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Optimization of a passive vehicle suspension system for ride comfort enhancement with different speeds based on design of experiment method (DOE) method

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This paper reported on an investigation to determine the spring and damper settings that ensured optimal ride comfort of vehicle in different speeds using design of experiment method (DOE). The extent to which the ride comfort optimal suspension settings vary for roads of different roughness and varying speeds and the levels of ride comfort that can be achieved, were addressed. Optimization was performed with the DOE method on a 7 DOF modeled in MATLAB software for speeds ranging from 60 to 90 km/h. Results indicated that optimization of suspension settings using the road and specified range of speed also improved the ride comfort on the same road at the different speeds. These settings also improved ride comfort for other roads at the optimization speed and other speeds, although not as much as when optimization has been done for the particular road. For improved ride comfort, damping generally has to be lower than the standard (compromised) setting, the rear spring as soft as possible and the front spring ranging from as soft as possible to stiffer depending on road and speed conditions. Ride comfort was most sensitive to a change in rear spring stiffness.

Key words: Ride comfort, optimization, design of experiment (DOE), suspension, road condition, spring and damper.

INTRODUCTION

Ride comfort is one of the most critical factors to evaluate the automobile performance and has been an interesting topic for researchers for many years. Automobile designers give an abundant attention to the isolation of vibrations in the car, in order to provide a comfortable ride for the passengers. Despite of all progresses in vehicle design profession, the determination of vibration comfort is still a subject of conflict between automobile designers, and so far no common standard has been developed on this matter (Kasaiezadeh et al., 2005). Two methods, that is, computer simulation and road experiment, are used to investigate ride comfort. Computer simulation method is based on the mathematical model of the vehicle vibration and power spectral density (PSD) of road surface (Demic et al., 2002; Gillespie et al., 1993; Cebon, 1993). Duncan (1982) used FEM models for suspension system to study the ride quality and using numerical optimization methods recommended some modifications on suspension system. Tamboli and Joshi (1999) tried to produce real data from road real profile and optimize the suspension system under real road inputs. One of the complete works in this area is the simulation done by Gobbi et al. (2001) at Fiat Company. They used a car model that was developed in ADAMS using real road inputs, and constructed an optimization scheme that was successfully implemented to their car. Hammond and Harrison (1981) considered a single degree of freedom

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quarter model for the vehicle dynamic system and have studied the response using a state space approach. Karnopp (1989) considered the vehicle model as a single degree of freedom quarter model with active and passive damping and linear stiffness subjected to white noise base velocity excitation. Sobczyk et al. (1977) have investigated stationary response to profile imposed excitation with randomly varying traverse velocity and showed that the change in velocity makes a proportional contribution to the response. Hac and Youn (1983) showed that incorporation of a time delay between the front and rear axles in controller design improve the dynamic behavior when road excitation is simulated by white noise and vehicle velocity is constant. Elbehiery and Karnopp (1996) have optimized suspension system parameters for five types of suspension systems to obtain constant root mean square suspension deflection. Marzbanrad et al. (2002, 2003) optimized an active controller for a vehicle suspension system including time delay. And finally the valuable work of Christensen et al. (2000), in ARC at University of Michigan must be mentioned. They used sensitivity analysis and DOE optimization techniques on an ADAMS car model to optimize car driving and ride performance. Keshavarz et al. (2011) tuned the stiffness of engine mounts of a passenger car in order to reduce the transmitted vibration to driver with regard to the permissible values of natural frequencies of engine using DOE method.

In this paper, the 7DOF vehicle model has been modeled and analyzed. To study the ride comfort of the vehicle, vertical acceleration and pitch angle have been calculated and optimized. For this case, the coefficients of front and rear (F/R) springs and dampers out of many other factors such as suspension geometry, mount and joint characteristics have been varied in the simulation. It is because of the dominant role of F/R springs and dampers.

Finally, DOE method is applied to decide the best values of selected parameters to fulfill the best ride comfort. A generic simulation procedure is shown in Figure 1.

RIDE COMFORT

The ride of a vehicle is the heaving, pitching and rolling motion in forced vibration caused by road roughness. The purpose of the suspension is to minimize the discomfort of the passengers, which obviously involves a minimization of some measure of the vehicle body motion, by choice of the springs and dampers as shown in Figure 2 (Dixon, 2007). Since passenger comfort is not a quantitative parameter, majority of research efforts have been focused on devising a qualitative measure of passenger comfort (also referred to as human response to vibration). Passenger comfort principally depends on magnitude and direction of acceleration and the frequency of vibrations acted on his body. Four methods

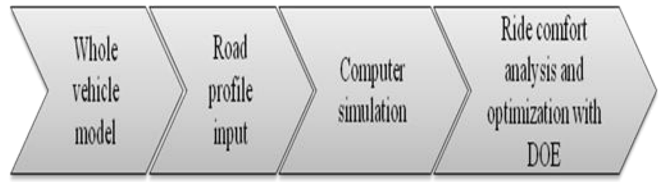


Figure 1. A generic computer simulation procedure**.**

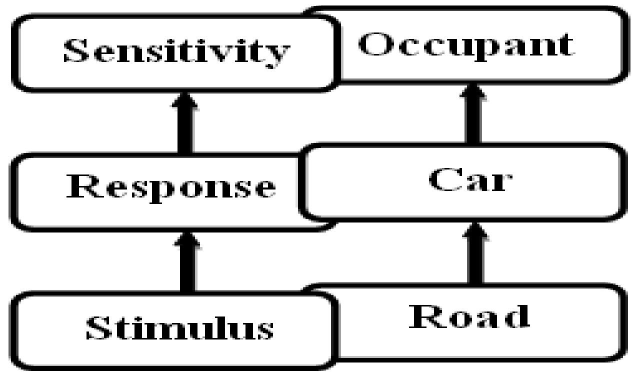


Figure 2. Ride system analysis.

