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Full Length Research Paper

Modeling of connection elements in static and dynamic analysis

C. Azoury[1]*, A. Kallassy[1], I. Moukarzel[1] and R. Boudet[2]

[1]Mechanical Engineering Department, Lebanese University, Lebanon.
[2]Mechanical Engineering Lab, Paul-Sabatier University, Toulouse (LGMT), France.

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This paper presents the proposal of a new linear finite element that can substitute for the three dimensional finite model. A study is conducted on a simple frame at the beginning in order to proceed to the crankshaft. First, a three-dimensional model is constructed, then an equivalent two-dimensional model. At a final stage, the results from static and dynamic analyses of many linear element models were compared to the results of the two-dimensional model. With a special treatment to the connection elements, the results of both the static and dynamic analyses of the new linear finite element model agree well with the results of a two-dimensional model.

Key words: Finite element analysis (FEA), Timoshenko beam, Euler-Bernoulli beam, connection element, static analysis, modal analysis, crankshaft.

INTRODUCTION

To perform a modal analysis of a crankshaft, usually, three-dimensional finite elements are used in the meshing process. A large number of elements are required, which in turn requires a great deal of work. The main idea is to develop a 1D model consisting of simple 1D elements to replace the three - dimensional model which consists of three- dimensional elements (tetrahedral or brick elements). To perform a modal analysis, the calculation of stiffness and mass matrices is required. These two matrices will be found for the new 1D finite element.

In order to find the best linear finite element meshing, a three-dimensional meshing is used, then a two-dimensional and finally, linear modeling is constructed.

All calculations are performed on Ansys 9 and Matlab 7. An application on the crankshaft is then performed.

*Corresponding author. E-mail: chadiazoury@gmail.com. Tel: +961 9 237844, 9613198644. Fax: +961 9 231835.

Notation: E, Young Modulus; ***v*,** Poisson coefficient; **rho,** density; **K,** stiffness matrix; **M,** mass matrix; ***t*,** *t*hickness of the frame; **EB,** Euler-Bernoulli; **sh,** shear deformation.

2D frame

A simple representative part of the crankshaft is taken. It consists of 3 frame elements (Figure 1). The section of the beams of the test structure is rectangular (20 × 20 mm^2). The analysis is linear. The mechanical characteristics of the test structure are those of the crankshaft:

Young Modulus: E = 184.052 GPa.
Poisson coefficient: v = 0.31.
Density: *rho* = 7800 Kg/m^3.

The test structure consists of 5 structural elements (Figure 2):

i. Left column.
ii. Right Column.
iii. Horizontal beam.
iv. Left corner.
v. Right corner.

The test structure is clamped at its base.

We will use different types elements in the meshing

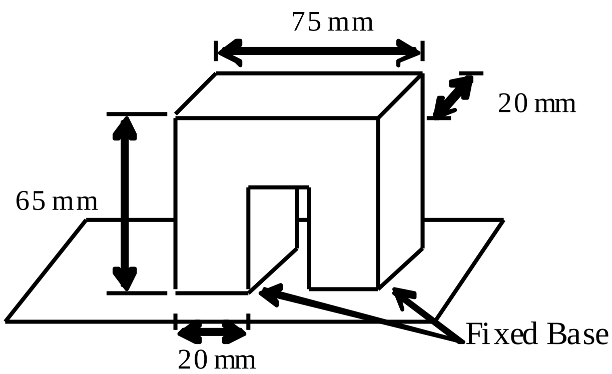


Figure 1. Test structure.

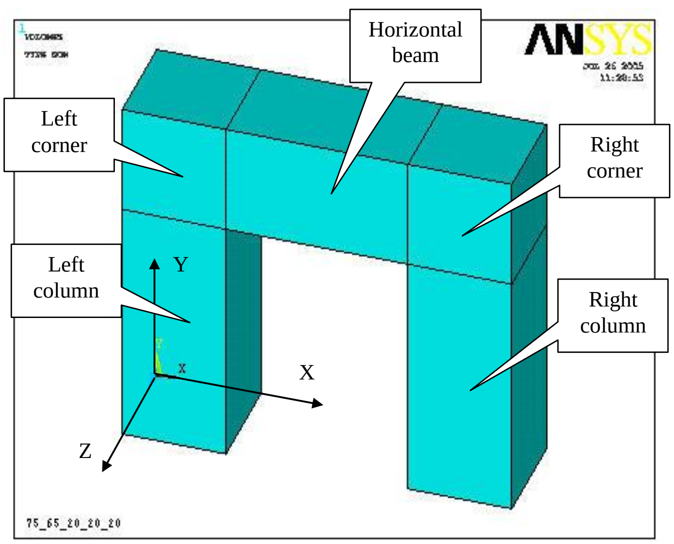


Figure 2. Test structure constructed in Ansys.

process and the models will be compared under 2 load cases:

Case 1: a body load of 78000 N/m^3 along the Y $^-$ axis of the horizontal beam (Figure 3).
Case 2: a body load of 78000 N/m^3 along the X $^+$ axis of the left column.

For the dynamical analysis, the total mass will be considered.

3D and 2D modeling

The test structure is meshed using H8 volumetric finite elements with 8 nodes. Each column is modeled using 8 × 8 × 16 = 1024 elements. Each corner is modeled using 8 × 8 × 8 = 512 elements. The horizontal beam is modeled using 8 × 8 × 16 = 1024 elements. In total: 2 ×1024 + 2 × 512 + 1024 = 4096 elements with 5265 nodes (Figure 4).

The test structure is meshed using Q4 plane finite elements with 4 nodes. The thickness of the test structure is t = 0.02 m. Each column is modeled using 8 × 16 = 128 elements. Each corner is modeled using 8 × 8 = 64 elements. The horizontal beam is modeled using 8 × 16 = 128 elements.
In total: 2 × 128 + 2 × 64 + 128 = 512 elements with 585 nodes (Figures 5 and 6).

The different models are compared using the displacements of 65 points on the central line of the longitudinal symmetrical plan of the structure. In order to establish a comparison between both FE models and using both load cases, UX and UY displacements of nodes of the central line are plotted on the same graph (Figures 7 to 11).

The results of the static analysis from both models H8 and Q4 are close enough, thus the Q4 model will be the reference model for the test structure. For comparison criteria of the results of different models, we have used the following formula for the error:

$$E_{\max} = \max_i \frac{|U_R(i) - U_C(i)| \times 100\%}{\left|\max_j(U_R(j)) - \min_j(U_R(j))\right|}$$

Where: U_R (i) = reference value of a displacement at point i; U_C (i) = calculated value of the same displacement and at the same point i of a new model.

The maximum error between both models (reference and other models), for all load cases, are listed in Table 1.

Dynamic

The criterion of comparison of different models is based on the calculation of relative error on the frequencies of vibration. The formula is given by:

$$E_i = \frac{|f_{R\,i} - f_{C\,i}|}{f_{R\,i}}$$

Where: E_i = relative error of the i th mode of vibration; $f_{R\,i}$ = frequency of vibration of the i th mode of the reference model; $f_{C\,i}$ = frequency of vibration of the i th mode of new model

STATIC ANALYSIS

One-dimensional modeling (linear elements)

We have shown that the plane model gives comparable

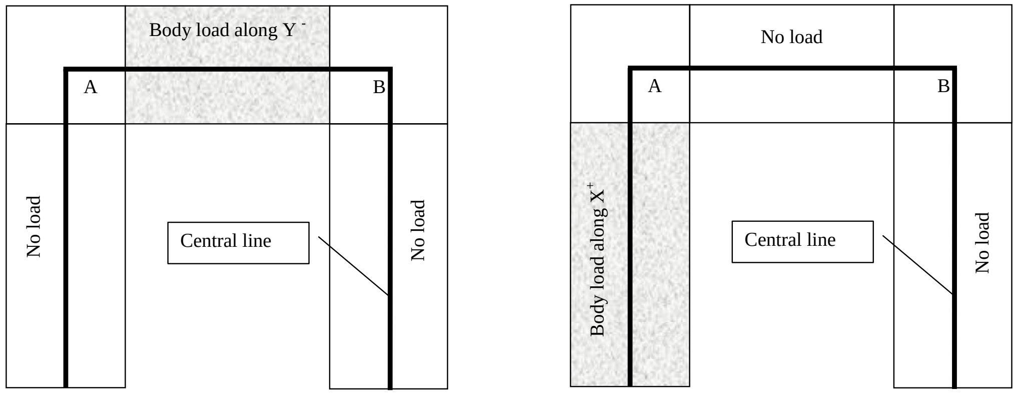
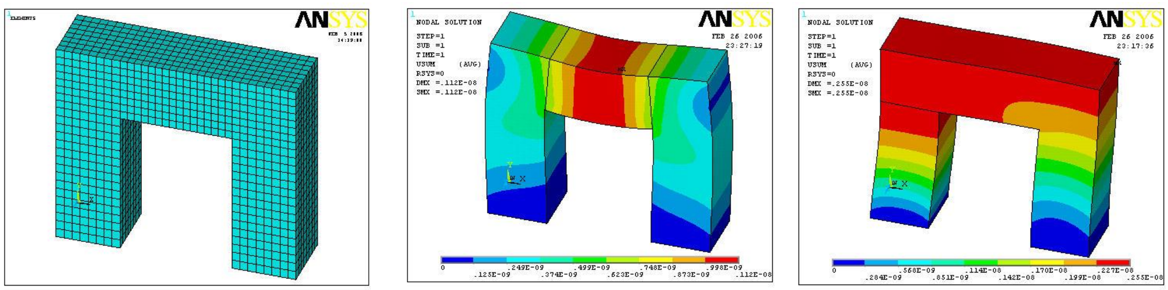
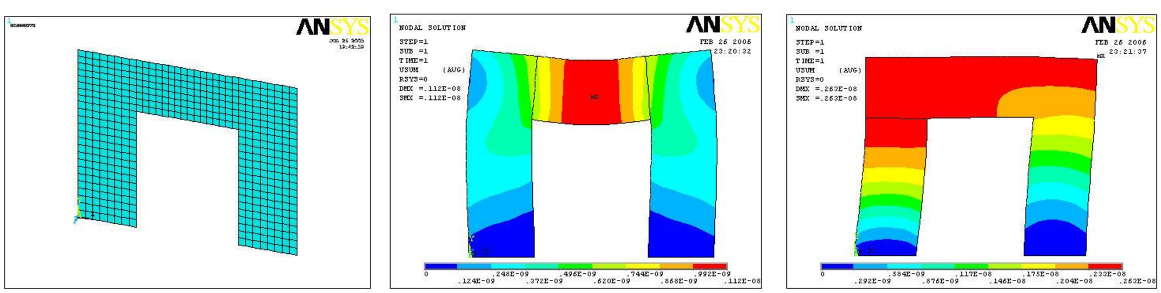


