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Full Length Research Paper

An upper-bound solution for forging load of an elliptical disc

K. S. Phogat[1]*, A. K. Shrama[2], R. K. Ranjan[2] and V. K. Bajpai[3]

[1]Department of Mechanical Engineering, IET, JK Lakshmipat University, Jaipur, Rajasthan, India.
[2]Department of Mechanical Engineering, Gyan Ganga Institute of Technology and Sciences Jabalpur, MP, India.
[3] Department of Mechanical Engineering, National Institute of Technology, Kurukshetra, India.

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The present work critically emphasize on the deformation limit of an elliptical disc. The upper- bound theorem has been used to determine the forging load. A suitable kinematically admissible velocity fields for three dimensional deformations have been chosen for analyzing deformation limit as well as the bulging of disc. During the analysis satisfactory friction factor, yield strength and other process parameter have been assumed and mathematically analyzed. It has been seen that parametric and experimental analysis makes very close agreement. This paper will be worthy for industrial applications in estimating forging load of such shapes.

Key words: Elliptical disc, deformation limit, upper- bound theorem.

INTRODUCTION

Forging process has becomes increasingly important in almost all manufacturing industries such as aerospace, steel plants, automobile applications. Upsetting with a flat die has great significance in metal forming applications and is particularly useful when the geometrical shape of billets is very complicated. As we know in many industrial metal forming processes, bulging of free work-piece surfaces occurs. At the same time these surfaces may fold over and come into contact with the dies. To understand the deformation characteristics of axi-symmetric forging operations, much effort is required to devote.

The technique (Baskaran et al., 2008) of forming process is widely and increasing accepted world-wide in the production industries, however a large number of process parameter still needs exact methodology to predict forging load. Among various theoretical methods available for metal forming problems, the upper-bound theorem is known to be a limiting approach to predict the maximum energy and assures a material to plastically deform into a desired shape. When this method is applied, an admissible velocity field that satisfies the incompressibility, continuity, and the velocity boundary conditions is usually an impart element to the upper-bound solution. Based on this velocity field and limit theorems (Yeh et al., 2005), the total forming energy and also the forming load can be computed to represent an upper-bound to the actual forming energy or actual forming load. For quite some time the studies made were for forging circular cylinder, ring and rectangular section in plane strain with the assumption that the plane sections remain plane during deformation. Avitzur (1968) has constructed upper bound solutions for both the circular discs and the rectangular plates taking into account the barreling along thickness. Alexander (1955) and Kudo (1960) have given slip line solutions for plane strain compression between rough dies. Kanacri (1972) modified the general method of analysis proposed by Hill (1963) and applied it to the compression analysis of rectangular blocks. In their study only the sidewise spread was considered without bulging along thickness under the assumption of small thickness. Juneja (1973a) proposed some appropriate velocity fields for the upper-bound analysis of polygonal disks under the assumption on that only the sidewise spread occurs during deformation. Juneja (1973b) also made a further analysis for upset forging of polygonal disks in

*Corresponding author. E-mail: Kapenderb143@gmail.com.