to objectively evaluate ride comfort are used throughout the world today. The ISO 2631 standard (2007) is used mainly in Europe and the British standard BS 6841 (1987) in the United Kingdom. Germany and Austria use VDI 2057 (Hohl, 1984), while average absorbed power or AAP is used by the United States of America and by NATO in the NATO reference mobility model (NRMM) (Pradko and Lee, 1966). Ride behavior may be analyzed in the time domain or in the frequency domain. Time-domain analysis predicts positions, velocities and accelerations as functions of time. Frequency domain analysis predicts the characteristics as functions of frequency, for example, the transmissibility, hence revealing resonances, etc.

DESIGN OF EXPERIMENT METHOD (DOE)

Design of Experiments or DOE method was first developed in the 1920s and 1930, by Sir Ronald A. Fisher, the renowned mathematician and geneticist (Antony, 2003). DOE is a collection of procedures and statistical tools for planning experiments and analyzing the results. Experiments are performed by investigators in virtually all fields of inquiry, usually to discover something about a particular process or system. Literally, an experiment is a test. More formally, an experiment can be defined as a test or series of tests in which purposeful changes are made to the input variables of a process or system so that one may observe and identify the reasons for changes that may be observed in the output response (Montgomery, 2001). In general, experiments are used to study the performance of processes and systems. The process or system can be represented by the model shown in Figure 3. Some of the process variables X_1, X_2,…,X_p are controllable, whereas other variables Z_1, Z_2,…,Z_q are uncontrollable. The objectives of the experiment may include the following:

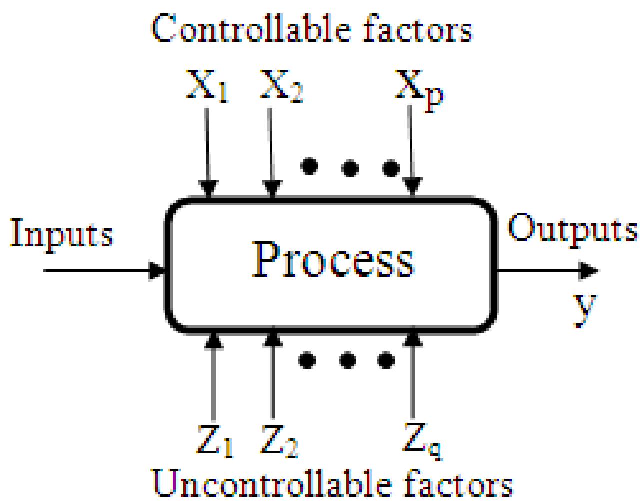


Figure 3. General model of a process.

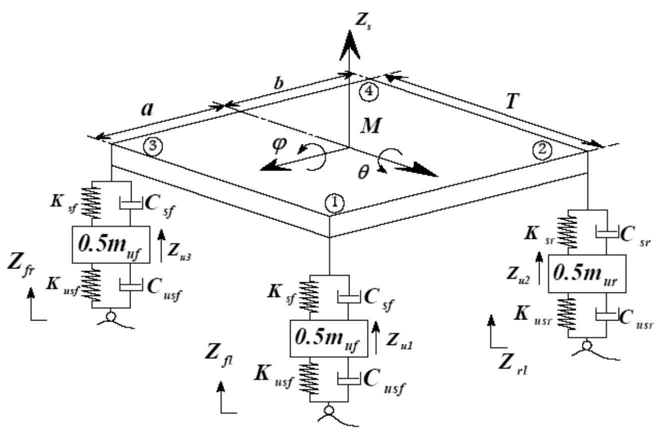


Figure 4. A full 7DOF vehicle mode.

(a) Determining which variables are most influential on the response y.
(b) Determining where to set the influential X's so that y is almost always near the desired nominal value.
(c) Determining where to set the influential X's so that variability in y is small.
(d) Determining where to set the influential X's so that the effects of the uncontrollable variables Z_1, Z_2,…, Z_q are minimized.

The main goal of experimental design is determining variations that have more effect on responses. After that, optimization of effective parameters could be done. Guidelines for Designing an Experiment are as follows:

(i) Recognition of and statement of the problem.
(ii) Choice of factors, levels, and ranges.
(iii) Selection of the response variable.
(iv) Choice of experimental design.
(v) Performing the experiment.
(vi) Statistical analysis of the data.
(vii) Conclusions and recommendations.

VEHICLE MODEL

A full car vibrating model has seven DOF according to Figure 4 with the equations of motion (1) to (7). The vehicle body is represented by three degree-of-freedom rigid cuboids with mass M. The heave, pitch and roll motions of the sprung mass are considered. The four unsprung masses (front-left, front-right, rear-left and rear-right) are connected to each corner of the rigid cuboid. It is assumed that the four unsprung masses are free to bounce vertically. Shock absorbers are far from being linear viscous dampers. In fact, most automotive shock absorbers are unsymmetrical, with a damping which is larger in the rebound stroke. The suspensions between the sprung mass and unsprung masses are modeled as nonlinear spring and nonlinear dampers elements, while the tires are modeled as nonlinear springs with viscous damping. It is assumed that the modeled nonlinear damper and spring of suspension have the curves as shown in Figure 5 and 6. Those figures are the spring and damper specifications that were used in this modeling as typical characteristics which were derived from ADAMS.