Figure 3. Test structure with load case 1 to left and 2 to the right.



a) Load case 1 b) Load case 2

Figure 4. Three-dimensional deformed test structure along 2 load cases.



a) Load case 1 b) Load case 2

Figure 5. Two-dimensional deformed test structure along 2 load cases.

results to the three-dimensional model. Here, we will use linear model for the test structure. Three types of elements are considered:

i. Euler-Bernoulli beam with 2 nodes per element and 3 dof per node (EB model).

ii. Euler-Bernoulli beam with 2 nodes per element and 3 dof per node with shear deformation (EB+sh model).

iii. Timoshenko beam with 2 nodes per element and 3 dof per node (Timoshenko model) (Figure 11).

The transformation from the Q4 model to the linear model is shown in Figures 12 and 13. Both columns (left

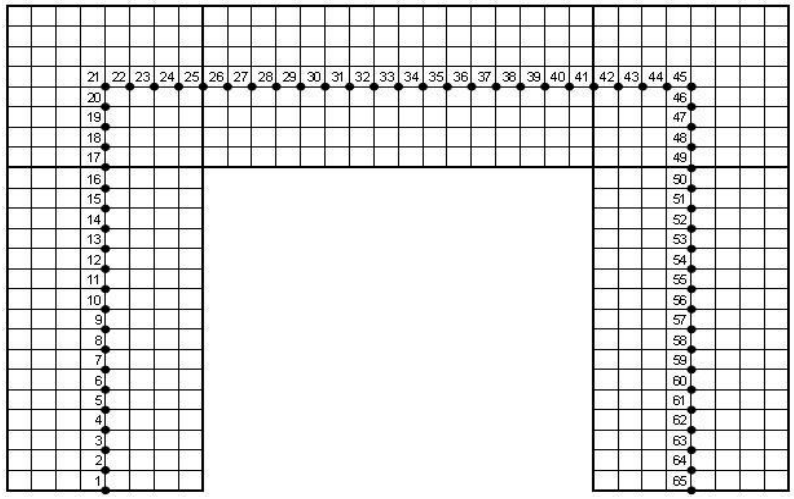


Figure 6. Longitudinal symmetrical plane of the structure showing the 65 points.

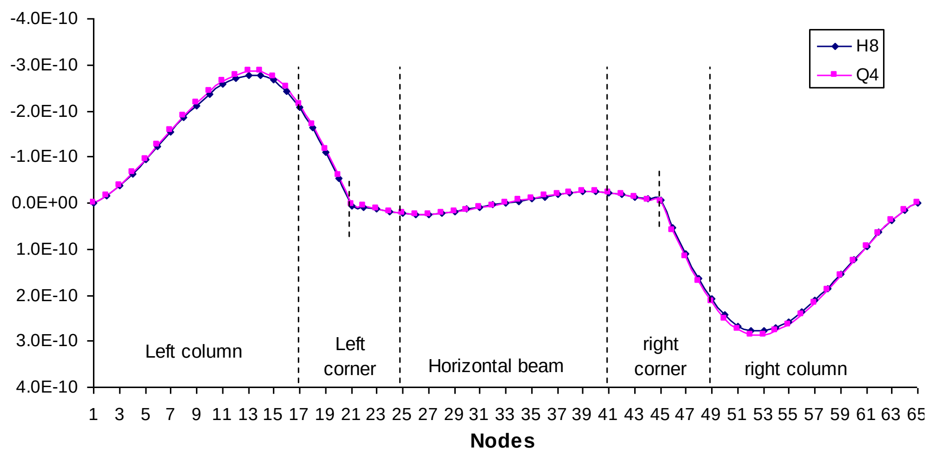


Figure 7. UX (m) displacement of the central line (load case 1).

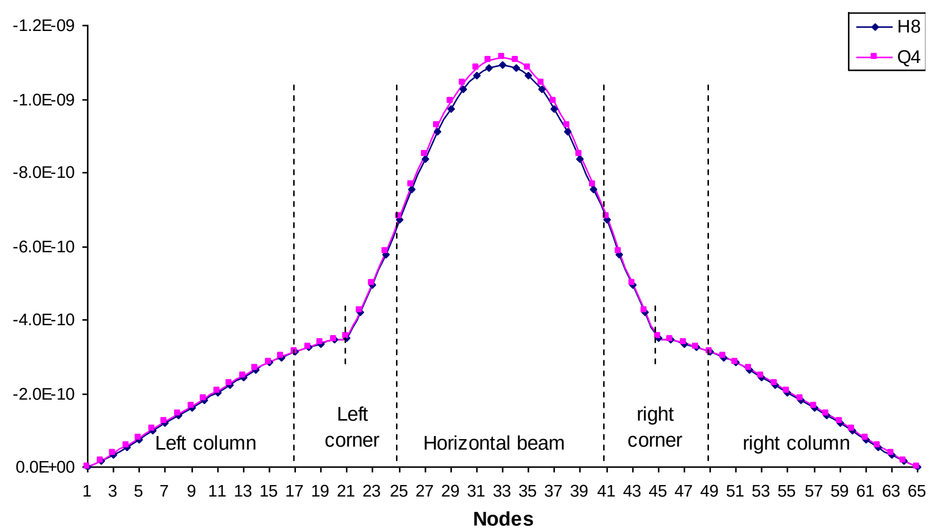


Figure 8. UY (m) displacement of the central line (load case 1).

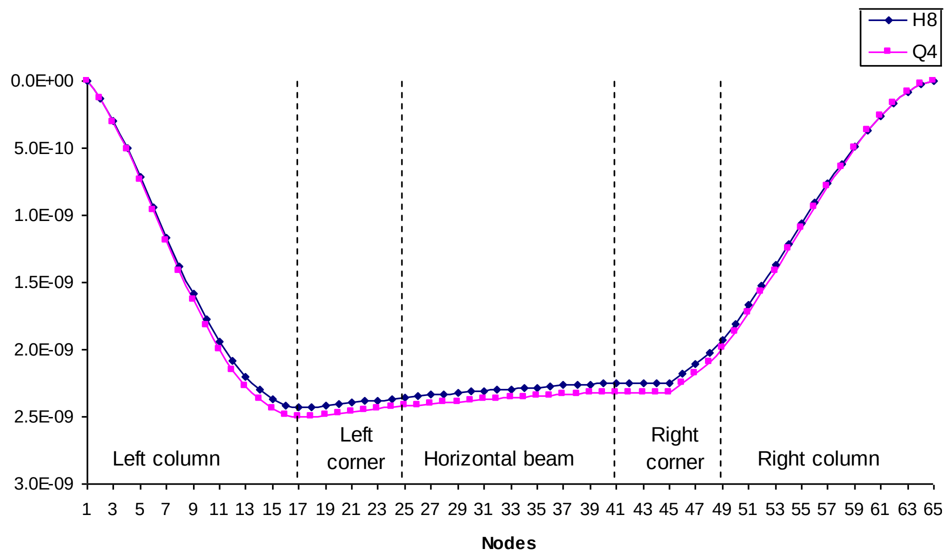


Figure 9. UX (m) displacement of the central line (load case 2).

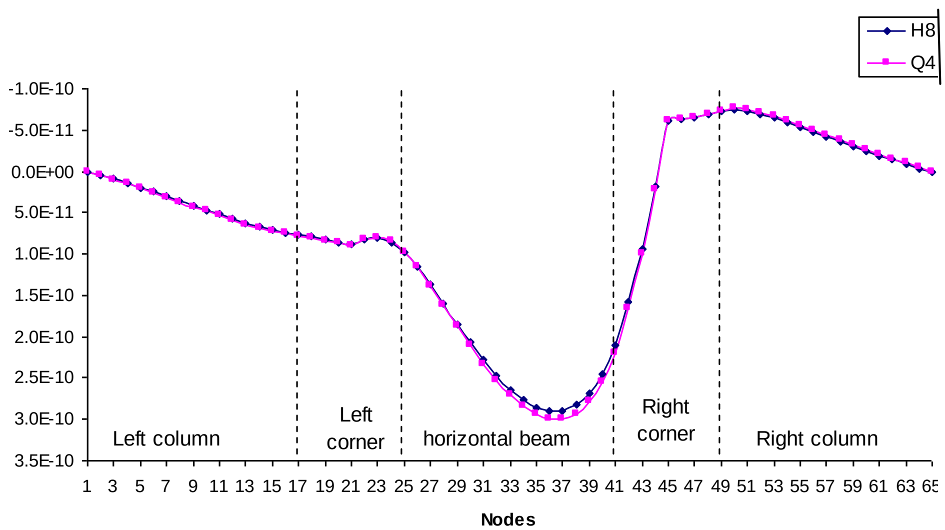


Figure 10. UY (m) displacement of the central line (load case 2).

