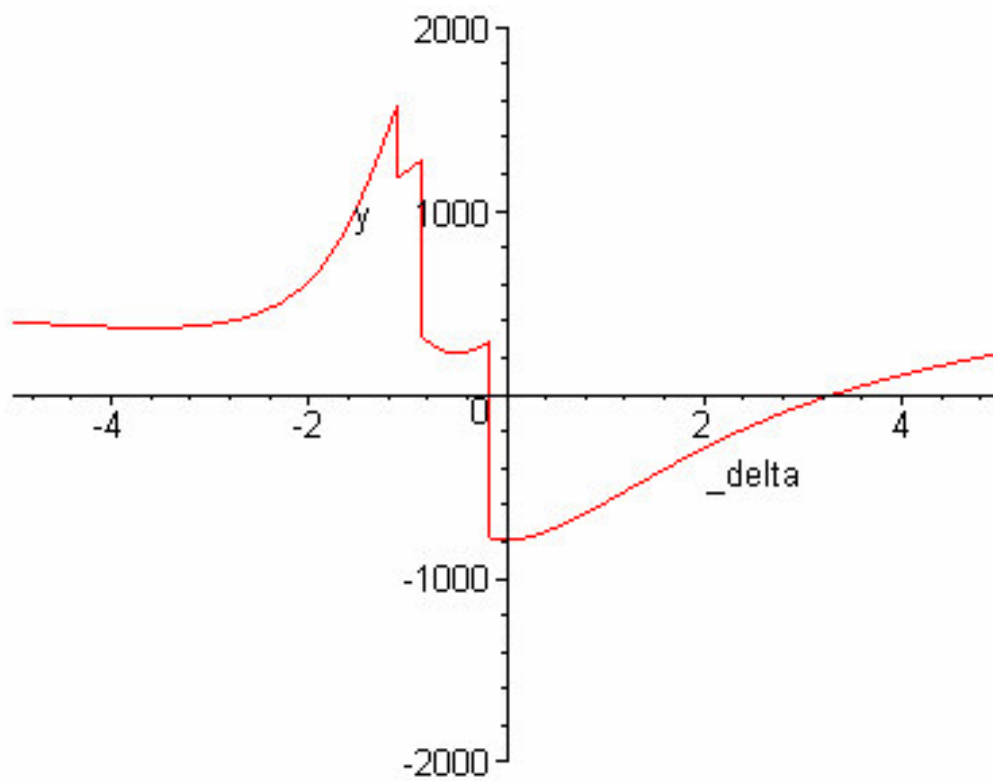
Addendum 1

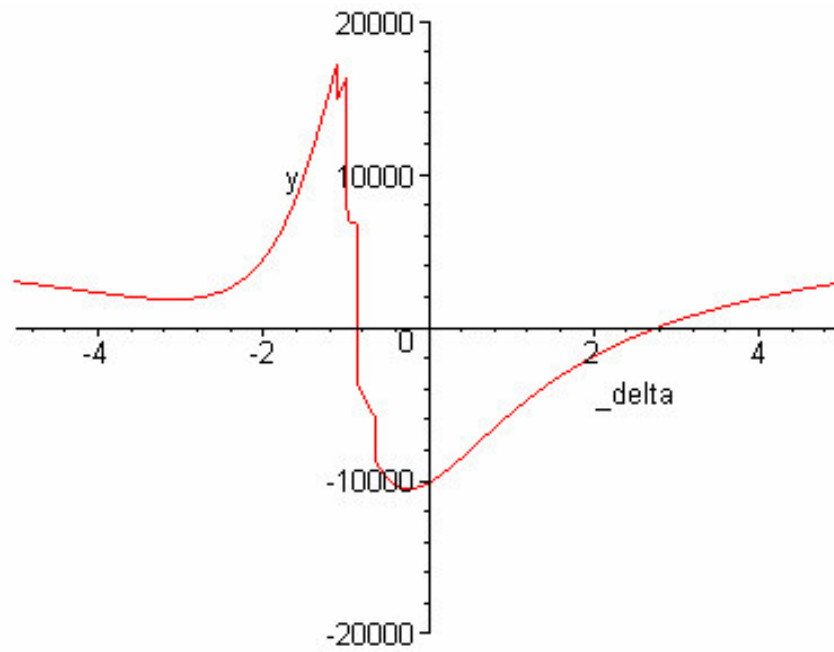**Optimization curves of member states of SACN**
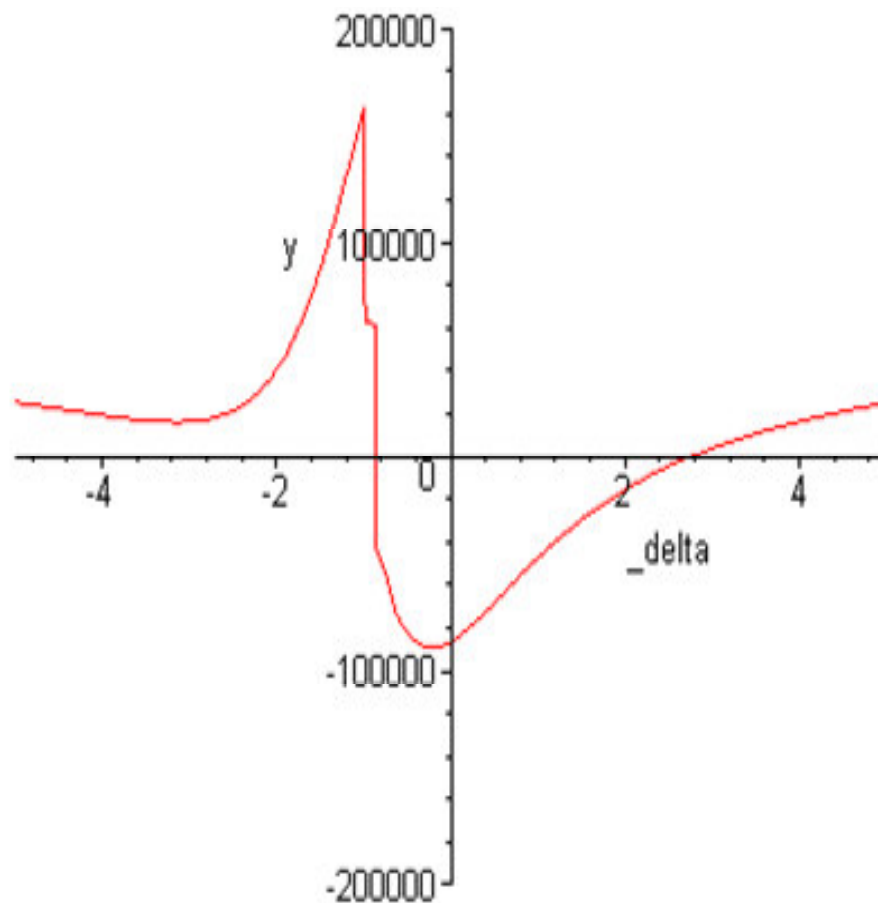
Argentina