The parameters of the vehicle model which are used in the numerical study are shown in Table 1.

$$M\ddot{Z}_s + F_{1s} + F_{1c} + F_{2s} + F_{2c} + F_{3s} + F_{3c} + F_{4s} + F_{4c} = 0 \tag{1}$$

$$I_\varphi \ddot{\varphi} + T/2\cos\varphi\left(F_{1s} + F_{1c} + F_{2s} + F_{2c} - F_{3s} - F_{3c} - F_{4s} - F_{4c}\right) = 0 \tag{2}$$

$$I_\theta \ddot{\theta} - a\cos\theta\left(F_{1s} + F_{1c} + F_{3s} + F_{3c}\right) + b\cos\theta\left(F_{2s} + F_{2c} + F_{4s} + F_{4c}\right) = 0 \tag{3}$$

$$0.5m_{uf}\ddot{Z}_{u1} - \left(F_{1s} + F_{1c}\right) + K_{usf}\left(Z_{u1} - Z_{fl}\right) + C_{usf}\left(\dot{Z}_{u1} - \dot{Z}_{fl}\right) = 0 \tag{4}$$

$$0.5m_{ur}\ddot{Z}_{u2} - \left(F_{2s} + F_{2c}\right) + K_{usr}\left(Z_{u2} - Z_{rl}\right) + C_{usr}\left(\dot{Z}_{u2} - \dot{Z}_{rl}\right) = 0 \tag{5}$$

$$0.5m_{uf}\ddot{Z}_{u3} - \left(F_{3s} + F_{3c}\right) + K_{usf}\left(Z_{u3} - Z_{fr}\right) + C_{usf}\left(\dot{Z}_{u3} - \dot{Z}_{fr}\right) = 0 \tag{6}$$

$$0.5m_{ur}\ddot{Z}_{u4} - \left(F_{4s} + F_{4c}\right) + K_{usr}\left(Z_{u4} - Z_{rr}\right) + C_{usr}\left(\dot{Z}_{u4} - \dot{Z}_{rr}\right) = 0 \tag{7}$$

where for i=1:4 the amount of F_{is} versus displacement and F_{ic} versus velocity is shown in Figures 5 and 6 numerically. Δ_i with i=1:4 the amount of spring displacement obtained from Equations (8) to (11):

$$\Delta_1 = Z_s - a\sin\theta + T/2\sin\varphi - Z_{u1} \tag{8}$$

$$\Delta_2 = Z_s + b\sin\theta + T/2\sin\varphi - Z_{u2} \tag{9}$$

$$\Delta_3 = Z_s - a\sin\theta - T/2\sin\varphi - Z_{u3} \tag{10}$$

$$\Delta_4 = Z_s + b\sin\theta - T/2\sin\varphi - Z_{u4} \tag{11}$$

In the above equations, Δ_i are the change of springs length equal to suspension system, which are used for forces, F_{ic} and F_{is} in Equations (1) to (7).

ROAD PROFILE

Knowledge of the excitation due to motion on uneven road is important for the study of riding comfort. Road excitation could enhance the impact to the automotive chassis and increase the applied vibration to the passengers.

Because such excitation cannot be studied with a deterministic approach, the methods used for random vibrations must be applied. A number of studies have been devoted to characterizing the road profiles experimentally and interpreting the results statistically.

For a single wheel track, the Sayers Roughness Model assumes that the power-spectral density (PSD) of the displacement

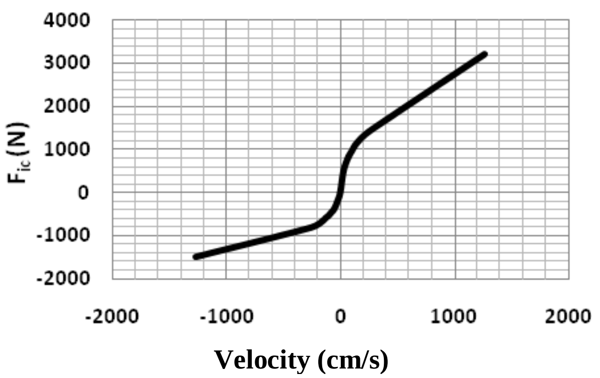


Figure 5. Damper characteristic curve.

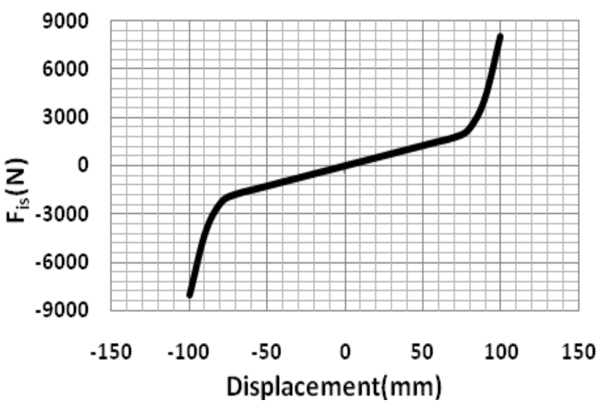


Figure 6. Spring characteristic curve.