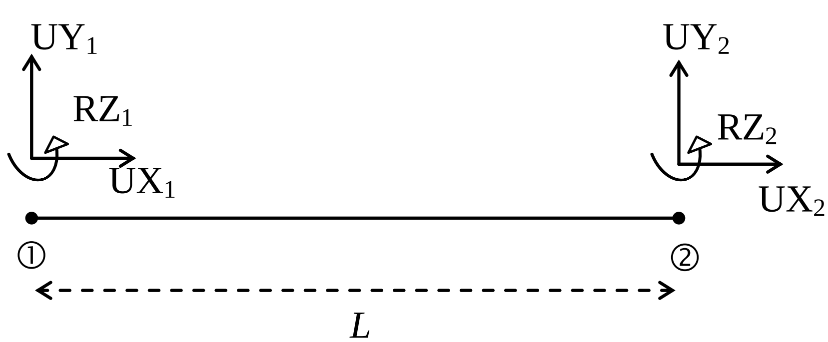


Figure 11. Beam with 2 nodes.

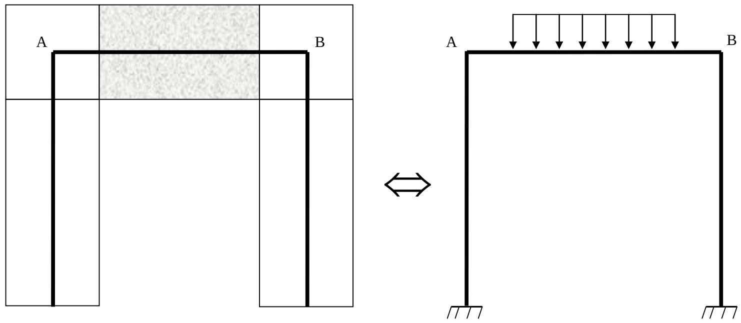


Figure 12. Transformation (Q4 model – linear model) (load case 1).

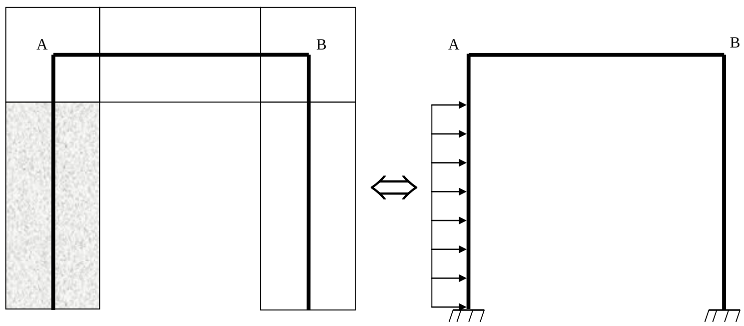


Figure 13. Transformation (Q4 model – linear model) (load case 2).

Table 1. Maximum error between both models.

Mode no.	H8	Q4	$E_i = \frac{\lvert f_{R\,i} - f_{C\,i}\rvert}{f_{R\,i}}$ (%)
1	4385.2	4375.6	0.22
2	12957	12950	0.05
3	17622	17558	0.36
4	18023	18009	0.08
5	22346	22432	0.38
6	26796	26894	0.37
7	32829	32996	0.51
8	40436	40761	0.80
9	47705	48173	0.98
10	51173	51532	0.70
Average			0.45

and right) are modeled using 16 elements each. The horizontal beam is modeled using 16 elements. Both corners (left and right) are modeled using 4 elements along the horizontal and 4 elements along the vertical. In total, we have 64 beam elements. Figures 14 to 17 show the deformed test structure for the 2 load cases for 4 models (the reference model: Q4 and the other three classical beam models).

We note that for frame structures, EB+sh or Timoshenko beams can be used. Linear models do not give proximate results compared to the Q4 model. Improved models are further shown.

First improved model (rigid links)

If the connection dimensions are not small, as compared to the member lengths, then their effect must be considered in the analysis (Kassimali, 1999). In our case, the connection is 0.02 m in width while the length of the

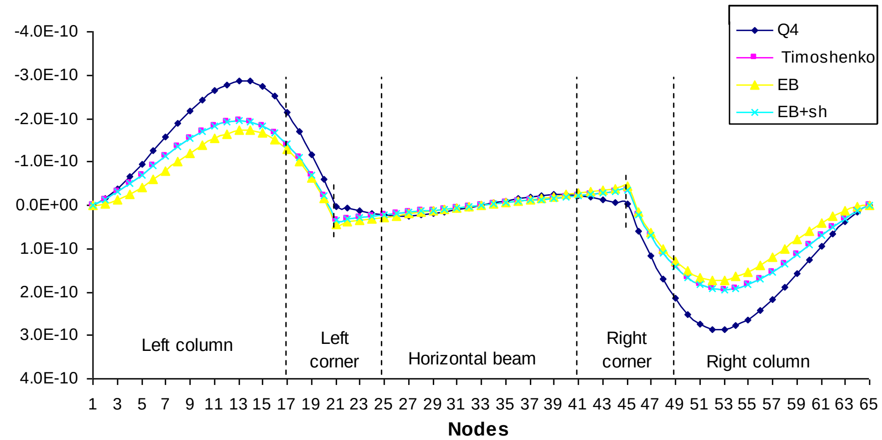


Figure 14. UX (m) displacement of nodes of the central line (load case 1).

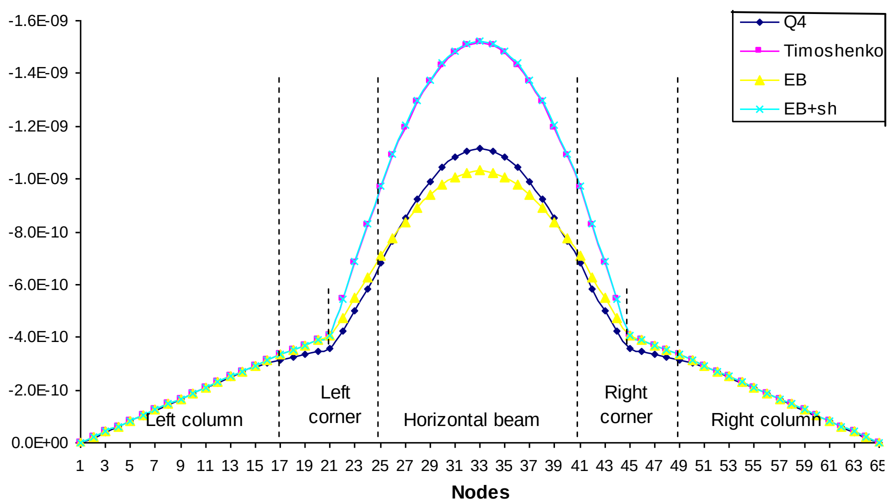


Figure 15. UY (m) displacement of nodes of the central line (load case 1).

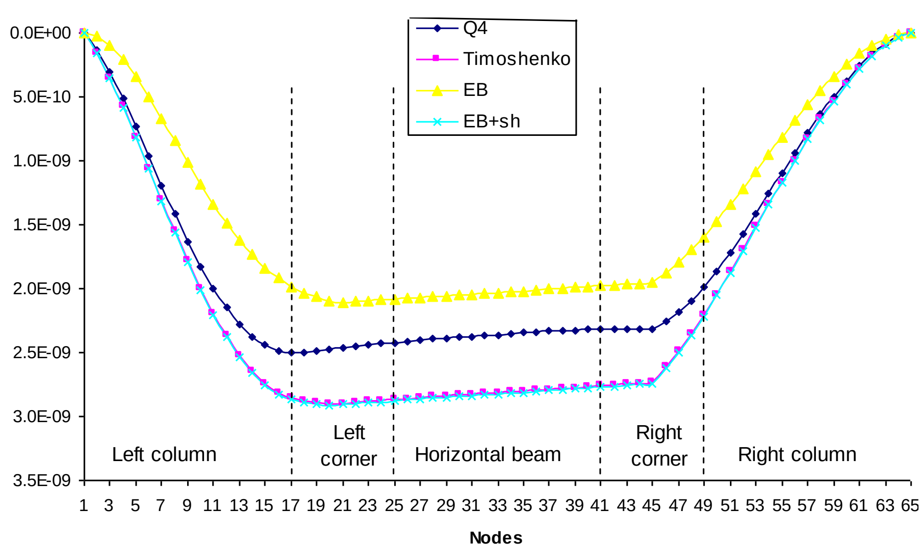


Figure 16. UX (m) displacement of nodes of the central line (load case 2).

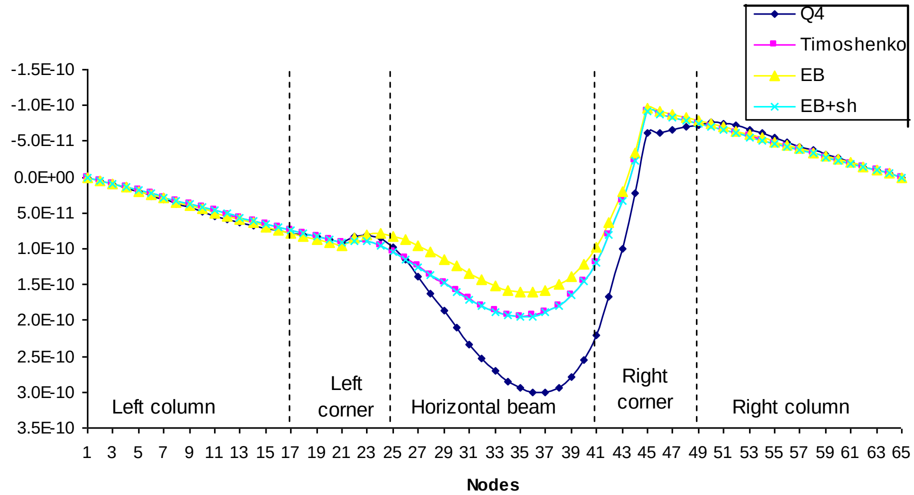


Figure 17. UY (m) displacement of nodes of the central line (load case 2).