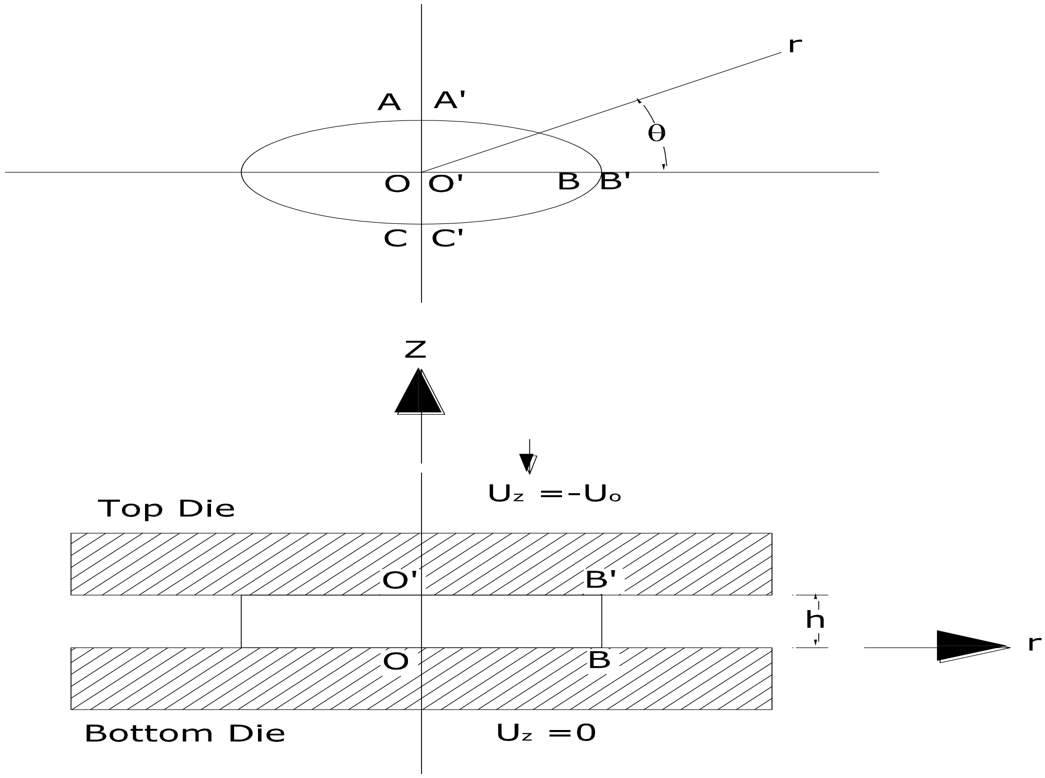

Figure 1. Upset forging of an elliptical disc.

consideration of only bulging along thickness and in comparison with his previous work could obtain better upper-bound solutions for some forging conditions. Park (1984) employed the three-dimensional finite element method which took eight node hexahedral elements in order to analyze the block compression. The numerical results were in good agreement with the experiment. But computation time should be reduced considerably in order to solve more complicated upset forging problems. Very recently Kim (1985) analyzed upset forging of square blocks by using a simple velocity field with in a reasonably short computational time. In this paper a kinematically admissible velocity field for the three-dimensional deformation in upset forging of elliptical disks is proposed which takes into account not only the sidewise spread in the plane perpendicular to the upsetting axis but also bulging along thickness. By optimizing the parameters in the upper bound formulation, the forging load and the deformed configuration of the billet are found at each step of height reduction.

METHODOLOGY

Velocity field

As shown in Figure 1, the cylindrical coordinates are taken as such that the origin lies at the center of the bottom surface of an elliptical disk. As the deformation proceeds in the upset forging of the disk, the side wise spread in the plane perpendicular to the Z-axis as well as bulging along thickness takes place due to non-uniform flow caused by friction at the die work piece interfaces.

As seen from the Figure 1, the disk is symmetrical about Z-axis and r-axis. Hence from the symmetry of the disk only a segment OABO' A' B' is considered for the velocity field. As from the assumption the plane section remain plane, here the geometrical symmetry demands that the surface OAA'O' and the surface OBB'O' must remain flat plane throughout the deformation. This means, no tangential velocity discontinuity can occur on these surfaces and also the normal velocity component across them must not exist.

The symmetric nature of the velocity field together with foregoing boundary conditions and the deformation characteristics of the sidewise spread and bulging along thickness lead to the selection of the following velocity field:

$$U_r = \frac{U_0}{h} Ar\left[\frac{1}{2} + \frac{a-b}{a+b}B\cos(2\theta)\right]\left[-\frac{4}{h^2}z^2 + \frac{4}{h}z + C\right] \qquad (1)$$

and

$$U_\theta = -\frac{U_0}{h} Ar\left[\frac{a-b}{a+b}B\sin(2\theta)\right]\left[-\frac{4}{h^2}z^2 + \frac{4}{h}z + C\right] \qquad (2)$$

Where 'a' and 'b' are half lengths of major and minor axes of the initial elliptical disc respectively. 'B' and 'C' are the parameters by which the amount of the sidewise spread and bulging along thickness can be determined through optimization at each step of deformation. 'A' is a parameter to be determined from the velocity boundary condition. As we know the incompressibility Avitzur (1968)