(elevation) of a road profile, G_d, is a function of wave nuber, ν, given by the equation (12) as stated in (Sayers MW and Karamihas SM, 1996):

$$G_d(\nu) = G_e + \frac{G_s}{(2\pi\nu)^2} + \frac{G_a}{(2\pi\nu)^4} \tag{12}$$

Therefore, it is assumed that roughness comes from three components. Each is obtained from an independent source of white noise, that is, random numbers: The first component, with amplitude G_e, is white-noise elevation. The second, with amplitude G_s, is white-noise slope (velocity) that is integrated once with respect to time. The third, with amplitude G_a, is white-noise acceleration that is integrated twice with respect to time.

The constants of the Sayers Roughness Model are listed in Table 2. Assuming a constant vehicle velocity of 28 m/s and referring to Table 2, the excitation of tires caused by road surface versus time is shown in Figure 7. It is clear that there is a phase difference between front and rear tires excitation depending on the vehicle velocity and an insignificant difference between left and right tires.

DETERMINING OPTIMAL SUSPENSION SETTINGS

Several questions have to be addressed when determining the optimal suspension settings. The first is to what extent does the optimal suspension settings vary if a road of given roughness is traversed at different speeds. Secondly, what levels of ride comfort can be achieved in these cases? Another question at hand is if an off-road vehicle travels over roads of different roughness with a suspension setting optimized for one road at a specified speed, what is the effect on ride comfort? Furthermore, what is the effect if a vehicle travels at a specified speed over a particular road profile with the suspension setting optimized for a different speed? Also how much will the ride comfort be affected and how much can the suspension settings vary so that the ride comfort is still within 5% of the optimal suspension value?

OPTIMIZATION DESIGN OF RIDE COMFORT BASED ON DESIGN OF EXPERIMENT METHOD (DOE)

Due to the fact that the geometry of a suspension system is optimized based on handling and road holding, the geometry of a suspension system has not been taken into account as a design variable. In fact, ride quality mostly depends on the characteristics of rear and front springs and dampers.

In this study, the 7DOF vehicle model as stated in (1) to (7) with numerical parameters as Table 1 is subjected to the road surface (14) in an asphalt profile according to Table 2. The vertical acceleration and pitch angle of the auto-body is calculated to be observed the effect of those two mentioned parameters in the forward and rearward springs and dampers. In Figure 8, the effect of variation of such characteristics on ride quality is illustrated. The optimization procedure is done according to a basic curve of spring and damper characteristics and a correction factor defined as the ratio of corrected to basic spring and damper coefficient. Four design parameters chosen for experiment are shown as vector V :

$$V = \{f_{sf}, f_{sr}, f_{cf}, f_{cr}\} \tag{13}$$

where: f_{sf}, f_{sr} are front and rear spring correction factors and f_{cf}, f_{cr} are front and rear damper correction factors.

Design variable are limited to the ranges defined by the bounds shown in Table 3. In practice, these ranges reflected technical abilities to manufacture theme in usual passenger cars.

Since ride comfort is a qualitative subject, suitable criteria shall be defined to quantify it. The objective function which is used here is designed based on the combination of vertical acceleration RMS (Root Mean Square) and pitch angle as:

$$S = w_1 \frac{a_{Z_{RMS}}}{a_{Z_{MAX}}} + w_2 \frac{\theta_{RMS}}{\theta_{MAX}} \tag{14}$$

where w_1, w_2 are weight factors and a_z, θ are vertical acceleration and pitch angle. There is no defined criterion in determination of w_1, w_2 weight factors. Vehicle industry professionals have offered some values for weight factors (Kasaiezadeh et al., 2005). Here, $w_1 = 0.75$, $w_2 = 0.25$, are chosen based on the range of their advice.

OPTIMIZATION RESULT

In this research, the optimization is accomplished in the speed range of 60 to 90 Km/h based on DOE method. Many experiments involve the study of the effects of two

Table 1. Numerical values of the system parameters.

Parameter	Value
Sprung mass, M	1295 Kg
Roll moment of inertia, I_{φ}	200 Kg.m^2
Pitch axis moment of inertia,I_{θ}	500 Kg.m^2
Front unsprung mass, M_{uf}	50 Kg
Rear unsprung mass, , M_{ur}	50 Kg
Tire spring stiffness, $K_{us,f}$, $K_{us,r}$	190,000 N/m
Damping coefficient of tire, , $C_{us,f}$, $C_{us,r}$	500 N.s/m
Length between the front of vehicle and the center of gravity of sprung mass, a	1.233 m
Length between the rear of vehicle and the center of gravity of sprung mass, b	1.327 m
Width of sprung mass, T	1.55 m

Table 2. Values for Sayers Roughness Model.