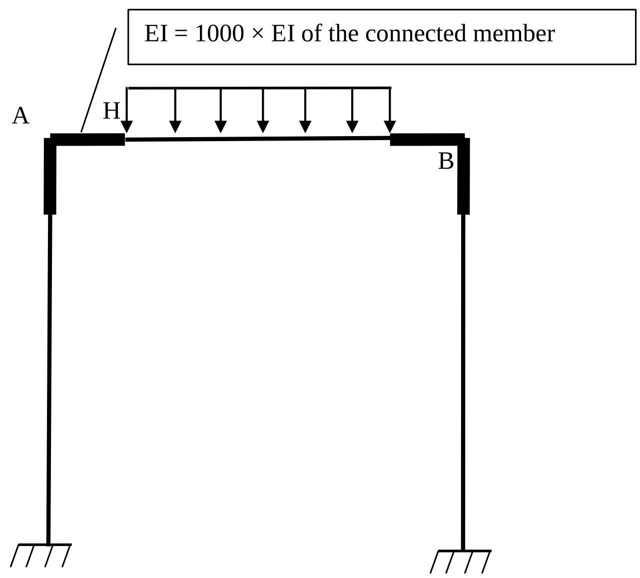


Figure 18. Analytical model considering connection sizes.

beam is 0.035 m which is a ratio of 1:75. A special approach is used to include the effect of offset connection.

Hoit (1995) suggests that the cross–sectional properties of an offset member be chosen so that its stiffness is 1000 times that of the connected member (Figure 18). So, we have increased the Young modulus of the corner beams to 1000×E. The results are shown in Figures 19 to 22.

When we increased the stiffness of corner beams, the general deformation has approached the deformation of Q4 model and in particular places, the model is stiffer. We notice that the EB 1000E model gives bad results. So, we will adopt the Timoshenko beam specially made for thick beams suitable for our study.

Second improved model (73E)

We tend to decrease the stiffness of the corner beams. Let Ev = Young modulus of the vertical beam, Eh = Young modulus of the horizontal beam at the corner.

Previously, the Young modulus was increased 1000 times which showed a rather rigid deformation. So we need to find nh and nv (Figure 23), the factors of Young modulus E of vertical and horizontal beams respectively of both corners.

Using trial and error method, we find that for Ev = 7×E and Eh = 3×E we get best results.

The results are shown in Figures 24 to 27. We notice that modeling corner beams using Timoshenko model leads to a flexible frame, using rigid beams (1000E) leads

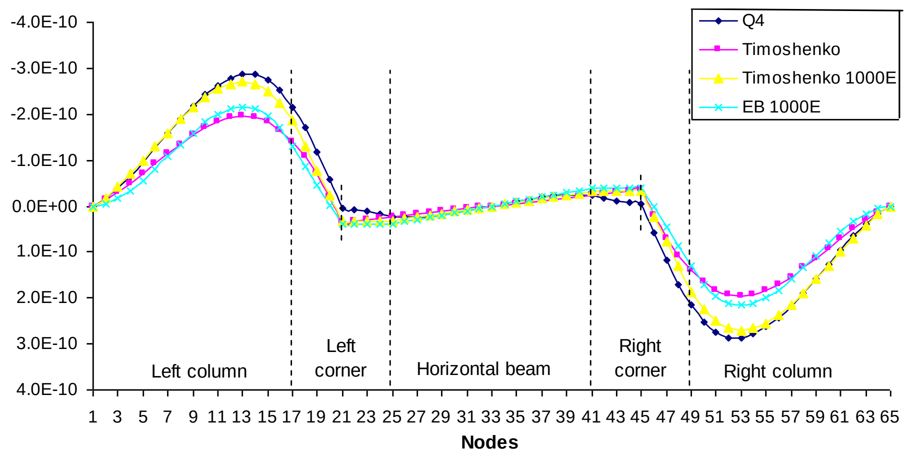


Figure 19. UX (m) displacement of nodes of the central line (load case 1).

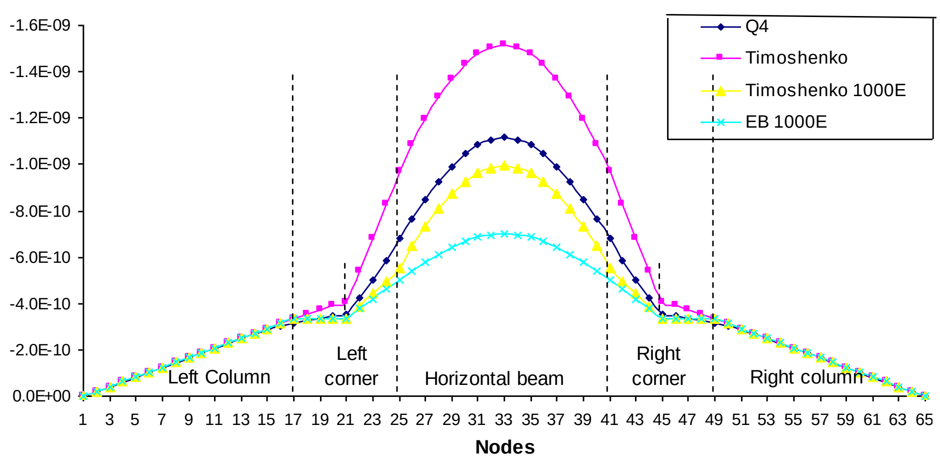


Figure 20. UY (m) displacement of nodes of the central line (load case 1).

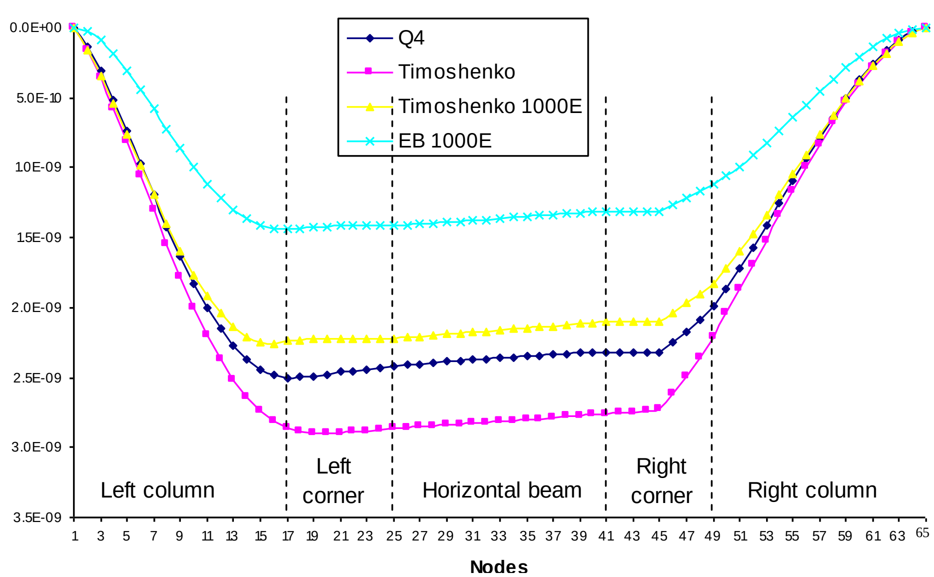


Figure 21. UX (m) displacement of nodes of the central line (load case 2).

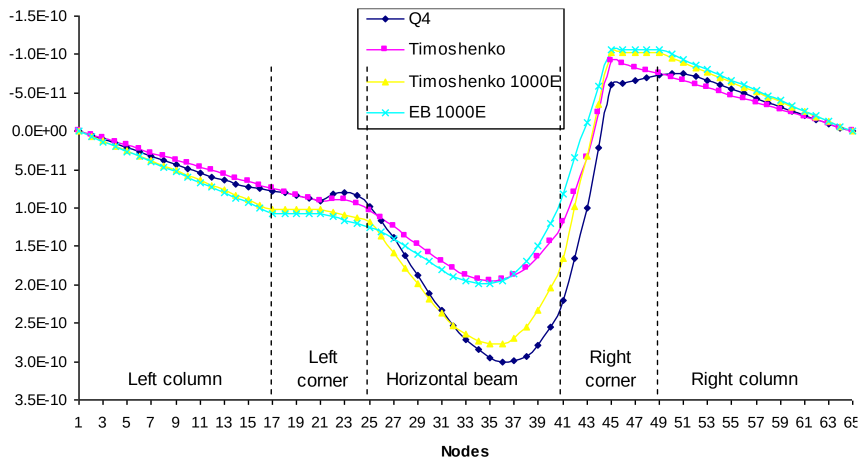


Figure 22. UY (m) displacement of nodes of the central line (load case 2).

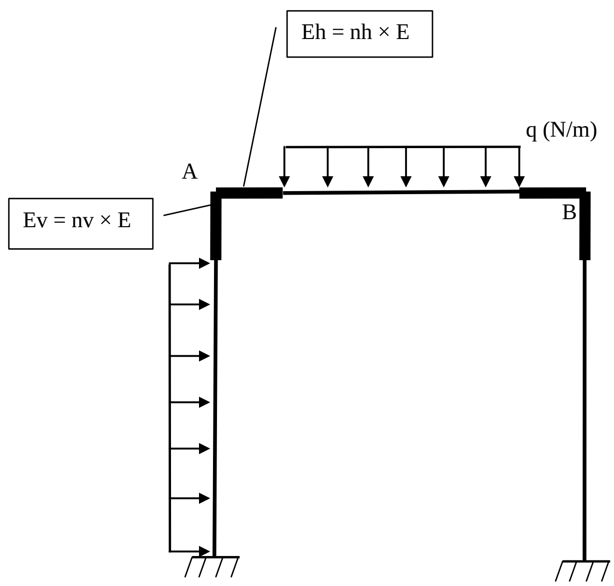


Figure 23. Analytical model considering connection size (Ev and Eh).