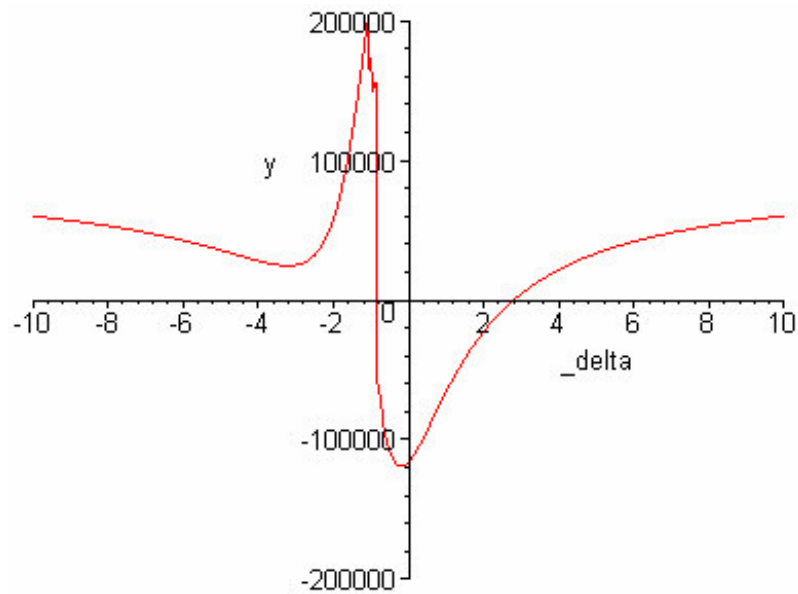
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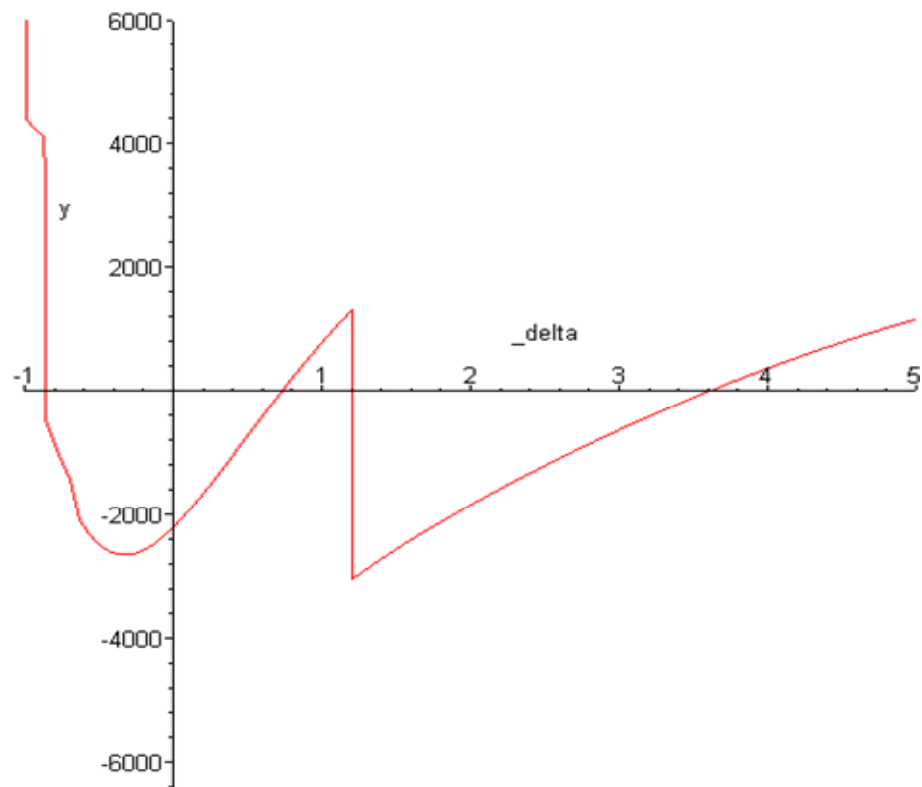
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