Parameter	G_e	G_s	G_a
Unit	$\left(\frac{m^3}{cycle}\times 10^{-6}\right)$	$\left(\frac{m}{cycle}\times 10^{-6}\right)$	$\left(\frac{1}{(m \times cycle)}\times 10^{-6}\right)$
Value	0.1	20	0.1

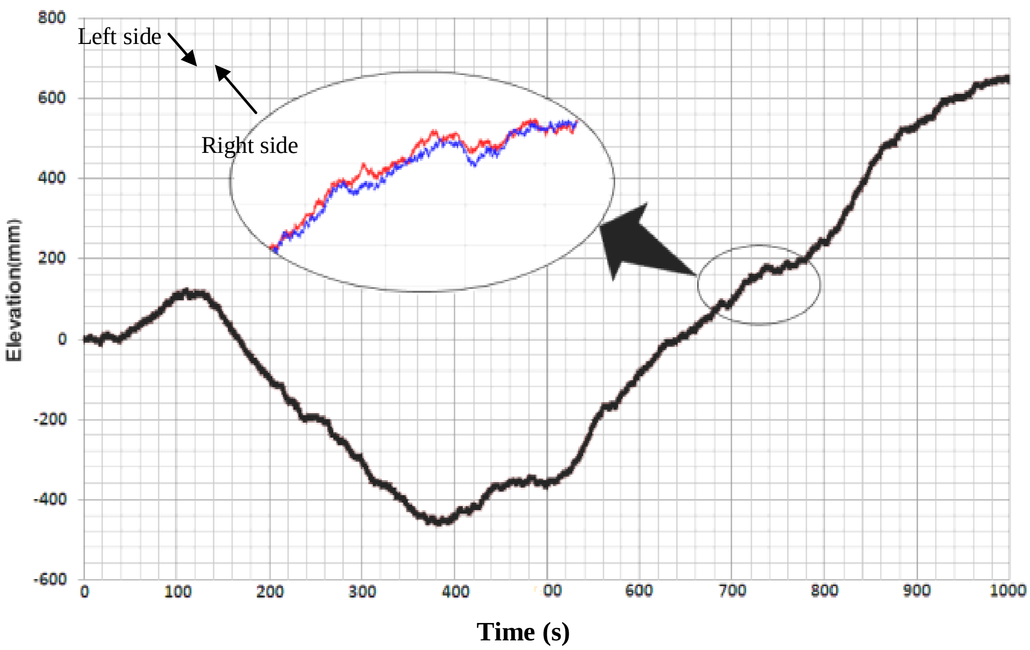


Figure 7. Time history of road irregularities and surfaces.

or more factors. In general, factorial designs are most efficient for this type of experiment. This is an experimental strategy in which factors are varied together, instead of one at a time. (Montgomery, 2001; Antony, 2003). The design variables are V as stated in Equation (13) and the objective function is S as expressed in Equation (14). Here, the final goal of optimization is estimating design variables. In this case, optimum values of design variables have been calculated By statistical analysis of the data and are shown in Table

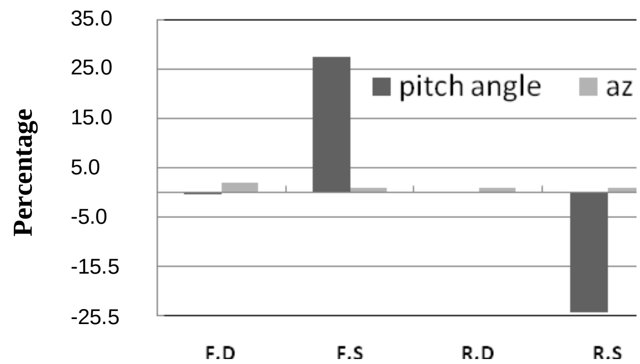


Figure 8. The effect of spring and damper characteristic on ride qualities.

Table 3. Correlation factor's bound limit.

Design variable	f_{sf}	f_{sr}	f_{cf}	f_{cr}
Lower bound limit	0.4	0.6	0.8	0.8
Upper bound limit	1.4	1.6	1.2	1.2

Table 4. Optimum value of Design variables.

Design variable	f_{sf}	f_{sr}	f_{cf}	f_{cr}
Optimized value	0.87	1.19	0.825	1.38

4. Increasing stiffness of rear spring and damper and decreasing stiffness of front spring and damper cause to improve ride comfort results as may be observed in Table 4.

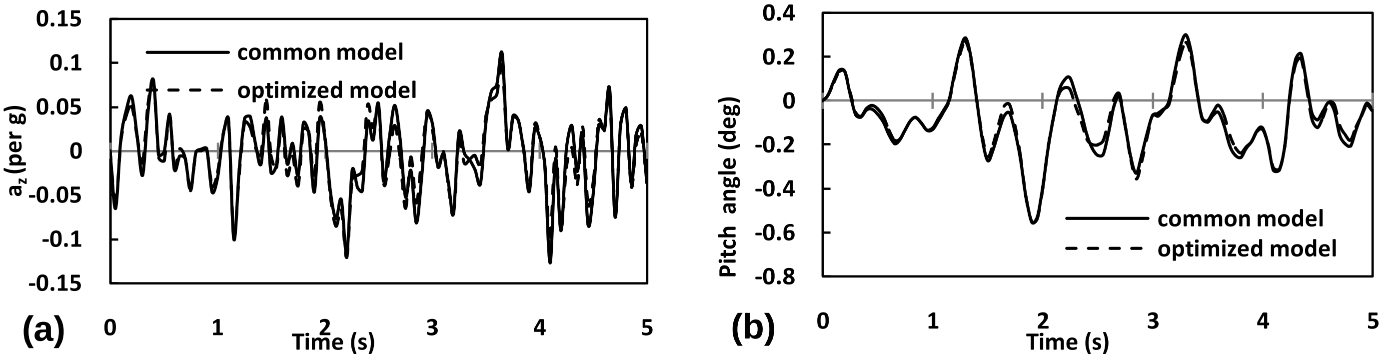
Vertical acceleration and pitch angles of the model (objective functions) are compared before and after optimization and the results may be seen in Figure 9 (a to l) for different velocities from 60 to 110 km/h. It is done to show that the optimization works acceptable in the range of 60 to 90 km/h that has been accomplished here. Also, RMS's of objective function before and after optimization are represented in Table 5 for better numerical comparison.