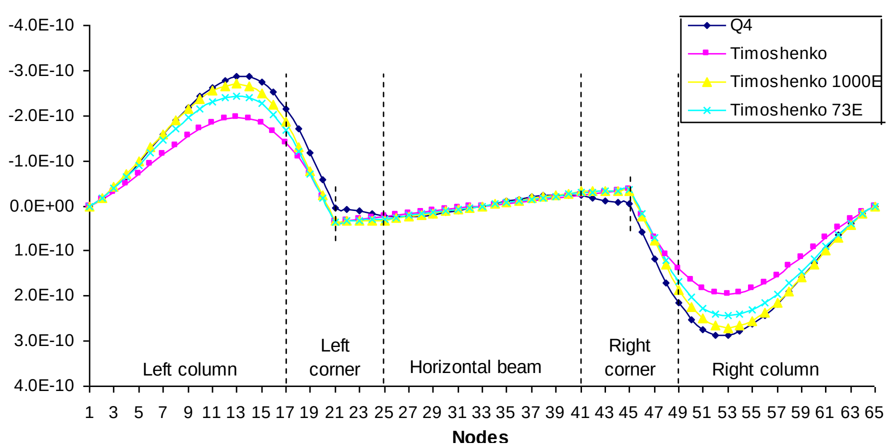


Figure 24. UX (m) displacement of nodes of the central line (load case 1).

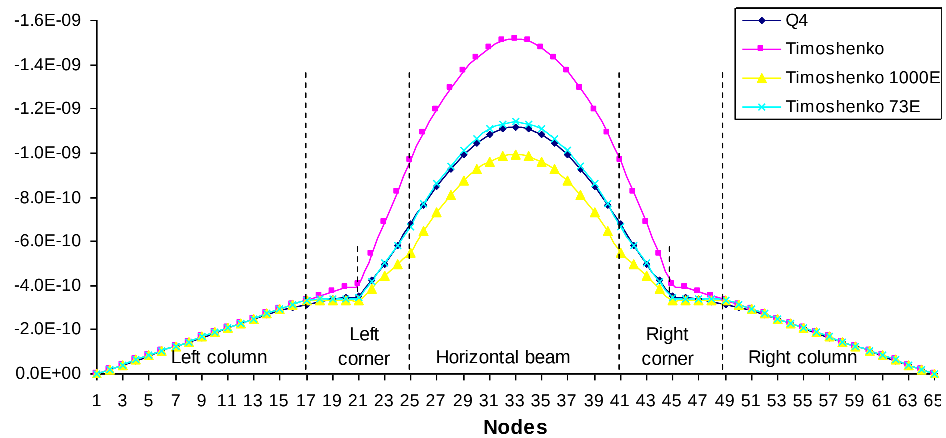


Figure 25. UY (m) displacement of nodes of the central line (load case 1).

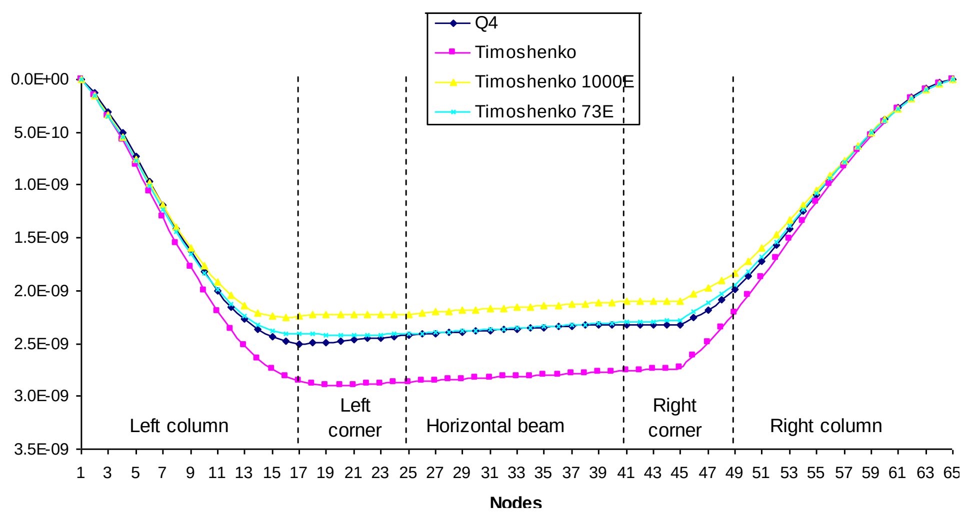


Figure 26. UX (m) displacement of nodes of the central line (load case 2).

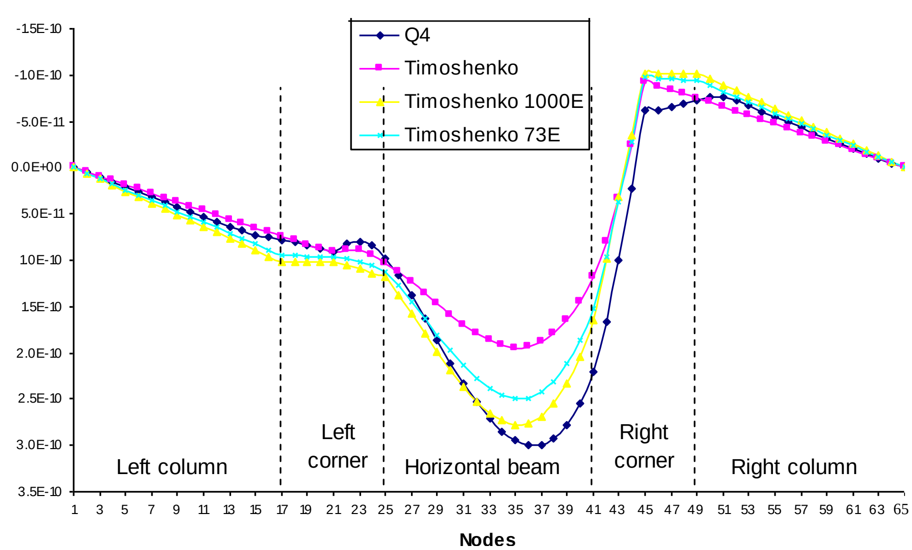


Figure 27. UY (m) displacement of nodes of the central line (load case 2).

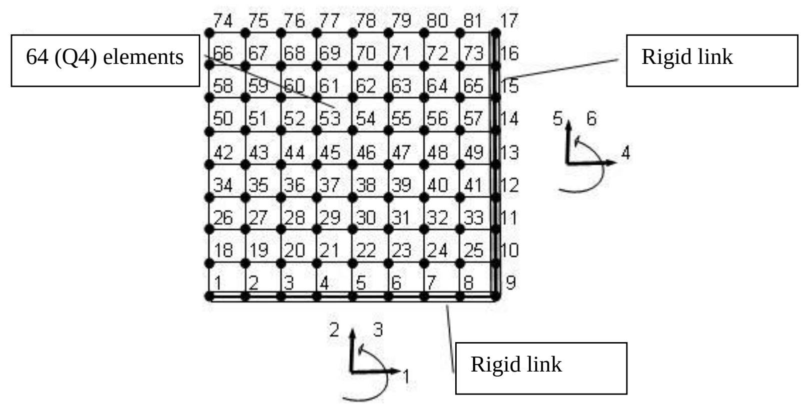


Figure 28. Corner with rigid links.

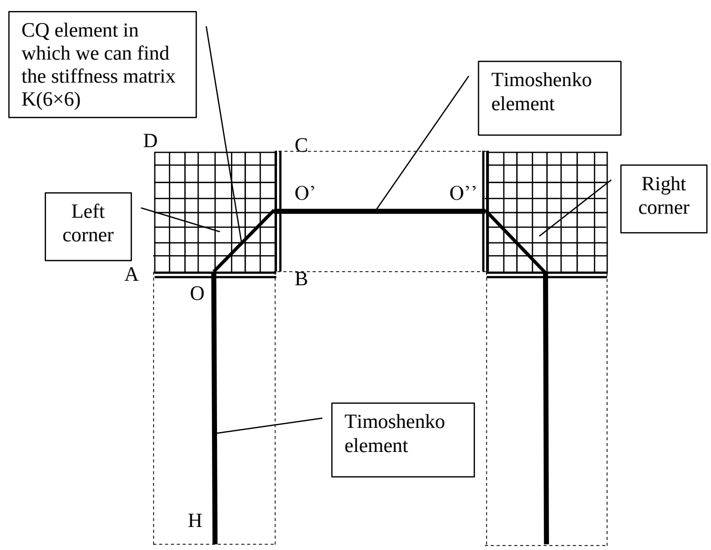


Figure 29. Description of the CQ model.

to a rigid frame; while using an average flexibility (73E), the results are acceptable when comparing to the Q4 model.

Third improved model (Corner Quad: CQ)

We suggest a new element for the corners, based on the utilization of Q4 surface elements. The interface between this new element and the connected horizontal and vertical beams is rigid.

The corner element of 0.02 × 0.02 m and 0.02 m in thickness is meshed using Q4 elements of 0.02/8 × 0.02/8 m which gives 64 elements and 81 nodes in total, hence 81×2 dof. The interfaces are rigid in nature, so 3 dof on every interface is sufficient to give dof values on nodes of these interfaces.

The stiffness matrix of this new element is obtained by assembling elementary matrices of the 64 elements. Using condensation, the dof of nodes that are not on the interfaces are eliminated. Finally, we get an elementary stiffness matrix K (6×6) for the corner (Figure 28).

For one Q4 element, the elementary stiffness matrix K is known. We find the assembled matrix KK (162×162), using the static condensation, K (6×6) is found using a Matlab® code. The stiffness matrix K is function of:

i. The length of side AB
ii. The length of side BC
iii. The Young modulus E of the material in part ABCD
iv. The Poisson coefficient *v*
v. The thickness of the frame *t* (Figure 29)

HO and O'O'' are Timoshenko beams. At the corner, there is a fictitious beam with stiffness matrix K (6×6). Kleft is for the left corner and Kright is for the right one.