(1968) condition in the cylindrical coordinates is given by:

$$\frac{\partial U_r}{\partial r}+\frac{U_r}{r}+\frac{1}{r}\frac{\partial U_\theta}{\partial \theta}+\frac{\partial U_z}{\partial z}=0 \tag{3}$$

$$\frac{\partial U_r}{\partial r}+\frac{U_r}{r}+\frac{1}{r}\frac{\partial U_\theta}{\partial \theta}+\frac{\partial U_z}{\partial z}=0$$
$$=\begin{Bmatrix}\frac{U_0}{h}A\left[\frac{1}{2}+\frac{a-b}{a+b}B\cos(2\theta)\right]\left[-\frac{4}{h^2}z^2+\frac{4}{h}z+C\right]+\\ \frac{U_0}{h}A\left[\frac{1}{2}-\frac{a-b}{a+b}B\cos(2\theta)\right]\left[-\frac{4}{h^2}z^2+\frac{4}{h}z+C\right]+\frac{\partial U_z}{\partial z}\end{Bmatrix} \tag{4}$$

This shows that

$$U_z=-\frac{U_0}{h}A\left[-\frac{4}{h^2}\frac{z^3}{3}+\frac{4}{h}\frac{z^2}{2}+Cz\right]+f(r,\theta) \tag{5}$$

Here $f(r,\theta)$ is a function of only r and θ to be determined from the boundary condition at the top and bottom surfaces.

as at z = 0 $\quad U_z=0$
and at z = h $\quad U_z=-U_0$

By putting these values in Equation (5) we get
When z = 0, $\quad U_z=0 \quad \Rightarrow \quad f(r,\theta)=0$
and
When z = h, $\quad U_z=-U_o$
We find the value of A as

$$A=\frac{1}{\left[\frac{2}{3}+C\right]} \tag{6}$$

Thus the velocity field includes only two free parameters ‘B’ and ‘C’. Finally the velocity field is written as

$$U_r=\frac{U_0}{h}Ar\left[\frac{1}{2}+\frac{a-b}{a+b}B\cos(2\theta)\right]\left[-\frac{4}{h^2}z^2+\frac{4}{h}z+C\right] \tag{7(a)}$$

$$U_\theta=-\frac{U_0}{h}Ar\left[\frac{a-b}{a+b}B\sin(2\theta)\right]\left[-\frac{4}{h^2}z^2+\frac{4}{h}z+C\right] \tag{7(b)}$$

and $$U_z=-\frac{U_0}{h}Az\left[-\frac{4}{3}\frac{z^2}{h^2}+2\frac{z}{h}+C\right] \tag{7(c)}$$

If half length of the major axis, ‘a’ is equal to the half length of minor axis, ‘b’ the above velocity field can also be applied to the analysis of axisymmetric upset forging problem.

Strain rates

The strain rates are obtained from the derived velocity field as:

$$\overset{0}{\varepsilon}_{rr}=\frac{\partial U_r}{\partial r}=\frac{U_0}{h}A\begin{bmatrix}\frac{1}{2}\\ +\frac{a-b}{a+b}B\cos(2\theta)\end{bmatrix}\begin{bmatrix}-\frac{4}{h^2}z^2\\ +\frac{4}{h}z+C\end{bmatrix} \tag{8a}$$

$$\overset{0}{\varepsilon}_{\theta\theta}=\frac{U_r}{r}+\frac{1}{r}\frac{\partial U_\theta}{\partial \theta} \quad \frac{U_0}{h}A\left[\frac{1}{2}-\frac{a-b}{a+b}B\cos(2\theta)\right]\begin{bmatrix}-\frac{4}{h^2}z^2\\ +\frac{4}{h}z+C\end{bmatrix} \tag{8(b)}$$

$$\overset{0}{\varepsilon}_{zz}=\frac{\partial U_z}{\partial z}=-\frac{U_0}{h}A\left[-\frac{4}{h^2}z^2+\frac{4}{h}z+C\right] \tag{8(c)}$$

$$\overset{0}{\varepsilon}_{r\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial U_r}{\partial \theta}+\frac{\partial U_\theta}{\partial r}-\frac{U_\theta}{r}\right)$$

$$\overset{0}{\varepsilon}_{r\theta}=-\frac{U_0}{h}AB\frac{a-b}{a+b}\sin(2\theta)\left[-\frac{4}{h^2}z^2+\frac{4}{h}z+C\right] \tag{8(d)}$$

$$\overset{0}{\varepsilon}_{\theta z}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial U_z}{\partial \theta}+\frac{\partial U_\theta}{\partial z}\right)=-2\frac{U_0}{h^2}ABr\left[\frac{a-b}{a+b}\sin(2\theta)\right]\left[1-\frac{2}{h}z\right] \tag{8(e)}$$

$$\overset{0}{\varepsilon}_{rz}=\frac{1}{2}\left(\frac{\partial U_r}{\partial z}+\frac{\partial U_z}{\partial r}\right)=\frac{2U_0}{h^2}Ar\left[\frac{1}{2}+\frac{a-b}{a+b}B\cos(2\theta)\right]\left[1-\frac{2z}{h}\right] \tag{8(f)}$$