The RMS of vertical acceleration and pitch angle of common and optimized suspension settings at different speeds is shown in Figure 10. As it was expected the RMS of acceleration after optimization is less than common case within the range of 60 to 90 Km/h, in which the problem was designed for. From this point onward the changes in suspension system are not performing well, since a_{RMS} that can be defined as $100\times\left(\left(a_{common}-a_{optimized}\right)/a_{common}\right)$ is higher than common as it shown in Figure 10a. As an example, at a velocity of 110 Km/h optimum a_{RMS} is 11.44% more than that of common system that means the optimization has not good performance if the designer does not consider the whole range in the time of optimality, so he/she should precisely choose the range of optimization. Meanwhile, the reduction percentage of a_{RMS} due to suspension modification is plotted versus velocity in Figure 11a which shows a declining trend. It means that the optimal design works best in the 60 km/h and has the worst case in 110 Km/h. Pitch angle has decreased substantially in and out of velocity range of 60 to 90 Km/h on contrary to a_{RMS}, meaning pitch angle of optimized system is still 24.96% less than normal one, in v=110 Km/h (Figure 11b).

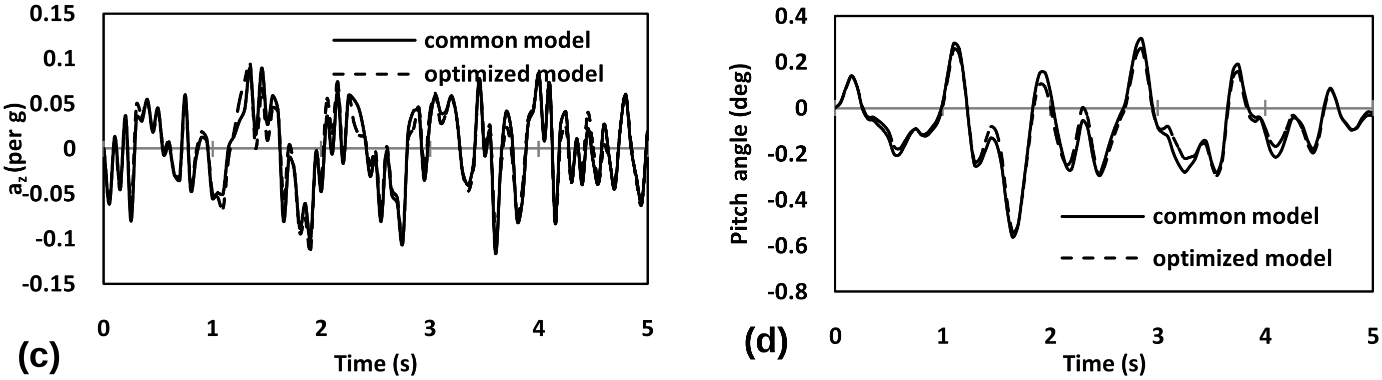
Conclusions

Conventional analytical studies on vehicle ride comfort in the low-frequency range are focused on the bounce and pitch behavior of vehicles. Due to high cost of active and semi-active suspension systems, passive ones can be used to determine proper stiffness coefficient of spring