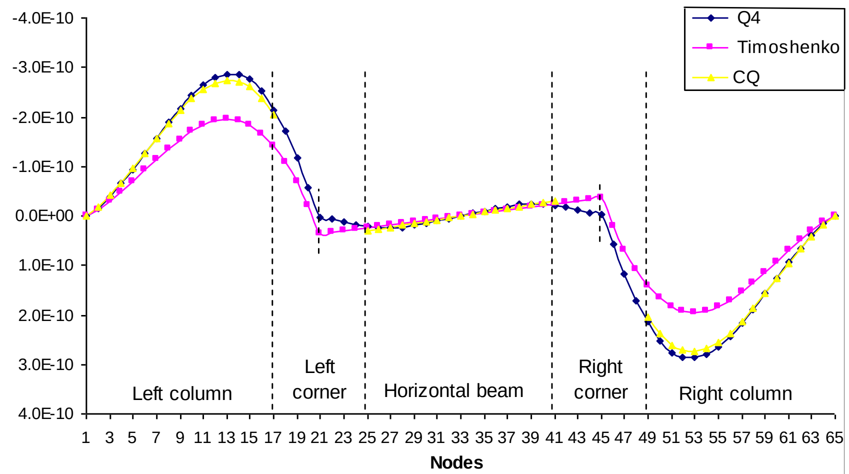


Figure 30. UX (m) displacement of nodes of the central line (load case 1).

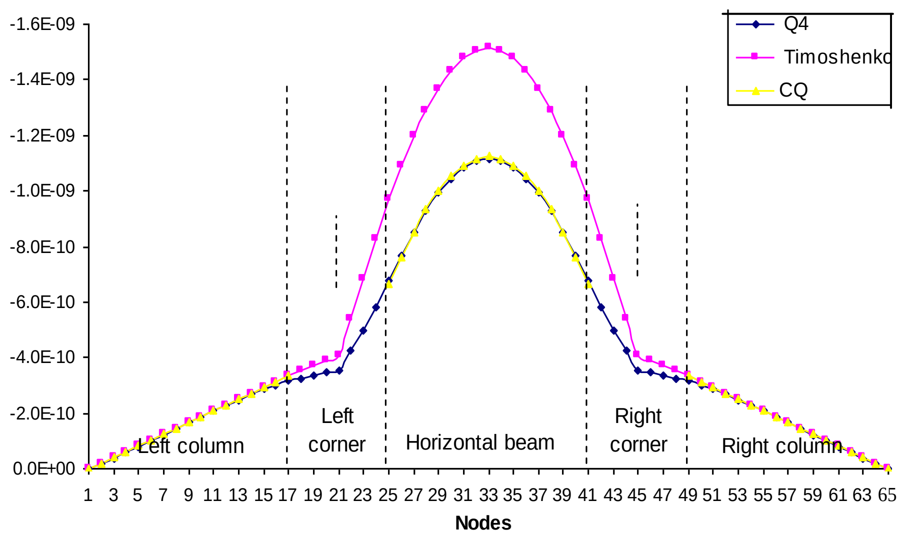


Figure 31. UY (m) displacement of nodes of the central line (load case 1).

And by transformation, we can find the stiffness matrix of the right corner. The results are shown in Figures 30 to 33. The graphs show that the results of the CQ model are very close to the Q4 model.

Table 2 shows the results of all models. From Table 2, we notice that the CQ model gives the best approximation to the Q4 model in a static analysis.

DYNAMIC ANALYSIS

The dynamical analysis in the (XY) plane requires the determination of mass matrices of different elements used in each model. Consistent elementary mass matrix is used in the calculations. The mass matrices of all beam elements (EB, EB+sh, Timoshenko) and of the Q4 element are known (Cook, 2002). As for the CQ model, using the Gyan-reduction method, we can find the mass matrix M of the element of the left corner with 2 nodes per element and 3 dof per node. Using transformation, we can find the mass matrix of the element of the right corner.

Table 3 shows the relative errors of frequencies (Hz) of vibration of all models with respect to the Q4 model. To compare different values, the average of absolute values of the first 10 modes is found. The results show that the CQ model is the best one.

Application on the crankshaft

Three – dimensional (TET10) volumetric elements are used to model the crankshaft using the FEA. Now, we try to model it using linear finite elements. The crankshaft is

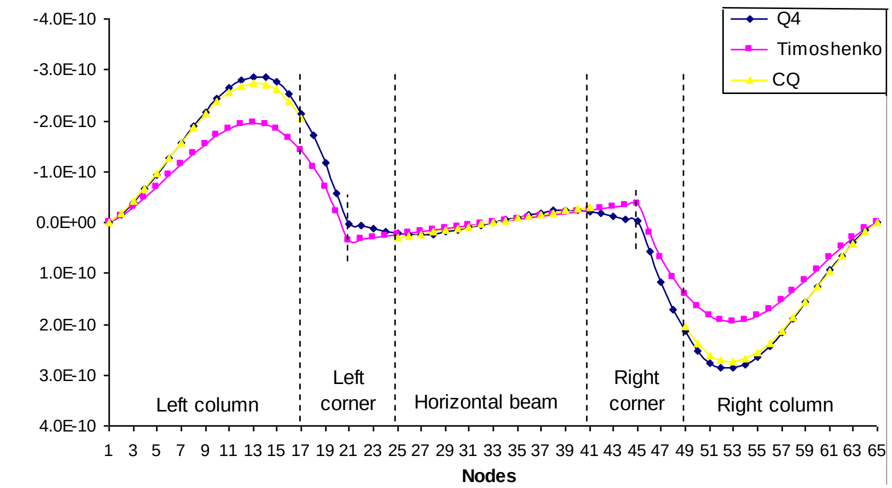


Figure 32. UX (m) displacement of nodes of the central line (load case 2).

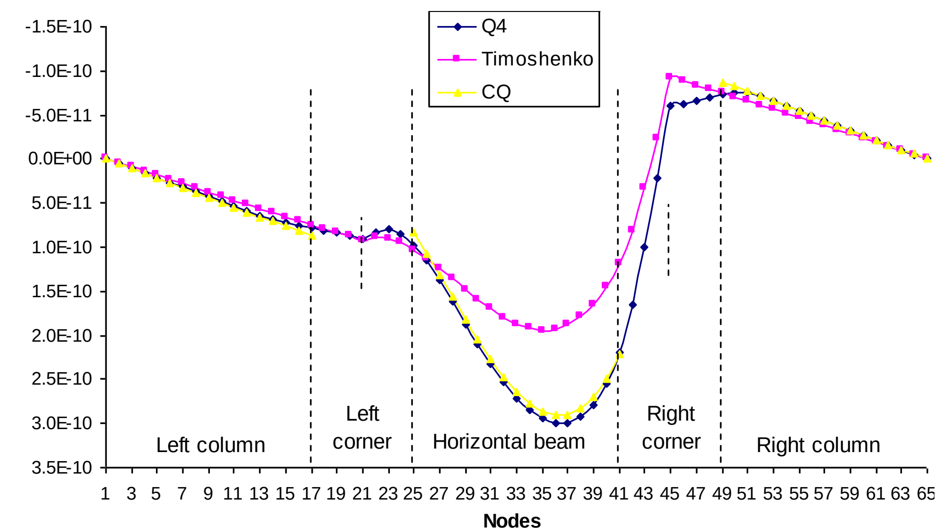


Figure 33. UY (m) displacement of nodes of the central line (load case 2).

constructed and sketched using Autocad® 2004 (Figure 34). The crankshaft has a very complex geometric form; therefore, each part is assigned a special name (Figure 34). By adding all parts of the crankshaft, we get a total mass of M = 10.37 Kg and total volume of V = 13.30×10^{-4} m^3.

Modeling the crankshaft using Timoshenko beam elements

At first, we will model the crankshaft using Timoshenko beam elements, with rectangular section, circular and taper. Figure 35 shows the model.

There are 54 nodes and 53 elements. We note that all nodes are in the XY plane. By adding all parts of the crankshaft from the Timoshenko model we get a total mass of M = 10.17 Kg and total volume of V = 13.04×10^{-4} m^3 which leads to a relative error of less than 2%.

Modeling using DBQ element

As previously seen, to improve the results of both static and dynamic analyses of a frame with thick elements, a particular modeling of connections is required. To test 2D frame, the CQ element is used at corners. Before applying a modification to the modeling of corner elements (connections) of the whole crankshaft, a part consisting of 3 cylinders and 2 elements pt1 and pt2 is analyzed. Let this part be called pt1pt2 as shown in Figures 36 and 37.

Table 2. Comparison of all models and all load cases.

Model	Node (where: $E = E_{max}$)	Load case	Displacement	E_{max} with respect to Q4 (%)
Timoshenko	14 or 52	1	UX	16.28
Timoshenko	33	1	UY	35.87
Timoshenko	36 or 37	2	UX	18.19
Timoshenko	38 or 39	2	UY	30.38
EB	13 or 53	1	UX	20.02
EB	33	1	UY	7.51
EB	12	2	UX	26.47
EB	38	2	UY	38.04
EB+sh	14 or 52	1	UX	16.19
EB+sh	33	1	UY	36.56
EB+sh	36 or 37	2	UX	18.59
EB+sh	39	2	UY	30.23
Timoshenko 1000E	19 or 47	1	UX	7.06
Timoshenko 1000E	25 or 41	1	UY	11.47
Timoshenko 1000E	18	2	UX	10.69
Timoshenko 1000E	42	2	UY	17.98
Timoshenko 73E	18 or 48	1	UX	8.83
Timoshenko 73E	33	1	UY	2.24
Timoshenko 73E	17	2	UX	3.80
Timoshenko 73E	42	2	UY	18.50
CQ	15 or 51	1	UX	2.59
CQ	17 or 49	1	UY	1.69
CQ	28	2	UX	2.70
CQ	25	2	UY	4.10

Table 3. Results (with respect to Q4) of the dynamical analysis of vibrations in plane (XY).