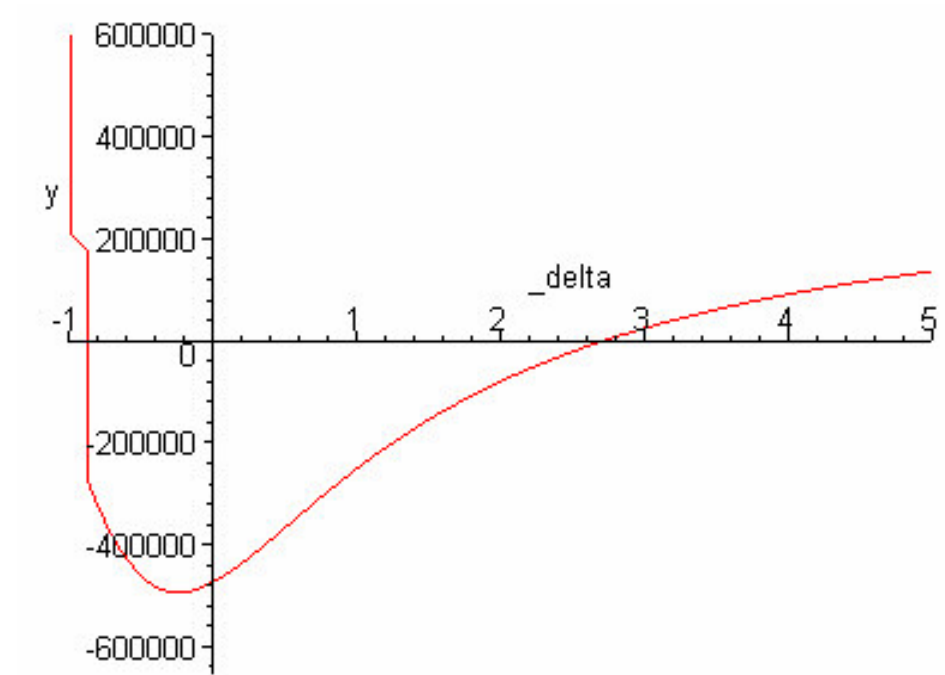
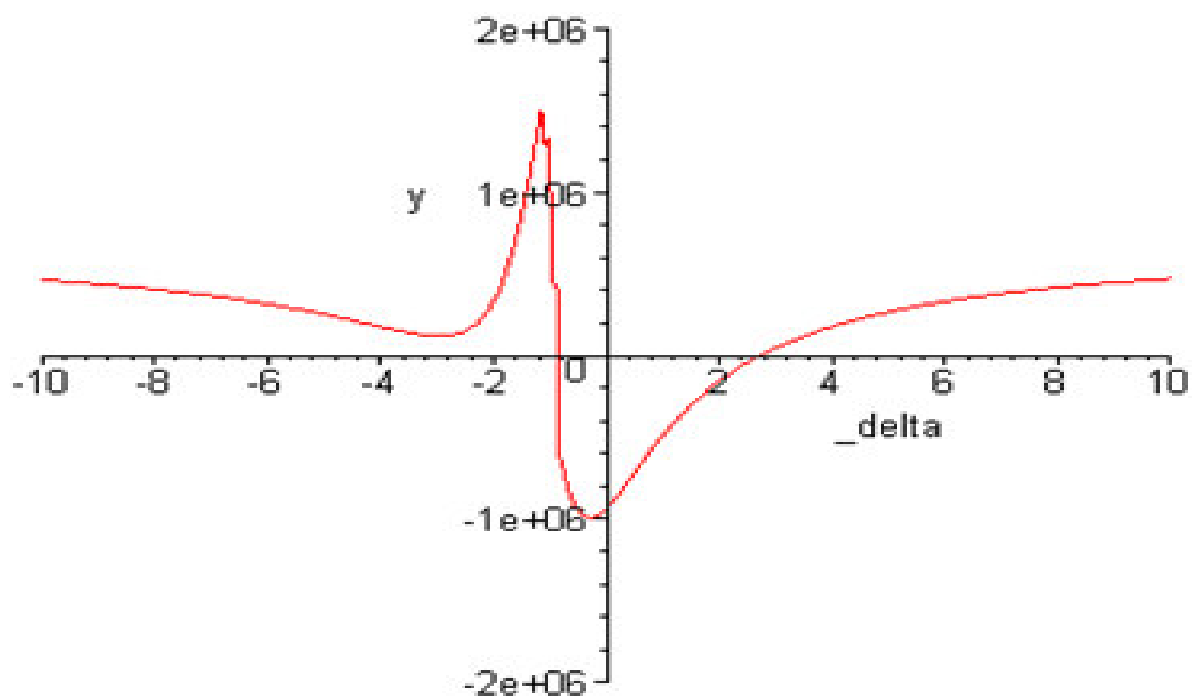**Chile**
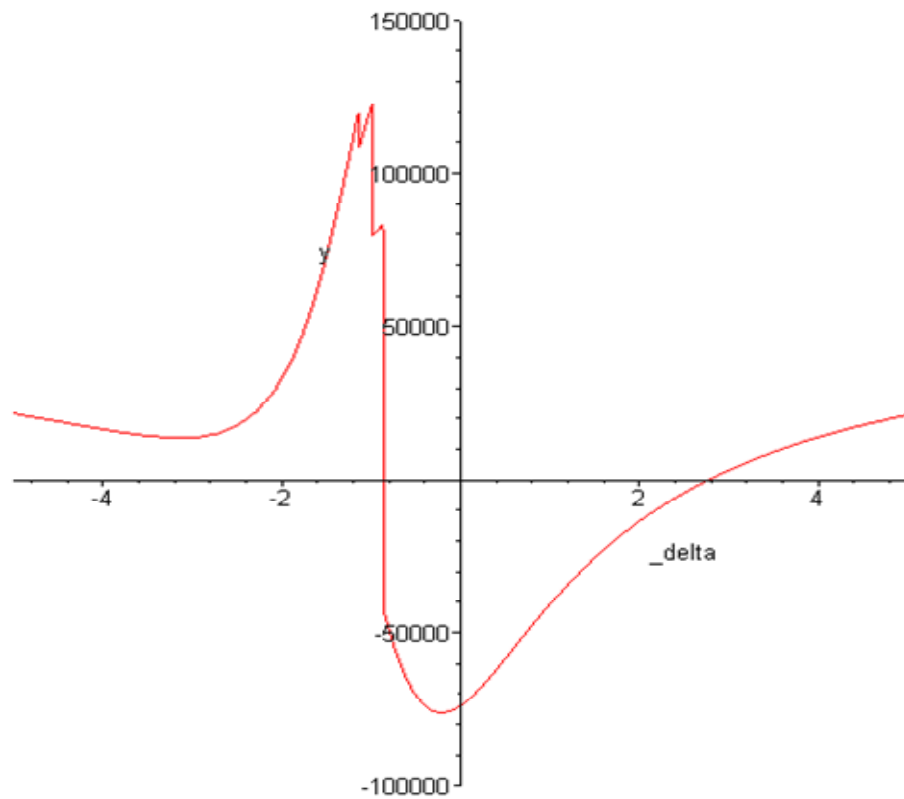
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