It can be easily seen that Equations 8(a) to 8(f) satisfy the incompressibility Equation (3) and the required velocity boundary conditions. Therefore, the velocity field given by Equations 7(a) to 7(c) is kinematically admissible and thus can be used for the upper – bound analysis of upset forging of elliptical discs.

Upper bound

The initial configuration in upset forging of an elliptical disc is shown in Figure 1. The top and bottom dies are assumed to be rigid. The top die moves downward with the velocity of $U_{0,}$ while the bottom die is stationary. The working material is assumed to be isotropic, incompressible and rigid plastic work-hardening. It also obeys von Mises’ flow rule and the frictional stress at the die-material interface is assumed to be constant.

The upper-bound theorem was formulated by Prager (1951) and was later modified by Drucker (1954) to include velocity discontinuities in the deforming region. It reads, among all the kinematically admissible strain rate fields the actual one minimizes the following expression.

$$J^*=W_d+W_f \tag{9}$$

Where J^* is an upper-bound energy on the total power consumption.
The Equation (9) can be written as the product of upper platen velocity and forging load applied by the upper platen as follows:

$$J^*=F_L\times U_0 \tag{10}$$

Energy consumed in deformation

The energy consumed in deformation, W_d, is calculated from the derived strain rate field.

$$W_d=\int_v \sigma_0\,\overset{0}{\varepsilon}\,dv \tag{11}$$

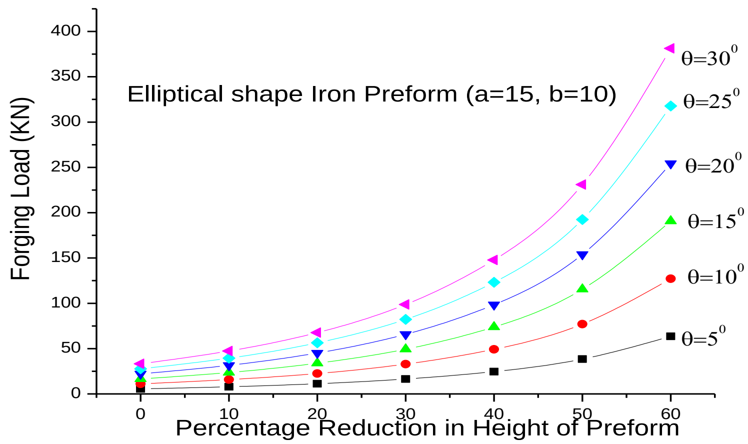

Figure 2. Shows variation of Forging Load Vs. % Reduction in the Height of the perform.

Where

$$\overset{0}{\varepsilon} = \frac{2}{\sqrt{3}}\left[\frac{1}{2}\left(\overset{0}{\varepsilon}_r{}^2 + \overset{0}{\varepsilon}_\theta{}^2 + \overset{0}{\varepsilon}_z{}^2\right) + \overset{0}{\varepsilon}_{r\theta}{}^2 + \overset{0}{\varepsilon}_{\theta z}{}^2 + \overset{0}{\varepsilon}_{rz}{}^2\right]^{\frac{1}{2}} \qquad (12)$$

The geometrical configuration of the working material changes continuously, as deformation proceeds. It is difficult to express the continuously changing configuration analytically and moreover to carry out the integration without resort to numerical calculation. Therefore, for integration the deformation region is divided into small quadrilateral elements and integration is carried out numerically for each elemental region. Since upset forging is a non-steady deformation problem, a slight reduction in height (Δh) should be applied for each step and each element has its own values for strain rates, strain and flow stress. After each step of deformation, new values of velocity and strain rate are calculated for each element. The strain is obtained by adding up the incremental strain for each step which is the strain rate multiplied by the time required for a deformation step. The flow stress is then obtained from the stress-strain relationship.
From Equations (11) and (12) we can find the value of W_d written as

$$W_d = \sigma_0 U_0 aA\left\{\begin{array}{l}\frac{b}{3}\theta\left[\frac{23}{48h^4C}+\frac{2}{5C}+\frac{2+3C}{2}\right]+\frac{a^2B^2}{9b}\theta\left[\frac{23}{4h^4C}+\frac{8}{5C}+4+3C\right]\\ -\frac{23aB}{36h^4C}\sin\theta.\cos\theta\end{array}\right\} \qquad (13)$$