Mode No	Q4	EB (%)	EB+sh (%)	Timoshenko (%)	CQ (%)
1	4375.6	-5.8	9.2	9.2	-0.1
2	12950	-2.3	8.9	8.8	-0.8
3	17558	-8.8	4.7	4.6	-2.2
4	18009	-19.6	-2.3	-2.4	-0.4
5	22432	-32.6	8.4	8.2	-1.8
6	26894	-14.9	3.1	3.0	-4.5
7	32996	-32.3	-1.0	-1.3	-6.2
8	40761	-31.9	0.5	0.2	-4.4
9	48173	-30.7	10.8	10.5	-7.5
10	51532	-27.8	4.4	4.1	-3.8
Average		20.2	5.6	5.4	3.0

The meshing of this part using Timoshenko beam elements is shown in Figure 38.

To mesh the corner of the test structure, we have previously used the CQ element consisting of 64 Q4 elements. As for part pt1pt2, the element linking 2 cylinders can be meshed using:

- Either 2 classic CQ corner elements (Figure 39)
- Or 1 DBQ element made of (8×16) Q4 elements (Figure 40).

Figure 41 shows part pt1pt2 model using Timoshenko beam elements and DBQ elements at corners. Table 4

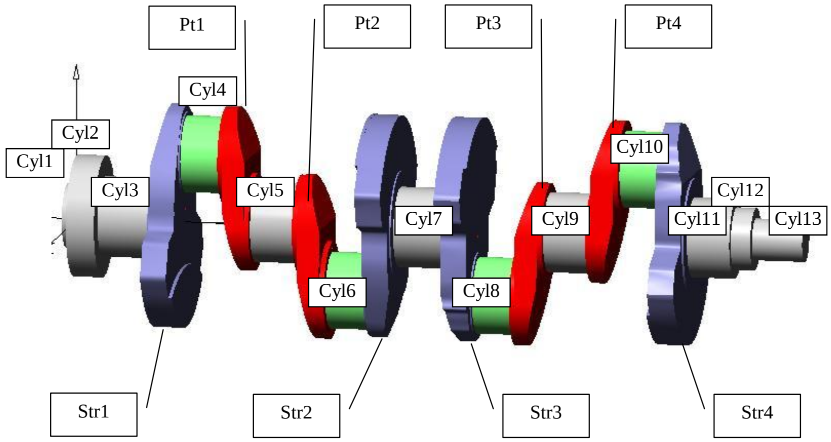


Figure 34. Parts of the 3D crankshaft.

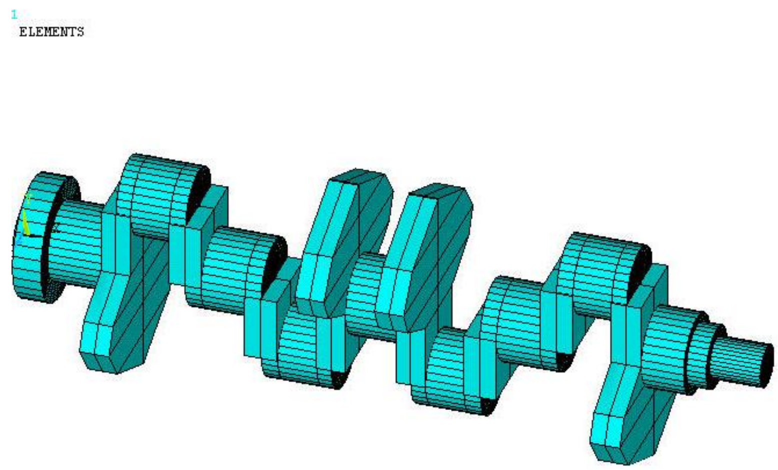


Figure 35. Crankshaft modeled using Timoshenko beam elements.

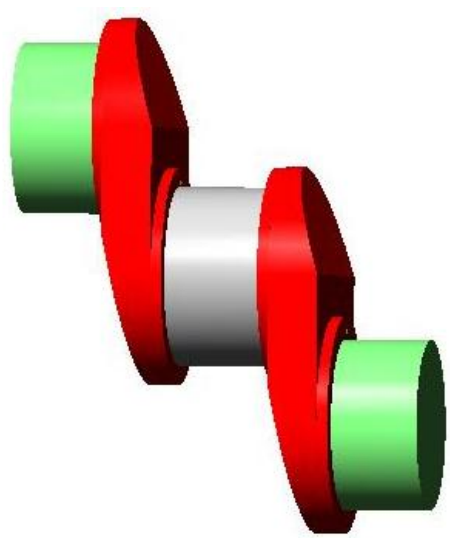


Figure 36. Part of the crankshaft (pt1pt2).

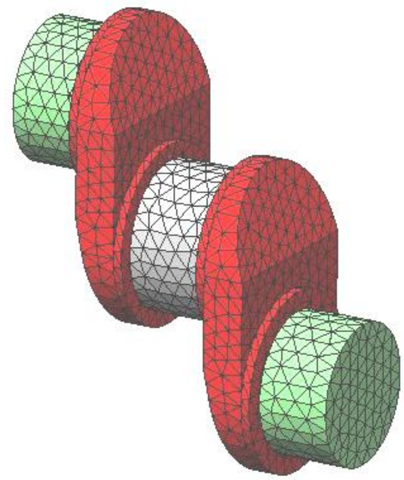


Figure 37. Part pt1pt2 modeled using volumetric elements TET10.

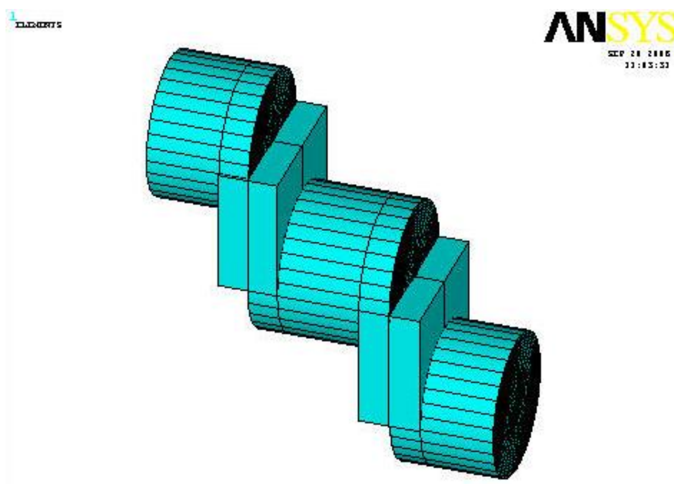


Figure 38. Timoshenko model of part pt1pt2.

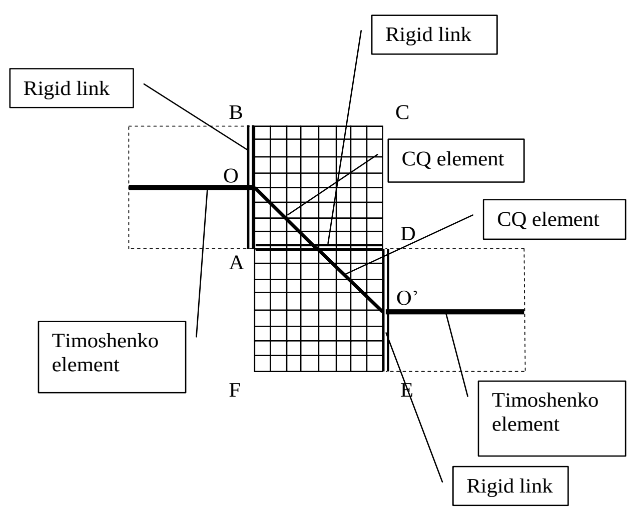


Figure 39. Modeling pt1pt2 using 2 CQ elements.

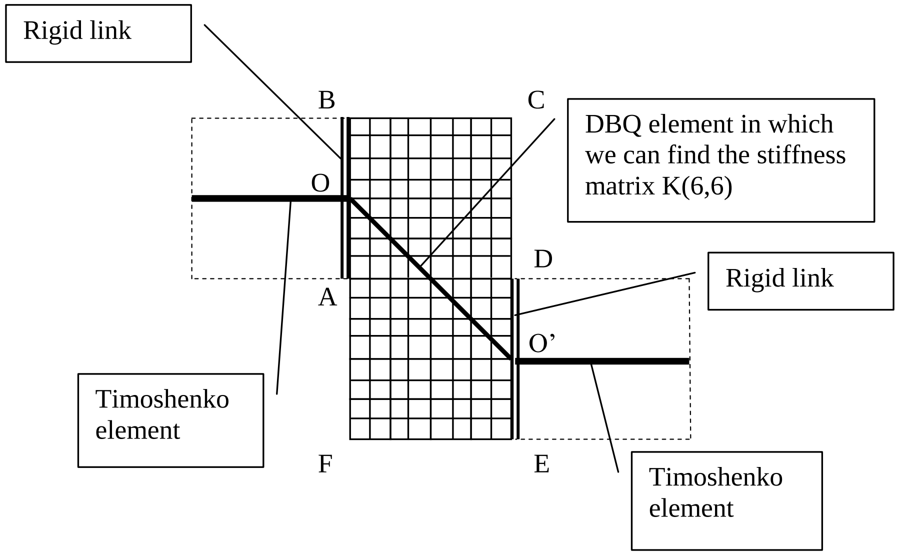


Figure 40. Modeling pt1pt2 using 1 DBQ element.