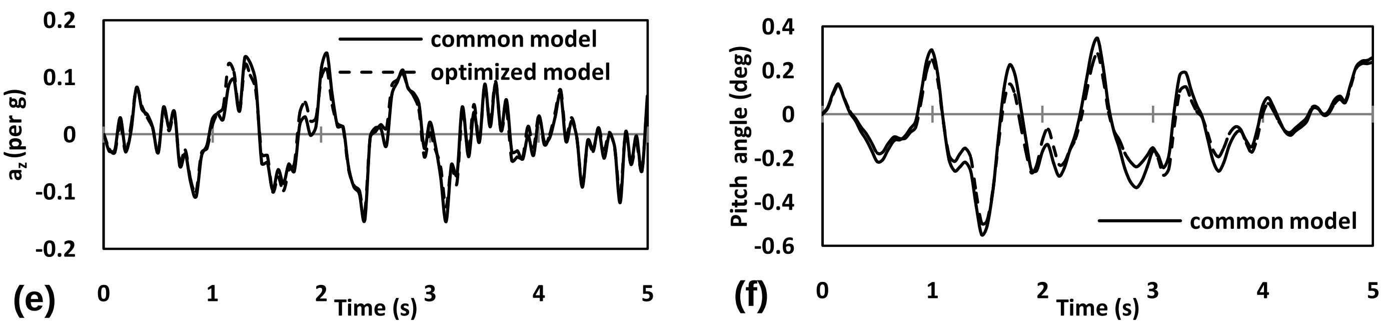
V = 60 Km/h



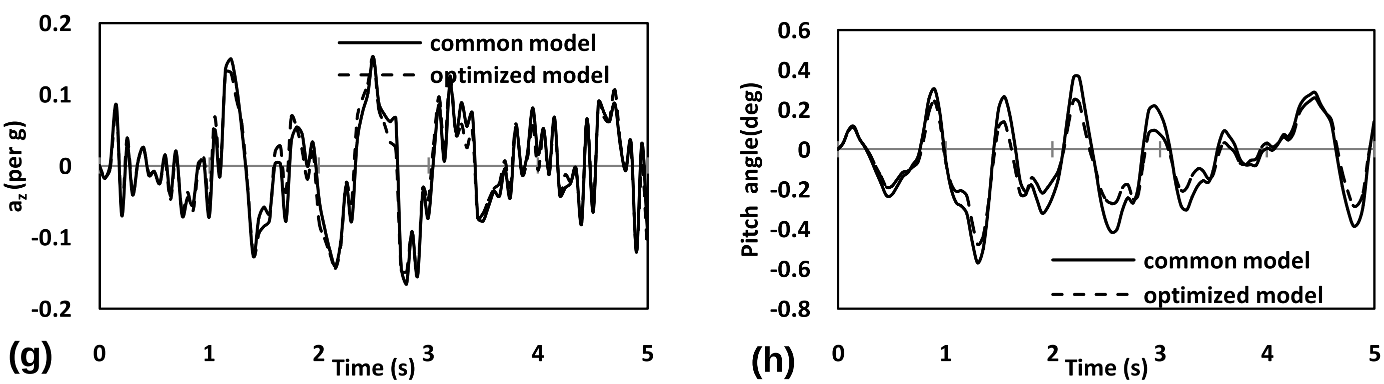
V = 70 Km/h



V = 80 Km/h



V = 90 Km/h



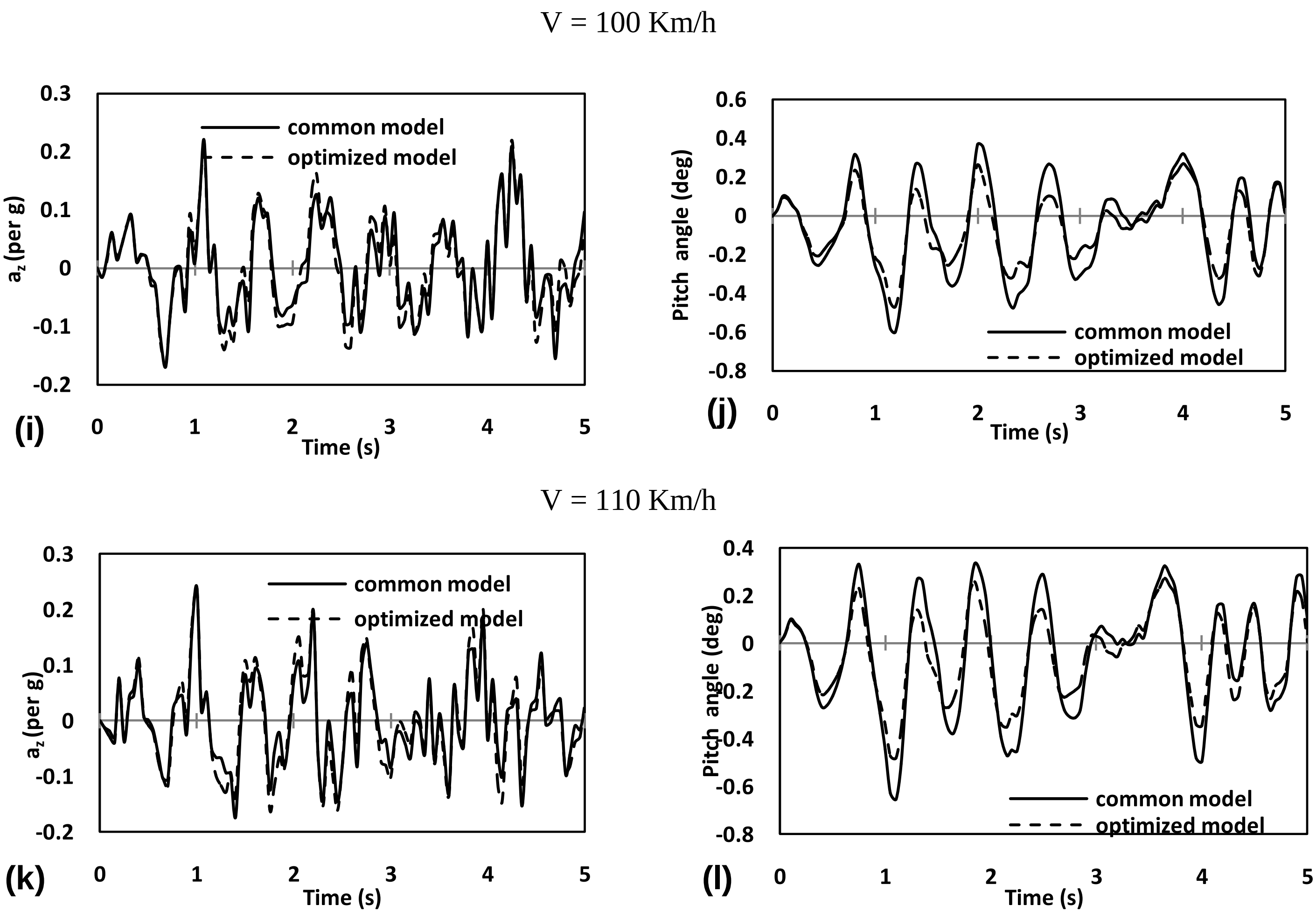


Figure 9. Effect of spring and damper characteristics on ride quality.