Energy consumed in overcoming friction

The frictional power, W_f, dissipated over the frictional boundary at the die-material interface is given by:

$$W_f = \frac{m}{\sqrt{3}}\int_s \sigma_0\left[\left|\Delta U\right|_{z=0} + \left|\Delta U\right|_{z=h}\right]ds \qquad (14)$$

The velocity discontinuities, $\left|\Delta U\right|_{z=0}$ and $\left|\Delta U\right|_{z=h}$, are determined from the Equations 7(a), 7(b) as follows.

$$\left|\Delta U\right|_{z=0} = \left|\Delta U\right|_{z=h} = \left[U_r{}^2 + U_\theta{}^2\right]^{\frac{1}{2}}_{z=0} \qquad (15)$$

From Equations (14) and (15)

$$W_f = \frac{2m}{\sqrt{3}}\sigma_0\left(\frac{U_0 aA}{h}\right)\left[\begin{array}{l}\theta\left(\begin{array}{l}\frac{4a^2B^2z}{3hb}+\frac{a^2B^2C}{3b}-\frac{4a^2B^2z^2}{3h^2b}\\ -\frac{2bz^2}{3h^2}+\frac{2bz}{3h}+\frac{bC}{6}\end{array}\right)\\ +\frac{1}{3}\sin\theta.\cos\theta\left(\frac{4aBz^2}{h^2}-\frac{4aBz}{h}-aBC\right)\end{array}\right]$$

(16)

From Equation (10), (13) and (16) we find:

$$F_L = \sigma_0 aA\left\{\begin{array}{l}\frac{b}{3}\theta\left[\frac{23}{48h^4C}+\frac{2}{5C}+\frac{2+3C}{2}\right]+\frac{a^2B^2}{9b}\theta\left[\frac{23}{4h^4C}+\frac{8}{5C}+4+3C\right]\\ +\frac{2m\theta}{3h\sqrt{3}}\left[\frac{4a^2B^2z}{hb}+\frac{a^2B^2C}{b}-\frac{4a^2B^2z^2}{h^2b}-\frac{2bz^2}{h^2}+\frac{2bz}{h}+\frac{bC}{2}\right]\\ +\frac{1}{3}\sin\theta.\cos\theta\left[\frac{2m}{h\sqrt{3}}\left(\frac{4aBz^2}{h^2}-\frac{4aBz}{h}-aBC\right)-\frac{23aB}{12h^3C}\right]\end{array}\right\}$$

(17)

RESULTS AND DISCUSSION

The upper-bound load given by Equation (17) was determined by numerical computation with the work hardening effect considered for each element. The computational results of forging load are shown in Figure 2 for different values of "θ" whereas the other parameters are assume to be constant i.e. a = 16mm, $h_0 = 10mm$ for the load calculations.

Figure 2 reveals that at initial conditions for all set of assumed **values of "*θ*" similar** deformation pattern follows and around 25-30% age reduction in height, a