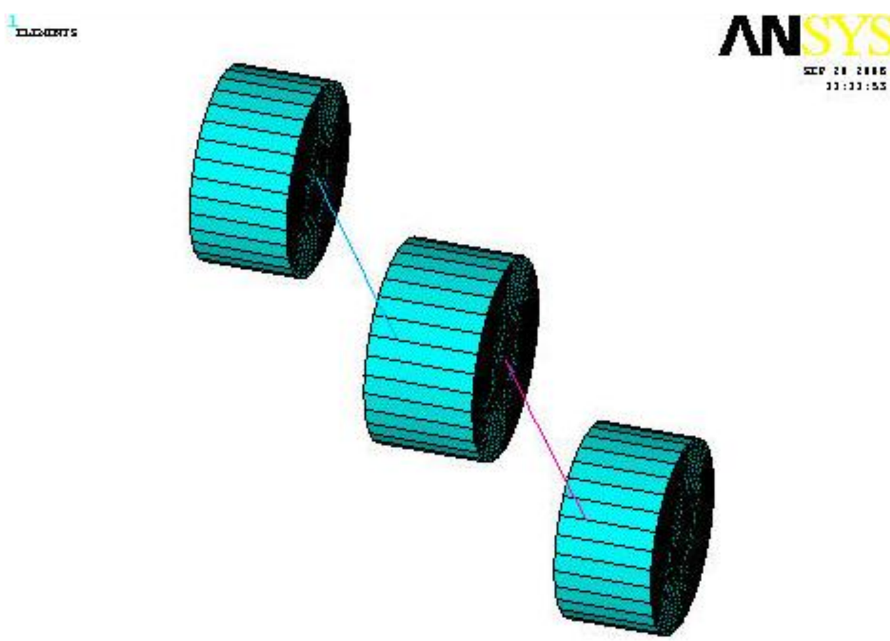


Figure 41. pt1pt2 (Timoshenko + DBQ) model.

lists the frequencies of the modes of vibrations of part pt1pt2 (in the XY plane) of the 3D model and the relative errors of the 3 models (Timoshenko, Timoshenko + 2 CQ and Timoshenko + DBQ)

We note that the results have improved when using the Timoshenko + DBQ model; therefore, this model will be generalized on the entire crankshaft. Also, we note that changing the meshing of corners is used to improve the results of both static and dynamic analyses in the XY plane of the crankshaft.

Modeling of the crankshaft

Figures 42 and 43 show the Timoshenko + DBQ model of the crankshaft. The mass of the crankshaft using this model is 11.06 kg and the volume is 14.18×10^{-4} m^3 which gives a 6.2% error in comparison with the 3D model.

Table 5 shows the frequencies (Hz) of the first 6 modes of vibrations of the entire crankshaft using a volumetric meshing (TET10 elements) and the relative errors using a Timoshenko model and a Timoshenko + DBQ model.

Table 4. Dynamic comparison of part pt1pt2.

Mode	3D (Hz)	Timoshenko (%)	Timoshenko + 2 CQ (%)	Timoshenko + DBQ (%)
1	4411	21.80	-4.01	-3.75
2	8084	30.42	-7.06	-6.86
3	14400	48.39	-5.52	-5.22
4	19337	48.40	-7.22	-6.97
Average		37.25	-5.95	-5.70

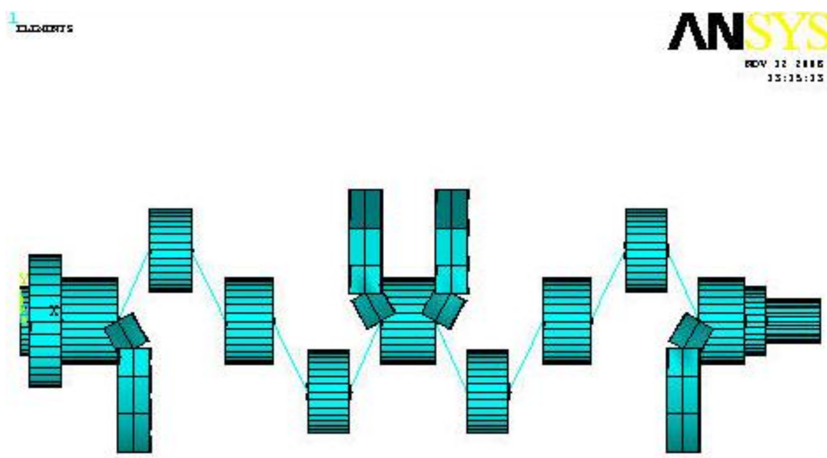


Figure 42. Crankshaft Timoshenko + DBQ model (side view).

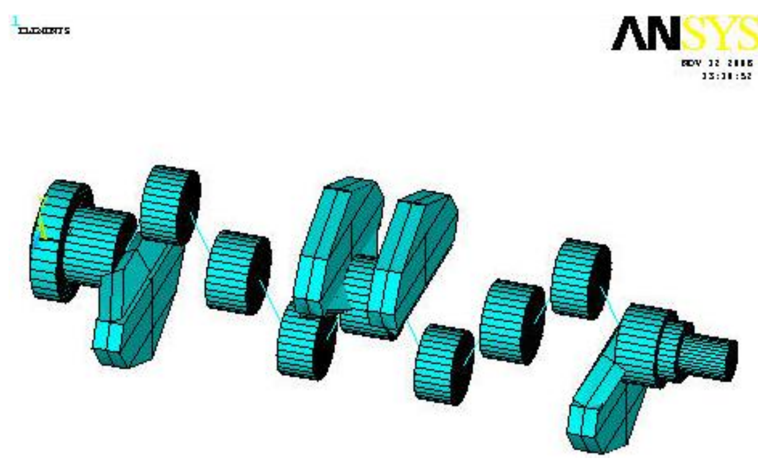


Figure 43. Crankshaft Timoshenko + DBQ model (isometric 3D view).

Imperfection in the meshing

Two sources of imperfection in the meshing process of the crankshaft contribute to the reason behind the relative errors obtained in the previous paragraph. They are:

1. In reality, cylinder cyl4 and cylinder cyl5 of part pt1pt2 are overlapped with a distance of 0.007 m along their extensions (Figure 44).

Points A and D do not lie horizontally. In our model, this overlapping is eliminated; that is, A and D lie horizontally. This modification is performed by increasing the height of BCEF element (Figure 45). This overlapping elimination is performed along all connections of cylinders and pt and Str parts.

2. The rounded shape of pt and STR parts can not be exactly modeled using linear finite elements with regular

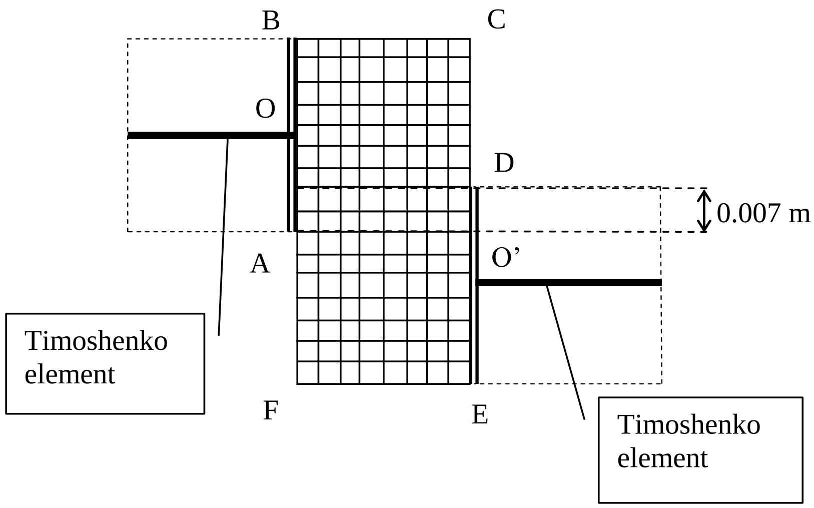


Figure 44. Overlapping of the 2 cylinders of pt1pt2.

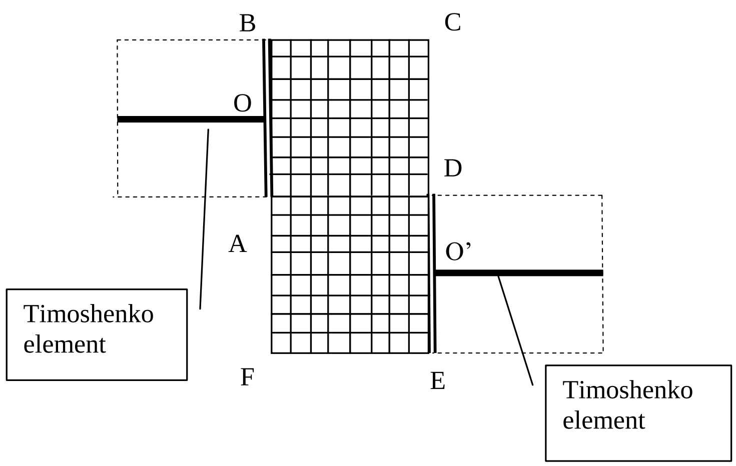


Figure 45. Elimination of the overlapping in part pt1pt2.

rectangular sections. Besides; parts pt and STR show a variation in their thicknesses that have been neglected in our models. All this explain the difference in the volume and mass of models with respect to the initial geometry and leads to a light modification of the stiffness of the crankshaft.

CONCLUSION

The results of the static and dynamic analysis of a 3D structure model using Q4 and H8 elements are close. Modeling 3D structures is possible using linear 2D elements with special consideration at the connections. In the study of frames with small dimensions, increasing the rigidity of the elements inside the connections gives better results in both static and dynamic analysis. Modeling corners with CQ elements and regular section elements with Timoshenko elements gives best results in both static and dynamic analysis.

Modeling the crankshaft by Timoshenko elements only, leads to an average error of 37.30% in the results of the analysis in the XY plane, which is a high error. While modifying the modeling of the connections using the Timoshenko + DBQ model reduces the error to 7.80%. This error is due essentially to geometric considerations.

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