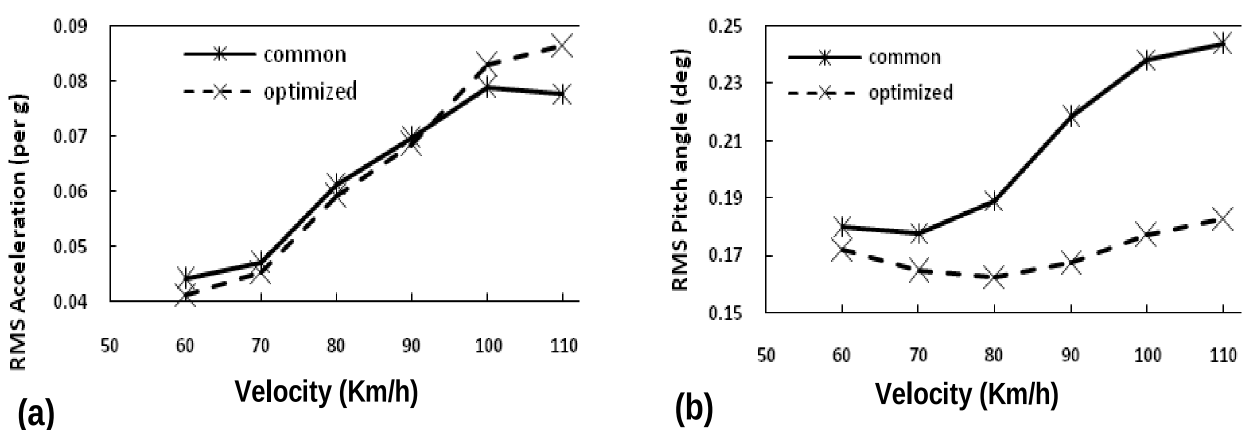


Figure 10. RMS vertical acceleration (a) and pitch angle (b) for optimized and common suspension settings at different speeds.

and damper in order to achieve the best ride comfort along with lowest expenses. DOE, as an efficient way of modeled parameter optimization using statistical method with saving cost and time, is a good approach for

Table 5. Vertical acceleration and pitch angle after optimization.

RMS	V=60	V=70	V=80	V=90	V=100	V=110
Common a_z (g)	4.4e-2	4.69e-2	6.11e-2	6.97e-2	8.3e-2	7.78e-2
Optimized a_z	4.13e-2	4.54e-2	5.9e-2	6.85e-2	8.25e-2	8.67e-2
Common θ (deg)	1.8e-1	1.78e-1	1.89e-1	2.18e-1	2.38e-1	2.44e-1
Optimized θ	1.72e-1	1.64e-1	1.62e-1	1.68e-1	1.77e-1	1.83e-1

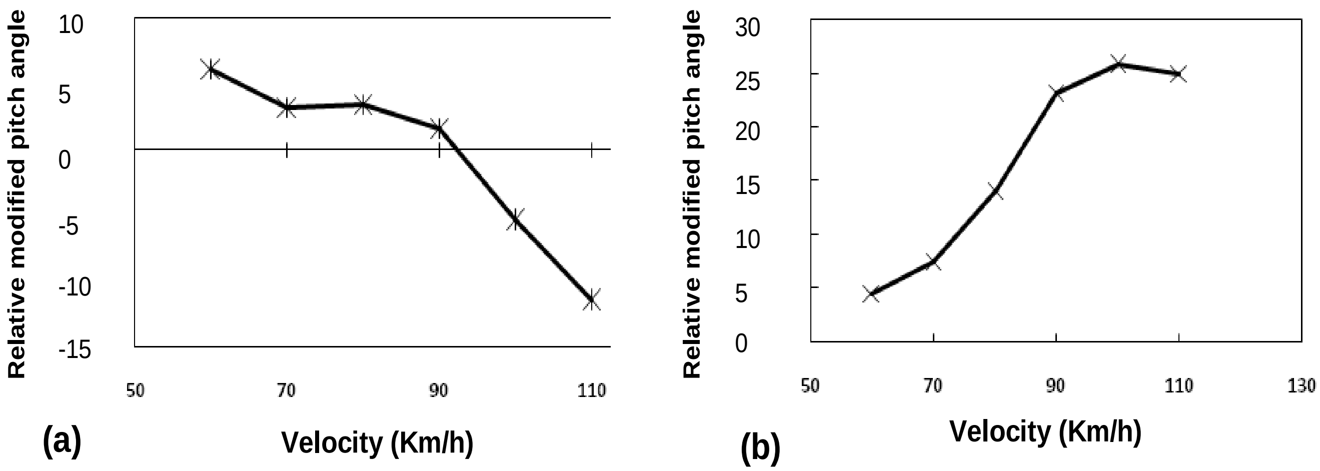


Figure 11. Relative RMS vertical acceleration (a) and pitch angle (b) at different speeds.

improving ride comfort. In this study, DOE has been applied on the results of a 7DOF model excited with a standard road to select F/R spring and damper coefficient resulting in minimum RMS of vertical acceleration and pitch angle for speed between 60 to 90 Km/h. The optimal designed method shows that the vertical acceleration works best in the 60 Km/h and will decrease the performance with increasing the automotive velocity, although the performance of pitch angle will improve from 5 to 25% with increasing the automotive velocity from 60 to 110 Km/h. Peresented methodology can be applied to any vehicle and optimum suspension parameter can be obtained.

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