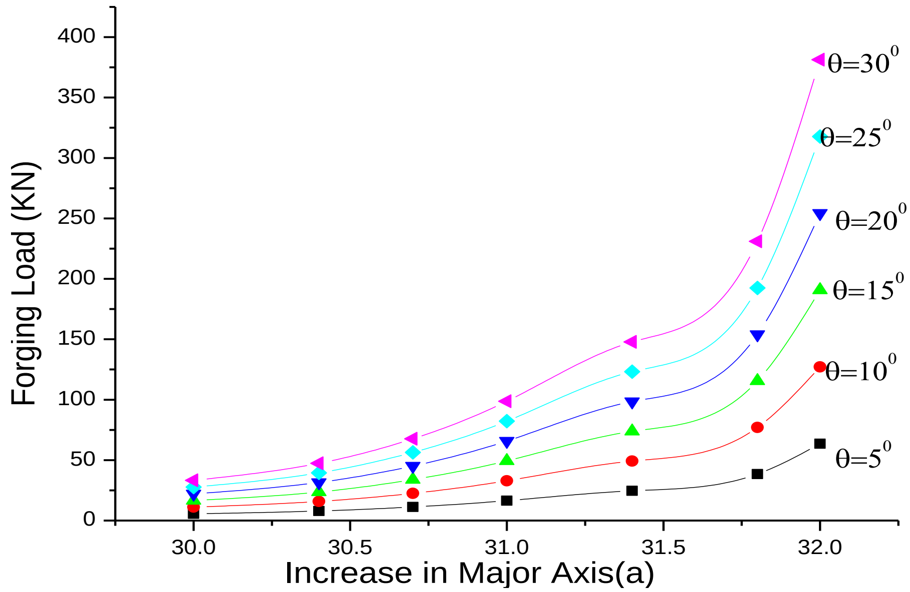

Figure 3. Shows Variation of Forging Load Vs. Increase in Major Axis.

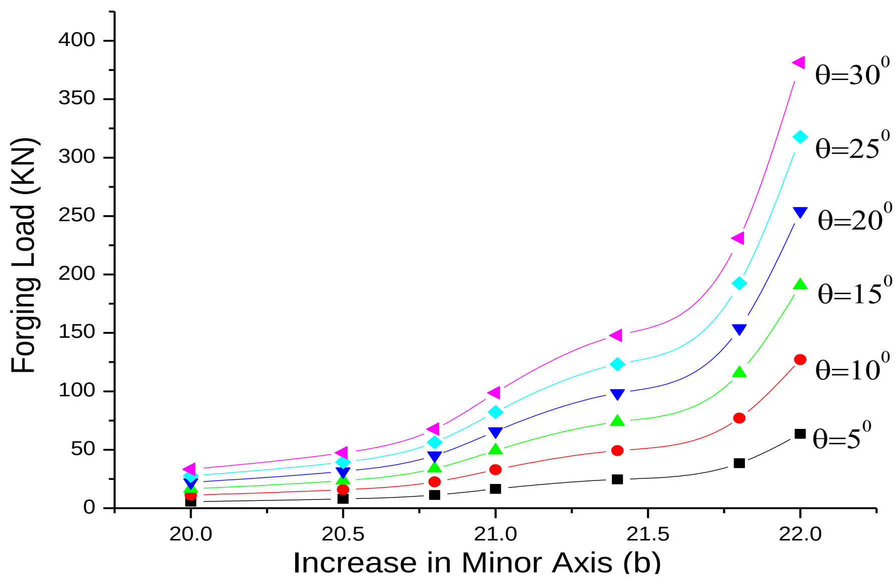

Figure 4. Shows Variation of Forging Load Vs. Increase in Min

sharp increase in the forging load is noticed comparatively less deformation. It may be due to the fact that sticking of pallet with platens takes place and consumes large amount of load.

Figure 3 and 4 reveal that at initial conditions for all set of assumed values of "θ" similar deformation pattern follows and after approximate 30 – 35 % reduction in height, it was noticed that the forging load increased rapidly as compared to the deformation of the billet. It may be due to the fact that sticking of billet with platens takes place and consumes maximum amount of load.

Conclusion

A simple kinematically admissible velocity field taking into account the sidewise spread as well as bulging along thickness is proposed for upset forging of elliptical disks. From the velocity field the forging load and the deformed configuration are determined by minimizing the total power with respect to two free parameters. The theoretical predication of the deformed configuration shows a good result. The velocity field proposed in the present work can be used conveniently for the predication of forging load and deformation in upset forging of elliptical disks and cylinders.

Nomenclature

U_r U_θ U_z Radial, circumferential and axial velocity components

U_0 Velocity of top die.

$|\Delta U|$ Magnitude of velocity discontinuity.

r, θ, z cylindrical coordinate system

m friction factor at the die-material interface

F_L forging load

J^* upper-bound forming energy

σ_0 Yield strength of material in tension

a, b lf length of major and minor axes of the elliptical disc

A, B, C optimization parameters

h_0 Initial height of the billet

h Current height of the deforming zone

H.R. Height reduction in percentage $\left(\frac{\Delta h}{h} \times 100\right)$

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