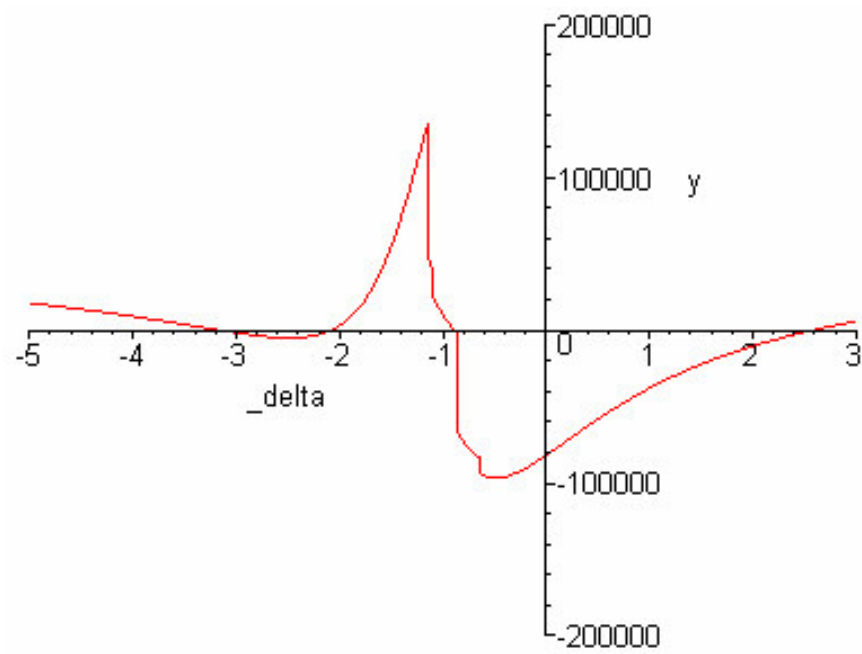
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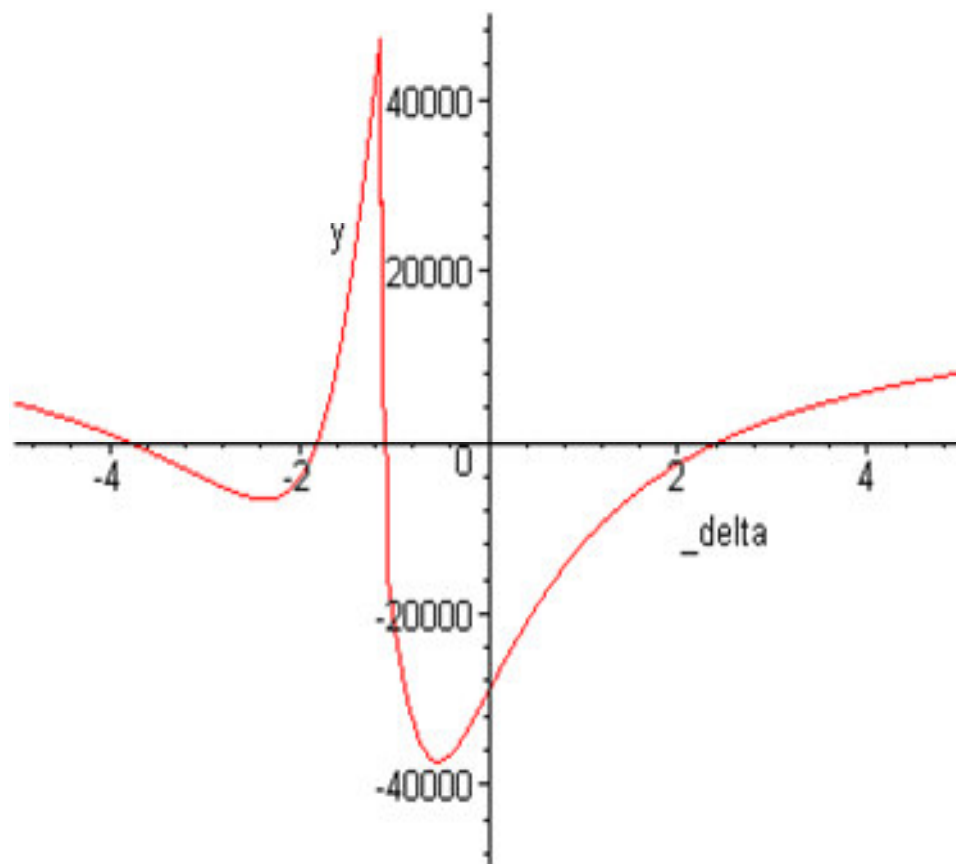
Guyana

Paraguay

Peru

Suriname

Uruguay

Venezuela

SACN

Addendum 2

The GDP and domestic savings rate of MERCOSUR and Andean Community, under formation of South American Community of Nations (2004).

Members	GDP (2004) blns. of US$[3]	GDP (2005) blns. of US$	Domestic savings rate (2004)[4]
MERCOSUR	775,817	997,518	26,03
Andean Community	313,271	376,405	25,98

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[3] IMF database
[4] USAID